Classical Cellular Automata

In the book we present some of results of the work we have done in theory of classical Cellular Automata (CA) and their appendices during 1969–2013 in truth with considerable pauses. These results at present form essential constituent of the CA problems. In particular, we have studied such problems as the nonconstructability problem in CA, decomposition of global transition functions in CA, extremal constructive possibilities, complexity of finite configurations and global transition functions, parallel formal grammars along with languages defined by CA, the modelling problem in the classical CA, computer simulation of CA, certain applied aspects of CA, etc. At present, the CA problems is a rather well developed independent sphere of the mathematical cybernetics which has considerable field of numerous appendices. At that, with the equal right the CA problems can be considered as a component of such fields as discrete mathematics, the discrete parallel dynamic systems, complex systems and some others. In our opinion the book will present an indubitable interest for students, post–graduates and persons working for doctor's degree of the appropriate faculties of universities.

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978-3-639-71345-9

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Impressum / Imprint
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Coverbild / Cover image: www.ingimage.com

Verlag / Publisher:
Scholar's Press
ist ein Imprint der / is a trademark of
OmniScriptum GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Deutschland / Germany
Email: info@scholars-press.com

Herstellung: siehe letzte Seite /
Printed at: see last page
ISBN: 978-3-639-71345-9

Contents

List of the adopted abbreviations and designations

∞–MEC	– infinite mutually–erasable configurations
AI	– Artificial Intelligence
APS	– algorithms of parallel substitutions
AS	– an algebraic system
C(A,d,ϕ)	– a set of finite ***d***–dimensional configurations defined in a state alphabet ***A***
C(A,d,∞)	– a set of infinite ***d***–dimensional configurations defined in a state alphabet ***A***
CA	– cellular automata ***(CA, CA–models)***
CAS	– computer algebra system
CF	– configuration(s)**;** depending on context
CHCS	– concentrated homogeneous computing systems
d–GDP	– ***d***–dimensional global decomposition problem ***(d ≥ 1)***
d–GLDP	– ***d***–dimensional generalized global decomposition problem ***(d ≥ 1)***
d–HS	– ***d***–dimensional homogeneous structure(s) ***(d ≥ 1)***
d–HSR	– ***d***–dimensional homogeneous structure(s) with refractority ***(d ≥ 1)***
d–HSS	– ***d***–dimensional homogeneous structure(s) with storage ***(d ≥ 1)***
DPDS	– discrete parallel dynamic systems
ESP	– elementary symmetrical polynomial(s)
FA	– functional algorithm(s)**;** depending on context
FGT	– formal grammar theory
GDP	– the global decomposition problem
GLDP	– the generalized global decomposition problem
GLHS	– the generalized linear classical structures ***d–HS (d ≥ 1)***
GTF	– global transition function(s)**;** depending on context
HCS	– homogeneous computing systems *(environments)*
HPDT	– heterogeneous periodically defined transformations
HS	– homogeneous structure(s) {***HS, HS–models***}
HSoS	– homogeneous structure(s) on splitting
HSR	– homogeneous structure(s) with refractority
IAN	– International Academy of Noosphere
IB	– an internal block
ISG	– isotonic structural grammar(s)
LBF	– local block function(s)
LRA	– locally realizable algorithms

LTF	– local transition function(s); depending on context
MEC	– mutually–erasable configurations
MEC–1	– the generalized mutually–erasable configurations
MTOHS	– mathematical theory of homogeneous structures ***(CA)***
NCF	– nonconstructible configuration(s) ***(Garden–of–Eden configurations)***
NCF–1	– nonconstructible configuration(s) of type ***1***
NCF–2	– nonconstructible configuration(s) of type ***2***
NCF–3	– nonconstructible configuration(s) of type ***3***
NI	– neighborhood index of a structure ***d–HS*** $(d \geq 1)$
NT	– neighborhood template of a structure ***d–HS*** $(d \geq 1)$
PADHS	– parallel algorithm(s) defined by classical ***d–HS*** $(d \geq 1)$
PBS	– parallel block substitutions
PC	– personal computer(s); depending on context
PCF	– passive configuration(s); depending on context
PCS	– parallel control system(s)
PDDS	– parallel discrete dynamic systems
PDT	– periodically defined transformations
PLG	– problem of limited growth
PSIP	– parallel system of information processing
RANS	– Russian Academy of Natural Sciences
SAA	– systems of algorithmic algebras
SPS	– systems of parallel substitutions
SRC	– self–reproducing configurations in the Moore's sense
SUDS	– sequence of uniquely defined sums
TRG	– Tallinn Research Group
TWPM	– two–way pushdown machine(s)
TM^s_q	– Turing machine with ***q*** states and ***s*** symbols on a tape
UCF	– universal configuration(s); depending on context
UHS	– universal homogeneous structure(s)
UMT	– universal Turing machine(s); depending on context
VCF	– vanishing configuration(s); depending on context

The above abbreviations are being understood in sole or in the plural depending on a context, others are introduced as required. The results presented here have mainly a descriptive character inasmuch as their proofs are lengthy enough and too technical to be included in the book In the meantime, the interested reader is referred to literature cited in appropriate places of the represented monograph that contain majority of the results proofs represented in the monograph.

Introduction

First of all, a few words about the terminology used below. Today, the problematics of *Cellular automata* ***(CA)*** well enough is advanced, being quite independent field of modern mathematical cybernetics, having own terminology and axiomatics at existence of a rather broad sphere of various appendices. At the same time, it is necessary to emphasize, that at the assimilation of the given problematics in the Soviet Union in Russian-lingual terminology, whose the basis for the first time have been laid by us at ***1970,*** for the concept «***Cellular automata***» the term «*Homogeneous structures*» ***(HS; HS–****models****)*** has been determined which nowadays is the generally accepted term together with a whole series of other notions, denotations and definitions [1,16,119]. So, during the present monograph along with the given term its well-known Anglo–lingual equivalent the «***Cellular Automata – CA***» is used too.

Homogeneous Structure ***(HS) –*** a parallel information processing system consisting of intercommunicating identical finite automata. Although *homogeneous structures* will be used throughout this monograph as the usual term, it is necessary to keep in mind that *cellular automata* ***(CA),*** *iterative networks* etc. are essentially synonyms. We can interpret ***HS*** as a theoretical basis of artificial parallel information processing systems. From the logical standpoint a ***HS*** is an infinite automaton with *specific* internal structure. The ***HS-****theory* can be considered as a structural and dynamical theory of the infinite automata. ***HS–***models can serve as an excellent basis for modeling of many discrete processes, representing interesting enough independent objects for research too. Recently, the undoubted interest to the ***HS–***problematics has arisen anew and in the given direction many remarkable results have been obtained.

So, the ***HS–***axiomatics provides such three fundamental properties as ***homogeneity, localness*** and ***parallelism*** of functioning. If in a similar computing model we shall with each elementary automaton associate a separate microprocessor then it is possible to unrestrictedly increase the sizes of such computing system without any essential increase of temporal and constructive expenses, required for each new expansion of the computing space, and also without any overheads connected to coordination of functioning of an arbitrary supplementary quantity of elementary microprocessors. Similar high–parallel computing models admit practical realizations consisting of rather large number of rather elementary microprocessors which are limited not so much by certain *architectural* reasons as by a lot of especially economic and technologic reasons defined by a modern level of development of microelectronic technology, however with the great potentialities in the future, first of all, in light of rather intensive works in field of nanotechnology [536].

The above three such features as *high homogeneity*, *high parallelism* and *locality* of interactions are provided by the ***HS***–axiomatics itself, while such property important from the physical standpoint as ***reversibility*** of dynamics is given by program way. In light of the listed properties even classical ***HS*** are high–abstract models of the real physical world, which function in a space and time. Therefore, they in many respects better than many others formal architectures can be mapped onto a lot of physical realities in their modern understanding. Moreover the ***HS***–concept itself is enough well adapted to solution of various problems of modelling in such areas as mathematics, cybernetics, development biology, theoretical physics, computing sciences, discrete synergetics, dynamic systems theory, robotics, etc. Told and numerous examples available for today lead us to the conclusion that the ***HS*** can represent a rather serious interest as a new *perspective* environment of modelling and research of many discrete processes and phenomena, determined by the above properties; in addition, raising the ***HS***–problematics onto a new interdisciplinary level and, on the other hand, as an interesting enough independent formal mathematical object of researches.

The base modern tendencies of elaboration of perspective architecture of high-parallel computer facilities, a problem of modelling of discrete parallel processes, discrete mathematics and synergetics, theory of the parallel discrete dynamic systems, problems of artificial intellect and robotics, parallel information processing and algorithms, physical and biological modelling, along with a lot of other important prerequisites in various areas of modern natural sciences define at the latest years a new ascent of the interest to the formal *cellular* models of various type that possess high-parallel manner of action; the *homogeneous structures* are some of major models of such type. During time which has passed after appearance of monographs and the collected papers [1,3,4,5,7-10, 13,15,45,53-57,75,82,83,143,145,146,152-157,171,173,175,197,304,408,467] that have been devoted to various theoretic and applied aspects of the ***HS***–problematics (*it first of all concerns the works* [1,5,8,9,90,134,141,144-146,164,169,186,293,464]), the certain progress has been reached in this direction, that is connected, above all, with successes of the theoretical character along with essential expansion of fields of appendices of the

HS–models, mainly, in computer science, cybernetics, physics, biology and substantial growth of quantity of researchers in given field. Along with that, in the USA, Japan, Germany, the Great Britain, Estonia, etc., a series of works summarizing the results of progress in those or other directions of the ***HS***–problematics including its numerous appendices in many areas has appeared. Our monographs at the substantial level have presented the reviews of the basic results received by the ***Tallinn Research Group*** on the ***HS***–problematics during the ***42***–year period of its creative activity, except twelve–year break ***(1990–2008, 2010–2013)*** caused by intense enough work with such systems of *computer* algebra as ***MathCAD***, ***Reduce***, ***Mathematica*** and ***Maple*** [1,3,5,7-10,631-636]. In the given period we prepared and published in Russia, Ukraine, USA, Lithuania, Estonia & Byelorussia a whole series of books in the stated directions along with development of software of all kinds for systems ***Windows*** and ***DOS.*** And what is more, in the same period the enough essential attention had been devoted to mathematics, statistics theory along with some other fields of natural sciences. Because of the above-mentioned reason, serious active investigations in the ***HS***-problematics were practically being not carried out, except programming of a whole series of procedures in the environment of computer algebra systems *Maple* and *Mathematica* for the purpose of experimental study of some dynamical aspects of the ***HS***–models. A whole series of procedures of the given type had been included in our large ***UserLib6789*** library for *Maple* [637], and two our packages ***AVZ_Package*** and ***AVZ_Package_1*** [634,638] for *Mathematica* system. These procedures allowed to receive a number of rather interesting experimental results of dynamics of ***HS***-models of dimensions one and two. The rather detailed description of these procedures with typical examples of their use can be found in our books and reports [99–118,631–636,640–643].

From the very outset of our researches on the ***HS***–problematics, above all, with application accent on mathematical development biology the informal ***Tallinn Research Group (TRG)*** consisting of the researchers of a few lead scientific centres of the former ***USSR*** has gradually been formed up. At that, the ***TRG*** staff was not strictly permanent and was being changed in broad enough bounds depending on the researched problems. In [1,7,9,10] the analysis of the ***TRG*** activity during ***30***–year period which to some degree can be instructive and for research of the dynamics of development of the ***HS***–problematics as an independent scientific direction as a whole is presented. Ibidem the basic directions of our researches can be found along with main received results.

Today, the homogeneous structures are being investigated from many standpoints and interrelations of the objects of such type with already existing problems are being discovered constantly. With the purpose of general acquaintance with extensive ***HS***–problematics as a whole, and with its separate basic directions specifically, it is recommended to address oneself to interesting and versatile surveys of such researchers as ***V.Z. Aladjev, V. Cimagalli, K. Culik, D. Hiebeler, A. Lindenmayer, A.R. Smith, P. Sarkar, M. Mitchell, T. Toffoli, R. Volmar, S. Wolfram,*** et al. [536]. A series of books and monographs of such authors as ***V.Z. Aladjev, A. Adamatzky, E. Codd, A. Ilachinskii, M. Garzon, M. Duff, P. Kendall, T. Toffoli, B. Voorhees, M. Sipper, O. Martin, K. Preston, V. Kudrjvcev, N. Margolus, R. Vollmar, B. Voorhees, S. Wolfram,*** and certain others contain rather interesting historical excursuses into the ***HS***–problematics; unfortunately, hitherto a common standpoint onto historical aspect in the given question does not exist [5,536,567,617]. In view of it, here is opportune moment to briefly emphasize once again our standpoint onto historical aspect of the ***HS***–problematics, namely: a brief historical excursus presented below make it one's aim to define the basic stages of becoming of the ***HS***-problematics, having digressed from numerous particulars. Having started own researches on the ***HS***-problematics in ***1969,*** we on base of analysis of large enough quantity of publications and direct dialogue with many leading researchers in this direction possess the certain information concerning the objective development of its basic directions, above all, of theoretical character. That allows us with sufficient degree of objectivity to time the pivotal stages of its development; at that, many details of historical character concerning the ***HS***–problematics the reader can find, for example, in a whole series of such works as [1,3,5,8-10,53-56,90,114,131,135,146,150, 161,163,179,186,230,264,265,271,536,567,617,639].

From the theoretical standpoint the concept of ***cellular automata (CA;*** *homogeneous structures)* has been introduced at the end of the forties of the past century by ***John von Neumann*** on ***S. Ulam's*** advice with the purpose of determination of more realistic and well formalized model for research of behaviour of complex evolutionary systems, including self–reproduction of alive organisms. While ***S. Ulam*** oneself has used ***CA***–like models, in particular, for researches of the growth problem of crystals and some other discrete systems growing in conformity with recurrent rules. The structures investigated by him and his colleagues were, mainly, ***1***– and ***2***–dimensional, however higher dimensions have been considered too. In addition, questions of universal computability together with some other theoretical questions of behaviour of *cellular* structures of this type also were kept in view. A little bit later also ***A. Church*** started to investigate the similar structures in connection with works in the field of *infinite* abstract automata and mathematical logic.

The ***J. Neumann's CA–***model has received the further development in works of his direct followers whose results together with the finished and edited work of the first one have been published by ***A.W. Burks*** in his excellent books [124,128], which have determined development of researches in this direction for several subsequent years. In process of researches on the ***HS–***problematics ***A. Burks*** has created at university of Michigan the research team ***«The Logic of Computer Group»,*** from which a whole series of the first–class experts on the ***HS–***problematics has come out afterwards ***(T. Toffoli, J. Holland, R. Laing, et al.).***

Meanwhile, considering historical aspect of the ***CA–***problematics, we should not forget an important contribution to the given problematics which was made by pioneer works ***Konrad Zuse*** and with which the world scientific community has familiarized oneself enough late and even frequently without his mention in the given historical aspect. At that, ***K. Zuse*** not only has created the first programmable computers ***(1935–1941),*** has invented the first high–level programming language ***(1945),*** but was also the first who has introduced idea of «***Rechnender Raum***» (*computable spaces*), or else ***– cellular automata (homogeneous structures)*** in the modern terminology [126,188,423]. At that, ***K. Zuse*** has supposed that physical processes in point of fact are calculations, whereas our universe is a certain «*cellular automaton*» [126]. In the late seventies of the last century such view on the universe was innovative while now the idea of the ***computing universe*** horrify nobody, finding a logical place in modern theories of some researchers working in the field of quantum mechanics [536]. Unfortunately, even at present the ***K. Zuse's*** ideas are unfamiliar even to rather meticulous researchers in this field. For exclusion of any speculative historical aspects existing occasionally today, in the future historical researches it is necessary to pay the most steadfast attention on the given essential circumstance. Namely therefore, only many years later the similar ideas have been republished, popularized and redeveloped in researches of other such researchers as ***T. Toffoli, E. Fredkin, S. Wolfram,*** et al. [5,536]. At that, the itself concept «***Cellular automata***» has been entered by ***John von Neumann.*** Perhaps, ***John Neumann,*** being familiar with ***K. Zuse*** ideas, could use cellular automata not only for simulation of process of self–

reproduction, but also for creation of high-parallel computing models.

From more practical standpoint and game experiment the ***CA***–model has notified about itself in the late sixties of the last century, when ***J.H. Conway*** has presented the now known game «***Life***». The given game became rather popular and has attracted attention to *cellular* automata of both numerous scientists from various fields and amateurs [10,239]. At present, the game, probably, is the most known ***CA***–model; at that, it possesses the ability to self–reproduction and universal computing. Modelling a work of an arbitrary Turing machine by means of spatial-temporal dynamics of such ***CA***–model, ***J. Conway*** has proved ability of the model to universal computability. Later a rather simple way of realization of any boolean function in configurations of the «***Life***» has been suggested [536]. Thus, even such very simple ***CA***–model turned out equivalent to the universal Turing machine. Furthermore, to this ***CA***–model the significant interest and till now does not vanish, above all, to its computer realization [5,54–56,79,90,536,567,617].

Thus, early ideas and researches of such first-rate mathematicians and cyberneticians as ***K. Zuse, John von Neumann, S. Ulam*** and ***A. Church*** along with their certain direct followers we can ascribe with complete reason to the *first stage* of formation of the ***HS***-problematics as a whole [125-127,129-133,288-299]. The necessity for a good enough formalized environment for modeling of processes of biological development and above all of the self-reproduction process was being as one of the base prerequisites stimulating the ***HS***–concept beginning. Thereupon, ***John Neumann*** and a whole series of his direct followers have investigated a series of questions of computational and constructive opportunities of the first ***HS***–models. The above works at the end of the fifties of the last century have attracted to the problematics a lot of researchers [1,5, 124,128]. At that, *homogeneous* structures were being rediscovered not once and under various names – in the electrical engineering they are known as *iterative networks*, in pure mathematics they are known as a section of *topological dynamics*, in biology – as *cellular structures*, etc.

As *second stage* in formation of the ***HS***–problematics it is quite possible to consider the publication of the widely known works of ***E.F. Moore*** and ***J. Myhill*** on the *nonconstructability* problem in *classical* ***HS***-models that along with solution of some mathematical problems have become in a sense by the *accelerators* of the activity which attracted a steadfast enough attention to the given problematics of a lot of mathematicians and researchers from other fields [123,274,275]. In particular, we have familiarized oneself with the given ***HS***–problematics in ***1969*** owing to Russian translation of the excellent proceedings edited by ***R. Bellman,*** that contain known articles of ***E.F. Moore, S. Ulam*** and ***J. Myhill*** [123]. Scientific groups on the ***HS***–problematics in the *USA*, *Germany*, *Japan*, *Hungary*, *Italy*, *France*, and *USSR* ***(TRG, 1969)*** are formed up. At that, the further development and popularization of the ***CA***–problematics can be connected with names of such researchers, as ***E. Codd, S. Cole, E. Moore, J. Myhill, E. Banks, H. Yamada, S. Amoroso, J. Buttler, V.Z. Aladjev, J. Holland, G. Herman, A.R. Smith, T. Yaku, A. Maruoka, Y. Kobuchi, G. Hedlund, M. Kimura, H. Nishio, T. Ostrand, A. Waksman*** and a whole series of others whose works in the sixties – the seventies of the last century have attracted attention to the given problematics from the theoretical standpoint; they have solved and formulated a lot of rather interesting problems [536]. Later, mathematicians, physicists, and biologists began to use the cellular automata with the purpose of simulation of own specific problems. So, in the early sixties – the late seventies of the last century the numerous researchers have prepared entry of the ***HS***–problematics into the current stage of its development being characterized by union of earlier *disconnected* ideas and methods on the general conceptual and methodological platforms, along with a rather essential expansion of fields of its appendices.

We can attribute the beginning of the ***third period*** to the early eighties of the last century when to ***HS***–problematics the special interest again has been renewed in connection with rather active researches on the problem of artificial intellect, physical modeling, elaboration of a new perspective architecture of high-parallel computer systems, and other important motivations. So, in our opinion namely since works of such researchers as ***Bennet C., Grassberger P., Boghosian B., Crutchfield J., Chopard B., Culik II K., Gács P., Green D., Gutowitz H., Langton C., Martin O., Ibarra O., Kobuchi Y., Margolus M., Mazoyer J., Toffoli T., Wolfram S., Aladjev V.Z., Bandman O.L.,*** etc. a new splash of interest to ***HS*** as an environment, above all, of physical modelling began [536].

At present, ***CA***–problematics are being widely studied from extremely various standpoints, and interrelations of such *homogeneous* structures with existing problems are constantly sought and discovered. A series of rather large teams of researchers in many countries and, first of all, in the USA, Germany, the Great Britain, Italy, France, Japan, Australia deals with the given problematics. Active enough scientific activity in this direction was carried out and in Estonia within of the ***TRG*** whose a whole series of results has received an international recognition and has made up essential enough part of the modern ***HS***–problematics.

Annual national and international scientific forums of a various level on ***HS***–problematics and its applied aspects are held. The number of publications in various periodic and nonperiodic editions **(***in the USA since* ***1987*** *on* ***HS***–*problematics the special journal* **«Complex Systems»** *is being issued***)** along with special monographs, books, proceedings and collections are annually published**;** the national programs on the ***HS***–problematics are developed, what with good reason allows to speak about constantly growing interest to the given problematics [121,536]**.** Since ***1987 HS***–problematics all is more widely represented at various international conferences on mathematical and computer modeling at level of sectional and plenary reports. Definition of a special index in the classification ***AMS*** for the ***HS***–problematics, assignment of special sections in mathematical encyclopedias, active attempts of creation of the advanced classification of directions in the ***HS***–problematics along with creation under the aegis of the ***IFIP*** of the international working group on *cellular automata* can be quite unambiguously considered as definition of a new independent scientific direction of researches [119, 129,443,536]**.** The users of ***Internet*** receive an access to the information on the given problematics by key phrases **«*homogeneous structures*»** and **«*cellular automata*»,** including such questions as research teams, the scientific forums, separate works, the bibliography together with a lot of many other important aspects of the ***HS***–problematics.

The modern standpoint on the ***HS (CA)*** theory has been formed under the influence of works of such researchers as ***Adamatzky A.I., Aladjev V.Z., Amoroso S., Arbib M., Bagnoli F., Bandini S., Bandman O., Bays C., Banks E.R., Barca D., Barzdin J., Binder P., Boghosian B., Burks A. W., Butler J., Cattaneo G., Chate H., Chowdhury D., Church A., Cole S., Codd E.F., Crutchfield J., Culik K.II, Das A.K., Durand B., Durret R., Fokas A., Fredkin E., Gács P., Gardner M., Gerhardt M., Griffeath D., Golze U., Grassberger P., Green D., Gutowitz H., Hedlund G., Honda N., Hemmerling A., Holland J., Ibarra O., Ikaunieks E., Ilachinskii A., Jen E., Kaneko K., Kari J., Kimura M., Kobuchi Y., Langton C., Legendi T., Lieblein E., Lindenmayer A., Maneville P., Margolus N., Martin O., Maruoka A., Mazoyer J., Mitchell M., Moore E.F., Morita K., Myhill J., Nasu M., Neumann J., Nishio H., Ostrand T., Pedersen J., Podkolzin A., Richardson D., Sarkar P., Sato T., Shereshevsky M., Sipper M., Smith A.R., Sutner K., Takahashi H., Thatcher J., Toffoli T., Toom A., Tseitlin G.E., Varshavsky V., Vichniac G., Vollmar R., Voorhees B., Waksman***

A., Weimar J., Willson S.J., Wolfram S., Wuensche A.A., Yaku T., along with other numerous researchers from many countries.

Along with our works in the ***HS*** theory, it is necessary to note a whole series of other Soviet researchers who have received in the given field both fundamental and considerable enough results at the sixties – the eighties of the last century. They are ***Adamatzky A.I.*** (*identification of* ***HS***), ***Bandman O.L.*** (*asynchronous* ***HS***), ***Blishun A.*** (*growth of patterns*), ***Bliumin S.L.*** (*growth of patterns*), ***Bolotov A.A.*** (*simulation among classes of* ***HS***), ***Varshavsky V.I.*** (*synchronization of* ***HS****, simulation of anysotropic* ***HS*** *on the isotropic ones*), ***Georgadze A., Matevosian A., Mandzhgaladze P.*** (*growth of the configurations; universal stochastic and deterministic* ***HS, HS*** *and parallel grammars*), ***Dobrushin R.L., Vasil'ev N., Stavskaya O., Mitiushin L., Leontovich A., Toom A.,*** (*probabilistic* ***HS***), ***Ikaunieks E.*** (*nonconstructible configurations*), ***Koganov A.V.*** (*universal* ***HS****, stationary configurations, simulation of* ***HS***), ***Kolotov A.*** (*models of excitable media*), ***Levenshtein V.*** (*synchronization in* ***HS***), ***Levin L.A.*** and ***Kurdiumov G.L.*** (*stochastic* ***HS***), ***Makarevskii A.I.*** (*implementation of boolean functions in* ***HS***), ***Petrov E.I.*** (*synchronization of* ***2D–HS***), ***Podkolzin A.S.*** (*simulation of the* ***HS****; asymptotic of the global dynamic; universal* ***HS***), ***Pospelov D.A.*** (*homogeneous structures and distributed AI in* ***HS***), ***Prangishvili I.V.*** (***HS*** *architectures of high-parallel processors*), ***Reshod'ko L.S.*** (***HS****–models of the excitable media*), ***Revin O.M.*** (*simulation of anisotropic* ***HS*** *on the isotropic* ***HS***), ***Solntzev S.*** (*growth of patterns*), ***Tzetlin M.*** (*collectives of automata, games in* ***HS***), ***Tzeitlin G.E.*** (*algebras of shift registers*), ***Scherbakov E.S.*** (*universal algebras of parallel substitutions*), and a whole series of others.

It is supposed that the ***HS***–models can play extremely important part as both conceptual and applied models of *spatially-distributed* dynamic systems among which first of all an especial interest the *computational*, *physical* and *biological* cellular systems represent. In the given direction already takes place a rather essential activity of a lot of the researchers who have received quite encouraging results [3-5,8,9,15,69,90,145,146, 153,155-157,161,162,197,269,275,355,536]. At last, theoretical results of the above-mentioned and of a lot of other researchers have initiated a modern mathematical ***HS***–theory evolved to the current time into an independent branch of the abstract automata theory having numerous interesting appendices in various areas of science and technics. Above all, in such fields as physics, development of perspective architecture of high–efficiency computer systems, parallel information processing, development biology, computing sciences and informatics, which are linked to mathematical and computer modeling, etc., by substantially raising the ***HS***–concept onto a new interdisciplinary level.

Meanwhile, the separate researchers in a burst of certain euphoria try to represent the ***HS***–approach as an universal remedy of the solution of all problems and knowledge of outward things, identifying it with a «***new kind***» of science of universal character. In this connection it is necessary to mark the vast and pretentious book of ***S. Wolfram*** [407]. Above all, the book contains many results which have been received much earlier by a whole series of other researchers on ***HS***-*problematics*, including the Soviet authors (*see* [1,3-5,8,9,53-57,80,82,83,127–135,137–142,150-161,169-171,175–179,182–191,195-201,230–233,240,241,536] *and many others*). Furthermore, many fundamental results in this direction belong to other researchers. The unhealthy vanity of the author of the given book does not allow him to look without bias on history of the ***HS***–problematics as a whole. In general, ***Wolfram*** enough frivolously addresses with authorship of the results received in ***HS***–problematics, therefore there can be a impression - everything made in this direction belongs basically to him. At that, the book contains basically results of computer experiments with very simple types of ***HS***–models, drawing the conclusions and assumptions on their basis with doubtful enough reliability and quality. In the book we can meet an irritating density of passages in which the author takes personal credit for ideas which are «*common knowledge*» among experts in the relevant fields. Seems, such ***Wolfram*** passages and inferences similar to them cause utterly certain doubts in the scientific decency and judiciousness of their author.

At last, we absolutely do not agree that ***Wolfram*** book presents a ***new kind*** of science, nevertheless his book would be more pleasant to read if he were more modest. In our opinion, this book represents in many respects a speculative sight both on ***HS***–problematics, and on science as a whole. Here we only shall note, contrary to the pursued purposes the given book not only was not revelation for the experts working in the ***HS***–problematics, but also to a certain extent has caused a little bit deformed representation about the researches area that is perspective enough from many standpoints. With relatively detailed standpoints concerning the above book the reader can familiarize oneself in works [536,567,617] and some others. Meanwhile, in spite of the told above relative to the given book, it can represent the certain interest, taking into consideration the marked and some other certain remarks. In our opinion, the given book doesn't introduce of anything essentially new in the ***HS***–models theory, above all into its mathematical component.

By a certain contraposition to the standpoint on the ***HS***–problematics that is declared by the above book [407] our vision of this question is being represented as follows. Our experience of researches in the ***HS***–problematics both on theoretical, and especially applied level speaks entirely about another, namely**:**

(1) HS–models (homogeneous structures, cellular automata) represent one of special classes of infinite abstract automata with the specific internal organization which provides extremely high–parallel level of the information processing and calculations; the given models form a specific class of discrete dynamic systems that function in especially parallel way on base of a principle of local short–range interaction;

(2) HS-models can serve as a quite satisfactory model of high-parallel calculations just as the Turing machines (Markov normal algorithms, Post machines, productions systems, etc.) serve as the formal models of sequential calculations; from this standpoint the HS–models it is possible to consider and as algebraical systems of processing of finite or/and infinite words, defined in finite alphabets, on basis of a finite set of rules of parallel substitutions; in particular, a HS–model can be interpreted as a some system of parallel programming where the rules of parallel substitutions act as a parallel language of the lowest level;

(3) principle of local interaction of elementary automata composing a HS–model which in result defines their global dynamics allows to use the HS and as a fine environment of modeling of a rather broad range of processes, phenomena and objects; furthermore, the phenomenon of reversibility permitted by the HS does their by very interesting means for physical modeling, and for creation of very perspective computing structures basing on the nanotechnologies;

(4) HS–models represent a rather interesting independent mathematic object whose essence consists in high–parallel processing of words in finite or infinite alphabets.

At that, it is possible to associate the ***HS***–approach with certain model analogue of the differential equations in partial derivatives describing those or another processes with that difference, that if the differential equations describe a process at the average, in a ***HS***–model defined in appropriate way, a certain researched process is really embedded and dynamics of the ***HS***–model enough evidently represents the *qualitative* behaviour of researched process. Thus, it is necessary to determine for elementary automata of the model the necessary properties and rules of their local interaction by appropriate way. The ***HS***–approach can be used and for research of processes described by complex differential equations which have not of analytical solution, and for the processes, that it is not possible to describe by such equations. Along with it, the ***HS*** represent a rather perspective modelling environment for research of those phenomena, processes, phenomena and objects for that there are no known classical means or they are difficult enough.

As we already noted, as against many other modern fields of science, the theoretical component of the ***HS***-problematics is no so appreciably crossed with its second applied component, therefore, it is possible to consider the ***HS***–problematics as two independent enough directions: ***(1) research of the HS as mathematical objects and (2) usage of the HS for simulating;*** at that, the *second* direction is characterized also by the wider spectrum. At that, the level of evolution of the second direction is appreciably being defined by possibilities of the modern computing systems since ***HS***–models, as a rule, are being designed on base of the *immense* number of elementary automata and, as a rule, with complex enough rules of local interaction among themselves. The indubitable interest to them amplifies also a possibility of practical realization of high-parallel computing ***HS***–models on the basis of modern successes of microelectronics and prospects of the information processing at the molecular level *(methods of nanotechnology)*; while the itself ***HS***-concept provides creation of both conceptual and practical models of *spatially–distributed* dynamic systems of which namely physical systems are the most interesting and perspective. Indeed, models which in an obvious way reduce ***macroscopic*** processes to rigorously defined ***microscopic*** processes, represent especial epistemological and methodical interest for they possess the great persuasiveness and transparency. Namely, from the given standpoint the ***HS***–models of various type represent a special interest, above all, from the applied standpoint at research of a lot of processes, phenomena and objects in different fields and, first of all, in physics, computer science and development biology.

The *first* direction enough intensively is developed by mathematicians while contribution to development of the second direction essentially more representative circle of researchers from various theoretical and applied fields (*physics*, *chemistry*, *biology*, *technics*, *etc.*) brings. Thus, if theoretical researches on the ***HS***-problematics in general are limited to *classical*, *polygenic* and *stochastic* ***HS***–models, then the results of second direction are based on essentially wider representation of classes and types of ***HS***–models. As a whole if classical ***HS***–models represent first of all the formal mathematical systems researched in the appropriate context, then their numerous generalizations represent a perspective enough environment of modeling of various processes and objects.

It is necessary to pay more steadfast attention and to a question of ***HS*** popularization and their applied possibilities in various fields. Above all, new successful examples of application of the concept, results and methods of the ***HS***–problematics in other areas will promote it. Thus, we come to necessity of essentially more active application of the ***HS***–concept for various fields of science and technics. And that, indeed, is a important enough, complex and multifold problem, promotional to properly the further development of the ***HS***-problematics. Once more, one should not to condescend both to exaggerated popularization and to obviously speculative representation of the ***HS***–concept as a certain all-powerful means of knowledge of complex enough outward things, and it would be rather naive to reduce it to some ***HS***–models. Or else, the serious scientific community will enough skeptically think about such potentially important direction of modern cybernetics, what will has a pernicious influence both upon the ***HS***–problematics, and upon its many existing and potential appendices [536,567,617]**.** Meanwhile, and as independent mathematical object the homogeneous structures **(***cellular automata***)** not carry in our opinion the fundamental character, making up one far from the very complex of subclasses of the class of all parallel discrete dynamic systems. The overwhelming majority of theoretical results on cellular automata is reduced to their research by means of known mathematical approaches, not introducing anything fundamental. Thus, objects of such type are of interest, above all, from the applied points of view in those fields where such essential factors play a principal part as uniformity, localness of interaction along with high–parallel principle of functioning. At that, it is necessary to mark once more what that is our private standpoint however formed under influence of a whole series of rather essential prerequisites.

In what follows**,** we present our main results of the research we have done in mathematical theory of the ***classical homogeneous structures (HS;*** *synonym* **«***Cellular Automata***»)** during ***1969 – 2011.*** Indeed, we have done much more, specifically, in such fields as mathematics, software, statistics, cybernetics, computer science, technology and mathematical theory of homogeneous structures. Much of these investigations have been stimulated by scientific program of *Tallinn Research Group* ***(TRG)*** and then of *International Academy of Noosphere* ***(IAN).*** Now, the *cellular automata* ***(CA)*** problems is a well enough developed independent field of the modern mathematical cybernetics that has considerable sphere of appendices. In this book a number of fundamental problems of the ***HS***–problematics is considered on the example of classical ***HS***–models forming up the certain base of the ***HS***-concept as a whole. At the same time, the themes considered in the book represent a rather transparent and simple model both for mastering of ***HS***–conception and principal concepts, and base results of the modern ***HS***–problematics as a whole.

The classical ***HS***–model is chosen as base since it is a basis or a direct prototype of all most known ***HS***–like models **(***homogeneous computing environments***,** *cellular processors and structures***,** *systolic structures***,** *neural and iterative networks***,** *etc.***),** not requiring special knowledge from a lot of sections of mathematics, cybernetics, etc. In addition, the theory of classical ***HS***–models for today is the most investigated and advanced as quite independent mathematical problematics in a whole.

In the conclusion once again it is necessary to note a rather important circumstance, at discussion of the *classical* homogeneous structures we emphasize the following rather essential moment. We considered the ***HS***–models that are a class of *parallel discrete dynamic systems* as formal *algebraic systems* of processing of finite words ***(configurations)*** in finite alphabets without any reference, as a rule, to their microprogrammed environment, i.e. without use of their cellular organization on lowest level inherent into them, what distinguishes our approach to research of the given objects from approaches of a lot of other researchers. Also, we consider ***HS***–models as formal mathematical object having specific inside organization without ascribing to them certain universality and generality in perception of the World.

At such approach the ***HS***–models are considered at especially formal level, not allowing in full measure to utilize their intrinsic property of high parallelism in area of computations, and information processing as a whole. Naturally, for solution of a lot of applied problems in the ***HS***–environment and obtaining of a series of thin results, first of all, of model character an approach at a microprogram level is needed when a researched process, algorithm or phenomenon is directly embedded into an environment, using its concrete parameters**:** a dimensionality, a neighbourhood index, a state alphabet and local transition function. In case of such approach it is possible to receive solutions for a lot of important concrete appendices along with generalizations of a rather high level of theoretical character. In particular, by direct embedding of the universal computing algorithms or logical elements into objects of the given kind, it is possible to constructively prove existence of the property of universal computability of homogeneous structures, etc.

Meanwhile, having a lot of cogent advantages, the above so-called the ***direct*** approach to research of structures does not allow to receive in a lot of cases the results describing them as the especially mathematical objects demanding the use of both methods and results of rather more abstract areas of modern mathematics. Thus, a reasonable combination of both specified approaches to investigation of such class of dynamic discrete systems, as homogeneous structures, seems the most natural, in our opinion.

The ***HS–***problematics questions considered in the present monograph, in many respects were conditioned by own interests and tastes of the authors along with traditions of creative activity of the *Tallinn Research Group* in the given direction. However, already on basis of the offered results, the reader can receive a quite definite situation of the modern state and problematics of some basic areas of the mathematical theory of ***HS-***models and some their applied aspects. At last, it is necessary to note, at present the ***HS–***problematics is in a stage of intensive enough development and the further work in the given direction is extremely desirable from many points of view and, above all taking into account interdisciplinary character of the general ***HS–***concept.

Contents of the proposed book can be briefly characterized as follows. The ***first*** chapter presents the basic conception of *classical homogeneous* structures, including the basic concepts, definitions and designations. The basic types of homogeneous structures **(*HS-*models)** are defined; at that, a draft architecture of the ***HS–***theory and its appendices together with basic means of research in this direction are briefly discussed. In this direction we proceeded from own experience of researches of ***HS–***models, therefore the presented block diagram of both theoretical and applied aspects carries in many respects a subjective character.

The ***second*** chapter deals with the nonconstructability problem for the classical homogeneous structures, beginning with certain preliminary information concerning the given problematics. Then, four main types of the nonconstructability in *classical* ***HS-***models are introduced; along with the basic nonconstructability type ***(NCF)*** additionally three types of the nonconstructability ***(NCF-1, NCF-2, NCF-3)*** are determined, that allow to investigate essentially more in detail the dynamics of *classical* structures. In this chapter such questions as criteria of existence of *base* types of *nonconstructability* in classical homogeneous structures along with algorithmic aspects of the nonconstructability problem, and the reversibility problem of dynamics of classical structures together with a whole series of the connected questions of dynamics of classical ***HS*–**models are considered. This problematics represents a very important component of the mathematical theory of classical ***HS*–**models.

The ***third*** chapter considers so–called extremal constructive resources of classical ***HS*–**models among which accent is done on universal and self–reproducing finite configurations. The given problematics plays an important enough part at researches of dynamic aspects of classical ***HS*-**models. Whereas in the ***fourth*** chapter the complexity problem of finite configurations in classical structures is discussed. At last, in the fifth chapter the parallel formal grammars and parallel languages that are defined by ***HS*–**models of the certain types are considered.

In the ***sixth*** chapter the modeling problem in classical structures along with related questions is considered. Here such questions as modeling of known *formal* algorithms by means of classical structures, modeling of classical structures by structures of the same class, formal parallel algorithms defined by classical one-dimensional structures along with some special questions of modelling in classical structures linked with their dynamics can be noted. The questions considered in this chapter play an important enough part at researches of ***HS*** as a certain formal environment for modelling. In this direction the computer modelling of ***HS*** plays an important enough part**;** a number of similar means can be found in library for *Maple* [637] and package for *Mathematica* [638]**.**

In the ***seventh*** chapter the decomposition problem of global transition functions in classical ***HS*-**models is considered. Here such questions as decomposition of special global transition functions, some approaches to solution of the general decomposition problem, certain questions of *solvability* of the decomposition problem of global transition functions, the complexity problem of global transition functions can be noted. In addition, some related rather interesting questions are discussed too. The mentioned questions allow more in detail to investigate such *basic* component of ***HS*–**models as global transition functions.

At last, the ***eighth*** chapter deals with the certain applied aspects of the ***HS*–**problematics. The questions of use of the ***HS*–**approach to solving of certain rather important problems in pure mathematics and biology of development are considered. So, the first direction is presented by a solution of a interesting enough combinatory problem of ***H. Steinhaus*** along with a ***S. Ulam's*** problem from theory of numbers. Whereas the second direction presents such aspects in biological sciences as formal discrete models of self-reproduction, modeling of processes of growth in homogeneous structures of various types along with ***HS***–models of differentiation, regulation and regeneration. Along with above themes the application of ***HS***–models of different types in computing sciences and a whole series of other fields is discussed. These applications once more illustrate modelling potentiality even of classical ***HS***–models. At the same time, outside the present book we considered a whole series of rather interesting appendices and of classical, and other types ***2–HS*** which illustrate potential opportunities of objects of the given kind.

We pass on now to the direct consideration of the basic aspects of the ***classical homogeneous structures***, having introduced previously the paramount concepts, definitions and designations. At that, the other basic elements will be introduced as required. It is necessary to mark, that with a view to decrease volume of the book by reason of essential extensiveness of the material in overwhelming majority the presented results carry in a great extent the intensional character which in most cases are supplemented with most essential requisite illustrations and clarifications. Whereas with rigorous proofs *(at times rather volumetric)* of the presented results the reader can familiarize oneself in the cited original sources. In addition, the examples quoted in the monograph for acknowledgement of corresponding assertions are simple enough, having much more complex analogues however they allow the reader enough simply to carry out all necessary verification. Unfortunately, restricted volume of the book had not allowed to us to consider a lot of interesting enough considerations on the given problematics, but they also can be found in the cited literature. In short, the represented material having brief character and concise considerations is intended above all for a wide enough circle of the readers which at times are far from any necessary grounding in the given problematics.

Extensive bibliography, in turn, contains numerous references to a lot of other sources in this direction, allowing to the more exacting reader to receive more detailed information on many important questions of the given problematics of both the considered here and the residuary ones for any reasons outside of the given monograph**;** in addition, the more comprehensive bibliography supplementing the presented one here can be found, for example, in vast bibliography [536,545]**.** Many rather useful references can be found and in the internet, in particular, by the key phrase ***"cellular automata".***

So, in conclusion it makes sense to dwell on a rather essential moment, namely. During numerous studies in the theory of *cellular automata* (*in generally accepted Russian terminology – so–called homogeneous structures*) extensive enough bibliography of original sources of a various level as directly in the theory, and in its numerous appendices in various fields had been collected by us. Quite naturally, the given bibliography is not exhaustive however it can present a certain interest for researchers in the given field, first of all, of beginners in this direction. Meanwhile, the interested reader has an opportunity to supplement the presented bibliography by the materials which are absent in it. We hope, that the given *bibliography* will allow to outline better both a circle of the active researchers in the given field, and breadth of a scope of the problems considered by them.

In addition, first of all, it concerns the Soviet and Russian researchers who have received a lot of priority results of fundamental character in homogeneous structures theory and their applications with which the community of English–speaking researchers are familiar insufficiently well or are not familiar entirely. Subsequently, certain of these results have been rediscovered by other researchers. That is especially topical and for the reason, that a whole series of the Soviet researchers directly stood at the beginnings of the making given direction of that class of parallel dynamical systems and abstract infinite automata within the theory of modern mathematical cybernetics. Basically, the represented bibliography is not annotated however headings of publications give the defined enough notion about the contents of the quoted material.

In particular, the pioneer results of the Soviet researchers were done in a whole series of directions in the homogeneous structures theory such as definition of Russian–language terminology and insertion of basics of homogeneous structures theory in text-books for university courses on informatics [1,94-96]. Along with our works [3-5,9,11,12] they gave a rather essential rise of interest to *homogeneous* structures in the USSR of both as interesting self–containded mathematical object of research, and a convenient enough environment of modelling of wide enough circle of processes, objects and phenomena. Furthermore, interesting enough results were obtained in a whole series of directions of theory of homogeneous structures such as: the nonconstructability problem, the universal and self-reproducing finite configurations, complexity of finite configurations, modelling problem in homogeneous structures, decomposition & complexity of global transition functions, etc. Some of these results were rediscovered by researchers of other countries.

Chapter 1. The basic concept of homogeneous structures *(HS–models, Cellular Automata)*

In conformity with afore-said, the ***homogeneous structures (HS)*** at all generality represent highly formalized models of the certain abstract Universes developing by simple rules and consisting of rather simple identical elements. The ***HS***–universes of such kind develop according to local and everywhere identical rules of interaction of the elements forming them ***(****laws****)***. In the given context we can consider ***HS***–models as a certain analogue of physical concept of «*field*»**.** The space of a ***HS***–universe represents a regular lattice whose each cell presents a certain identical element ***(****an elementary particle****)*** that receives a finite number of states. Development of a similar ***HS***–universe acts in a discrete time scale ***(t=0,1,2,3, ...)*** according to a finite set of instructions of change of states of its elements during each moment of time ***t > 0*** as a function of states of the element and finite number of its nearest elements during the previous moment of time ***(t – 1).*** Change of configurations of such *universe* under the action of global transition function defines *dynamics* of operating of a ***HS***–model with time**;** this aspect plays the basic part in researches of its behavioural ***(****dynamical****)*** properties.

At that, states of elementary automata of ***HS***-models can be associated with such various concepts as commands of cellular microprocessors, characteristics of points of an abstract field, symbols of certain parallel formal systems, states of biological cells, etc. Whereas the histories of finite configurations in a certain ***HS***-model associate with dynamics of various sort of discrete processes, objects and phenomena, embedded in such model. Similar models can be successfully applied in the very various areas. We can interpret ***HS***–models not only as an abstraction of biological cellular systems, but also as a theoretical basis of artificial parallel systems of the information processing or as a environment of presentation of conceptual and practical models of *spatially–distributed* dynamic systems. Furthermore, from logic standpoint the ***HS***–models are infinite abstract automata with specific internal structure defining a series of important enough properties allowing to use their as a new perspective environment of modelling of different discrete processes utilizing a mode of maximal paralleling. In toto, the ***HS***–problematics can be considered as ***structural*** and ***dynamic*** component of the theory of infinite automata with the specific internal organization carrying an qualitative character having very important applied aspects.

In spite of such simple organization and principle of the functioning, ***HS***–universes admit complex enough behaviour ***(****dynamics of behavior of configurations of states of elements forming them****)***, providing modelling a plenty of objects, processes, and phenomena in multifarious areas of science, technics, etc. For more objective consideration of the concept of ***HS***–universes ***(****models****)*** we need a whole series of basic concepts and definitions, allowing at a formal level to investigate the opportunities of homogeneous structures as a perspective environment of modeling in a series of strategically important directions of the modern natural sciences along with other applied fields. In addition, such ***HS***–models presents the certain interest as an independent object of researches.

1.1. The basic concepts, definitions and designations

In the present section the basic concepts, definitions and designations which are connected to the concept of a classical ***HS***–model and used during all our further consideration are introduced. Detailed enough discussion of the basic concepts of the ***HS***–problematics and questions linked with them will allow the reader to understand more deeply the *basics* of the given field of general theory of infinite abstract automata.

First of all, we note, the basic consideration of the material is based on so–called ***classical*** concept of ***d–dimensional homogeneous structures (d-HS; HS-****models****; d-CA; d≥1)*** concerning which a whole series of *basic* definitions is introduced and certain results concerning the important enough question of generality degree of *classical* concept are totalized. An axiomatic definition of an arbitrary *classical homogeneous* structure ***d–HS (d≥1)*** is introduced as follows.

1. A classical ***d***–dimensional homogeneous structure ***(d–HS)*** is defined as the ordered tuple of the following four components, namely**:**

$$d\text{–}HS \equiv \langle Z^d, A, \tau^{(n)}, X\rangle$$

where ***A*** is a finite, non-empty set called the *state alphabet*, and it is the set of states which each of the individual finite–state automata in the structure can assumes. The state alphabet ***A*** contains a distinguished element called the ***quiescent*** state, that is designated by «***0***» ***(****at that, for convenience in a number of cases the symbol «****0****» will be identified by symbol* « »***)***; essence of the given special state will be elucidated a little later. We, without loss of generality, shall use the set ***A***={***0,1,2,…, a–1***} which contains ***a*** elements – integers from ***0*** up to ***a–1*** as a state alphabet.

Component Z^d is a set of all ***d***–dimensional tuples – *integral coordinates* of points in Euclidean space E^d, i.e. Z^d is integer lattice in E^d, whose elements serve for spatial identification of individual automata of a ***d-HS***. Thus, component Z^d defines homogeneous space of the structure in which it functions. It is possible to show [1,3,5], that other types of regular lattices as a homogeneous space mathematically do not bring anything new concerning dynamic properties of ***HS***–models, i.e. it is quite enough to be limited oneself to space Z^d; thus, the integer lattice is entirely sufficient. So, Z^d is a set of all ***d***–tuples of integers which is used to name the cells where ***Z*** is a set of integers, and is called ***space***, i.e. space in which all processed elements are identical. An automaton in Z^d can be thought of as a ***name*** or ***address*** of a particular finite-state machine which occupies this *position* in the space Z^d. It will frequently be rather convenient to identify an automaton located at a cell ***j*** with ***j***–cell itself. Naturally, in a lot of applied aspects of ***d–HS (d ≥ 1)*** their geometry plays a rather essential role (*so, question relative to geometry of a homogeneous space gets special significance in the structural theory, when properties of* ***d–HS*** *are investigated depending on the internal organization*), however in the present book, the given question is not considered and the reader is referred to the works presented in bibliography [536,640].

Dimensionality ***(d)*** of homogeneous space of a ***HS***–model also plays a rather essential role, differentiating all set of models into two various subsets: one-dimensional models ***(d = 1)*** and ***d***–dimensional models ***(d ≥ 2)***. At that, transition already from ***1***–dimensional to ***2***–dimensional case not only sharply changes the dynamics of the ***HS***–models that is caused by increase of dimensionality but and increases the complexity of most of problems being solved concerning them. So, below we shall show that certain problems of the dynamics for case of classical ***1–HS*** and ***d–HS (d≥2)*** are solvable and unsolvable accordingly; i.e. generally speaking, for their solution there is not any decision algorithm in our modern understanding.

In the given attitude ***1-HS*** represent a special subclass of class of all ***d–HS (d ≥1)***, researched effectively enough. And if in the plan actually of modelling ***1–HS*** special prospects, in our opinion, have not, however represent a certain interest as an independent mathematical object. At that, on the example of ***1–HS*** it is much easier to master the concept of classical ***HS***–models, what and will be widely used by us hereinafter. Because of what the ***1–HS*** are much easier even than the class of ***2–HS*** (*much smaller set of possible rules of transition for an individual automaton* *of* ***1–HS; solvability of a series of problems, etc.),*** a lot of types of ***1-HS*** was the most intensively investigated from the theoretical point of view**;** in addition, the majority of both theoretical publications, and computer simulation their for the purpose of research of those or other dynamic properties have been devoted just to the given class of ***HS–models.***

Into each point of space Z^d a copy of the finite automaton of Moore is placed, whose state alphabet is ***A.*** Well is known, Moore's automaton represents a finite automaton, whose output at the time ***t*** depends on its internal state at the same time ***t*** only and does not depend on value of its inputs. A state S_t of such automaton at the time $t > 0$ is a certain function $S_t=F(I_1,...,I_n,t-1)$ of its inputs in moment ***(t-1);*** at that, output of the automaton at the time ***t*** is identical to its internal S_t state. In this case each point of Z^d will define a ***name*** (*coordinate*) of the elementary automaton of Moore placed in the given point. For convenience below we shall identify points of space Z^d with elementary automata located in them. Thus, two terms «*automaton z*» and «*automaton with coordinate* $z \in Z^d$» we shall assume as identical ones.

Further we assume, the component ***X,*** named as ***neighbourhood index*** of ***d–HS,*** is an ordered tuple of ***n*** elements from Z^d, which serves for definition of the automata neighboring for any elementary automaton of ***d-HS*** $(d \geq 1)$, i.e. those its automata with which the given automaton is directly interlinked by information channels, i.e. communicates. In an elementary example of structure ***2–HS*** we can present space Z^2 in the form of cellular paper in each cell of which a copy of some Moore automaton is located. Then, $X_N=\{(0,0),(0,1),(1,0),(0,-1),(-1,0)\}$ and $X_M=\{(i,j)\}\ (i,j \in \{0,1,-1\})$ are called by neighbourhood indexes of ***J. Neumann*** and ***E.F. Moore*** accordingly**.** The given neighbourhood indexes ***X*** long ago became classical, and are widely used in researches of theoretical and applied aspects ***d–HS,*** whereas the ***neighbourhood templates (NT)*** determined by them have rather transparent geometrical image (*fig.* **1**)

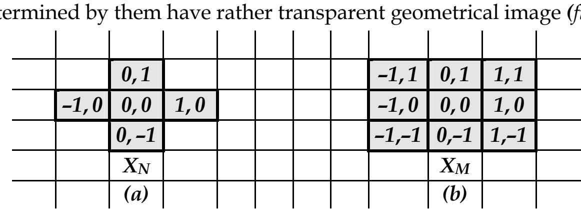


Fig. 1. Neighbourhood templates of J. Neumann (a) and E.F. Moore (b)

In one-dimensional case both these types of neighbourhood indexes ***X*** coincide. So, the reader can enough easily generalize two-dimensional neighbourhood indexes X_N and X_M onto general ***d***–dimensional case ***(d ≥ 3).*** Generally speaking, neighbourhood template of a ***HS***–model is arbitrary and can accept rather exotic kind defined by applied aspects of the ***HS***–model. The following neighbourhood indexes ***(****together with neighbourhood templates corresponding to them****)*** in the aggregate with the above-mentioned neighbourhood indices of ***J. Neumann*** and ***E. Moore*** we can ascribe to the most frequently used ones, namely**:**

	(0,1)			
0 | 1	(0,0) | (1,0)	–1 | 0 | 1	0 | 1 | ... | n–1	
X_1={0,1}	X_2={(0,0), (0,1), (1,0)}	X_3={–1, 0, 1}	X_4={0, 1, 2, ..., n–1}	

Thus, neighbourhood indexes X_1 and X_2 are elementary for structures ***1–HS*** and ***2–HS*** accordingly. Whereas for ***d***–dimensional classical ***HS–*** models an elementary neighbourhood index ***X*** assumes the next form**:**

$$X=\underbrace{\{\underbrace{(0,0,\ldots\ldots,0)}_{d},\ \underbrace{(1,0,0,\ldots,0)}_{d},\ \underbrace{(0,1,0,\ldots,0)}_{d},\ \ldots\ldots,\ \underbrace{(0,0,\ldots,0,1)}_{d}\}}_{d+1}$$

i.e. one automaton z_o of the elementary ***NT*** is central one and from it along each axis of E^d-coordinates is spread out strictly one elementary automaton of ***d–HS (d ≥ 1).*** Meanwhile, in spite of universality of the simplest neighbourhood indexes ***(****every classical* ***structure d–HS*** *can be modeled by a certain structure of the same dimension but with an elementary neighbourhood index****)*** for concrete applications and the many researches more complex indexes ***(****for example****,*** *for case of* ***1–HS*** *– the neighbourhood index* ***X=***{***0,1, ..., n-1***}***)*** with special geometry of ***NT*** are used. In a whole series of practically important cases the mentioned approach allows to simplify essentially a process of embedding into classical ***HS***–models of concrete processes, objects, phenomena and algorithms. At that, the approach is enough effectively applied at theoretical researches of the ***d–HS (d ≥ 1)*** which are based, in particular, on computer modelling.

Thus, the ***neighborhood index*** of ***d–HS*** is ***n-****tuple* of different ***d-****tuples* of integers and is used to define the neighbors of any cell, i.e., those cells from which the cell directly receive information. Then, ***n*** neighbours of cell *z* are cells $z+\alpha_o, z+\alpha_1, \ldots, z+\alpha_{n-1}$, where $X=\{\alpha_o, \alpha_1, \ldots, \alpha_{n-1}\}$. The neighborhood index ***X*** describes the uniform interconnection pattern among elementary automata in ***d–HS.*** It represents positions relative to the given cell *z* of all cells whose automata are directly connected to cell ***z*****.** If index ***X*** contains the point 0^n=**{*0,0,0, ..., 0,0*}**, every automaton is contained in its own neighborhood template, where neighborhood template consists of all neighbours of cell. Without loss of generality, we shall, as a rule, assume that ***X*** contains the point 0^n**,** which defines ***central*** automat of an arbitrary neighborhood template.

It is proved the dynamics of a ***d-HS (d≥1)*** does not depend on a choice of an arbitrary automaton of ***NT*** as central [1,3]**.** Thus, subdivision of ***d–HS*** onto structures with strongly expressed gradient of information transfer, caused by a choice of the central automaton of ***NT*****,** does not change dynamic and computing opportunities of ***HS*****–**models in the time attitude, but influences onto their constructive characteristics, i.e. onto characteristics dependent on geometry of space of a model. In a constructive attitude, generally speaking, such ***HS*****–**models differ. The interesting enough examples of such sort along with considerations in this context can be found, in particular, in [54–56,79,88,90,567,617,640]**.**

Among all neighbourhood templates ***(NT)*** distinguish connected and disconnected ones**;** this parameter generally essentially influences the dynamics of ***d-HS (d ≥ 1).*** At that, a certain ***NT*** is called by ***connected*** if area occupied with it is connected in topological sense**;** otherwise, ***NT*** is called ***disconnected.*** So, for example, two ***1-HS*** with neighbourhood indexes ***X*****=**{***0,1,2,3,4,5,6,7***} and ***X'*****=**{***0,1,2,4,5,7,8,10***} have the connected and disconnected ***NT*** accordingly what even for case of identical local transition functions $\sigma^{(8)}$ can cause rather essential distinctions in their dynamics. Detailed enough analysis of both types of neighbourhood templates in context of their influence on dynamics of ***HS*****–**models can be found in [567,617]**.** Further, we shall deal, as a rule, with connected neighbourhood templates, keeping in mind the circumstance what an arbitrary disconnected ***NT*** always it is possible to replace with certain connected equivalent ***NT*** of the same maximal size, having included in appropriate ***NT*** insignificant elements. Meantime, the trends of the majority of the major dynamic characteristics of ***d–HS (d ≥ 1)*** with the connected and disconnected neighbourhood templates are kept under the condition of identity of local transition functions $\sigma^{(n)}$**.** In particular the dynamic property of universal *reproducibility* in the Moore's sense of finite configurations is kept with change only of certain numerical characteristics of reproduction process of finite configurations [54–56, 79,88,545,567,617]**.** The given question will be detailed a little below.

2. Three first components of an arbitrary ***d–HS (d≥1)*****,** namely, the *state* alphabet ***A***, the space Z^d, and neighbourhood index ***X*** form so–called ***homogeneous space.*** Homogeneous space is static part of structures ***d–HS (d ≥ 1).*** That describes physical structure of ***d–HS (d ≥ 1),*** however it does not specify interactions that will take place among elementary automata in Z^d, i.e., strictly speaking, the above three components do not determine the dynamics of ***HS***–models.

In order to define and to study the operation *(dynamics)* of a ***d-HS,*** it is necessary to have means for a describing of the current state of entire homogeneous space at any given time $t > 0$. A state of the entire space is called a *configuration* ***(CF)*** of the space and it is simply the complete set of current states of each of elementary automata in the ***d–HS.*** So, a *configuration* is any mapping $CF: Z^d \rightarrow A$; $C(A,d)$ denotes the set of all *configurations* with respect to Z^d and ***A***, i.e. $C(A,d) = \{CF \mid CF: Z^d \rightarrow A\}$. By special symbol « ^{d}» is denoted completely *zero* configuration ***(CF);*** $^d: Z^d \rightarrow 0$, i.e. when all elementary automata in ***d–HS (d ≥ 1)*** are in the *quiescent* state «***0***». Identifying both states {«***0***», « »}, we shall use the second of them for designation of infinite areas of space Z^d filled with automata only in the quiescent state «***0***». This state has numerous and natural enough interpretations from the applied standpoint, above all. It is necessary to have in mind, that all results presented in this book and formulated relative to quiescent state «***0***» are valid for the general case of quiescent state $h \in A$, i.e. for all classical structures ***d-HS (d ≥ 1).***

The set of configurations ***C(A,d)*** is nonuniform relative to dynamics of functioning of ***d–HS*** because of presence of the selected *quiescent* state, therefore we determine two its basic subsets of configurations – ***finite*** configurations $C(A,d,\phi)$ and ***infinite*** configurations $C(A,d,\infty)$. On fig. **2** two simple examples of ***1***–dimensional finite & infinite configurations in alphabet A={ ,0,1,2,3} ***('0' ≡ ' ')*** are represented; at that, « » denotes an infinite chain of states «***0***» to the left or/and to the right.

a.1	...		2	1	3	2	0	1	3	0	1	2	3		...
a.2	...	1	2	1	0	1	2	2	1	0	3	2	1	3	...

Fig. 2. Examples of finite (a.1) and infinite (a.2) configurations.

Under ***finite*** configuration $c \in C(A,d)$ we shall understand a ***CF*** which contains only finite number of elementary automata in states distinct from a quiescent state «***0***»; otherwise, the configuration is considered the ***infinite.*** Formally, the given definition is formulated as follows.

Let ***c(z)*** be the current state of an automaton ***z*** located at ***z***–cell of Z^d.

Support of a configuration ***c*** (*denoted by* [***c***]) is the set of all such cells ***z*** that ***c(z)≠0;*** i.e., the support is a nonquiescent part of the configuration ***c.*** Configurations with *finite support* are of particular interest**;** the set of all such ***finite*** configurations is denoted as ***C(A,d,ϕ); C(A,d,∞)*** denotes the set of all configurations with *infinite* support and ***C(A,d) = C(A,d,ϕ) ∪ C(A,d,∞);*** in addition ***C(A,d,ϕ) ∩ C(A,d,∞) = ∅,*** where ***∅*** denotes the empty set, while dimensionality ***d*** of configurations is determined by dimensionality of an arbitrary classical structure ***d-HS (d≥1).*** Here and further we shall use standard set–theoretical and logic designations.

Special denotation is used to present the sets of all ***1***–dimensional ***CF*** with finite and infinite support in a ***1-HS,*** namely**: *C(A,1,ϕ)*** & ***C(A,1,∞)*** accordingly**; *C(A)=C(A,1,ϕ)∪C(A,1,∞).*** At that, any finite configuration ***c*∈C(A,1,ϕ)*** is represented as $x_1x_2 \ldots x_m$, where indicates a string of unbounded length of elementary automata in state «***0***», i.e.,

$$c^* = \; x_1\, x_2 x_3 \ldots x_m \;\; \equiv \ldots 000\, x_1 x_2 \ldots x_m\, 000 \ldots; \;\; x_j \in A; \;\; j=1..m$$

Taking into account the specificity of classical structures that is caused by presence of quiescent state «***0***» along with a lot of other important enough reasons considered below we shall ascribe the completely null configuration c_o = ' ' to the set ***C(A,d,ϕ).*** The given approach allows to receive a lot of interesting enough results concerning dynamics of the classical structures. That concerns the problems of nonconstructability and reversibility in the classical homogeneous structures considered below. So, for a structure ***d–HS (d ≥ 1)*** the set ***C(A,d,ϕ)***\{ } can consists solely of nonconstructible configurations such as ***NCF. Diameter*** of a finite ***d–***dimensional configuration ***c*** is defined as distance between its two extreme elementary automata in the non-quiescent states of ***A***\{***0***} and will be denoted as **|*c*|.** For ***1–***dimensional case a diameter will be associated with ***length*** of a finite configuration defined analogously.

Along with configuration of all space Z^d, a configuration c_b of a finite ***d–***dimensional hypercube ***(block)*** $b \subset Z^d$ of elementary automata of the structure ***d–HS*** is defined too**;** a set of all such configurations we shall denote as ***C(A,d,B).*** The concept of ***block*** configurations plays a rather important role, for example at investigation of the nonconstructability problem in a classical ***d–HS (d ≥ 1).*** The given question further will be detailed. Inasmuch as further speech will go, at a great extent, about one-dimensional ***HS (1–HS)*** for designation of one–dimensional finite, block and infinite configurations the designations ***c**** = $c_1c_2c_3c_4 \ldots c_k$,

$c_b=b_1b_2 \ldots b_p$ and $c_\infty=\infty c_1c_2c_3c_4 \ldots c_k\infty$ will be used accordingly, where $c_1, c_k \in A\backslash\{0\}$, and $c_j, b_q \in A;\ j=2..(k-1);\ q=1..p$. At that, length of a finite configuration c^* equals k, and is designated as $|c^*|$ ($|c^*| = k$); length of a block configuration c_b is defined analogously.

Sets of one–dimensional *finite*, the *block* and *infinite* configurations we shall designate as $C(A,1,\phi)$, $C(A,1,B)$ and $C(A,1,\infty)$ accordingly. Thus, directly from definitions of sets $C(A,1,\phi)$ and $C(A,1,B)$ follows, the first is closed concerning the global transformation $\tau^{(n)}$ determined below, whereas the second is closed concerning the concatenation. Moreover, the following two relations are obvious, namely:

$$(\forall c_b \in C(A,1,B))(c = \quad c_b \in\in C(A,1,\phi))\ \&\ (\forall c^* \in C(A,1,\phi))([c^*] \in C(A,1,B))$$

Zero configuration $c = '\quad '$ may be presented as a finite configuration of the zero size. In this context with good reason we can ascribe it to the set $C(A,d,\phi)$. This assumption is expedient because of a whole series of rather important reasons being discussed below in detail. We go on to the description of principle of acting of classical ***HS***–models.

3. The operation of ***d–HS*** $(d \geq 1)$ occures in discrete time $t=0,1,2,\ldots$ and is specified by a ***local transition function (LTF)*** $\sigma^{(n)}$ which sets a state of each elementary automaton in the current moment $t>0$ on the basis of states of its neighbor automata (*according to a neighbourhood index* ***X***) in previous moment $(t-1)$. In other words, the local transition function $\sigma^{(n)}$ is an arbitrary mapping $\sigma^{(n)}: A^n \rightarrow A$; further for ***LTF*** $\sigma^{(n)}$ of ***HS***–models we shall use the following basic designations, namely:

$$\sigma^{(n)}(a_1, a_2, \ldots, a_n) = a^*_1; \qquad a_j, a^*_1 \in A \quad (j=1 \,..\, n) \qquad (1)$$

$$a_1a_2 \ldots a_n \Rightarrow a^*_1 \text{ – a set of parallel substitutions} \qquad (2)$$

where a_j – states of any z-automaton of ***d-HS*** $(d \geq 1)$ and its neighbours (*relative to neighbourhood index* $X = \{x_1, x_2, \ldots, x_n\}$) at moment $(t-1)$, and a^*_1 – a state of the same z-automaton at the following moment $t>0$. At that, the considered z-automaton of ***d–HS*** $(d \geq 1)$ is assumed the central concerning its neighbourhood template. In particular, arbitrariness of choice of the central automaton of a neighbourhood template allows to speak about admissibility of one–side universal classical structures ***1–HS***. The detailed explanation of a question of arbitrariness of choice of the central automaton can be found in works [1,3,5,54,536,567,617]. Whereas in each case as the *central* one the most acceptable automaton of the neighbourhood template is chosen.

Formula representation of a ***LTF*** when calculation of the subsequent state of an arbitrary ***z***-automaton in a structure is made on the basis of the formula ***(1)*** is the most convenient. So, in many rather interesting cases such approach is possible, however in a lot of cases the use of a local transition function $\sigma^{(n)}$ in the form of set of parallel substitutions ***(2)*** is necessarily required. The set of parallel substitutions ***(2)*** defines the program ***(****parallel algorithm****)*** of functioning of classical ***HS***–models**;** parallel substitutions ***(2)*** represent the low-level *parallel* programming language in the environment of classical ***HS***–models.

In particular, if at moment ***t*** the current configuration ***c**** of ***1–HS*** with alphabet ***A*** **=** {***0,1,2,3,4,5***}**,** neighbourhood index ***X*** **=** {***0,1,2,3***} **&** ***LTF*** $\sigma^{(4)}$ determined by the parallel substitution ***1023*** **⇒** ***2*** {*or in equivalent form* $\sigma^{(4)}(1,0,2,3)=2$} has kind **(φ)** then in the next moment ***t+1*** it passes into a configuration **φ*,** which contains the distinguished ***S***–automaton of the structure in a new state received on basis of ***LTF*** $\sigma^{(4)}$ ***(****parallel rules of substitution****)*** of such ***HS***–model, namely**:**

t	**....**	*1*	*3*	*0*	*1*	*0*	2	*3*	5	2	*4*	*3*	*3*	**....** : φ
					⇓									
t+1	**....**				2									**....** : φ*

S *–* *distinguished elementary automaton of* ***1–HS***

Formula presentation of ***LTF*** $\sigma^{(n)}$ is especially preferable at computer realization of ***d–HS*** whereas parallel substitutions are irreplaceable on a stage of programming of certain concrete ***HS***–models. Questions of formula representations ***(1)*** of parallel substitutions ***(2)*** are considered enough in detail in [1,3,5,54]**.** Meanwhile, by far not all local transition functions $\sigma^{(n)}$ can be presented in a formula form, ensuring the work with ***HS***–models only at a level of systems ***(2)*** of parallel substitutions determining them local transition functions ***(LTF)*** $\sigma^{(n)}$**.**

In the given monography the ***HS***–models ***LTF*** $\sigma^{(n)}$ of which satisfy the determinative condition $\sigma^{(n)} : 0^n \Rightarrow 0$ or $\sigma^{(n)}(0, ..., 0) = 0$ are considered, i.e. structures with *restriction* on speed of information transfer in them ***(****a certain analogue of a limit of speed of pass of light according to the modern physical conception****).*** The given assumption plays a rather essential role at researches of dynamic properties of ***d–HS*** $(d \geq 1)$ and well meets the requirements of usage of structures as a basis of modelling of parallel dynamic systems of the various nature and assignment.

The above determinative condition not only introduces restriction on speed of information distribution in ***HS***–models, but also defines *space (certain peculiar formal **vacuum**)* in which dynamics of development of the researched discrete objects, processes and phenomena occurs. At that, an arbitrary element of alphabet ***A*** of structure can be chosen as the quiescent state, however according to a whole series of reasons for this purpose the element {**'0' | ' '**} seems to us the best.

The structures satisfying the above determinative condition, we shall name ***stable*** ones, otherwise **– *unstable.*** In context of research of ***HS***–models as an independent mathematical object, a certain interest the unstable structures represent too. Meanwhile, unstable structures can present interest also from the point of view of research in them of the models based on the concept of instant information transfer upon any distances. Similar examples of usage of unstable ***HS***–models for us for the present are not known, however the works in the given direction are being represented to us as interesting enough ones.

At definition of classical ***d–HS*** which are basic subject of consideration of the given monography, we shall constantly appeal to determinative condition $\sigma^{(n)}(0,0,\ldots,0) = 0$ for their local transition functions $\sigma^{(n)}$; this condition defines the given important class of *homogeneous* structures whose specificity was considered earlier. Meanwhile, it is necessary to have in mind, the determinative condition for local transition function of an arbitrary classical structure ***d–HS*** $(d \geq 1)$, generally speaking, can has the following kind, namely**:**

$$(\exists h \in A)\left(\sigma^{(n)}(h,h,h,\ldots,h) = h\right) \qquad (3)$$

i.e., an arbitrary element of alphabet ***A*** of internal states of a structure can be the quiescent state. The given definition of concept of classical ***d–HS*** enough appreciably expands a class of such type of structures**;** the majority of the results, presented below, which are conditioned by presence for similar homogeneous structures ***(HS)*** of the most typical determinative condition $\sigma^{(n)}(0,0,0,\ldots,0) = 0$, quite are naturally spread and onto ***d–HS*** $(d \geq 1)$ of the general class of structures whose ***LTF*** $\sigma^{(n)}$ satisfy the determinative condition ***(3).***

From definition of the classical ***HS***–models follows, that for them the instant transition {*during one step of **GTF*** $\tau^{(n)}$} of a finite configuration into infinite configuration *(infinite growth of the finite configurations is possible only potentially)* is impossible, whereas instant transition of an infinite configuration into finite configuration *(theoretically allowable by axiomatics of classical **HS–models**)* is deprived, in the certain degree, of a natural sense. Though in some individual cases it has a certain sense [88,90,536]. Thus, dynamics of ***finite*** configurations of the set ***C(A,d,φ)*** presents the basic interest for the purpose of research of both applied and theoretical aspects of the theory of homogeneous structures in all their generality and extensiveness of spheres of application [536,617].

Now, let's estimate a number ***N*** of all *classical* structures with alphabet ***A***={***0,1, ..., a–1***} and neighbourhood index ***X***={***0,1,2, ..., n–1***}, depending on the *determinative* condition ***(3)*** defining the given class of ***d–HS***. On basis of simple enough combinatorial considerations we determine a number ***N*** of all classical structures ***d–HS (d≥1)*** as follows [567,617]:

$$N = \sum_{j=1}^{a} C_a^j (a-1)^{a-j} a^{a^n-a} = (a-1)^a a^{a^n-a} \sum_{j=1}^{a} C_a^j (a-1)^{-j} = a^{a^n}\left(1-\frac{1}{a}\right)^a \sum_{j=1}^{a} C_a^j (a-1)^{-j} =$$

$$a^{a^n}\left(1-\frac{1}{a}\right)^a \left[\sum_{j=1}^{a} C_a^j (a-1)^{-j} - 1\right] = a^{a^n}\left(1-\frac{1}{a}\right)^a \left[\left(1+\frac{1}{a-1}\right)^a - 1\right] = a^{a^n}\left(1-\frac{(a-1)^a}{a^a}\right)$$

or on basis of more simple approach we can easily receive quantity of structures relating to the «***classical***» type, namely:

$$N = a^{a^n} - (a-1)^a a^{a^n-a}$$

Thus, quota ***(Δ)*** of all classical structures concerning all possible ***d–HS***, including unstable ones, does not depend on dimensionality and size of neighbourhood template, and is defined by the following relation:

$$\Delta = \frac{N}{a^{a^n}} = 1-\left(\frac{a-1}{a}\right)^a$$

depending only on cardinality of alphabet ***A*** of a structure under the following condition, namely:

$$\lim_{a\to\infty}\left[1-\left(\frac{a-1}{a}\right)^a\right] = 1-\frac{1}{e} \approx 0.63212$$

i.e. quota of all classical ***d–HS*** for arbitrary neighbourhood index and dimension makes up the majority lying over the range [***0.63 .. 0.75***]. At that, it is necessary to have in mind, that the case when all states from alphabet ***A*** are quiescent states, is *singular* in a certain sense relative to a series of results received on classical structures for which alphabet ***A*** contains the states distinct from quiescent states too. Whereas in other cases many important enough results that have been received for case of a single quiescent state have been spread on the classical structures also in their generalized understanding.

For example, among the structures having more than ***1*** *quiescent* state, i.e. structures relating to the «***classical***» type, the basic results on self–reproducing finite configurations in Moore's sense are valid also. So, it is possible to show the *classical* structures ***1-HS*** with neighbourhood index ***X***={***0,1***}, alphabet ***A*** = {***0,1, ..., a–1***} and symmetric local transition function $\sigma^{(2)}$ which obey the following condition $(\forall b\in A)(\sigma^{(2)}(b,b) = b)$ have all finite configurations as self–reproducing in the Moore's sense. As an example, the $\sigma^{(2)}(x,y) = (a+1)(x+y)/2 \ (mod\ a)$ of the generalized class of linear structures can serves. Can be shown that the next result takes place: ***For prime numbers a≥3 and n≥3 the classical structures 1–HS with an alphabet A = {0,1, ..., a–1} and symmetrical LTF $\sigma^{(n)}$ which satisfy condition $(\forall b\in A)(\sigma^{(n)}(b,...,b)=b)$ there are classical structures possessing the universal reproducibility in the Moore's sense; however by far not all classical structures with such local transition functions possess the universal reproducibility.***

This result confirms also the fact of existence of classical structures for which the pure linearity of their ***LTF*** $\sigma^{(n)}$ is not compulsory, i.e. of the more wide class of classical structures possessing attribute of *universal reproducibility* in Moore's sense of finite configurations (*about such type of classical structures will go speech a little below*). The classical structures possessing property of «***universal reproducibility***» in certain respects determine extreme properties of the structures. For empirical research of the phenomenon a number of procedures has been programmed in the environment of systems *Mathematica* and *Maple*, that have allowed to research the phenomenon from various standpoints [545,640,641].

Meanwhile, the existence of several quiescent states in the majority of appendices does not find adequate interpretation. Therefore, further we (*as a whole, without loss of generality, and by some conceptual reasons*) shall consider only classical structures with a single quiescent state as the most typical and widely used. The rather interesting philosophical comprehensions of classical ***HS***–models of such type, though in a lot of cases and not entirely indisputable ones, can be found in [55,568].

For the further consideration quite opportunely will make one remark carrying conventional character. This agreement is used by us already more than ***35*** years for description of local transition functions given by systems of parallel substitutions of ***1***–dimensional ***HS***–models. For convenient representation of *binary classical* ***1-HS*** with neighbourhood index of Neumann–Moore, since ***1971*** [19,20,22], we used a simple and rather natural reception (*as a whole, it can be generalized to* ***1–HS*** *of more* *general kind)* when ***LTF*** $\sigma^{(3)}$ of the structures of such type are defined by parallel substitutions of the following kind, namely:

$$000 \to x_1 = 0 \quad 010 \to x_3 \quad 100 \to x_5 \quad 110 \to x_7$$
$$001 \to x_2 \quad 011 \to x_4 \quad 101 \to x_6 \quad 111 \to x_8$$

Obviously, quantity of all structures ***1–HS*** of such type equals 2^7***=128.*** Ignoring the condition $000 \to 0$ which determines type of the ***classical*** structures we receive equally ***256*** various structures, of which half are classical ones. Now we enumerate the given structures by the integers corresponding to their binary representations $<x_1\,x_2\,x_3 \ldots x_8>$, i.e. from ***0*** up to ***127 (255).*** In our opinion, the above approach is most natural; moreover, in the early eighties ***S. Wolfram*** proposed other principle of numbering of such type of structures when numbering of structures is based on application of the reverse order of the parallel substitutions determining ***LTF*** $\sigma^{(3)}$, namely:

$$111 \to y_1 \quad 110 \to y_3 \quad 101 \to y_5 \quad 100 \to y_7$$
$$011 \to y_2 \quad 010 \to y_4 \quad 001 \to y_6 \quad 000 \to y_8$$

and ***discriminating*** number of a structure ***1–HS*** is defined by decimal equivalent of binary tuple $<y_1 y_2 y_3 \ldots y_8>$. Naturally, this distinction is not essential, however this circumstance is necessary to have in view when the above–mentioned principle of identification of structures ***1–HS*** of the given type is used. Thus, the classical structure with ***LTF*** $\sigma^{(3)}$ that has the following formula representation:

$$\sigma^{(3)}(x_o, x_1, x_2) = \sum_{j=0}^{j=2} x_j \pmod 2; \quad x_j \in B = \{0,1\}$$

in our classification will have the discriminating number ***105,*** whereas in ***Wolfram's*** classification – number ***150.*** Therefore for the purpose of simplification of translation of ***1–HS*** of the above type from ***Aladjev's*** numbering into ***Wolfram's*** numbering rather simple program ***AW(N),*** that has been programmed in ***Maple*** and operates by switch principle can be used: if ***N*** is a discriminating number of ***1–HS*** in numbering of ***Wolfram (Aladjev),*** then ***AW(N)*** will be the appropriate *discriminating* number in numbering of ***Aladjev (Wolfram)*** [545,617]. Here only our numbering of binary classical structures ***1–HS*** of such type is used.

Naturally, it is possible to investigate also and *unstable* ***d-HS*** for which there is no specially chosen *quiescent* state; i.e. the structures which are not satisfying specified determinative condition $\sigma^{(n)}(h,h,h, \ldots, h,h) = h$. However, such structures carry speculative enough character and, in our opinion, they do not represent any serious interest from practical point of view. Discussion of the given question can be found in [617].

4. So, dynamics of a classical ***d–HS*** is completely determined in terms of ***LTF,*** i.e. local interactions of automata of neighbourhood template of each elementary *z*–automaton, whereas itself ***LTF*** $\sigma^{(n)}$ is the typical example of local algorithm which is carried out by especially parallel manner on the basis of configuration of states of elementary automata of a local neighbourhood determined by neighborhood index ***X*** of the current *z*-automaton of space Z^d of a certain *classical* ***HS***–model. Thus, simultaneous applying of local transition function to neighbourhood of every *z*–automaton of entire homogeneous space defines the ***global*** *transition function* $\tau^{(n)}$ ***(GTF)*** that transforms the ***current*** configuration $c \in C(A,d)$ into the ***next*** configuration $c\tau^{(n)} \in C(A,d)$. Formal definition of the configuration $c\tau^{(n)}$ can be presented as follows.

Let ***C(A, d)*** denotes the set of all configurations with respect to Z^d and ***A.*** If $s[z]$ denotes the current state of an elementary *z*–automaton, then formally ***GTF*** $\tau^{(n)}$ with neighbourhood index $X = \{x_1, x_2, x_3, x_4, ..., x_n\}$ is determined by the following formal condition, namely:

$$c\tau^{(n)} = c^{**} \leftrightarrow (\forall z \in Z^d)(s^{**}[z] = \sigma^{(n)}(s[z+x_1], s[z+x_2], ..., s[z+x_n]))$$

From this definition immediately follows, that every ***LTF*** $\sigma^{(n)}$ defines an ***unique*** global transition function $\tau^{(n)}$, and $\tau^{(n)}$ cannot be defined by two different local transition functions $\sigma^{(n)}$. There is, in other words, biunique correspondence between set of all global transition functions $\tau^{(n)}$, and the set of all local transition functions $\sigma^{(n)}$ for the given state alphabet ***A,*** dimensionality ***d*** of space Z^d and neighbourhood index ***X.*** Thus, it is possible to speak about ***GTF*** $\tau^{(n)}$ defined by ***LTF*** $\sigma^{(n)}$, and vice versa. It is proved [1,3,5], that an arbitrary ***GTF*** in classical ***d–HS*** is *primitive recursive function*. The given result defines not only place of ***GTF*** $\tau^{(n)}$ in hierarchy of all recursive functions, but also together with other components determines simplicity of such mathematical objects as homogeneous structures ***d–HS (HS–****models)*. Meantime, such simple structures admit a complex enough dynamics of both *finite* and *infinite* configurations, including opportunity of universal computability.

It turned out that the family of global transition functions of classical ***d–HS*** represents excellent means for solution of rather broad range of problems of modelling in a mode of maximal paralleling. In addition, the global parallel transformations determined by *classical* ***HS***-models, in our opinion, can be used effectively enough and widely similarly to other well-known mathematical transformations *(Fourier, Laplace, etc.)*.

The ***fourth*** component of structures ***d–HS*** $(d \geq 1)$ can now be defined. For the given A, Z^d and X, a set of admissible transformations T is any nonempty subset of the complete set of all global transition functions $\tau^{(n)}$ which are determined by three parameters A, Z^d and X. If the set T contains single global transition function $\tau^{(n)}$, then the object ***d–HS*** $= \langle Z^d, A, \tau^{(n)}, X\rangle$ is said to be ***monogenic*** or ***classical d–HS*** $(d \geq 1)$. The operation of an arbitrary classical ***d–HS*** $(d \geq 1)$ is particularly simple, namely: if $c=c_o$ is an initial configuration of homogeneous space Z^d at the time $t = 0$, configuration at the time $t=m$ is $c^*=c_o\tau^{(n)m}$, the result of applying of a global transition function $\tau^{(n)}$ to configuration c_o of the homogeneous space m times.

Let $\langle c_o\rangle[\tau^{(n)}]$ designates ***configurations sequence*** generated by some ***GTF*** $\tau^{(n)}$ from an initial configuration c_o. Then for an arbitrary finite configuration $c_o \in C(A,d,\phi)$ the given sequence represents a history of configuration c_o in a classical ***d–HS*** $(d \geq 1)$ which plays the basic part in researches of dynamic properties of ***HS***–models. Under ***dynamics*** is understood a functioning of either type of ***d–HS*** consisting in change in the course of time of configurations of the structure as function of its initial configuration and ***LTF (GTF)***. Thus, dynamics of a classical ***d–HS*** $= \langle Z^d, A, \tau^{(n)}, X\rangle$ {*configurations sequence* $\langle c_o\rangle[\tau^{(n)}]$; *the history of development of objects, embedded in the structure*} is determined by quite uniquely its considered base components d, Z^d, A, X, and $\tau^{(n)}$ $\{\sigma^{(n)}\}$.

The ***direct predecessor*** of an arbitrary configuration $c \in C(A,d)$ is each configuration $c^{-1} \in C(A,d)$ such that the condition $c^{-1}\tau^{(n)}=c$ takes place. Each configuration $c \in C(A,d)$ can has a single predecessor, their finite or infinite number, or have none. The *direct predecessors* for *block*, *finite* and *infinite* configurations in classical structures ***d–HS*** $(d \geq 1)$ also are quite naturally determined [1,3,5,536,545,567,617,640–643].

Let ***1–HS*** has alphabet $A=\{0,1,2,3,4\}$, neighbourhood index $X=\{0,1,2,3\}$ and ***GTF*** $\tau^{(4)}$ whose ***LTF*** $\sigma^{(4)}$ is defined, in particular, by set of parallel substitutions in the form of $\{1023 \Rightarrow 2,\ 1101 \Rightarrow 4,\ 1010 \Rightarrow 3,\ 0102 \Rightarrow 2,\ 0232 \Rightarrow 1,\ 2322 \Rightarrow 3,\ 3221 \Rightarrow 2,\ 2213 \Rightarrow 2,\ 2133 \Rightarrow 4,\ \ldots\}$. Then, if at the current moment t a configuration of ***1–HS*** looks like $(\varphi 1)$, then at the following moment ***t* + *1*** under the influence of ***GTF*** $\tau^{(4)}$ it passes into configuration **φ*1**** of the following kind.

***c*:**	**....**	*1*	*1*	*0*	*1*	*0*	2	3	2	2	*1*	3	3	**....**	***t* : φ*1***
					⇩									
$c\tau^{(n)}$:	**....**	*4*	3	2	2	*1*	3	2	2	*4*	**....**	**....**	**....**	**....**	***t* +*1* : φ*1****

The problem of finding of ***LTF*** whose appropriate ***GTF*** generates the certain history ***(dynamics)*** of configurations of structure, is similar to an inductive problem of finding of the laws underlying an observable phenomenon. This analogy underlies modelling in ***HS***–environment of various natural and artificial systems, first of all, of cellular nature, and also at a lot of other motivations caused by research problems. In the general case the given problem of the full and exact description of dynamics even of rather simple *classical* ***HS***–models is one of the most complex in the ***HS***–problematics, and numerous attempts existing in this direction still are insufficiently effective.

It is possible to ascertain, in spite of all *simplicity* of such mathematical object as classical ***HS***–models their dynamics carries complex enough character, and its research presupposes generally speaking significant efforts, and in a whole series of cases also *nonconventional* approaches. For this reason in the given direction there are relatively a few results received by theoretical methods whereas rather considerable part of them has empirical character, and many of them had been received by means of computer modelling.

A rather interesting and instructive example of research of ***HS***-models by means of computer simulation is well–known game «***Life***» [54,239] which represents a classical binary ***2–HS*** with Moore's neighbourhood index and local transition function $\sigma^{(9)}$ of the following kind, namely:

$$\sigma^{(9)}(x_o, x_1, x_2, \ldots, x_8) = \begin{cases} 1, & if\ \ x_o = 0 \ \& \sum_{j=1}^{j=8} x_j = 3 \ \ or \ \ x_o = 1 \ \& \sum_{j=1}^{j=8} x_j = \{2\,|\,3\} \\ 0, & otherwise \end{cases}$$

where x_o – the current automaton of the model, and x_j ***(j=1 .. 8)*** – eight its immediate neighbours. Now the given type of ***2–HS*** is investigated enough intensively from the various points of view [3,150,168,172,409, 429,536] whereas in works [523–531] game «***Life***» has been considered in various interesting contexts as a classical binary ***2–HS*** with Moore's neighbourhood index. Along with structures ***2–HS*** of various purpose the reader can familiarize oneself with rather interesting programmed implementations of games similar to ***HS***–models more thoroughly, for example, in works [1-3,5,6,15,19,20,53-56,85,88,90,150,159-161,167,536]. Interesting enough thinking concerning the given problematics can be found in works [79,90,545,567,617,640-643].

Meanwhile, in case of simulation of a ***d–HS*** on computers of *traditional* consecutive Neumann architecture [6,54] we are compelled to use the following scheme of simulation *(on example of classical* ***1–HS)***, namely:

←~~~~ ***NT*** ~~~~→

t :	....	CF_t	....	a_{-j}	....	a_o	a_1	....	a_n	....	a_j	....	M_o
	....	⇩	....	⇩	....	⇩	⇩	....	⇩	....	⇩	....	
t+1 :	....	CF_{t+1}	....	a^*_{-j}	....	a^*_o	a^*_1	....	a^*_n	....	a^*_j	....	M_1

Two arrays M_o and M_1 of the same size where the first defines a basic Z^1 space mapping dynamics of a configurations sequence of a certain simulated *classical* structure ***1–HS*** are defined. Taking into account the consecutive principle of processing of the information by a traditional computer, on some step t on the basis of consecutive processing of ***NT*** for any element of M_o array containing the current *finite* configuration ***(CF)*** c_t, the array M_1 is formed which will contain the next ***CF*** c_{t+1} of a history of the simulated structure ***1–HS.*** After, both arrays logically exchange the places or contents of M_1 array is written into array M_o, identifying the next configuration c_{t+1}, whereupon the above process is repeated onto the given depth of dynamics of the simulated ***1–HS.*** The described algorithm *(or its modifications)* of a computer realization of dynamics of classical and many other types of ***HS***–models has been used by a whole series of researchers and for research of properties of structures themselves, and for their applied aspects in various areas of natural sciences [1,4,5,8,9,15,54–56,74,78,79,73,94–96,137,165,166,408].

Hence, already in case of rather small neighbourhood templates ***(NT)*** and significant depth of simulated dynamics of structures ***2–HS*** the time expenses for w steps of the structure are increased very quickly, being appreciable even on modern computers. Naturally, use of more powerful clasters and supercomputers, essentially improving similar situation, if necessaryly simulation of enough complicated ***d–HS*** onto the large depth of dynamics, does not liberate from arising of similar *temporal* problems [5,54,536,567,617]. So, only high-parallel computing systems of non–Neumann architecture allow to solve the problems of computer simulation of ***d–HS*** in real time, which by maximal manner reflect specificity of operating of ***HS***–models. Today, a whole series of the computers basing on such architecture is being created.

In particular, ***N. Margolus*** & ***T. Toffoli*** have offered an interesting and perspective approach to practical realization of computing ***HS–****models*, having created a series of so–called *Cellular Automata Machines* ***(CAM)*** [150,165,376,394]. Use of ***CAM*** appeared rather effective at modelling of a whole series of problems of hydrodynamics, ecology, studying of mathematical properties of ***HS***–models of various types, obtaining of effects of a various sort, physical modelling, images processing, etc. In Italy the computing ***CAMEL*** system ***(C****ellular* ***A****utomata environ****M****ent for syst****E****ms mode****L****ing****)*** has been created, using a ***HS***–model as theoretical basis; it is successfully applied to modelling of broad enough range of applications from various practically important areas [410,411]. Thus, researchers of ***Cellular Automata Group*** (University of Calabria, *Italy)* are characterized by multifarious applications of ***CAMEL*** system for a modelling of various phenomena, objects and processes [397–402,410–414]. At that, there is a series of other rather interesting projects in the given direction too [366]. Whereas of the Soviet and Russian practical projects of realization of computing ***HS***–models the numerous results of groups of researchers from Novosibirsk, Leningrad, Moscow, Kiev, Taganrog, and Kishinev can be marked [176–178,536,593]. As a whole, with development prospects of the given direction in ***HS–****problematics* it is possible to acquaint oneself in the interesting monography [366].

Thus, *language of parallel substitutions* ***(LPS)*** can serve as an interesting example of algebraic language for description and analysis of parallel microprograms [5,12,54,173] which is a linguistic formalism of narrow enough class of computing ***HS***–models and which is based on concept of systems of *parallel substitutions* determining local transition function $\sigma^{(n)}$ of the *classical* ***HS***–models. Meanwhile, ***LPS*** is of interest and from more general position at empirical research of structures because not each ***LTF*** $\sigma^{(n)}$ can be represented in the form of formula. We suppose that *complexity* of parallel programming in the environment of ***d-HS*** of complicated enough problems is much more than their programming on computers of especially sequential models. To motivate it possibly by a series of circumstances discussed in [3,54-56,567,617]. In the same place it is possible to find reasonings concerning influence of parallel principle of information processing on development of many aspects of human activity, including thinking. Philosophic and gnosiological aspects of the given problem most complicated in every respect claim the most steadfast attention and serious study.

Thus, the concept of the classical ***d–HS*** is being intuitively represented rather simple and in the given connection exists a question concerning *degree* of its *generality* i.e. as far as widely the given concept admits the expansions which not exceed the limits of some studied phenomenon or limits of the given criterion of equivalence **(***a certain kind of property of stability of concept***).** With this purpose the detailed *analysis* of a series of expansions of the *classical* concept of ***d–HS*** relative to their dynamic properties had shown that in spite of rather strict criteria of *equivalence* of dynamics of two ***d–HS*** **(***which were based on the comparative analysis***)** the *classical* concept of ***d–HS*** possesses a sufficient degree of generality that allows to consider the given concept as one of basic, constituting the certain basis of the ***HS***–concept in all its generality [38,54-56,88,90]**.**

We considered only the generative power of the classical structures ***d–HS*** **(*$d \geq 1$*)** and have proved that a whole series of *widenings* of classical concept of the ***d–HS*** reveal, that even with respect to a narrow enough concept of equivalence of two structures ***d–HS,*** the concept of classical structures possesses a quite sufficient degree of generality [1,3,5,8,54]**.**

From definition of classical ***d-HS*** **(*$d \geq 1$*)** we can simply make sure, that these structures represents formal parallel algorithms of processing of finite configurations of the set ***C(A, d, ϕ)*** by means of global transition functions which may be considered as functions everywhere defined on the set ***C(A, d, ϕ).*** Of the above follows, that the concept of classical ***d–HS*** = $\langle Z^d, A, \tau^{(n)}, X \rangle$ possesses quite acceptable degree of generality for many rather important applications **(***in spite of all its simplicity***);** it represents rather significant interest as the independent mathematical object being the important component of both theoretical and applied models of parallel processing of information and computations.

So, if three components Z^d**,** ***A*** & ***X*** of structures ***d–HS*** **(*$d \geq 1$*)** are rather simple and transparent, any ***GTF*** $\tau^{(n)}$ is *primitive recursive* function [5]**.** Hence, such simple objects as the classical ***d–HS*** possess considerable enough degree of generality and by quite complex dynamics allowing to model extensive enough class of objects, processes and phenomena having a place in a lot of fields of science and technics. Along with it, these objects present appreciable interest for research as independent formal model of parallel processing. Meanwhile, within of classical ***d–HS*** the special subclasses of the structures with specific characteristics such as ***HS*** with refractority, memory and a series of others allowing to more effectively simulate a lot of interesting enough processes and objects are chosen. Some of these types of structures are considered a little below**,** other interesting types can be found in bibliography [536]**.**

Meanwhile, complexity and variety of the real world are not inscribed in no way in *Procrustean* bed of the ***HS***-concept without its any serious complicating, by influencing seriously attractiveness of its simplicity in its primordial concept. In our opinion, for today, the homogeneous structures are of interest in ***2 basic*** natural–science directions, namely**:**

(1) An environment of modelling and realization of various processes, phenomena and objects (first of all of those, which with difficulty or impossibly to describe by other means, in particular, by means of the differential equations in partial derivatives); that is to say, for today in this direction we have the greatest number of researches;

(2) An independent mathematical object for researches (high–parallel dynamic discrete systems; formal high–parallel calculators similarly to machines of Turing and Post, Markov systems of substitutions etc. for sequential computations and information processing; algebraical processing systems of words with parallel rules of substitutions).

In our researches ***d-HS*** are mainly considered as a formal *mathematical* object. At that, if in the first quality the above determinative condition ***(3)***, specifically $\sigma^{(n)}(0,0,0,\ldots,0,0) = 0$, is the essential feature of classical ***HS***-models, then in the second it is possible to do without it, however real advantage of such assumption seems to us not quite persuasive. So, let's introduce the certain types of homogeneous structures which, however, were being investigated by us essentially less actively, than classical ones. Meanwhile, for many rather interesting features of ***HS***–models the above determinative condition ***(3)*** is very essential.

1.2. The basic types of homogeneous structures

Today with different degree of intensity a whole series of expansions and generalizations of the concept of classical ***HS***–models determined above is used. However not each expansion of *classical* concept of ***HS***–models leaves us within of chosen criteria of equivalence. Specifically, the *polygenic deterministic*, *nondeterministic* **&** *stochastic* ***HS***–models are essential generalizations of the classical concept, whereas widely-used expansions **–** ***HS***–models with Margolus's neighbourhood index, with memory, with refractority, etc. In this part we shall briefly dwell on ***5*** types of ***HS***-models representing rather significant interest from many points of view, namely**:** *polygenic* ***HS***, ***HS*** with *memory* and *refractority*, ***HS****–structures, and structures on the *splitting* ***(****with neighborhood index of Margolus****)***, which are oriented, mainly, onto modelling of physical applications, first of all, because of their possibility to programme the reversibility of the processes embedded to them.

For the above components ***A***, Z^d and ***X***, the set ***G*** of allowable parallel transformations is any nonempty subset of set of all global transition functions $\tau^{(n)}$, determined by these three components. If the above set ***G*** contains only one ***GTF*** $\tau^{(n)}$, then the structure $d\text{–}HS = \langle Z^d, A, \tau^{(n)}, X\rangle$ is called ***monogenic,*** otherwise **–** ***polygenic.*** Monogenic ***d–HS*** is called also ***classical*** structure in the presence of the determinative condition ***(3)*** for it, and more concretely $\sigma^{(n)}(0, ..., 0) = 0$. Whereas for a ***polygenic d–HS*** instead of a single function more than one ***GTF*** $\tau^{(n)}$ is used, and at each discrete moment $t > 0$ to the current configuration $c^{**} \in C(A,d)$ one of functions of the set ***G*** of allowable global transition functions is applied. In this book polygenic structures are considered in passing.

Among *polygenic* ***d–HS*** *three* basic subclasses of structures are differed**:** ***deterministic*** *ones* **(***at each moment* $t > 0$ ***GTF*** $\tau^{(n)}$ *are applied according to a certain determined algorithm***),** ***nondeterministic*** *ones* **(***at absence of such algorithm***)** and ***stochastic*** **(*GTF*** *are applied under a certain stochastic law***).** At present the theory of ***deterministic*** structures ***d–HS*** $(d \geq 1)$ is most advanced, therefore further the given class of classical structures ***d-HS*** is most in detail considered.

Monogenic $\langle Z^d, A, \tau^{(n)}, X\rangle$ and *polygenic* $\langle Z^d, A, G, X\rangle$ structures will be the basic object of discussion and, first of all, the *monogenic* or *classical* structures ***d–HS*** $(d \geq 1)$. Moreover, the paramount attention is devoted to questions of the theory of classical ***1–HS,*** having meanwhile in view that base parameters ***d, #(A)*** & ***X*** for ***HS***–models are essential, forming hierarchy of properties concerning certain phenomenons or dynamics of the models. For example, classes of structures ***1–HS*** and ***d-HS*** $(d \geq 2)$ are not equivalent already concerning constructive opportunities, the nonconstructability problem, algorithmic solvability, etc. The similar situation takes place as well for other basic parameters of ***HS***–models, and in the given direction exists a whole series of interesting enough results [3,5,9], however general picture in the given direction remains yet up to the end not quite transparent [5,54–56,79,88,90,536,567,617].

1. In case of so–called ***polygenic*** structures ***d–HS*** an explicitly written–out sequence of global transition functions is required for specifying any particular computation of $d\text{–}HS \equiv \langle Z^d, A, G, \omega\rangle$, where ω is function describing the order of applying of global transition functions of ***G*** to any initial configuration of homogeneous space Z^d. So, if $c_o \in C(A,d)$ is an initial configuration and $\omega = \tau_1^{(n_1)}\tau_2^{(n_2)}\tau_3^{(n_3)}...\tau_k^{(n_k)}...$ is a sequence of global transformations of ***G***, then configurations sequence $<c_o>[\omega]$:

$$<c_o>[\omega] = c_o\tau_1^{(n_1)}\tau_2^{(n_2)}\tau_3^{(n_3)} ..... \tau_j^{(n_j)} ..... ; \quad \tau_j^{(n_j)} \in G \quad (j = 1..m)$$

$$<c_o>[\omega] = \{c_o, c_k = c_{k-1}\tau_k^{(n_k)} | k = 1..\infty\}$$

is called the *configurations sequence* generated by means of some rule ω from an initial configuration c_o, if for any integer $k\geq 1$ the next relation $c_{k-1}\tau_k^{(n_k)} = c_k$ takes place. Let $<c_o>[\omega]$ and $<c_o>[\tau^{(n)}]$ denote sequences of configurations generated by the rule ω of a polygenic ***d–HS*** and by global transition function $\tau^{(n)}$ of a ***classical d–HS (d ≥ 1),*** accordingly. These sequences of configurations defines the dynamics of the ***d–HS,*** which is main object of our researches in the given monograph.

The set ***G*** contains finite quantity of ***d***–dimensional ***GTF,*** determined in the same alphabet ***A*** however each of the above ***GTF*** $\tau_k^{(n_k)}$ generally speaking can have own neighbourhood index. Depending on type of used algorithm ω distinguish the ***deterministic, not deterministic*** and ***stochastic*** polygenic ***HS***–models. So, for the ***deterministic polygenic*** structure is typically presence of strictly determined algorithm, which at each moment $t > 0$ determines the order of application ***GTF*** $\tau_k^{(n_k)}$ of the specified finite set ***G*** of allowable global transition functions to the current configuration $c_t \in C(A,d)$ of ***HS***–model. ***Deterministic*** *polygenic* structures have a rather wide sphere of applications of theoretical and applied character [536]. In particular, structures of the given class play an essential role in the problem of configurations complexity.

The following example represents rather simple *deterministic polygenic* structure $1\text{–}HS \equiv <Z^1, A=\{0,1\}, G=\{\tau_1^{(2)},\tau_2^{(3)}\}, \omega=\tau_1^{(2)}\tau_2^{(3)}\tau_1^{(2)}\tau_2^{(3)} ...>$, where ***GTF*** $\tau_1^{(2)}$, $\tau_2^{(3)}$ are defined by ***LTF*** σ_1, σ_2 with neighbourhood indexes $X_1=\{0,1\}$ and $X_2=\{-1,0,1\}$ accordingly, namely:

$$\sigma_1^{(2)}(s_1,s_2)=s'_1=s_1 + s_2 \ (mod\ 2) \qquad \sigma_2^{(3)}(s_1,s_2,s_3)=s'_2=s_1 + s_2s_3 \ (mod\ 2)$$

The order of application of ***GTF*** of the set ***G*** of all allowable functions to an arbitrary initial configuration $c^{***} \in C(A, d)$ is strictly determined and defined by the algorithm ω whose essence of special explanations does not demand. Then the sequence $<c^{***}>[\omega]$ *(history of configuration* c^{***}*)* by depth onto ***16*** steps on condition that $c^{***} = 11$ obtains the following rather simple kind, namely:

*c*** =*	***11***	*c8 =*	***10100011***
	⇓ $\tau_1^{(2)}$		⇓ $\tau_1^{(2)}$
c1 =	***101***	*c9 =*	***111100101***
	⇓ $\tau_2^{(3)}$		⇓ $\tau_2^{(3)}$
c2 =	***101***	*c10 =*	***1001100101***
	⇓ $\tau_1^{(2)}$		⇓ $\tau_1^{(2)}$
c3 =	***1111***	*c11 =*	***11010101111***
	⇓ $\tau_2^{(3)}$		⇓ $\tau_2^{(3)}$
c4 =	***10011***	*c12 =*	***111010110011***
	⇓ $\tau_1^{(2)}$		⇓ $\tau_1^{(2)}$
c5 =	***110101***	*c13 =*	***1001111010101***
	⇓ $\tau_2^{(3)}$		⇓ $\tau_2^{(3)}$
c6 =	***1110101***	*c14 =*	***1010011010101***
	⇓ $\tau_1^{(2)}$		⇓ $\tau_1^{(2)}$
c7 =	***10011111***	*c15 =*	***11110101111111***
	⇓ $\tau_2^{(3)}$		⇓ $\tau_2^{(3)}$
c8 =	***10100011***	*c16 =*	***1001101101100000011***

Generally speaking, a deterministic polygenic ***d–HS*** is determined by an algorithm **ω,** i.e. some recursive function ***N=ω(t) (t=1,2,3,...),*** which returns number of ***GTF*** $\tau_k^{(n_k)}$ from the set ***G*** with elements numbered by a certain manner. For example, for the above *deterministic* ***1–HS*** the **ω–**function obtain rather simple kind, namely**:** ***ω(t) = 1+t (mod 2) + 1.***

As a **ω–**function is recursive, it is computable by appropriate Turing machine ***(MT^s_q)*** [180,181]**.** Taking into account partial recursiveness of every ***GTF*** from the set ***G*** [3,5,53]**,** it is simple to be convinced that for an arbitrary deterministic polygenic structure ***1–HS*** exists ***MT^s_q*** which simulates it. However, as an arbitrary ***MT^s_q*** is simulated in real time by means of a classical ***1–HS*** with neighbourhood index of Neumann-Moore, then it is possible to receive rather interesting corollary ***(****with a quite natural generalization to the* ***d–****dimensional case****)****,* namely [54–56]**:**

Every deterministic polygenic d–HS ≡ <Z^d,A,G,ω> can be simulated by means of an appropriate classical structure d–HS (d ≥ 1).

The given approach allows us to receive non-constructive proof of the corollary whereas in the *constructive* way it is possible to receive some quantitative estimations of basic parameters of classical structure that models a deterministic polygenic structure [3,5,54–56]**.**

Nondeterministic polygenic ***d–HS*** along with independent interest as the important mathematical object appear rather useful at research of methods and principles of increase of reliability and survivability of complicated cellular systems of both animate nature, and abiocoen. It is possible to familiarize oneself with interesting enough discussion of the given problematics in works [5,53-56]. Here the given the question is briefly considered only in connection with concept of generality of the classical ***HS***–models.

Let's name a structure $SH(d,a,n,N) \equiv \langle Z^d, A, G, X\rangle$ *nondeterministic*, if at any moment $t>0$ to the *current* configuration $c \in C(A,d)$ of the structure are successively applied all ***GTF*** of a set G of cardinality N $(\#G = N)$ of possible global transition functions; at that, the order of application of these functions strictly is not determined. It is shown that ***an arbitrary non–deterministic structure SH(d,a,n,N) defined thus is simulated by means of an appropriate classical structure d–HS (d ≥ 1) with Moore's neighbourhood index; at that, an arbitrary nondeterministic structure SH(1, a, n, N) is simulated by means of an appropriate classical 1–HS with Moore's neighbourhood index and an alphabet A* of cardinality*** $N(a^{n+1} - 1)/(a - 1) + a + n$, once again confirming quite sufficient degree of generality of classical concept of ***HS***–models [53–56].

With generalization of concept of ***d–HS*** $(d \geq 1)$ and onto the stochastic case not only new applied aspects are being discovered, but also very interesting mathematical objects of *stochastic* and *quasistochastic* nature are offered for research. Thus, if the *deterministic* and *nondeterministic* polygenic ***HS***–models are characterized accordingly by existence of **ω**-algorithm and by its absence for use on each step of ***HS*** of appropriate ***GTF*** $\tau_k^{(n_k)}$ of the set G of allowable global transition functions, then in case of ***stochastic*** *polygenic* ***HS***-models the ω-algorithm is determined by a certain distribution function which ascribes to each function $\tau_k^{(n_k)}$ of the set G the probability p_k of its applications at any moment $t>0$ to the current configuration of the structure ***d–HS*** $(d \geq 1)$, namely:

GTF:	$\tau_1^{(n_1)}$	$\tau_2^{(n_2)}$	$\tau_3^{(n_3)}$	....	$\tau_j^{(n_j)}$	....	$\tau_m^{(n_m)}$	
P:	p_1	p_2	p_3	....	p_j	....	p_m	$\sum p_j=1$

In a whole series of cases there is possibility to draw quite satisfactory analogies between stochastic ***d-HS*** $(d \geq 1)$ and various multicomponent objects of stochastic or quasistochastic nature. In the given connection we have determined two types of stochastic ***d–HS (heterogeneous and homogeneous structures)*** and investigated their asymptotic behaviour [1,3,5,21,54–56]. In particular, the condition of ***asymptotical*** approach of dynamics of *heterogeneous* ***d–HS*** to classical ones has been received. A whole series of interesting enough results concerning the stochastic ***HS***–models is received by many other researchers too.

So, on the basis of a special approach to modelling of some stochastic structures ***d–HS*** by other structures of the same class ***A. Matevosjan*** has proved [182] the existence of universal stochastic structures, and investigated estimations of the corresponding costs of the modelling, whereas ***A. Adamatsky*** has studied the interesting enough questions of identification of stochastic finite ***HS***–models [161,183]. Discussion of problematics of stochastic ***HS***–models along with a whole series of interesting results in the given direction the reader can find in rather interesting works [1,3,5,9,15,55,90,143,147,149,157,160,161,166,308,536] containing references to big enough number of original sources. Now, stochastic ***HS***–models of different type are being investigated enough intensively from theoretical and applied standpoints.

Till now, we considered ***(****and later also we shall consider****)*** exceptionally synchronous ***HS***–models for which on each step $t > 0$ to all space Z^d is applied a single ***GTF*** from a fixed set ***G;*** for classical ***HS***–models the set ***G*** contains a single ***GTF*** $\tau^{(n)}$. Meanwhile, both from the theoretical point of view and some appendices the ***asynchronous HS***–models that admit use on any step $t > 0$ of a structure in different areas of space Z^d of different functions ***GTF*** of a certain fixed set ***G*** are of interest too. In our opinion, *asynchronous* structures can represent the certain interest from the standpoint of *cosmogeneous* models of the Universe, however today any interesting results concerning structures of the given class are not known to us. More detailed discussion of asynchronous ***d–HS*** $(d \geq 1)$ can be found, for example, in works [54–56,79,90,545,567,617].

2. At the latter decades significant enough interest to development of models of new generations of the computing systems basing on non–Neumann parallel model of calculations has appeared [6]. Extremely interesting practical elaborations in the given direction that are based on the computing ***HS***-models ***(****some of them were mentioned earlier****)*** have been realized [6,136,150,156,165,169,178,366,394]. Meanwhile, classical ***d–HS*** are uncomfortable enough for a adequate simulation of a whole series of complex parallel computing devices and algorithms. In this connection we have determined a special subclass of structures ***d–HS*** with storage ***(d–HSS)*** which in the certain measure are resembling the parallel computing devices with storage [1,3-5,68-72,79,88,90,536,593].

Each elementary automaton in an arbitrary structure ***d–HSS*** has finite storage with depth ***P*** and can receive information directly from one's neighbours according to some neighbourhood index ***X.*** Storage of an elementary automaton is a ***P***–position register whose positions states are elements of a certain finite alphabet ***M*** = $\{m_1, m_2, ..., m_k\}$ ***(fig. 3).***

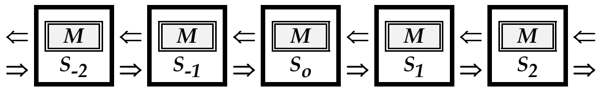


Fig. 3. Base organization of 1–dimensional HS with storage (1–HSS).

Each automaton in ***d–HSS*** can synchronously change own state and contents of storage ***M*** at the discrete moments ***t > 0*** as functions of the previous states ***(F)*** and storage contents ***(R)*** of elementary automata of neighbourhood template defined by a neighbourhood index ***X.*** Thus, ***d–HSS*** is an ordered septenary tuple ***<Z^d,A,M,P,X,F,R>***, where ***Z^d, A, X*** are defined analogously to case of a classical ***d–HS, M –*** alphabet of register storage of ***P***–depth, whereas ***F*** and ***R –*** the mentioned discrete transition functions. Functioning of an elementary automaton of such structure ***d–HSS*** is made at the discrete moments ***t=1,2,3, ...*** according to the following discrete equations, namely:

$$\begin{cases} S(z,t+1) = F(S_z^X(t), M(z,t)) & S(z,t+1) \in A \\ M(z,t+1) = R(S_z^X(t), M(z,t)) & M(z,t+1) \in M \end{cases} \qquad (4)$$

where $z \in Z^d$ – arbitrary elementary automaton of structure; ***S(z,t)∈A –*** a state of *z*–automaton; $S^X_z(t)$ – a configuration of the neighbourhood template ***(NT)*** of *z*-automaton and ***M(z,t) –*** contents of its storage ***M*** at the time ***t > 0.*** An arbitrary automaton of a structure ***d–HSS*** discretely and synchronously changes state ***S*** and state of storage ***M*** depending on states of its storage ***M*** and a configuration of states of automata of its neighbourhood template at previous discrete moment ***(t–1).*** In our opinion, structures such as ***d–HSS (d ≥ 1)*** can quite serve as a certain formal basis for a modelling in their environment of a whole series of parallel discrete processes and algorithms, furthermore they represent interesting enough independent formal object of research. At that, it is possible be convinced that structures ***d–HSS*** that are determined thus represent a *subclass* of all classical ***HS***-models for which the *appropriate* two–level structurization of states of elementary automaton – internal ***S*** and state of its register storage ***M*** is determined. Really***, an arbitrary structure d–HSS ≡ <Z^d,A,M,P,X,F,R> whose functioning of elementary automata is determined by system of discrete equations (4), is strictly equivalent to a classical d–HS≡<Z^d,A*,$\tau^{(n)}$,X> (d≥1) appropriate to it***

for which conditions $A^* = A \times M^p$ and $\#(A^*) = ak^p$ take place [5,53].

Meanwhile, similar *structurization* of states of elementary automata in ***d–HSS*** allows to solve enough effectively on ***HSS***–like models a series of specific applied tasks, in the first place, connected with research of computer facilities of high-parallel acting. And if within of the general theory of *classical* ***d–HS (d≥1)*** the ***HSS***–models do not represent special interest, whereas *structurization* of states of their elementary automata determines for them interesting enough new applied possibilities. But for today the advanced applied theory of the ***d–HSS*** does not exist.

An interesting subclass of ***d–HSS*** are the structures determined by the following conditions, namely**:** ***(1)*** alphabet of *storage* ***M*** coincides with alphabet ***A*** of states of an elementary ***z***–automaton of the structure**;** ***(2)*** storage ***M*** of depth ***P*** has stack organization**;** and ***(3)*** updating of the current state of ***z***–automaton is accompanied by simultaneous putting of the updated state into the stack ***M*** of the automaton with pushing out the earliest state in stack**;** ***(4)*** state of ***z***–automaton at time ***t > 0*** is a function of configurations of states of its neighbourhood template and the stack ***M*** at time ***(t-1)***. Functioning of structures ***d–HSS*** of such type can be formally described by a certain system of discrete equations of the following general kind, namely**:**

$$\begin{cases} \sigma^{(n+p)}(x_1, x_2, x_3, \ldots, x_n | m_1, m_2, m_3, \ldots, m_p) = x^*_1 \\ R^{(p)}(m_1, m_2, m_3, \ldots, m_p) = Sh(x^*_1, m_1, m_2, m_3, \ldots, m_{p-1}) \\ \sigma^{(n+p)}(0, 0, 0, \ldots, 0 | 0, 0, 0, \ldots, 0) = 0; \quad x_n, x^*_1, m_p \in A \\ R^{(p)}(0, 0, 0, \ldots, 0) = Sh(x^*_1, 0, 0, 0, \ldots, 0); \quad (j = 1..n; k = 1..p) \end{cases} \qquad (5)$$

Of the equations ***(5)*** directly follows, the ***LTF*** of ***d–HSS*** has relative to classical structure the expanded set of variables on depth of the stack of ***M*** whereas function ***R*** determines *updating* of the stack ***M*** by means of its shift with the subsequent placing into its beginning a new state of a ***z***–automaton. At last, last two equations of the system ***(5)*** specify determinative conditions for quiescent state «***0***» of the classical ***d–HS*** with storage. A series of interesting enough examples of modelling of structures ***d–HSS (d ≥ 1)*** by means of classical structures is considered in [54,79,90,567,617]. On the basis of such modelling it is rather simple to formulate the following result.

***Theorem 1**. A structure with storage 1–HSS≡<Z^1,A,M,P,X,F,R> under the condition X={0,1,...,n–1} where Sh & R are determined by a system of equations in the form (5) is modeled strictly real time by means of*

an appropriate classical structure 2–HS≡<Z^2,A∪{b},$\tau^{(2n+p)}$,X> (b∉A). The given result is generalized to general ***d***–dimensional case whereas a series of interesting enough examples of appendices of structures of such type can be found in our works [54–56,79,88,90,640–643].

3. Along with the above structures ***d–HSS*** within of *classical* structures ***d-HS*** can be chosen a special subclass of ***HS***–models with ***refractority.*** The structure ***d-HS (d ≥ 1)*** with *refractority* ***(d-HSR)*** represents classical ***d–HS*** whose states alphabet ***A*** is structured by some *particular* manner ***A***={***0,1***}∪{$\omega_1,...,\omega_r$} and whose ***LTF*** $\sigma^{(n)}$ is determined by the following system of parallel substitutions ***(6);*** their sense is transparent enough and of any special explanations does not demand, namely**:**

$$\begin{cases} \sigma^{(n)}(\omega_k, x_2, ..., x_n) = \omega_{k+1} \\ \sigma^{(n)}(\omega_r, x_2, ..., x_n) = 0 \\ \sigma^{(n)}(1, x_2, ..., x_n) = \omega_1 \\ \sigma^{(n)}(0, x_2, ..., x_n) = 1 \Leftrightarrow \left(\sum_{x_j=1} x_j \geq P \right) \end{cases} \qquad (6)$$

$$x_j \in A,\ k = 1..(r-1);\ j = 2..n$$

Structures of the given type we shall designate as ***d–HSR(r, P),*** where ***r –*** a refractority depth, ***P –*** a threshold of excitability of an elementary ***z***–automaton of structure. Alphabet ***A*** of states is structured by means of assignment for its elements of the following characteristics of state**:** *rest* ***(0),*** *excitation* **(*1*)** and *refractority of* ***r***–*depth* {$\omega_1, \omega_2, ..., \omega_r$}**.** If size ***r*** is constant, the structures with the *fixed depth* of refractority, otherwise **–** with *variable depth* of refractority take place. In a whole series of rather important applications it is natural to consider the depth of a variable *refractority* of ***d-HSR(r,P)*** depending on size of *threshold* ***P*** of excitation of ***z***-automata of the structure, i.e. ***r=r(P).*** A series of rather interesting results in the given direction can be found, for example, in works [79, 88,90,536] and references specified in them. Structures with *refractority* ***d–HSR(r, P)*** play important enough part in a lot of applied aspects of ***HS***–models, defining own subclass of the class of all structures ***d–HS.***

The essence of *refractority of* ***r***-*depth* consists in the following**:** an *excited* elementary ***z***–automaton of structure *(at state «****1****»)* at time ***t ≥ 0,*** during the next ***r*** steps *(period of refractority)* will be at the state of refractority, by passing from one ***r***–level of refractority onto another. After period of refractority the ***z***–automaton passes into state of rest «***0***»**,** becoming susceptible for the subsequent excitation. The general object of study in similar structures is investigation of the dynamical distribution of elementary automata in ***excitation*** states in conformity with an initial configuration, neighbourhood index ***X***, threshold of excitation ***(P)*** and ***r***–depth of refractority. A whole series of interesting enough examples of use of models ***(****excitable environments*, *some chemical systems*, *etc.****)*** on basis of structures ***2–HSR*** is considered in [5,33,54–56,79,567,617]. The main principle of functioning of the ***HSR***–models is well illustrated by a whole series of rather simple examples [545,567,617,618,640–643].

The modified ***HSR***–models are enough successfully used in a series of medical and biologic researches, in particular at development of some ways of processing of temporal and spatial characteristics of processes in some medical diagnostic systems [536,545].

For analysis of ***HSR*** their computer modeling can be considered as an effective enough research method; for that purpose a whole series of interesting software already exists; certain programs of such type are considered in the book a little lower. In this connexion a some interest the program ***VR*** represents, that is intended for modeling of excitable environment of ***Wiener-Rozenbluth*** formed by ***HSR***-models [536,593].

Of the fundamental properties of ***HSR***–models it is necessary to note existence in them of *nonconstructible configurations* ***(NCF)*** considered a little lower more in detail. It is simple to make sure, the ***1***-dimensional ***HSR(r,P)*** possess ***NCF*** of the kind $c=1^m$ $(m \geq n-P+1)$, where ***n*** – size of neighbourhood template and ***P*** - excitation threshold of an elementary automaton of the structure. At that, it is necessary to have in view that definition, in particular, of ***1***–dimension ***HSR(r,P)*** assumes correctness of relation $P \leq n-1$ otherwise existence of excited automata ***(****in state* ***«1»)*** in ***1-HSP(r,P)*** will be impossible, excepting the initial moment ***t=0***. So, in ***d–HSR(r,P)*** rather large continuous blocks of elementary automata in the excited condition can not be. In addition, with growth of value ***P*** of an excitability threshold the feasible size of blocks of the excited automata in a ***HSR***–model decreases. This result is easily generalized to a case of the higher dimensions of the ***d–HSR(r,P)***, allowing rather interesting applied interpretations for such formal model of «*excitable tissue*». With the certain classes of ***HS***-models a whole series of special questions of *dynamics* is connected. For example, in class of structures ***d–HSR(r,P)*** with refractority the basic object of researches is dynamics of distribution of excitability as a function of an initial configuration, a neighbourhood index, *depth* of *refractority* ***(r)***, *threshold* of *excitation* ***(P)***. We shall speak, in a structure ***d–HSR*** the ***sustained*** excitations exist, if for some initial configuration $c_o \in C(A,d)$ at an arbitrary moment $t>0$ a configuration $c_t = c_{t-1}\tau^{(n)}$ $(t=0,1,...)$ will contain at least one elementary automaton of structure in the excited condition. In addition, existence of these excitations in ***HSR***–models is enough closely connected to the threshold of excitability. Moreover, the excitation areas of an arbitrary configuration in ***HSR***–models can not exist at a moment $t>0$, namely, the following general result takes place [1]**:**

In a structure d–HSR(r, P) an excited area identical to neighbourhood template of the structure is unattainable; in such structure d-HSR(r,P) (d≥1,r≥1) with Moore's neighbourhood index the sustained excitations exist if the relation $P < 3^{d-1}$ ***takes place.***

Hence, the existence problem of the *sustained excitations* in an arbitrary ***d–HSR(r,P)*** $(r \geq 1, r \geq 1)$, as it is simple to make sure, is algorithmically solvable and in such ***HSR***–model do not exist large enough *continuous* excitation areas at a moment $t>0$; i.e. configurations of the model with the excited conditions have a sparse enough kind. This fact has rather interesting applied interpretations. Perhaps, the given phenomenon is connected to reliability control of functioning of neuro–like networks defined by ***HSR***–models, and ways of information processing in them [1]**.** In such context the *nonconstructability* concept in *classical* structures can receive new rather interesting interpretations. Some results in this direction can be found in the extended bibliography [536,545,640]**.**

At that, the second part of the above result allows to define the certain hierarchy in the class of all structures ***d–HSR(r,P)*** $(r \geq 1, r \geq 1)$ relative to possibility of generating of configurations of excited states as a certain function of ***d***–dimensionality and ***P***–threshold. Of the given result, for example, follows, under the condition $P \geq 3^{d-1}$ in an arbitrary structure ***d–HSR(r,P)*** any initial configuration $c_o \in C(A,d,\infty)$ will be vanishing. In case of an arbitrary neighbourhood index, a value of the ***P***–*threshold* of *excitation* is a function of ***r***-depth of *refractority* and the neighbourhood index. Discussion of a lot of interesting enough questions concerning the general type of structures ***d–HSR(r,P)*** $(d \geq 1)$ which represent quite certain interest for formal research of a class of networks of threshold type can be found, for example, in works [5,54,79,88,90,536,567,617]**.**

A lot of rather interesting results relative to the given subclass of ***HS***–models, received on basis of theoretical and computer research can be found in works [3,5–9,53–58,88,536]**.** Discussion of interesting enough examples concerning this theme can be found in [55,536,567,617]**.** The introduced structures ***d–HSR (d ≥ 2)*** can serve as a formal model for a lot of interesting enough objects, processes and phenomena, including studying of processes of self–organizing in systems of cellular nature and distribution of excitations in formal neural networks of different kinds. Along with that, the more special appendices of the structures ***d–HSR*** have been demonstrated [3,5,33,54–56,79,88,90,641,644]**.**

However the greatest interest the researchs of excitations distribution depending on such parameters as *refractority* type, its *depth***,** *excitability* threshold, existence of *defective* automata in ***d–HSR(r,P),*** and an initial configuration of its *active* area represent. In the given direction a series of interesting enough theoretical results of general character had been received [3,54]**,** however the more thin properties of ***d–HSR(r,P)*** have remained unexplored. Therefore, for the empirical research of certain dynamic properties of ***2–HSR(r,P)*** special modeling program has been created [6,9,15,46]**,** allowing to receive a series of interesting empirical results among which it is possible to note the following ones, namely**:**

♦ ***In 2-HSR(r,P) the self-reproducing areas in Moore's sense of excited elementary automata of a rather complex configuration exist;***

♦ ***Depth (r) of refractority greatly appreciably influences excitability distribution in 2–HSR(r, P). In particular, many of self–reproducing areas of excitability in case of P = 2, r = 2, 3, in case of r = 4 become the vanishing ones, by passing into zero configurations;***

♦ ***In 2–HSR(r,P) from a whole series of initial finite configurations in case of P = 2, r = 2, 3 are generated self–complicated excitability areas which for r = 4 become the vanishing ones;***

♦ ***Fronts of excitability distribution in 2–HSR(r, P) are being stopped by «sieve» of defective automata whose size of cells is less P however they continue spreading if size of cells not smaller than P;***

♦ ***In 2–HSR(r,P) the certain excitability areas densely enough packed by automata in the excited condition can, meanwhile, broaden out on the borders at damping of excitability in the interior of the area;***

♦ ***In a class of all periodic excitability areas exist initial excitability areas which starting with a certain step k become the periodical with period c = 3 and k ≥ h, where h – the size of side of the minimal square containing initial excitability area of automata in 2–HSR(r,P).***

In case of variable depth of refractority the situation with excitability distribution in ***2–HSR(r,P)*** essentially become complicated and for the effective research the use of high–performance computers of a class of super–computers or computers, basing on computing ***HS***–models, in particular, such as ***CAM*** [96,150,165] is demanded. Certain theoretical results concerning class of structures ***d–HSR*** will be presented below. ***d–HSR*** models find more and more wide practical application. With a lot of the interesting questions concerning the ***HSR***–problematics it is possible to familiarize oneself in works [9,54–56,88,90,134,159,160,166, 167,187,198,201,205,255,301,308,333,346,360,536,545,567,617,618,640].

To structures ***HSR*** the so-called structures with a delay ***(HSD)*** enough closely adjoin, which are defined as follows. A structure ***d–HSD*** $(d\geq 1)$ with delay ***(d–HSD)*** represents a classical ***d–HS*** whose states alphabet ***A*** is associated with an integer vector ***D*** of delays; at that, between sets ***A*** and ***D*** is determined one-to-one correspondence. Let without loss of generality $A=\{0,1,2,\dots,a-1\}$ and $D=\{0,d_1,d_2,\dots,d_{a-1}\}$; $0\leq d_j\leq a-1$ $(j=1..a-1)$.

An initial finite configuration in such ***HSD*** structure is associated with corresponding delays configuration in concordance with the scheme

$$\begin{bmatrix}\dots\\\dots\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}a_1\\d_1\end{bmatrix}\begin{bmatrix}a_2\\d_2\end{bmatrix}\begin{bmatrix}a_3\\d_3\end{bmatrix}\dots\begin{bmatrix}a_{m-1}\\d_{m-1}\end{bmatrix}\begin{bmatrix}a_m\\d_m\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}\dots\\\dots\end{bmatrix}$$
$$a_j\in A;\ d_j\in D;\ j=1..m$$

Each step of such structure ***HSD*** reduces by ***1*** each delay except for the case, namely: if under the influence of local transition function a state a_j turns to a state a_j', for it the appropriate delay d_j' is established $(a_j', a_j\in A;\ d_j'\in D)$. Procedure ***HSD*** simulates *dynamics* of structures ***1–HSD.***

```
In[70]:= HSD[A_List, dl_List, C_List, L_List, n_Integer, p_Integer] :=
                              Module[{a = Partition[Riffle[A, dl], 2],
                          b = Table[{0, 0}, {n – 1}], c, d = {}, k, F, t, h, g},
                   F[x_List] := Part[Select[L, Part[#, 1] == x &][[1]], 2];
  g = Map[{#, dl[[Flatten[Position[a, #]][[1]]]]} &, C]; c = Join[b, g, b];
  Do[c = Map[{#[[1]], If[#[[1]] == 0 && #[[2]] == 0, 0, #[[2]] – 1]} &, c];
                         For[k = 1, k <= Length[c] – n + 1, k++, t = c[[k]];
               If[t[[2]] != 0, d = AppendTo[d, t], h = c[[k ;; k + n – 1]];
                                                h = F[Map[#[[1]] &, h]];
             d = AppendTo[d, {h, dl[[Flatten[Position[A, 1]][[1]]]]}]]];
                c = Join[b, d, b]; d = {}, {p}]; Map[#[[1]] &, c[[n ;; –n]]]]

In[71]:= L = {{0, 0} –> 0, {0, 1} –> 1, {0, 2} –> 2, {1, 0} –> 1, {1, 1} –> 2,
              {1, 2} –> 0, {2, 0} –> 2, {2, 1} –> 0, {2, 2} –> 1};
In[72]:= HSD[{0, 1, 2}, {0, 1, 2}, {1, 1, 2, 1, 2}, L, 2, 20]
Out[72]= {1, 0, 2, 2, 2, 2, 2, 0, 0, 2, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 2, 2, 2, 2, 2}
```

The call **HSD[*A, dl, C, L, n, p*]** returns the configuration in the list form which is result of generation from configuration ***C*** on step ***p*** of a local transition function ***L*** given by the list of substitutions of a ***1–HSD*** with alphabet ***A*,** the delays ***d*** and neighbourhood template size ***n*.** Multiple computer experiments along with theoretical research of structures of the given type allow to obtain a number of interesting enough results concerning their dynamical properties [638,641–643].

In particular, in such **HSD** structures the self–reproductivity problem of finite configurations has been researched both theoretically and by means of computer simulation. So, for computer simulation of ***1–HSD*** has been used the procedure **ReprodHSD,** whose source code along with example of application the following fragment represents.

```
In[2448]:= ReprodHSD[A_List, dl_List, Cf_List, ltf_List, n_Integer,
                                                   p_Integer, v_Integer] :=
                         Module[{a = Partition[Riffle[A, dl], 2], c, d = {}, k,
                                           F, t, h, g, s, b = Table[{0, 0}, {n – 1}]},
                       F[x_List] := Part[Select[ltf, Part[#, 1] == x &][[1]], 2];
 g = Map[{#, dl[[Flatten[Position[a, #]][[1]]]]} &, Cf]; c = Join[b, g, b];
 Do[ c = Map[{#[[1]], If[#[[1]] == 0 && #[[2]] == 0, 0, #[[2]] – 1]} &, c];
                            For[k = 1, k <= Length[c] – n + 1, k++, t = c[[k]];
                      If[t[[2]] != 0, d = AppendTo[d, t], h = c[[k ;; k + n – 1]];
                                                          h = F[Map[#[[1]] &, h]];
          d = AppendTo[d, {h, delays[[Flatten[Position[A, 1]][[1]]]]}]]];
                         t = Map[#[[1]] &, d]; If[s = ListCount[t, Cf]; s >= p,
                                   Return[{p, s}]]; c = Join[b, d, b]; d = {}, {v}]]
In[2449]:= ltf = {{0, 0} –> 0, {0, 1} –> 1, {0, 2} –> 2, {1, 0} –> 1, {1, 1} –> 2,
                    {1, 2} –> 0, {2, 0} –> 2, {2, 1} –> 0, {2, 2} –> 1};
In[2450]:= ReprodHSD[{0, 1, 2}, {0, 2, 1}, {1, 2, 1, 2, 1}, ltf, 2, 25, 10000]
Out[2450]= {25, 31}
```

Procedure **ReprodHSD** allows experimentally to study in a structure ***1–HSD*** with alphabet ***A*,** the delays ***d*,** neighbourhood template size ***n*** and a local transition function ***ltf*** given by the list of substitutions the self–reproductivity of an initial configuration ***Cf*** given in the list form. The procedure call **ReprodHSD[*A,dl,Cf,L,n,p,v*]** returns the **2-**element list whose first element defines the demanded quantity of copies ***p*** of a finite configuration ***Cf*** given in the list form, and the second element defines the quantity of copies in the Moore sense of the configuration ***Cf*** that have been really obtained in process of generation of ***1-HSD*** on interval out of no more than v steps. As a result, numerous computer experiments with ***ReprodHSD*** procedure along with theoretical study allow to show, structures of such type possess the property of *essential* or *full* self–reproductivity in the Moore sense of finite configurations. Along with that a number of rather interesting results concerning the structure of configurations containing self-reproducing configurations have been obtained too [638,641–643].

4. According to the aforesaid, the classical structures ***d-HS*** $(d \geq 1)$ have quite satisfactory degree of generality, suppose wide enough range of appendices, and are enough widely used for modeling of dynamics of many parallel discrete processes [1,3,5,8–10,12,15,23,27,32-34,46-58,64-67,79-90,121,135,138,146,147,149–152,154,156,157,159-161,165–167,175–179,184–188,201,213–219,285,298,308,318,329,354,360,366,378,384–388]. In this respect the dynamical behaviour of model of a certain process, object or phenomenon, embedded into a classical ***d–HS*** $(d \geq 1)$, can be represented by own dynamics of development of certain set of initial configurations $c_j \in C(A,d,\phi)$ $\{j=1..h\}$, where dynamics is determined by the sequences $<c_j>[\tau^{(n)}]$ of finite configurations. At that, the classical ***d–HS*** are the formalized enough objects of researches as by means of methods of modern and classical mathematics, and by methods of the arsenal of own formed apparatus.

Meanwhile, in the applied attitude the classical concept of ***d–HS*** $(d \geq 1)$ appears inconvenient in a lot of cases at modelling of the complicated enough discrete processes and objects. Such modelling (*as a matter of fact undermost level of parallel symbolic programming*) becomes complex, not sufficiently visual and inefficient. In addition, the essence of some modelled processes urgently demands definite *modification* of classical concept of ***d–HS***. With this purpose we have determined a *special* class of structures (*further designated as* ***d–HS****) [3,5,15,41] which are to some extent similar to neural networks or nervous tissues; they well enough reflect a principle of functioning of many types of electronic systems.

Structures ***d–HS**** well enough meet the *base* general requirements and aspects in biology of development at the cellular level, along with *base* principles of functioning of parallel computing systems. All aspects of development of multicellular systems contain *intercellular* interactions whose mechanism in own basics is complex enough and many–sided. However, a series of its very important phenomena can be modelled quite well by a spreading of special control impulses in the structures such as ***HS****. On basis of these structures ***d–HS**** it is possible to model essentially more adequately also phenomena of morphogenetic fields which now are very actively investigated in various aspects. At that, modern ideas and hypotheses in biology of development [5,33,90,264, 330-334] enough definitely specify perspective of use of the structures ***d–HS* (d≥1)*** as a formal simulation environment of many phenomena from the given area and applied areas adjoining to it [88,90,536,567].

In particular, structures ***d-HS* (d≥1)*** is rather convenient environment of modelling of a whole series of biological processes and phenomena such as *neural* networks along with processes in molecular liquids and membranes, development of populations at both *cellular* and *individual* levels; migrations, simulation of cooperative phenomena of various types, excitable environments of various sort, etc. [4,27,31,134,137,159, 167,172,190,198,201-205,264,330-334]. So, many of the listed aspects lay also in a basis of creation of neuro–computers [5,8,9,354,374], forming one of the major components of the interface between biological and computing sciences [4,15,45,47,50,90]. Perspective use of ***HS****–models is supposed in a series of other important areas [1,3-5,7,8,9,11,26,27,31, 33,36,40,45,46,90,134-139,146-152,155-167,176-179,184-189,330-334]. As a model of excitable environments the structures ***d–HS**** provide their major characteristic feature – the opportunity of transfer of the control impulses upon distances of any length and with the necessary speeds, allowing enough simply to create *wave-fronts* of *excitations* distribution of various sorts in a modelling environment. More detailed discussion of the given class of structures can be found in our books [1,3–5,8,9,15, 88,90,536,567,617]. Character of functioning of a structure ***HS**** defines more individual behaviour of an elementary automaton in structure, allowing better to realize the principle of local action ***(PLA)***, it in more significant degree reminds of functioning of homogeneous computing systems and environments ***(HCS)*** if to draw an association of states of elementary automata of the structure with run programs in nodes of ***HCS***, whereas impulses of the structure – with control streams of the information in such computing system ***(****environment****)***. In terms of such structures a series of formal models of the ***HCS*** has been represented.

Nonformally structures ***HS**** of such type are defined as follows. Each individual automaton in ***d–HS**** can receive information directly from the nearest neighbours and can synchronously change own state, and emit control impulses at discrete moments ***t>0*** as a certain function of the current state and incoming control impulses. Certain advantages of structures of the given class have been considered in [56,567,617] in detail enough. Without loss of generality and more formally, we shall define structures ***d–HS**** for the most simple one-dimensional case ***(fig. 4).*** Meanwhile, structures of the given class can be considerably more easily generalized to an arbitrary ***d*–**dimension than the others ***(d≥2).***

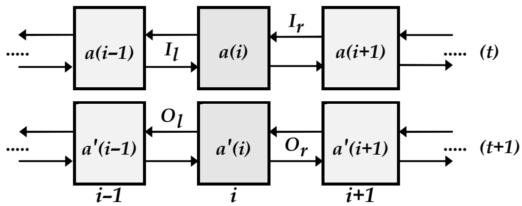


Fig. 4. The principal scheme of functioning of a structure 1–HS*

HS*, by definition, is a four-element tuple ***HS* ≡ <Z¹,A,I,φ>***, where Z^1 and ***A*** are defined just as for *classical* ***1–HS, I*** is a set of impulses and **φ** is a *functional algorithm* ***(FA)*** of ***HS*.*** We shall associate an elementary automaton with each point ***i*** of Z^1, and shall identify the automaton with ***i. FA* φ** is defined by the following discrete equations, namely:

$$\begin{cases} a'(i)_{t+1} = S[i_r, a(i), i_l]_t \\ (j'_r)_{t+1} = R[i_r, a(i), i_l]_t \\ (j'_l)_{t+1} = L[i_r, a(i), i_l]_t \end{cases} \quad a'(i), a(i) \in A; \quad j'_r, j'_l, i_r, i_l \in I; \ t = 0,1,2,\ldots \qquad (7)$$

where ***a'(i), a(i) –*** states of ***i*–**automaton; i_r, i_l – right and left ***incoming*** impulses of ***i*–**automaton accordingly; whereas j'_r, j'_l are right and left ***output*** impulses of ***i*–**automaton accordingly; at last, ***S, R*** & ***L –*** choice functions determining the next state, output impulse to the right, and output impulse to the left accordingly. Thus, essence of functioning of ***1-HS*,*** determined thus, is rather simple and consists in following ***(fig. 4).*** Being in discrete moment ***t≥0*** in a state ***a(i)*** and receiving on input the control impulses j_r *(on the right)* and j_l *(at the left)*, at the following moment ***(t+1)*** the ***i*-**automaton passes into state ***a'(i)*** and emits control impulses j'_r *(to the right)*, j'_l *(to the left)*, that are determined according to the equations system ***(7).*** Thus, the output impulses of an arbitrary ***i*–**automaton are input impulses for all its direct neighbours.

Obviously, if *input* impulses of an ***i*-**automaton coincide with states of its nearest neighbours ***(i-1,i+1)*** whereas *output* impulses coincide with its state, the structures ***1–HS**** and classical ***1–HS*** with neighbourhood index of Moore are identical and condition ***I≡A & I∪A=A*** takes place. Hence, ***d–HS**** is equivalent modification of classical structures ***d–HS*** which is considerably more adapted for research of a whole series of applied aspects of ***HS***–problematics. Numerous examples of concrete use of models ***HS**** have confirmed their high enough efficiency, first of all, from the applied point of view [5,54–56,79,88,90,536,567,617].

It is shown that ***d–HS**** it is possible to exploit successfully as an quite satisfactory intermediate stage at modelling in classical structures and at researches of some questions of their dynamics [1,3,5,9,12,88]. This fact is put in the basis of that approach what an arbitrary ***d–HS**** ***(d ≥ 1)*** can be constructively embedded into classical structure. In particular, it is shown that: ***An arbitrary 1–HS**** $\equiv$ ***<Z^1, A, I, Fa> is equivalent to a classical structure 1–HS*** $\equiv$ ***<Z^1,A∪I,$\tau^{(7)}$,X> with neighbourhood index X*** ={***–3,–2,–1,0,1,2,3***} [1,3,5]. Using enough simple approach [5,567,617] and representing states of a modeling classical ***1–HS*** in the structured kind, it is rather simple to be convinced of validity of following result.

<u>*Theorem 2*</u>. ***An arbitrary structure 1–HS**** $\equiv$ ***<Z^1,A,I=$0_l \cup 0_r$,Fa> is being modeled strictly real time by means of an appropriate classical 1–HS with neighbourhood index X=***{***-1,0,1***} ***and state alphabet A****=A$\cup 0_l \cup 0_r$ ***where 0_l and 0_r – sets of output impulses of automata of the 1–HS**** ***to the left and to the right accordingly.***

A series of other results about equivalence ***(****including strict equivalence****)*** of models ***HS**** and classical ***HS***–models has been received [3,54-56]. In any case quite pertinently to notice the following circumstance – for a theoretical research of a formal cellular model are more preferable the classical structures ***d–HS*** whereas structures ***d–HS**** represent in many respects more acceptable environment for modelling concrete objects, i.e. both classes of structures represent as if two different sides of the classical cellular model of parallel information processing [5,88,90,536, 567,617]. Let's illustrate a series of opportunities of models ***HS**** by the example of solving of known enough ***P****roblem of* ***L****imited* ***G****rowth* ***(PLG)*** which is the typical representative of *minimax* problems in ***HS***–theory.

In a whole series of cases the research of sequences **<c_o>**[$\tau^{(n)}$] includes important enough question such, as existence in such sequences of so-called *passive configurations* ***(PCF),*** i.e. configurations ***c***** for which the following condition $c^{**}\tau^{(n)} = c^{**}$ takes place. So, certain authors have investigated a problem consisting in definition of classical ***HS***–models allowing to generate from simple enough initial finite configurations the ***PCF*** of greatest possible size depending on size of neighbourhood template [1,3,5,54–56,79,88,90,131,190,536,545,640].

As a kind of the extremal problem connected with ***PCF,*** the problem of ***Gaisky-Yamada*** consisting in ascertainment of the greatest possible size ***PCF,*** generated by a classical ***d–HS (d≥1)*** from some simple initial configuration, but without *emphasis* on connection of its size with size of neighbourhood template of the structure has been considered [5,88, 191]. A series of interesting enough questions of growing of chains of automata of the given length can be found also in works [3,5,195,536].

Below, we shall consider so–called *Problem of Limited Growth* ***(PLG)*** in the classical ***HS***–models, most directly related to the above problem of ***Gaisky–Yamada,*** with class of minimax problems in ***HS***–problematics and representing doubtless gnoseological interest from standpoint of developing cellular systems of various nature. Taking into account the difficulties of technical character arising at embedding of complicated enough algorithms into classical ***HS***–models, we for solution of ***PLG*** chose class of the structures ***HS**** defined above. Our considerations in favour of the given solution along with some accompanying thoughts can be found in our works [5,54–56,79,88,545,567,617,640].

Without loss of generality we determine the ***PLG*** concerning a class of the simplest structures such as ***1–HS*.*** A finite configuration c_o of the following kind $c_o =$ *SSS...SSS* {where $|c_o| = r$} of length ***r*** of states ***S*** of elementary ***z***–automata in ***1–HS**** is being set. Then ***PLG*** is reduced to definition of a functional algorithm ***Fa*** of the ***1-HS**** which allows to grow from the initial configuration c_o a passive configuration of kind ***c= FFF ... FFF*** of the greatest possible size $L=L(c_o,Fa)$. The following basic result is the best solution of ***PLG*** known to date [3,5,9,41,56,88].

<u>Theorem 3</u>. For structures 1–HS* ≡ <Z^1,A,I,P> with values #A=12 and #I=4m+17, where m – possibly minimal speed of spreading of control impulses in the structure, there is a functional P–algorithm allowing to grow configurations c** of elementary z–automata of length L in a state «F» from an initial finite configuration c_o of length r where size L is determined by the following recursive formulas, namely:

$$L = r(2m+1)^{\sum_{j=0}^{n} \varpi_j^2 + 2(2^r+1)}, \quad \varpi_0 = 2^{4rm(m+1)}, \quad \varpi_j = 2(L_j - r)$$
$$L_1 = r(2m+1)^{\varpi_0^2+2}, \quad L_j = L_{j-1}(2m+1)^{2^{2L_{j-1}+2}+2} \qquad (8)$$

For a growing of the final configuration of chain of z–automata of the given length L for functional P–algorithm is required t=]1/2*m+3/2[*L steps of the modeling structure 1–HS*.

The essence of realization of one such functional algorithm that solves the ***PLG*** in ***1–HS**** is represented, for example, in [41,567,617]. On the basis of such offer it is possible to investigate its various modifications allowing substantially to improve the result represented in theorem **3** [5,41,88,90,536]. Thus, time of growing of chains of *z*–automata of the specified fantastic length does not exceed their *double* length and with increase of size ***m***, which very essentially influences the length of the growing chain, asymptotically approaches the limit ***t*** =]***3/2****L*[.

Obviously, theoretical limit of time of growing of chain of *z*–automata of length ***L*** in structure ***1-HS**** is ***t***=]***L/2***[, however the above functional ***Fa***–algorithm does not allow to achieve the given limit. Meanwhile, a modification of the ***Fa***–algorithm used for the above solution of ***PLG***, allows to grow a chain of *z*–automata at the same initial preconditions for time, equal asymptotically ***t*** =]***1/2+1/2m***[****L***, and with length of the following size, namely:

$$L = r*(2m+1)^{4^{r+1}+3} - 2m$$

The executed analysis of the functional algorithms that solve the ***PLG*** allows to partition them into two large classes which essentially differ among themselves [3,5,41,54,79,88,536,545], namely:

(1) Algorithms, whose essence consists in constant backing of growth of a figure before reception of the control stopping impulse (signal);

(2) Algorithms, whose essence consists in the preliminary marking of contours of a growing figure with the subsequent filling of it by some final F–symbols (fillers).

The functional algorithm underlying the above of the first solution of the ***PLG***, concerns to the second class, whereas algorithm *optimal* with respect to time – to the first. Meantime, the further complication of the given functional algorithm allows to improve a certain limit of sizes of configurations, growing in structures ***HS**** [3] and in this connexion an interesting enough question arises, namely:

Whether exist functional Fa–algorithms utilizing any other ideas and allowing to obtain the essentially better results concerning growing of configurations of the maximal size other things being equal?

Detailed enough discussion of the ***PLG***, its applied aspects along with a series of other related problems (*Firing squad synchronization problem, problem of the French Flag, etc.*) can be found in the following works [3, 5,15,54-56,88,90,157,196,536,567,617]. The above concept of ***HS**** can be easily generalized to the general case of ***d***–dimensionality ***(d ≥ 2)***.

Each classical ***d–HS*** ***(d≥1)*** is characterized by an *unique* neighbourhood index ***X*** that defines the neighbourhood template corresponding to it. Meanwhile, there are a rather interesting class of the structures which are based on concept of classical structures for which within a general neighbourhood template of the *limited* size the current neighbourhood index for an elementary automaton is defined by a configuration of its previous neighbourhood template. In this case ***LTF*** $\sigma^{(n)}$ of a structure will have the following kind, namely:

$$\sigma^{(n)}(x_1,x_2,\dots,x_n)=x^*j;\quad j=j(x_1,x_2,\dots,x_n);\quad x^*_j,x_k\in A;\quad k=1..n;\quad j\in\{1,2,\dots,n\}$$

i.e., on the basis of configuration $<x_1,x_2,\dots,x_n>$ is evaluated the state of a ***j–****automaton* of neighbourhood template according to neighbourhood index $x_j=\{-(j-1),\dots,-2,-1,0,1,2,\dots,n-j\}$; $1\leq j\leq n$. However at such definition of structure quite possibly appearance of ambiguities at evaluation of states of elementary automata of structure in the course of time ***t.*** For elimination of similar ambiguity it is necessary to define some choice function which is unambiguously defining a state for each elementary automaton of the structure. In particular, logical choice function ***XOR*** can be used. Many interesting enough examples of use of structures of this kind can be found in [99,90,114,642].

It is possible to be convinced quite simply that such class of structures defined by us as $\mathfrak{R}$–class expands the *classical* structures already at the level of generative opportunities relatively some fixed neighbourhood index within the same neighbourhood template [88,90]. In this context it has been shown that the criterion of existence of the most common type of nonconstructability ***NCF*** based in the classical ***d–HS*** on such concept as mutually erasable configurations ***(MEC)*** remains valid and for $\mathfrak{R}$–class of the structures, i.e. the following interesting enough and useful result takes place [88,90], namely:

The homogeneous structure d–HS (d ≥ 1) defined by the above $\mathfrak{R}$***–class with a variable index neighborhood within the general neighborhood pattern possesses the noncoconstructability of NCF–type if and only if for such structure the couples of MEC exist.***

The result can be extended on similar type of structures with variable neighborhood index without any limitation by fixation of the general neighborhood template. In detail, the questions of applications of the structures of $\mathfrak{R}$–class, and their generalizations along with a number of their interesting properties had been considered in [77,88,90,567].

5. For a great many problems of physical modelling and, first of all, of the processes reversible at microlevel, so-called ***HS*** on the splitting are represented effective enough [150]**;** these structures are equivalent to asynchronous ***HS*–**models and are defined below as follows. Without loss of generality, and also on account of the greater transparency the consideration of the concept of ***HS-***models of the similar type we shall carry out concerning their one–dimensional case.

A ***homogeneous structure on splitting*** **(*HSoS*,** *in English terminology* **– *HS*** *with Margolus index***)** can be defined as the ordered tuple of **5** basic component of the following general kind, namely**:**

$$1\text{–}HSoS \equiv \langle Z^1, A, m, \Psi^{(m)}, \Xi \rangle$$

where two first components Z^1 and A are similar to a case of classical ***HS*–**models**;** m **–** size of a block into which space Z^1 of the structure is divided**;** $\Psi^{(m)}$ **–** local block transition function**;** Ξ **–** rules of switching of blocks *(over–marking)* of space Z^1 of the structure. Thus, the similar organization of structure ***1–HSoS*** excludes its from the class of typical ***HS*–**models. Meanwhile, functioning of the structure ***1–HSoS*** is simple enough and occures as follows.

At the initial moment $t=0$ space Z^1 of the structure ***1–HSoS*** is divided into blocks equal by length m of elementary automata**;** i.e. all adjacent automata are grouped into blocks of length m ***(m*–*blocks*).** The splitting is determined by a j**–**parameter which without loss of generality, we suppose equal $j=0$**.** Hence, ***m*-**blocks are formed by adjacent *elementary* automata of the structure ***1–HSoS*** with coordinates of the next general kind, namely**:**

$$m\text{–block: } [x_{j+1}x_{j+2}x_{j+3} \ldots x_{j+m}] \qquad j \in \{0; \pm m; \pm 2m; \pm 3m; \ldots\}$$

(α) Then simultaneously to configurations of all ***m*–**blocks of ***1–HSoS*** the *parallel* ***block substitutions (PBS)*** of the following general kind are applied, namely**:**

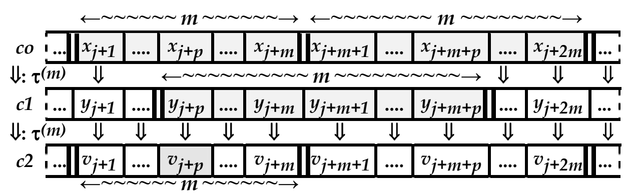
$$\begin{array}{l} x_{j+1}x_{j+2}x_{j+3} \ldots x_{j+m} \Rightarrow y_{j+1}y_{j+2}y_{j+3} \ldots y_{j+m} \quad 0^m \Rightarrow 0^m;\ x_{j+p}, y_{j+p} \in A \\ (j = 0; \pm m; \pm 2m; \ldots; p = 1..m) \quad N = a^{m(a^m-1)} \text{ – quantity of the local function} \end{array} \qquad (9)$$

which define the *local block function* **(*LBF*)** $\Psi^{(m)}$ of transition. Function $\Psi^{(m)}$ maps an arbitrary configuration of a certain ***m*–**block into a new configuration of the same ***m*–**block**;** i.e. the mapping $\Psi^{(m)}: A^m \Rightarrow A^m$ takes place. Simultaneous application of the ***LBF*** to all ***m*-**blocks of the space Z^1 of structure ***1–HSoS*** defines the *global transition function* $\tau^{(m)}$

which maps an initial configuration c_o of all space of the ***1–HSoS*** into the following global configuration c_1 ***(fig. 5)***, i.e. $c_o\tau^{(m)} = c_1$. However, as against *classical* ***1–HS*** in which switching of elementary automata is constant and is determined only by neighbourhood index, in ***1–HSoS*** at the following moment ***t=1*** is being applied the re–commutation of ***m***–blocks *(over-marking of space* Z^1*)*. With this purpose the ***j***–parameter determining *block* marking of space Z^1 accepts a value $j=p$ $(0<|p|<m)$, by providing a mode of *continuous overlapping* of both block markings. So, ***m–blocks*** of the 2^{nd} marking of space Z^1 of the structure are formed by the adjacent elementary automata with the following coordinates:

$$m\text{–block: } [y_{j+p}y_{j+p+1}y_{j+p+2} \cdots y_{j+m+p}] \qquad j \in \{0; \pm m; \pm 2m; \pm 3m; \ldots\}$$

i.e. shifting to the left or to the right the borders of the block marking concerning previous marking. Again to ***m***–blocks of a new marking the ***PBS (8)*** are simultaneously applied, completing on it the second step of ***1–HSoS*** and receiving configuration c_2 of the structure at the output ***(fig. 5)***. Then again we come back to the marking ***(j=0)*** of space Z^1, and process of functioning of the ***1–HSoS*** recurs since item ***(α)***. Of the represented definition of structure ***1–HSoS*** and the principle of its functioning, it is simple to receive a whole series of rather interesting theoretical and applied *conclusions* whose enough detailed discussion can be found in [54–56,79,90,536,545,567,617,640].



$$x_{j+p}, y_{j+p}, v_{j+p} \in A; \quad j = \{0; \pm m; \pm 2m; \ldots; p=1 \,..\, m\}$$

Fig. 5. An illustration of principle of functioning of structures 1–HSoS

The above organization of *commutation* of elementary automata of the structures ***HSoS*** allows to programme quite simply their ***reversibility*** property, provided by ***LBF*** $\Psi^{(m)}$, which can easily realize function of simple rearrangement in configurations of above ***m***–blocks of states of elementary automata. At the made assumptions we can consider the ***HSoS***–model as an enough essentially expanded variant of a classical ***HS***–model for which the individual step is divided into certain more elementary substeps; at that, the ***HS***–model should be provided with possibility of *variable commutation* of neighbourhood index dependent on position of elementary ***z***–automaton of the model, i.e. to represent one of elementary versions from the class of asynchronous ***HS***-models [3,5,88,90,536,567]. Already simple enough ***HSoS***–models can be used quite successfully, for example, in physical modelling. Constructions of similar models are adequate to so-called «***billiard automaton***» and are based on the ***Margolus*** neighbourhood template [150,268,273,376]. The reader can receive more detailed information, having addressed to the sources represented in the extended bibliography [536,545,640].

However between *classical* ***HS***-model and ***HSoS***–model exists the *basic* distinction, namely: if for the first each elementary ***z***-automaton of the ***HS***–model has several inputs (*depending on neighbourhood index* ***X***) and only one output (*determined by the its current state*), then for the model ***HSoS*** a block *splitting* of space Z^d provides *identical* amounts of inputs and outputs for the blocks of elementary ***z***–automata in the presence of independence of blocks at each moment $t \geq 0$. In the given direction the following basic result characterizing interrelation of classical ***HS***–models and models ***HSoS*** takes place [54–56,79,88,90].

<u>Theorem 4</u>. An arbitrary structure d–HSoS $\equiv <Z^d,A,m,\Psi^{(m)},\Xi>$ ***can't be modeled by a classical structure d–HS*** $\equiv <Z^d,A,\tau^{(n)},X>$***, and vice versa.***

Thus, the ***HSoS*** concept is rather essential modification of the classical ***HS***–models that allows easily to programme the reversibility of their dynamics. Discussion of this question can be found in [56,88,90,567].

On the other hand, due to increase of the ***A***–alphabet of classical ***d–HS*** only by one symbol it is possible to model by means of it an arbitrary ***d–HSoS*** in real time. Below, we shall briefly consider the problem of modelling of an arbitrary ***d–HSoS*** $\equiv <Z^d,A^*,m, \Psi^{(m)}, \Xi>$ by means of a *classical* structure on an example of ***1***-dimensional case what generally speaking does not break the generality of consideration.

A structure ***1–HSoS***$\equiv<Z^1,A,m,\Psi^{(m)},\Xi>$ with the following parameters, namely: $A=\{0,1, ..., a\text{-}1\}$, ***LBF*** – a mapping $\Psi^{(m)}: A^m \Rightarrow A^m$, that defines global transition function $\tau^{(m)}$, Ξ–rule of block over–marking consists in alternation of {***0,1***} for ***j***–parameter at the ***even*** and ***odd*** moments ***t*** is chosen as an initial structure. At the made presumptions a classical ***1–HS*** that models the ***1–HSoS***$\equiv<Z^1,A,m,\Psi^{(m)},\Xi>$ is defined as ***1–HS*** $\equiv$ $<Z^1,A,\tau^{(n)},X>$, where $A^*=\{b\}\cup A$ $(b \notin A)$, $\tau^{(n)}$ – composition of four ***GTF*** in the form of $\tau^{(n)} = \tau^{(2m)}\tau_l^{(3)}\tau^{(2m)}\tau_r^{(3)}$ $(n = 4m + 9)$.

At that, ***GTF*** $\tau^{(2m)}$ is defined by means of ***LTF*** $\sigma^{(2m)}$ which is given by the following formulas, namely:

$$\sigma^{(2m)}\begin{pmatrix} x\dots xb_m x_1\dots \mid x_k \mid \dots x_m b_1 x\dots x \\ \leftarrow\ m-1\ \rightarrow \quad\quad \leftarrow\ m-1\ \rightarrow \end{pmatrix} = x_k^*;$$

$$b_p = \begin{cases} \Delta, & if\ p = k \\ b, & otherwise \end{cases}; \quad p = 1,m;\ x, x_j \in A;\ 1 \le k \le m;\ j = 1..m;$$

$$x\dots x - \text{tuple of any elements from } A;\ \Delta - \text{empty symbol}$$

For support of block over-marking Ξ of space Z^1 *(shift to the left and to the right onto one automaton)* on steps **(2, 4)** of the simulating structure, the ***GTF*** $\tau_l^{(3)}$ and $\tau_r^{(3)}$ are applied whose ***LTF*** $\sigma_l^{(3)}$ & $\sigma_r^{(3)}$ are defined accordingly by the following simple functions, namely:

$$\sigma_l^{(3)}(x_1, x_2, x_3) = \begin{cases} x_1x_2b & \rightarrow\ b \\ x_2bx_3 & \rightarrow\ x_2 \\ x_1x_2x_3 & \rightarrow\ x_2,\ otherwise \end{cases}$$

$$\sigma_r^{(3)}(x_2, x_3, x_4) = \begin{cases} bx_3x_4b & \rightarrow\ b \\ x_2bx_3 & \rightarrow\ x_3 \\ x_2x_3x_4 & \rightarrow\ x_3,\ otherwise \end{cases}$$

$$x_j \in A = \{0, 1, 2, \dots, a-1\};\ b \notin A;\ j = 1..4$$

At that, if ***LTF*** $\sigma^{(2m)}$ operates with neighbourhood index of kind $X = X=\{-m, \dots, -1, 0, 1, \dots, m\}$, whereas local transition functions $\sigma_l^{(3)}$ & $\sigma_r^{(3)}$ operate with neighbourhood index of Neumann–Moore $X=\{-1,0,1\}$. It is shown [54,567,617] that the composition of above three ***GTF*** of kind $\tau^{(n)} = \tau^{(2m)}\tau_l^{(3)}\tau^{(2m)}\tau_r^{(3)}$ $(n = 4m + 9)$ provides simulating of one cycle (*application* ***LBF*** $\Psi^{(m)}$ *to the current configuration of* ***1–HSoS*** $\rightarrow$ *shift onto automaton to the left, new application of* $\Psi^{(m)}$ $\rightarrow$ *shift onto automaton to the right)* of the structure ***1–HSoS*** determined above. Thus, aforesaid brief considerations can be summarized by the following rather interesting result [5,54–56,79,88,90,536,545,567,617,640].

<u>Theorem 5</u>. An arbitrary structure 1–HSoS≡<Z^1,A,m,$\Psi^{(m)}$,Ξ> with the simplest Ξ–rule of block over–marking is simulated strictly real time by means of the classical structure 1–HS determined above with state alphabet A'={b}∪A (b∉A) and GTF $\tau^{(n)} = \tau^{(2m)}\tau_l^{(3)}\tau^{(2m)}\tau_r^{(3)}$ (n=4m+9).

The given result can be spread and onto structures ***HSoS*** of the more general types, in particular, ***d–HSoS ≡ <Z^d,A,m,$\Psi^{(m)}$, Ξ>***. In the given attitude the following rather interesting result takes place [53,54,88].

<u>Theorem 6</u>. An arbitrary structure d–HSoS determined in a finite state A–alphabet is modeled real time by means of an appropriate classical structure d–HS (d ≥ 1) with state alphabet A* = A∪{#} (# ∉ A).

On the other hand, an arbitrary classical structure ***1–HS*** is modeled by means of structure ***1–HSoS;*** a whole series of algorithms of simulation has been submitted, for example, in works [54–56,88,545,567,617,640].

<u>*Theorem 7*</u>***. An arbitrary classical structure 1–HS with neighbourhood index X={0,1, ..., n–1} is simulated with delay 2(n–1) by an appropriate structure 1–HSoS≡<Z^1,A*,2,$\Psi^{(2)}$,Ξ>. An arbitrary structure 1-HS with a state alphabet A={0,1, ..., a–1} and the simplest neighbourhood index is 3–modeled by an appropriate irreversible 1–HSoS≡<Z^1,A',2,$\Psi^{(2)}$,Ξ> with blocks of size m = 2 of over–marking of homogeneous space Z^1 of the structure 1–HSoS≡<Z^1,A',2,$\Psi^{(2)}$,Ξ> along with a state alphabet A' of cardinality #A' = a(a+1). An arbitrary structure 1–HS with a state alphabet A={0,1, ..., a–1} and the simplest neighbourhood index X={0,1} is 2–modeled by an appropriate irreversible 1–HSoS≡<Z^1,A',3,$\Psi^{(3)}$,Ξ> with blocks of size m = 3 of over–marking of homogeneous space Z^1 of the structure 1–HSoS≡<Z^1,A',3,$\Psi^{(3)}$,Ξ> along with a state alphabet A' of cardinality #A' = 2a, where #U is cardinality of a finite set U.***

Here we shall present a sketch of proof of the second part of the above assertion. It is well-known the an arbitrary structure ***1–HS*** with a state alphabet ***A***={***0,1,...,a-1***} and neighbourhood index ***X***={***0,1,...,n-1***} can be modeled by means of an appropriate structure ***1–HS*** with an alphabet ***A**** and the simplest neighbourhood index ***X***={***0,1***} under the condition ***#A* > #A.*** By a modelling structure for such structure ***1–HS*** a structure ***1–HSoS ≡ <Z^1,A*,2,$\Psi^{(2)}$,Ξ>*** is chosen with blocks of size ***m = 2*** of over–marking of homogeneous space ***Z^1*** with a structured state alphabet ***A**** in the form of $A^* = \left\{\begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x \\ b \end{bmatrix}\right\} (x, y \in A, b \notin A)$**;** a Ξ–rule of over–marking consists in alternation of {***0,1***} for ***j***-parameter relative to even and odd moments ***t,*** and a local block function $\Psi^{(2)}$ is defined by parallel block substitutions in the following form $\psi^{(2)}\!: \begin{bmatrix} x \\ p \end{bmatrix}\begin{bmatrix} y \\ q \end{bmatrix} \to \begin{bmatrix} x' \\ p' \end{bmatrix}\begin{bmatrix} y' \\ q' \end{bmatrix}$ where ***x,x',y, y'∈A; p,p',q,q'***∈***A***∪{***b***}; ***b∉A.*** Then, in moment ***t = 0*** an arbitrary initial configuration $c_0 = \ldots\ \ x_0 x_1 x_2 \ldots x_9\ \ \ldots$ of the above simulated structure is embedded into the modeling structure ***1–HSoS*** as shown below. It is simple to be convinced that the diagram represented below enough well illustrates a principle of such modeling**;** in addition, a little more detailed analysis definitely shows irreversibility, generally speaking, of such modeling structure ***1–HSoS.*** The scheme represented below to some degree explains the principle of such modeling.

$t=0\ldots$	$\begin{bmatrix}0\\b\end{bmatrix}$	$\begin{bmatrix}x_o\\b\end{bmatrix}$	$\begin{bmatrix}x_1\\b\end{bmatrix}$	$\begin{bmatrix}x_2\\b\end{bmatrix}$	$\begin{bmatrix}x_3\\b\end{bmatrix}$	$\begin{bmatrix}x_4\\b\end{bmatrix}$	$\begin{bmatrix}x_5\\b\end{bmatrix}$	$\begin{bmatrix}x_6\\b\end{bmatrix}$	$\begin{bmatrix}x_7\\b\end{bmatrix}$	$\begin{bmatrix}x_8\\b\end{bmatrix}$	$\begin{bmatrix}x_9\\b\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	...
$t=1\ldots$	$\begin{bmatrix}0\\b\end{bmatrix}$	$\begin{bmatrix}x_o\\x'_o\end{bmatrix}$	$\begin{bmatrix}x_1\\b\end{bmatrix}$	$\begin{bmatrix}x_2\\x'_2\end{bmatrix}$	$\begin{bmatrix}x_3\\b\end{bmatrix}$	$\begin{bmatrix}x_4\\x'_4\end{bmatrix}$	$\begin{bmatrix}x_5\\b\end{bmatrix}$	$\begin{bmatrix}x_6\\x'_6\end{bmatrix}$	$\begin{bmatrix}x_7\\b\end{bmatrix}$	$\begin{bmatrix}x_8\\x'_8\end{bmatrix}$	$\begin{bmatrix}x_9\\b\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	...
$t=2\ldots$	$\begin{bmatrix}x_{-1}\\x'_{-1}\end{bmatrix}$	$\begin{bmatrix}x_o\\x'_o\end{bmatrix}$	$\begin{bmatrix}x_1\\x'_1\end{bmatrix}$	$\begin{bmatrix}x_2\\x'_2\end{bmatrix}$	$\begin{bmatrix}x_3\\x'_3\end{bmatrix}$	$\begin{bmatrix}x_4\\x'_4\end{bmatrix}$	$\begin{bmatrix}x_5\\x'_5\end{bmatrix}$	$\begin{bmatrix}x_6\\x'_6\end{bmatrix}$	$\begin{bmatrix}x_7\\x'_7\end{bmatrix}$	$\begin{bmatrix}x_8\\x'_8\end{bmatrix}$	$\begin{bmatrix}x_9\\x'_9\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	...
$t=3\ldots$	$\begin{bmatrix}x'_{-1}\\b\end{bmatrix}$	$\begin{bmatrix}x'_o\\b\end{bmatrix}$	$\begin{bmatrix}x'_1\\b\end{bmatrix}$	$\begin{bmatrix}x'_2\\b\end{bmatrix}$	$\begin{bmatrix}x'_3\\b\end{bmatrix}$	$\begin{bmatrix}x'_4\\b\end{bmatrix}$	$\begin{bmatrix}x'_5\\b\end{bmatrix}$	$\begin{bmatrix}x'_6\\b\end{bmatrix}$	$\begin{bmatrix}x'_7\\b\end{bmatrix}$	$\begin{bmatrix}x'_8\\b\end{bmatrix}$	$\begin{bmatrix}x'_9\\b\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	...
	$m=2$		$m=2$		$m=2$		$m=2$		$m=2$		$m=2$		...

$$\Psi^{(2)}: \begin{bmatrix}x\\b\end{bmatrix}\begin{bmatrix}y\\b\end{bmatrix}\rightarrow\begin{bmatrix}x\\x'\end{bmatrix}\begin{bmatrix}y\\b\end{bmatrix}\quad \begin{bmatrix}x\\b\end{bmatrix}\begin{bmatrix}y\\z\end{bmatrix}\rightarrow\begin{bmatrix}x\\x'\end{bmatrix}\begin{bmatrix}y\\z\end{bmatrix}\quad \begin{bmatrix}x\\z\end{bmatrix}\begin{bmatrix}y\\h\end{bmatrix}\rightarrow\begin{bmatrix}z\\b\end{bmatrix}\begin{bmatrix}h\\b\end{bmatrix};$$

$$\sigma^{(2)}(x,y)=x';\quad x,x',y,z,h,x_j,x'_j\in A=\{0,1,\ldots,a-1\};\quad b\notin A;\ j=\{0,\pm1,\pm2,\pm3,\ldots\};$$

In the second case of simulating, as a modelling structure for similar structure ***1–HS*** the structure ***1–HSoS*** $\equiv <Z^1,A^*,3,\Psi^{(3)},\Xi>$ is chosen with blocks of size $m=3$ of over–marking of homogeneous space Z^1 with a structured state alphabet $A^*=A\cup B$ where $B=\{b_o,b_1,\ldots,b_{a-1}\}$ and among elements of sets A and B the bijection is determined in the form $x\equiv b_x$, $x\in A$; while a Ξ–rule of over-marking consists in alternation of $\{0,2\}$ for j–parameter relative to even and odd moments t together with a local block function $\Psi^{(3)}$ which is defined by parallel block substitutions in the following form $\Psi^{(3)}: b_x xy\rightarrow b_x x'y$ and $\Psi^{(3)}: xb_y y'\rightarrow x'b_{y'}y'$, where $x,x',y,y'\in A$; $b_x,b_y,b_{y'}\in B$; $A\cup B=A'$, $A\cap B=\varnothing$. Then, in moment $t=0$ an arbitrary initial configuration $c_o=\ldots b_{x_1}x_1x_2b_{x_3}x_3x_4b_{x_5}x_5x_6b_{x_7}\ldots b_{x_j}x_j\ldots$ of the above simulated structure is embedded in a modeling structure ***1–HSoS*** as shown below. In addition it is simple to be convinced, that the diagram represented below enough well illustrates a principle of such modeling and a little more detailed analysis shows irreversibility of the modeling structure ***1–HSoS.***

$t=0$	...	$0\ 0$	0	$b_{x_1}\ x_1$	x_2	$b_{x_3}\ x_3$	x_4	$b_{x_5}\ x_5$	x_6	$b_{x_7}\ x_7$	x_8	...	$0\ 0$	0	...
$t=1$	...	$b_o 0$	0	$b_{x_1}\ x'_1$	x_2	$b_{x_3}\ x'_3$	x_4	$b_{x_5}\ x'_5$	x_6	$b_{x_7}\ x'_7$	x_8	...	$b_o 0$	0	...
$t=2$	...	$b_o 0$	x'_o	$b_{x'_1}\ x'_1$	x'_2	$b_{x'_3}\ x'_3$	x'_4	$b_{x'_5}\ x'_5$	x'_6	$b_{x'_7}\ x'_7$	x'_8	...	$b_o 0$	0	...
		$m=3$		$m=3$		$m=3$		$m=3$		$m=3$			$m=3$		

$$000\rightarrow b_o00\qquad b_{x_j}x_jx_{j+1}\rightarrow b_{x_j}x'_jx_{j+1}\qquad x_jb_{x_{j+1}}x'_{j+1}\rightarrow x'_jb_{x'_{j+1}}x'_{j+1}$$

$$hpq\rightarrow hpq\ -\ \textit{on other tuples}\ <h,p,q>;\quad b_o,b_{x_j},b_{x'_j}\in B;\quad 0,x_j,x'_j\in A$$

$$\sigma^{(2)}(x,y)=x';\ h,p,q\in A\cup B;\ (j=0,1,\ldots);\ A=\{0,1,\ldots,a-1\},\ B=\{b_o,b_1,\ldots,b_{a-1}\}$$

Thus, an arbitrary structure ***1–HS*** can be simulated by an appropriate structure ***1–HSoS*** only with double increase of cardinality of alphabet of the modelling structure**;** that was achieved at the expense of using of a special coding of an arbitrary initial configuration of a simulated structure ***1-HS.*** The given result under the certain prerequisites can be generalized also to the general case of structures ***d–HS*** ***(d ≥ 1).*** A proof of this assertion can be based oneself on above modificated approach.

Let's consider now the question of modelling of an arbitrary structure ***HSoS*** $\equiv <Z^1, A, 2, \Psi^{(2)}, \Xi>$ by a classical structure $<Z^1, A, \tau^{(n)}, X'>$ with a variable neighbourhood index ***X'*** when application of its subindices is determined by coordinates of an automaton of a modelling structure. Without loss of generality, let with taking into consideration of block over–marking two consecutive steps of a modelled structure ***1–HSoS*** which simulate one step of the modelling structure with an arbitrary configuration ***c**** assume generally the following simple kind, namely**:**

Marking	*Dynamics of c*, subjected to j–marking of* Z^1*–space*	*t*
....	....	...
$j = 0$	... ▌ s_{k-2} s_{k-1} ▌ s_k s_{k+1} ▌ s_{k+2} s_{k+3} ▌ s_{k+4} s_{k+5} ▌ ...	0
$j = 1$	... s'_{k-2} ‖ s'_{k-1} s'_k ‖ s'_{k+1} s'_{k+2} ‖ s'_{k+3} s'_{k+4} ‖ s'_{k+5} ...	1
$j = 2$	... ▌ s''_{k-2} s''_{k-1} ▌ s''_k s''_{k+1} ▌ s''_{k+2} s''_{k+3} ▌ s''_{k+4} s''_{k+5} ▌ ...	2
....	....	...

$$s_{k+p}, s'_{k+p}, s''_{k+p} \in A \ (p = 0; \pm 1; \pm 2; \pm 3; \pm 4; \ldots)$$

$$\begin{cases} x'_{k-1} = f^1_{k-1}(x_{k-2}, x_{k-1}) & x'_k = f^1_k(x_k, x_{k+1}) \\ x'_{k+1} = f^1_{k+1}(x_k, x_{k+1}) & x'_{k+2} = f^1_{k+2}(x_{k+2}, x_{k+3}) \\ x'_{k+3} = f^1_{k+3}(x_{k+2}, x_{k+3}) & x'_{k+4} = f^1_{k+4}(x_{k+4}, x_{k+5}) \end{cases}$$

$$\begin{cases} x''_k = f^2_k(x'_{k-1}, x'_k) = f^2_k(f^1_{k-1}(x_{k-2}, x_{k-1}), f^1_k(x_k, x_{k+1})) = \\ \qquad F^2_k(x_{k-2}, x_{k-1}, x_k, x_{k+1}) \\ x''_{k+1} = f^2_{k+1}(x'_{k+1}, x'_{k+2}) = f^2_{k+1}(f^1_{k+1}(x_k, x_{k+1}), f^1_{k+2}(x_{k+2}, x_{k+3})) = \\ \qquad F^2_{k+1}(x_k, x_{k+1}, x_{k+2}, x_{k+3}) \end{cases}$$

In view of told and of the definition of ***HSoS***–models **(***section* **1.2)** it is simple to be convinced that states ***s, s', s''*** of an elementary automaton ***(k+p)*** in the modelled ***HSoS***–structure are connected by the following system of functional discrete equations. Of the system it is easy to see that the modelling structure $<Z^1, A, \tau^{(n)}, X'>$ should possess a variable neighbourhood index $X' = <X_k = \{-2, -1, 0, 1\}, X_{k+1} = \{-1, 0, 1, 2\}>$ and whose components $\{X_k \mid X_{k+1}\}$ are used depending on a coordinate of ***(k+p)–*** automaton of ***1–HSoS (p=0; ±1; ±2; ...).*** So, the next result takes place.

Any structure HSoS≡<Z^1,A,2,$\Psi^{(2)}$,Ξ> is 1–modeled by an appropriate structure <Z^1,A,$\tau^{(4)}$,X'> with a variable neighbourhood index, whose components are applied only depending on a coordinate of the current elementary automaton of the modelled structure 1–HSoS.

This result is enough naturally generalized to case of the more general types of ***HSoS.*** In some concrete cases a variable neighbourhood index of a modeling structure will consist of much more simple subindices.

A generalization of one principal scheme to the general ***d***-dimensional case allows to formulate the following result useful in many respects.

<u>*Theorem 8*</u>***. An arbitrary classical structure HS ≡ <Z^d,A,$\tau^{(d+1)}$,X> with the simplest neighbourhood index is modeled with delay (d+1) by an appropriate HSoS ≡ <Z^d,A*,2,$\Psi^{(d+1)}$,Ξ> whose block of over–marking of space Z^d is identical to a neighbourhood template of the modelled structure and #A* = a(a+1), where #U is cardinality of a finite set U.***

Meanwhile, on the basis of use of some expansion of classical concept ***d–HSoS*** possibly to solve the above problem without expansion of an alphabet ***A*** of a simulated classical structure ***d–HS (d ≥ 1).*** As a simple example illustrating the given approach, we shall consider modelling of a classical structure ***1–HS*** with alphabet ***A***={***0,1,..., a-1***} and simplest neighbourhood index ***X***={***0,1***}. It is known [1,3], that the any structure ***1–HS*** is simulated by the above structure ***1–HS*** owing to expansion of state alphabet ***A*** of a simulated structure. Here we shall present only a sketch of proof of the result of simulation of such structure ***1–HS*** by a modification of structure ***1–HSoS*** with the same state alphabet ***A.***

For our case by a modeling structure for such classical structure ***1–HS*** a structure ***1–HSoS**** ≡ <Z^1, A, 4, {$\Psi_1^{(4)}$, $\Psi_2^{(4)}$}, Ξ> is chosen with blocks of size ***m = 4*** of over–marking of space Z^1 with a state alphabet ***A.*** The space Z^1 is divided by a grid, whose cells have length ***m = 4;*** the given grid has a reference point ***j=0*** and on each step of ***1–HSoS**** it is shifted of two positions to the left. On each step to configurations of all cells of the current grid an appropriate ***LBF*** is simultaneously applied:

$$\Psi_1^{(4)}: \ \left|x_j y_j x_{j+1} y_{j+1}\right| \rightarrow \left|x_j x_j x_{j+1} x_{j+1}\right|; \quad x_j, y_j \in A; \quad j = 0, \pm 1, \pm 2, \ldots$$

$$\Psi_2^{(4)}: \ \left|x_j\, x_j\, x_{j+1}\, x_{j+1}\right| \rightarrow \left|\sigma^{(2)}(x_j, x_{j+1})\, x_j\, x_{j+1}\, x_{j+1}\right|$$

where $\sigma^{(2)}$ – local transition function of a simulated ***1-HS.*** Functioning of a modeling structure is defined as follows. An initial configuration $c_o = \ldots x_j x_{j+1} x_{j+2} \ldots$ of a simulated structure ***1–HS*** in modeling ***1–HSoS**** is defined as $b_o = ...x_j y_j x_{j+1} y_{j+1} x_{j+2} y_{j+2} ...;\ x_j, y_j \in A$. At that, step-by-step functioning of the modelling ***1–HSoS**** is described by a program **Π** of application of an appropriate ***LBF*** $\Psi^{(4)}$ to the current grid, namely:

$$\prod: b_o \psi_1^{(4)} (-2) \psi_2^{(4)} (-2) \psi_2^{(4)} (-2) \psi_1^{(4)} (-2) ...$$

where ***(-2)*** defines shift of the current grid of **2** positions to the left. It is easy to be convinced, that since a configuration b_o on the third step we receive a configuration $b_3 = ...z_j x_j z_{j+1} x_{j+1} z_{j+2} x_{j+2} z_{j+3} x_{j+3} ...$ which is structurally similar to the configuration b_o, where $z_j = \sigma^{(2)}(x_j, x_{j+1})$.

The above approach extends to high dimensionality $d \geq 2$, representing each automaton by a ***d***–block and the shifts along all dimensions what easily enough allows to formulate the following result [54].

A structure d–HS with the simplest neighbourhood index is simulated by an appropriate structure d–HSoS* with the same state alphabet.

A whole series of interesting questions of *mutual* modeling of classical structures ***d–HS*** and ***d–HSoS*** $(d \geq 1)$ with useful enough consequences which follow from it along with discussion can be found in [9,90,617]. Thus, many questions of dynamics of ***HSoS***-models can be considered enough effectively within the advanced theory of classical ***HS***-models, taking into account the specificity of the first [5,88,90,536,567,617]. The class of ***HSoS****-models* is of indubitable interest not only as independent formal object of parallel information processing, but, first of all, as the good environment of physical modelling and research of calculations having the important property of reversibility. At that, it is necessary to note, that the theory of the given type of ***HS***–models practically is not developed *(in contrast to the classical **HS**–models, for example)*; and the overwhelming quantity of the results about them carry empirical character. It is caused by wide enough use of the ***HSoS***–approach for problems, first of all of physical modeling on basis of the above ***CAM***–computers of ***T. Toffoli*** which are suitable hardware-software support of ***HSoS***–models [150,165,430,536,545,567,617,618,640–643].

In ***CAM***–computers the isomorphism of the general ***HS***–concept and hardware–software support that provides simplicity and efficiency of programming of ***HS***–models of both the classical, and the ***HSoS*** which admit constructive definition of *reversibility property* has been realized. In works [150,160,273,318,536,567,617] it is possible to find interesting enough discussion of reversibility property of dynamics of the ***HSoS***–models of various physical processes and phenomena at microscopic level, and physical interpretation of the given important property too. In view of definition of the ***HSoS***–models, it is simple to be convinced, that ***the problem of reversibility of their dynamics is algorithmically solvable.*** However for its solution the question of one–oneness of the mapping $\Psi^{(m)}: A^m \Rightarrow A^m$, which defines ***LBF*** of ***HSoS***–model, along with Ξ–rule of block over–marking of space Z^d of the model are both equally important ones. So, ***HSoS***–models as against the classical ones are characterized by solvability of the reversibility problem.

Indeed, any ***HSoS***–model is considered quite determined if it is given by a set of parallel block substitutions ***(8)*** for its ***LBF*** $\Psi^{(m)}$ along with Ξ–rule of block over–marking of homogeneous space of the model. At the same time, the dynamics reversibility of an arbitrary ***HSoS***–model directly follows of mutual one-valuedness of mapping $\Psi^{(m)}:A^m \Rightarrow A^m$, that is algorithmically solvable. So, having defined a concrete ***HSoS***–model, we at once receive the precise answer concerning reversibility of its dynamics whereas for case of a classical ***HS***–model this question, generally speaking, is algorithmically unsolvable. It is simple to make sure that among diverse structures on *splitting* $<Z^d,A,m,\Psi^{(m)},\Xi>$ exist exactly $R = (a^m - 1)!$ with reversible dynamics, where ***m*** – quantity of elementary automata forming the block of splitting of the space Z^d. In principle, in case of *irreversibility* of dynamics of the ***HSoS***–models for pairs of block configurations, mapped by means of ***LBF*** $\Psi^{(m)}$ into the same block configuration we can draw an analogy with pairs of ***MEC*** of the classical ***HS***–models, while on basis of block configurations not having predecessors concerning mapping $\Psi^{(m)}$ we can obtain a set of all nonconstructible configurations such as ***NCF.***

At the same time, on basis of definitions of structures ***d–HS*** and ***HSoS*** it is simple to make sure that converse monitoring of dynamics of an arbitrary configuration ***c***** for both structures are essentially different, demanding various information starting points. More precisely, direct and reverse dynamics of the ***HSoS***–models demand essentially more of information, than for case of the classical ***HS***-models, thus, at times, enough essentially complicating the modeled object [567,617]. At last, the *nonconstructability* problem of the ***HSoS***–models is associated, first of all, with the reversibility problem of their dynamics which is briefly discussed in the present book, and is in detail considered in a series of works [54,79,90,150–152,157,160,204,268,273,318,378,386–388,536,617].

In conclusion it is necessary to note that term «***structures on splitting***» used by us is equivalent to the term «*neighbourhood index of Margolus*». Meanwhile, our term seems the more adequate to essence of concept reflected by it that is linked with over–marking of space Z^d, i.e. fact of dynamics of alternate choice of neighbourhood index, while the term «*neighbourhood index of Margolus*» to some degree associates more with static neighbourhood indexes of Neumann–Moore, Moore etc. Along with the represented types of ***HS***–models for a whole series of special theoretical, and applied purposes other classes of structures are used, many of which are considered in works [1,3,5,8,9,54,88,90,135,146,147, 150,160,161, 166, 175–179,187,201,214, 252,285,308,360, 366,381,536,617]. Having introduced general definitions, concepts and designations, the others we shall introduce in process of their necessity. In the sequent chapters of this monography the most fundamental problems of the mathematical theory, first of all, of the classical ***d–HS*** are considered.

And yet it is necessary to focus our attention on certain moments. For the first time in world practice into textbooks on information science for the universities that have sustained **2** editions [94-96] we included the separate chapter «***General formal computing models***» in which the Turing machine as a formal model of classical consecutive evaluations and processing together with ***d–HS*** as a formal model of nonclassical highly–parallel calculations and processing are considered.

In our opinion, such innovation is currently very opportune especially in light of the model perspectives in different fields, along with rather intensive research in the field of non–traditional perspective computer facilities of parallel architecture. Moreover, the growing interest in the ***HS*** issues more insistently requires promoting its concept, *fundamental* and applied aspects. In addition, is a very topical work to consolidate the efforts of a large number of individual researchers in this and the adjacent fields of modern cybernetics. In the proposed monograph the ***d–HS***, basing on classical models, are discussed the most fundamental problems of their dynamics, forming a foundation of the mathematical theory of ***HS***–models dynamics ***(behavioral** aspects**)***, without affecting the structural problems of their organization.

Simplicity and transparency of the classical ***HS***–models, not reducing, meanwhile, degree of *community* of the presented material, at the same time, allow the reader essentially easier to perceive an offered material to that promotes and informal level of discussion, and quite sufficient choice of an illustrative material.

1.3. Architecture of the theory of HS-models and their appendices. Means of research in this problematics.

At present, the ***HS*** theory is a highly enough advanced independent field of the modern cybernetics that has considerable enough sphere of applications in different branches of science and engineering. The architecture of the ***HS*** theory and its applications from our standpoint has been presented in our monographs [3,5,8,9,567]. The architecture takes into account our previous attempts in this direction along with the most basic recent applied and theoretical results in the ***HS*** theory. We hope that the architecture will be described in detail and will be verified in the relevant scope, since its analysis can be useful enough in choice of subsequent directions for investigations in this field.

Naturally, the represented standpoint upon *architecture* of the modern ***HS***–problematics, including its theoretical and applied aspects along with the basic components of the apparatus of researches in the given area, appreciably carries subjective character, allowing meanwhile at the certain presumptions to receive quite definite general presentation about a state of the given problematics as a whole.

HS–models well enough reflect specific features of the systems basing exclusively on local interaction of elements and providing computing universality on the assumption of maximal parallelism of functioning. At the same time, the applied aspects of modelling have been widely investigated from the theoretical point of view. Applied aspects of the ***HS***-*problematics* are rather extensive, covering such sections of modern natural sciences, as modelling of dynamics of liquids and gases, many physical, chemical, biological and geological processes, processing of images; computing sciences, artificial intellect, robotics, modelling of climatic processes, social processes, etc. For this reason, we undertook attempt to define an architecture of ***HS***–problematics from the present point of view. The offered architecture carries appreciably subjective nature and does not pretend to exhaustive completeness. At the same time, in the architecture a series of remarks and offers, received after discussion of the given question along materials of some our previous publications [5,39,54-56,88,90], have been taken into account. A whole series of questions concerning the architecture of classical ***HS***–models and their appendices representing the certain *gnosiological* interest are considered in our books [5,8,54,617]. Here we shall only schematically present architecture of the ***HS***–problematics as a whole *(fig.* ***6).***

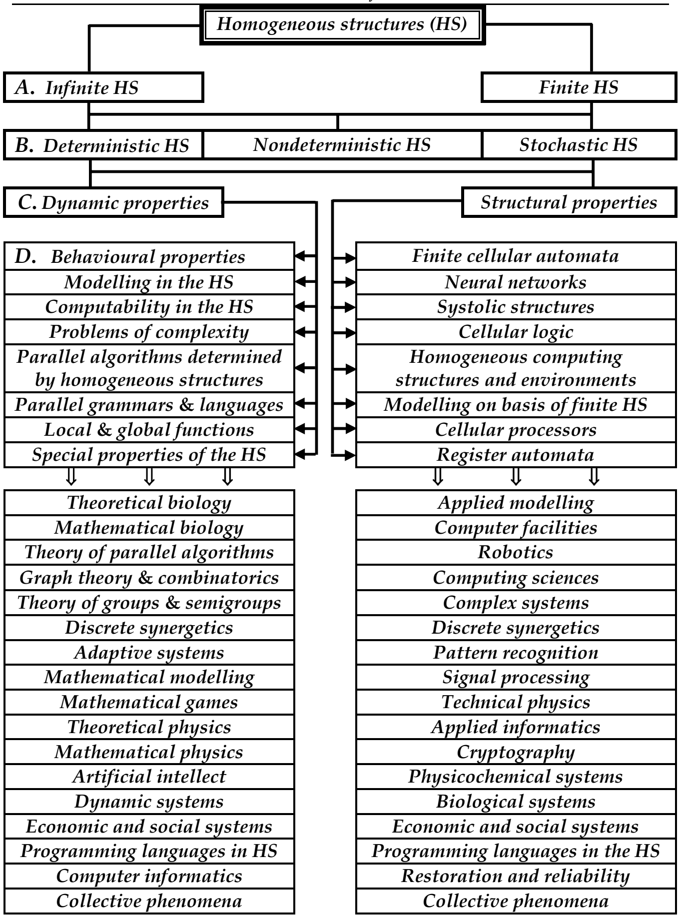


Fig. 6. Principal architecture of the theory of classical HS–models and their appendices.

At last, representing the given architecture of ***HS***–models along with their appendices, it is necessary to make one rather essential remark. At the level ***A*** *homogeneous structures* are subdivided onto ***infinite*** and

finite ones**.** At that, ***finite HS*–**models represent usual finite automata having the specific cellular organization which promotes occurrence in them of qualitatively new properties inaccessible to finite automata of the traditional organization. Now, the theory of finite ***HS*–**models enough intensively develops by a whole series of researchers [135,145, 148-150,158–160,162,164–168,172-178,536]**.** Detailed enough and actual information about finite ***HS*–**models, first of all, of applied nature can be found in works [10,108,148-150,155,174-176,349,370,536,543-545,567, 617]**,** and also in the Internet by a key phrase «*finite cellular automata*»**.**

Infinite HS*–**models are infinite cellular automata consisting of infinite set of elementary *identical* automata interconnected among themselves by a local manner, and whose global dynamics is determined by local interactions of all neighbour automata according to a neighbourhood index. The ***HS*–**models of such type appear rather convenient when it is necessary to investigate dynamics of behaviour of a various type of objects, processes, algorithms and phenomena in whose basis a series of principles ***(*high level of parallelism****;** discrete behaviour**,** unboundedness of resources**,** locality of interactions**,** reversibility**,** etc.***)** lay which are quite inherent to functioning of infinite classical ***HS*–**models. Thus, detailed enough discussion of infinite classical ***HS*–**models within of the given architectural level can be found in [3,5,54,56,567,617]**.** The basic part of the present monography is devoted to the class of infinite ***HS*–**models.

At that, if *infinite* and *finite* ***HS*–**models composing its ***A*–**level exhaust the given level of differentiation of structures, then already at ***B*–**level that includes three groups of the structures**,** namely**:** *deterministic***,** *non–deterministic and stochastic* ones, meanwhile, all variety of kinds of the ***HS*–***models* existing for today does not become completely covered. At the same time, these ***3*** groups of structures form a basis of the modern ***HS*–**problematics and concerning them the lion's share of results both theoretical, and applied character has been received. At that, making the architecture, offered above as a basis, it is possible to substantially expand the architecture, and concretely to detail its separate elements. However we, first of all, have wanted to introduce easily foreseeable architecture reflecting a certain basis of the modern ***HS*–**problematics, not paying attention to its specific aspects may even very interesting ones. Researches of the given classes of ***HS*–**models are conducted, as a rule, along two main directions **–** *dynamic* **(***behavioural***)** and *structural* properties ***(****the level* ***C;*** *fig****.*** ***6*).** In the present monograph the dynamics of classical ***HS*–**models is mainly considered, whereas their structural aspects are enough skin–deeply considered.

In most cases, as a rule, the applied aspects of ***HS***–models demand the *precise* enough specification of structure of elementary automata along with their interrelations. Thus, structural aspects of *infinite **HS**–models***,** as a rule, are being considered in close interrelation with their *dynamic* properties when it is necessary to establish or correlate the dynamics of an arbitrary ***HS***–model with structure of its elementary automata.

Dynamic properties as against structural ones suppose investigations of ***HS***–models, as a rule infinite, in the main, from the point of view of histories of configurations ***(****both finite and infinite ones****)*** in the course of time. At such approach we disengage oneself from internal structure of elementary automaton of a ***HS***–model, and the attention is accented only on its basic parameters, namely**:** dimensionality, neighbourhood index, alphabet of states of its elementary automaton, local and global transition functions, along with certain types of sets of the researched configurations determining *dynamics* of the model in context of *history* of its initial configurations. Researches of dynamics of the infinite ***HS***–models represent the big interest, first of all, from the theoretical point of view when *fundamental* or *global* properties of certain processes and phenomena in structures are studied. As a researched variable at the given approach a configuration ***(****finite or infinite****)*** is, and homogeneous space is used only for embedding into it of histories of configurations.

At last, at the ***D***–level ***(****fig****. 6)*** the basic groups of *dynamic* and *structural* aspects in the ***HS****–problematics* are represented. Meantime, intensity of researches in the field of ***HS***–problematics is non-uniform enough and in a certain extent it can be reflected in table ***1*** [617]**,** carrying to some extent subjective character. At that, despite of a certain subjectivism of the represented analysis, it by a series of estimations reflects the basic distribution of efforts of researchers in the ***HS***–problematics for today [5,32,108,110,111,149,150,155,201,349,536,545,567,617,618,640–643]**.**

Finishing section, it is necessary to make some remarks of the general nature. First, the represented architecture of structures ***HS*** and their appendices has substantially subjective character though reflects the basic tendencies of development of the ***HS****–problematics* as a whole. At that, it is based, mainly, on researches first of all in the field of *classical* ***HS***–models. Here with a view of the book compactness a sketch of the architecture is presented only, therefore the interested reader can find its detailed analysis in our books [567,617]**.** Moreover, if the presented architecture of the ***HS***–problematics carry a subjective character, then it with not the smaller basis it is possible to tell and about the offered subject classification concerning the given problematics [110].

Second, representing in the present monography basically own results we shall strictly not adhere to the architecture. That is caused by that that vast scope of the ***HS*–**problematics and area of our interests have not allowed to cover completely any section of the architecture by the received results though their some subsections are developed enough in detail. Therefore the subsequent consideration will base, mainly, on the description of *subsections* or *groups* of closely connected problems.

Today, the mathematical theory of ***HS*** presents well enough advanced *subsection* of the abstract automata theory with own problematics and with own methods of researches together with numerous appendices. Below we represent a brief *sketch* of the basic methods and approaches which make up a *core* of the apparatus of researches of various aspects of the ***HS*–**problematics**;** at that, the basic accent is done on the *classical* ***HS*–**models being the major object of our research. In table ***1*** our view on the basic areas whose methods and approaches the researchers and first of all we have used at researches of multifarious questions of the ***HS*–**problematics is represented. The offered discussion, whose rather detailed contents can be found in [5,88,567,617] pursues the following two basic purposes, namely.

*First***,** we wanted to show, that ***HS*–**models is a very much perspective field for applications in it of results and methods of a whole series of both the classical, and modern fields of pure mathematics, cybernetics and computer sciences along with other fields of the modern natural science. *Second***,** today as a whole the ***HS*–**problematics is still no to the end the formed field with deficiency of the standard, settled methods and approaches, without own advanced apparatus of researches. The majority of the researchers, actively working in the ***HS*–**problematics, apply own favourite methods of researches of those or other problems which are being determined, in the main by their previous experience, their interests and tastes.

Here at once quite pertinently to emphasize, that a whole series of the considered methods forming a modern apparatus of researches in the ***HS*–**problematics, were used and for obtaining of the results discussed in the present monograph, however in view of nature ***(****in many respects a review nature****)*** we have decided by expedient in appropriate places of the monography to designate only references to original works where the discussed theoretical questions are considered in more details and with required stringency of formalism.

In the very beginning of the making the ***HS–**problematics* has consisted of a series of relatively independent problems, whose solutions have demanded working–out of own approaches. For solution of the given problems the *classical* apparatus of *discrete* mathematics has been used taking into account the specificity of researched object ***– homogeneous structures (HS).*** It is necessary to have in mind, the namely specificity of researched properties of ***HS***-models was here as determinative base whereas use of methods and results of *discrete mathematics* had carried appreciably the subordinated nature. In addition, usage of methods of traditional discrete mathematics and, first of all, of combinatorics so runs through majority of used receptions, methods and approaches at researches in ***HS***–problematics, that these fields to a basic level of the apparatus of the ***HS***–theory can be quite pertinently ascribed [1,3–5,8, 9,19,22,34–38,53–56,79,88,90,536,545,567,617,618,640–643].

Table 1

Level	*Apparatus of researches in the HS–problematics*
16	***Approaches on basis of the general systems theory***
15	***Approaches on basis of the complexity theory***
14	***Methods of the theory of formal grammars and languages***
13	***Methods of the theory of recursive functions and algorithms***
12	***Statistical and stochastic approaches***
11	***Structural approaches***
10	***Methods of the shift dynamic systems***
9	***Methods on basis of graph–topological approach***
8	***Methods of groups, semigroups and algebras***
7	***Computer methods of research, including genetic algorithms; evolutionary methods and programming***
6	***Methods on basis of modelling (modelling approach)***
5	***Approaches on the basis of theory of abstract automata***
4	***Methods of the classical theory of numbers***
3	***Combinatorial technique***
2	***Apparatus of Boolean algebra and a–valued logics (a ≥ 3)***
1	***Basic level of the apparatus of researches in HS–problematics***

As a result of the solution of rather wide range of problems had arisen the ***first*** level of apparatus of researches in the ***HS–****problematics*, which had been focused, mainly, on solution of especially specific questions of dynamics of configurations in classical ***HS***–models. The methods of investigation along with obtained results concerning such *fundamental* problems, as nonconstructible configurations, modelling of one ***HS*** by another, complexity of finite configurations, decomposition of global transition functions and a whole series of others have formed the first ***(basic)*** level of the apparatus of researches.

Thus, the ***first (basic)*** level of such apparatus of researches caused by needs of the ***HS–****problematics* has initially been focused on studying of special enough properties of dynamics of ***HS***–models. In addition, in process of development of mathematical theory of ***HS***–models with its numerous appendices the arsenal of means and methods of the basic apparatus of researches in the ***HS–****problematics* ***(****table* ***1)*** constantly had been extended, become today impressive enough. Now, the first level constantly expands thanks to new results. Detailed enough discussion of the presented levels of the apparatus of researches in the ***HS***-theory ***(****table* ***1)*** can be found in [54,567,618]. Ibidem the numerous references to original sources directly connected to filling of levels of *apparatus* of researches in ***HS***-problematics are represented. Acquaintance with the given discussion seems to us useful enough in many respects. Along with research of ***HS***-models by methods and means of ***HS-****problematics* ***(****its base apparatus****)*** and of fields of mathematics and cybernetics which are presented above, the research of dynamics of ***HS–****models* by means and methods from some other fields seems to us as a rather promising direction. Furthermore, it is supposed that own theory of ***HS***–models will be developed, promoting expansion of the apparatus of research of ***HS***–models as a whole.

Finishing the brief review of *basic* methods and means which are used at different researches in the ***HS–****problematics* and whose summary has been represented in *table* ***1*** it is necessary to have in mind a number of essential enough moments. First, the basic attention has been devoted to dynamic aspects of the theory of classical ***HS***–models.

Meanwhile, *structural* aspects of ***HS–****models* have appreciably the more applied character and have demanded the attracting of other methods having especially *constructive* features though some of the enumerated approaches and methods are applicable too.

Second, the converse effect of application of the enumerated methods and their influence on stimulation of the further development of these methods has been noted only skin-deeply on a few concrete examples. Naturally, being one of types of dynamic systems and formal systems of processing of words, the ***HS***–models suppose for own research the much wider spectrum of research means from areas, which have been not mentioned. With them the interested reader can acquaint oneself in the extended bibliography [536] along with bibliography to the book.

Chapter 2. Nonconstructability problem in classical homogeneous structures *(Cellular Automata)*

2.1. Preliminary information on the problematics

Above all, hereinafter as the concepts «***nonconstructible configuration (NCF)***» and «*nonconstructability*» (*at that, this term in a certain degree can be associated with the term «non–constructivity»*) as a rule is understand the block configurations such as «***Garden–of–Eden***» and the existence in ***d–HS*** ***(d ≥ 1)*** of configurations of such type accordingly, i.e. concept of the nonconstructability defines one of fundamental characteristics of structures ***d–HS*** that consists in presence for them of configurations which cannot be generated in moment ***t > 0*** of any configuration given in the initial moment ***t = 0.*** However, the nonconstructability problem has more wide comprehension that is briefly characterized as follows.

First of all, concerning the classical structures we deal with two sets of essentially different configurations: ***finite*** configurations ***C(A,d,ϕ)*** and ***infinite*** configurations ***C(A,d,∞);*** in the aggregate these sets constitute the set ***C(A,d)*** of all configurations, i.e. ***C(A,d)=C(A,d,ϕ)∪C(A,d,∞).*** The generally accepted nonconstructability concept directly relates to the impossibility of generating from an arbitrary configuration ***c*∈C(A,d)*** by means of global transition function of some classical structure of a configuration containing a certain block configuration.

Meantime, the principal difference of finite and infinite configurations in case of classical structures allows quite naturally to differentiate the above nonconstructability concept what provides more detailed study of dynamics of *classical* structures along with receiving of a number of results which bear fundamental character.

In particular, along with nonconstructible block configurations quite appropriately makes sense investigation of the nonconstructability of finite configurations concerning as the set ***C(A,d)*** as a whole, and the set ***C(A,d,ϕ).*** The given approach allows naturally to introduce **2** new nonconstructability concept, namely ***NCF–1*** and ***NCF–2*** which are not equivalent as between themselves and to the standard concept ***NCF.*** Below, along with the generally accepted nonconstructability concept, a series of other important enough nonconstructability concepts will be defined and considered, including the above-mentioned. Moreover, below, the above-mentioned term «***HS***» along with term «***HS-models***» we shall assume as the identical concepts.

Generally speaking, the reversibility is a multiaspect enough concept. For classical structures being a subclass of parallel discrete dynamical systems the research of the reversibility of dynamics *(trajectory)* of the *finite* configurations seems both interesting and natural. It is natural to assume that a configuration $c \in C(A,d,\phi)$ has reversible dynamics if for it any configuration c^p *(direct or indirect predecessor)* will be sole, where $p \in \{-1, -2, -3, \ldots\}$ and $c^p\tau^{(n)} = c^{p+1}$, $c^p \equiv c$. However under such condition we have **2** alternatives**:** ***(1)*** c^p should belong only to the set $C(A,d,\phi)$, or ***(2)*** to the set $C(A,d,\infty)$**.** Therefore in view of that below we shall define a series of concepts of reversibility, what allows to consider the given concept relative to classical ***HS***-models more comprehensively. So, we shall define the concepts of *formal* and *real* reversibility because of two types of the *nonconstructability* for classical ***HS***-models ***(NCF** & **NCF-1)*****.**

Questions of nonconstructability are fundamental in the mathematical theory of ***HS***–models and their numerous appendices especially at use their as conceptual and applied models of spatially-distributed *discrete* dynamic systems from which real *physical* systems are most preferable prototypes. For this reason the given problematics opens questions of consideration of theoretical aspects of classical ***HS***–models.

The *nonconstructability* problem represents rather serious *gnoseological* interest in case of embedding into ***HS***–models of cosmological objects and phenomena. Here that can be associated with various aspects of problem of reachability of those or other conditions or aggregations at formation of special cosmological objects. For example, reversibility of the basic physical processes and phenomena can be as an analogue of absence of certain types of nonconstructability in classical ***HS***–models [1,3,5,30,53-56,90,150-152,157,268,273,536]**.** This problematics becomes more and more actual both in view of formation of modern existential physical theories, and in connexion with a whole series of attempts of interpretation of the different phenomena of abnormal character from the traditional point of view.

In this connection the problem of central importance is *characterization* of global behaviour *(dynamics)* of ***HS***-models as effect of *local transition function* ***(LTF).*** Since study of behaviour of configurations in ***HS*** plays a basic part at investigation of ***HS*** dynamics**,** it is extremely interesting to find conditions of the existence of ***nonconstructible*** configurations ***(NCF)*****,** i.e. constructive limitations of ***HS***–models. Along with that, the *nonconstructability* problem presents rather considerable gnoseological interest. The problem takes place both for *monogenic*, and for *polygenic*

d-HS and also to some extent for finite structures [1,3,5,8,9,30,43,53-57, 61–64,66, 68–74,80,90,128, 131,135,138,150,239,244,248,255,269,307,314]. At the same time, along with aforesaid the *nonconstructability* problem can be considered as important enough component of own apparatus of researches concerning dynamics of classical ***HS***–models. Below, we will consider a series of features of the given problematics in a class of *finite* ***HS***–models while to the finite ***HS***–models large enough attention is devoted, first of all, by the Japanese research school [135,185,230].

Whereas for polygenic ***d-HS*** this problem is known as the *completeness* problem: ***Whether can an arbitrary finite configuration be generated from the given initial primitive configuration by means of some finite sequence of global transition functions of a polygenic HS–model?*** A series of the researchers was engaged in research of the completeness problem, and they have received a lot of interesting enough results in this direction [3,5,90,184–187,240,248,256,294,304]. For solution of this problem the technics for the first time suggested by ***Yamada-Amoroso*** [256] along with application of graph theory by ***Nasu-Honda*** [248] has been used. So, ***H. Yamada*** and ***S. Amorozo*** have proved, that: ***Exists a certain binary finite configuration c* that can't be generated from the primitive configuration c_p by means of application of a certain finite sequence of global transition functions of binary polygenic structures 1–HS with neighbourhood template of size n = 2.*** Omitting a series of intermediate results, the final solution of the given problem has been received by ***A. Maruoka*** and ***M. Kimura*** which have proven the next important enough result in the general case, i.e. without restriction on size of neighbourhood template [244,245,526,640].

<u>*Theorem 9*</u>***. An arbitrary finite configuration c can be generated from a primitive configuration $c_p \in C(A,d,\phi)$ by some finite sequence of global transition functions of an appropriate polygenic structure d–HS (d≥1) given in the same finite alphabet A of inner states.***

The theorem ***9*** gives the full solution of the completeness problem for polygenic ***HS***–models. However, along with this problem the problem under condition of monotonous generation of finite configurations is considered also [1,20,248,256,536,567]. The completeness problem to a certain extent characterizes constructive opportunities of the *polygenic* ***HS***-models and its positive solution proves rather wide possibilities of the given type of homogeneous structures for a generating by them of finite configurations. Actually, on basis of result of theorem ***9*** we had shown, that: ***An arbitrary preassigned finite configuration $c \in C(A,d,\phi)$***

can be generated from an arbitrary nonzero configuration $c_o \in C(A,d,\phi)$ by means of application to it of a finite sequence of global transition functions of a certain polygenic structure d–HS (d ≥ 1).

Proof of the given assertion is rather simple. Thus, our assertion about full constructibility for case of the polygenic ***HS***–models easily follows from the afore–said and theorem ***9.*** In spite of direct connection of the *completeness* problem with other questions of dynamics of ***HS***–models, in more details it here is not considered**;** however, the separate results on it will be discussed below in a context of other questions of the ***HS***-problematics. For more detailed information, the interested reader is being referred to literature quoted above, and to bibliography [536]**.**

Entirely other picture takes place for a case of classical ***HS***–models. In the network of the given problematics are investigated also problems of *surjectivity* and *injectivity* of global mappings induced by the global transition functions of classical ***HS***–models. Detailed enough analysis of results in the given direction is presented in works [3,5,54,88,90,138, 156,184–187,270–282]**;** some from them are given below.

The first researches on the nonconstructability problem **(***in Russian but already good settled terminology***)** go back to known works of ***E.F. Moore*** and ***J. Myhill*** that have performed a whole series of rather interesting researches and formed the given direction [1,3,5,123,128,230,274,275]**.** Meanwhile, in a certain sense we can note that properly speaking, the mathematical theory of homogeneous structures has grown from the above problematics which till now keeps and urgency, and appeal. In the present chapter the most considerable results **(***our results***,** *mainly***),** and a modern situation of the nonconstructability problem in classical ***HS***–models along with discussion of the further ways of researches in the given direction are represented.

First of all with the purpose of more profound coverage of all types of nonconstructability, the four classes of ***NCF*** are entered, and relations between them are established, expanding the results received today in the given direction. At that, a series of criteria of existence in classical structures of various types of the nonconstructability is established. In addition, some of these criteria are more convenient for the theoretical qualitative research, while others allow to receive more *comprehensible* estimations for the basic numeric characteristics of classical structures.

Within the given problematics special attention is given to algorithmic aspect of the nonconstructability problem along with its interrelations with other questions of dynamics of *classical* ***HS***-models. In the further presentation of the ***NCF***–problematics if the opposite is not stipulated, the basic discussion will be carried out for case of classical structures ***1–HS*** though the majority of results here are being generalized to case of the classical structures of the supreme ***d***–dimensions too ***(d ≥ 2).***

Results presented in the chapter solve the *nonconstructability* problem as a whole, whereas single–purpose questions considered in the given direction allow to investigate the problem in details. Along with that, the received results on the ***NCF***–problematics allow to form effective enough apparatus of research of dynamics of the classical ***HS***–models.

2.2. The nonconstructability types for HS–models

Representation of all types of *nonconstructability* in *classical* ***HS***-models we shall begin with the concept going back to ***E. Moore*** and ***J. Myhill,*** on basis of which a lot of basic results on dynamics of ***HS***–models had been received, appreciably stimulating theoretical investigation in the ***HS***–problematics as a whole [128,274,275].

Definition 1. A configuration $c_b \in C(A,d,B)$ of a finite d-dimensional B-hypercube (block) of automata in d–HS is nonconstructible CF (NCF) if and only if does not exist CF $c \in C(A,d)$ such that $c_b \subset c\tau^{(n)}$ $(d \geq 1)$.

From the given definition follows, a configuration c_b of a finite block of individual automata will be for a ***HS***–model as ***NCF*** if and only if it cannot be as subconfiguration of a certain configuration of the model at the time moment ***t > 0.*** We shall name similar nonconstructability as ***block nonconstructability*** (*or* ***NCF***–*type*)***.*** If a block configuration c_b is ***constructible,*** then it obviously will have ***c***-predecessors $\{c_b \subset c\tau^{(n)}\}$ as from the set ***C(A,d,ϕ),*** and from the set ***C(A,d,∞).*** The given concept of nonconstructability is the strongest ***(****to a certain degree it can be called the* ***«absolute»).*** Meantime, earlier it has provoked a series of discussions and misunderstandings, therefore the nonconstructability concept in classical ***HS***–models has been analysed in detail and differentiated by us in terms of the essence of classical ***HS***–models [1,3,5,9,30,43,53–56].

Choosing a certain set ***C* ⊂ C(A)*** as allowable configurations, we have a possibility to define so–called *relative* nonconstructability in contrast to ***absolute*** *nonconstructability* ***(****definitions* ***1 & 4)*** which allows not only to research essentially more in detail the nonconstructability *essence* in ***HS***–models, but also to receive powerful enough means of research of many dynamic properties of classical ***HS***–models.

Definition 2. A configuration B_s of a finite d–dimensional hypercube of automata in a structure d–HS ($d \geq 1$) is called the nonconstructible relative to some set S (NCF_s) only if does not exist such configuration $c^* \in S \subseteq C(A,d)$ that the following relation $B_s \subset c^*\tau^{(n)}$ takes place.

Obviously, in case of identity ***S ≡ C(A,d)*** the concepts of ***absolute*** and ***relative*** nonconstructability coincide. Otherwise, each ***NCF*** will be in the same ***HS***–model as NCF_s relative to any preassigned set ***S⊂C(A,d),*** but not vice versa. So, the concept of *relative* nonconstructability in the classical ***HS***–models has a series of interesting interpretations of both theoretical, and applied character, stimulating its next research which now is enough being activated [54–56,79,88,90,536,545,567,617,640].

Strictly speaking, a reason of differentiation of the nonconstructability concept in classical ***HS***–models is being defined by differentiation of the set of all configurations ***C(A,d)*** into two noncrossing subsets of the *finite* ***C(A,d,ϕ),*** and *infinite* ***C(A,d,∞)*** configurations which concerning parallel mappings $\tau^{(n)}$ (*global transition functions*) are non–equivalent along with the used definitions of configurations [***C(A,d,ϕ)∪C(A,d,∞)= C(A,d) & C(A,d,ϕ)∩C(A,d,∞)=Ø,*** *where* ***Ø*** *is the empty set*].

So, if the set ***C(A,d,ϕ)*** is closed relative to a mapping $\tau^{(n)}$ of a classical ***d–HS ($d \geq 1$),*** then the set ***C(A,d,∞),*** generally speaking, is non–closed. That is caused by presence of a special ***quiescent*** state that satisfies the condition $\sigma^{(n)}(x,x,\dots,x) = x$ $\{x \in A\}$ for local transition function of a ***d–HS ($d \geq 1$).*** Thus, the classical ***HS***–model of such type can serve as a certain formal analogue of physic reality and during further consideration (*if the opposite was not stipulated*) we shall consider the classical structures, mainly, considering briefly and some other interesting types.

Use of sets of *final*, *block* and *infinite* configurations allows to advance appreciably both the differentiation, and the specification properly of concept of *nonconstructability* in classical ***d–HS*** relative to the previous state of this question [1,3,5,7]. Enough simply we can make sure, that the nonconstructability concept such as ***NCF*** relates, first of all, to the ***block*** configurations, allowing us to consider two, generally speaking, nonequivalent classes of nonconstructability, namely ***(1)*** the ***block*** and ***(2)*** the ***configuration*** nonconstructability in classical structures.

Indeed, let for a ***1–HS*** exists a *block* configuration c^*_b, being for it ***NCF.*** Then in conformity with definition ***1*** a configuration $c_b = c \in C(A,d,\phi)$

also will be ***NCF;*** the opposite is generally incorrect, about it a rather simple example testifies. We consider a classical ***1-HS*** with alphabet ***A*** **=** {***0,1,2***}**,** primitive neighborhood index ***X*** **=** {***0,1***} and ***GTF*** $\sigma^{(2)}$**,** that is determined by parallel substitutions of the following simple kind**:**

$$00 \to 0 \quad 01 \to 1 \quad 02 \to 1 \quad 10 \to 1 \quad 11 \to 2 \quad 12 \to 1 \quad 20 \to 2 \quad 21 \to 1 \quad 22 \to 1$$

Obviously, at condition $c_o = \ 11$ we have $c_o\tau^{(2)} = \ 121$, i.e. the block configuration $c_b = 2$ is not ***NCF*** in such ***HS***–model. On the other hand, finite configuration $c_1 = \ 2$ will be as ***NCF*** in the ***HS***–model [567,618]**.** Thus, the *block* nonconstructability of type ***NCF*** provokes *configuration* nonconstructability, while the opposite is, on the whole, incorrect. In view of told, it is possible to define a new type of nonconstructability, that arises on the border of *block* and *configuration* nonconstructability, admitting its qualitative expansion. A nonconstructible configuration such as ***NCF–3*** is defined as follows.

<u>*Definition 3*</u>***. A configuration*** $c^* = \ c_b \in\in C(A,d,\phi)$ ***is nonconstructible of type NCF–3 if and only if the block configuration*** c_b ***of d–dimensional hypercube B of automata in a d–HS is constructible but configuration c* is nonconstructible, where – edging of a block configuration*** c_b ***by infinite number of states «0»; in other words on other automata of the classical structure d–HS (d ≥ 1), i.e. outside of the block B.***

Obviously, a configuration $c \in C(A,d,\phi)$ being ***NCF-3*** is as well the ***NCF*** however it cannot be neither ***NCF-1,*** nor ***NCF-2.*** At that, it is simple to show, exists not less $N=(a-2)^{a-1}[(a-1)^{a^n-a} - (a-2)^{a^n-a}]$ of the classical structures ***1–HS*** which possess the nonconstructability such as ***NCF–3.*** At the heart of that very simple combinatorial considerations lay.

In view of the remarks made above, in addition to **2** considered types of nonconstructability ***(NCF*** *and* ***NCF–3)*** we determine in addition *two* important types of nonconstructability in classical ***d–HS (d ≥ 1).*** These types are conditioned, first of all, by the feature of ***HS***–models, which allows to ascribe their to the special class mentioned above and which naturally allows to introduce for the set ***C(A,d)*** of all configurations its differentiation into **2** noncrossing subsets of *finite* ***C(A,d,ϕ)*** and *infinite* ***C(A,d,∞)*** configurations accordingly.

<u>*Definition 4*</u>***. A configuration*** $c^* \in C(A,d,\phi)$ ***is nonconstructible such as NCF–1 for a classical d-HS if and only if*** $(\exists c' \in C(A,d,\infty))(c'\tau^{(n)} = c^*)$ ***and*** $(\forall c \in C(A,d,\phi))(c\tau^{(n)} \neq c^*)$***. On the other hand, configuration*** $c^* \in C(A,d,\phi)$

is nonconstructible such as NCF–2 for a classical d–HS, if and only if $(\exists c' \in C(A,d,\phi))(c'\tau^{(n)} = c^*)$ ***and*** $(\forall c^{\infty} \in C(A,d,\infty))(c^{\infty}\tau^{(n)} \neq c^*)$ $(d \geq 1)$.

Obviously, if in ***d–HS*** $(d\geq 1)$ exist ***NCF–1***, then $(\exists c \in C(A,d,\infty))(c\tau^{(n)} = \ )$. At that, in a series of cases the distinctions (*prima facie frequently hardly perceptible*) between ***four*** types of *nonconstructability* determined by us introduce rather essential *qualitative* influence on dynamics of classical ***HS***–models. With a view of more precise concentration of attention on the above ***4*** concepts of *nonconstructability* by the words of more clear terminology it is possible to establish, that:

(1) A *block nonconstructability* (*nonconstructability such as* ***NCF***) is being characterized by impossibility of generating a block configuration c_b from any initial configuration, i.e. $(\forall c^* \in C(A,d))(c_b \not\subset c^*\tau^{(n)})$; naturally, an arbitrary configuration containing such block configuration, itself will be nonconstructible such as ***NCF*** too;

(2) Nonconstructability such as ***NCF–1*** {***NCF-2***} is defined by existence of a configuration $c^* \in C(A,d,\phi)$ that has predecessors (*i.e. configurations* c' *such that* $c'\tau^{(n)} = c^*$) only from the set $C(A,d,\infty)$ $\{C(A,d,\phi)\}$;

(3) At last, nonconstructability of type ***NCF–3*** is characterized by the condition that a finite configuration $c = \ \ c_b \in\in C(A,d,\phi)$ constructed on basis of block constructible configuration c_b is not a constructible ***CF***, i.e. the configuration c has not predecessors from the set $C(A,d)$ of the finite and the infinite configurations.

In a series of our works [54,640] the problem of existence of ***NCF–1*** for elementary types of ***1-HS*** with minimal neighbourhood index $X = \{0,1\}$ has been considered. In this direction some interesting enough results have been received; of them we can mention the following, namely:

1–HS with ***LTF*** $\sigma^{(2)}(x,y) = x^* = x+y \ (mod\ 2)$ $\{x,x^*,y \in \{0,1\}\}$ not possesses ***NCF*** and possesses the finite configurations such as ***NCF–1*** containing odd number of states «*1*» only; quota of total ***NCF–1*** relative to the set $C(B,1,\phi)$ of all binary finite configurations equals ***1/2***.

Now, we will consider the problem of existence of ***NCF–1*** for models such as ***1–HS*** with the general neighbourhood index $X = \{0,1,2, \ldots, n-1\}$, a state alphabet $A = \{0,1,2, \ldots, a-1\}$, and ***LTF*** $\sigma^{(n)}$ defined as follows:

$$\sigma^{(n)}(x_o, x_1, \ldots, x_{n-1}) = x^*_o = \sum_{j=0}^{n-1} b_j x_j \ \ (mod\ a);\ \ b_j, x_j, x^*_o \in A;\ j = 0..n-1$$

As is shown, for such type of structures called «***linear***» structures the following relation takes place, namely [545,567,617,618,640–643]:

$$\bigcup_j \{c_j\tau^{(n)k} \mid k = 0,1,2,3,\ldots\} = C(A,1,\phi)\setminus\{c_j\}; \qquad c_j\tau^{(n)0} \equiv c_j,$$

where $\{c_j\}$ *– a set of all finite configurations such as NCF-1*

Hence, in structures of such type the set of all finite configurations is being generated from set of configurations of type ***NCF–1,*** including ***NCF–1*** themselves. In other words, the set of ***NCF–1*** is that basis from which all set ***C(A,1,ϕ)*** is generated. This result can be enough simply generalized to general case of linear classical structures ***d-HS (d≥1).*** At that, the above result is generalized to the classical structures ***d-HS*** for which the relation $(\forall c\in C(A,d,\phi)(|c\tau^{(n)}| > |c|)$ takes place ***(d ≥1)*** where $|h|$ is maximal diameter of an arbitrary finite configuration ***h.***

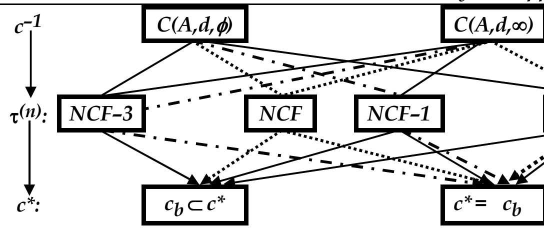
In addition, presence in ***HS–***models of *nonconstructible* configurations such as ***NCF–3*** defines to a certain extent a rather unexpected result, namely: ***At presence of a constructive core (a nonzero part) for a finite configuration, the configuration can be absolutely nonconstructible.*** Thus [54-56,640], presence for a ***HS–***model of nonconstructability such as ***NCF–3*** necessarily entails also presence of nonconstructability such as ***NCF*** for the model; whereas converse assertion, generally speaking, incorrectly. In particular, simple ***HS–***model $<Z^1, B=\{0,1\}, \tau^{(2)}, X=\{0,1\}>$ with ***LTF*** $\sigma^{(2)}$, defined by the following formula:

$$\sigma^{(2)}(x,y) = \begin{cases} 1, & \text{if } x = y = 1 \\ 0, & \text{otherwise} \end{cases}$$

possesses the nonconstructability such as ***NCF*** *(for example,* ***CF*** c_b***=101)*** whereas in the model the nonconstructability such as ***NCF–3*** is absent. At that, it is possible to show, what in a class of very simple structures $<Z^1,A,\tau^{(2)},X=\{0,1\}>$ there are ***HS-***models having ***NCF-3*** of *minimal* size $m = a-1$ [10]. Other interesting enough estimations and characteristics for structures of similar type take place too [1,3,5,54-56,88,90,536,618].

Four types of the *nonconstructible* configurations ***(NCF, NCF–1, NCF–2, NCF–3)*** introduced above are pairwise nonequivalent and allow more in detail to investigate the nonconstructability problem in the classical homogeneous structures *(models)* ***d–HS (d ≥ 1).*** So, nonconstructability such as ***NCF–1*** allows the more religiously to investigate the question of dynamics *reversibility* of finite configurations in the classical ***d–HS.***

The following diagram illustrates interconnections of the above four types ***(NCF, NCF–1, NCF–2, NCF–3)*** of the nonconstructability *(fig.* ***7).***



where $c^{-1}\tau^{(n)} = c^*$; c^{-1} – a predecessor for a finite configuration c^*

Fig. 7. Diagram illustrating the essence of basic four types of the nonconstructability (NCF, NCF–1, NCF–2, NCF–3) in a classical d–HS

So, on fig. **7** the essence of the concepts determined by us above of the *nonconstructible block* configurations and *finite* configurations c^* which are defined by absence for them of predecessors from sets $C(A,d,\phi)$ or/ and $C(A,d,\infty)$ is schematically submitted. Thus, decomposition of the diagram (*fig.* **7**) onto more detailed components (*fig.* **8**) allows to make the interrelation picture between introduced four fundamental types of nonconstructability considerably more transparent.

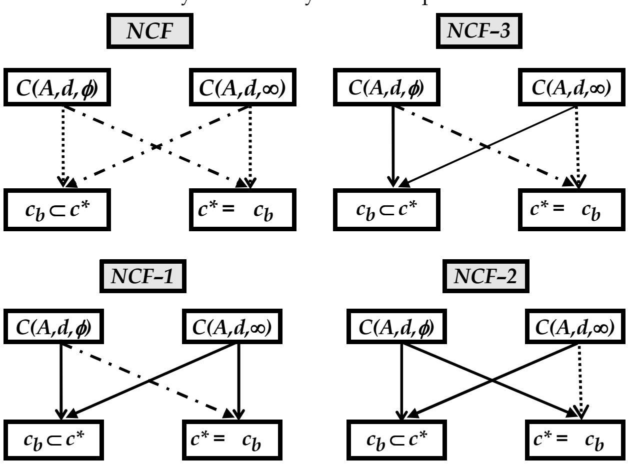


Fig. 8. Diagrams representing possibilities of existence of predecessors in sets $C(A,d,\phi)$, and $C(A,d,\infty)$ for configurations $\{c_b \mid c = c_b\}$ relative to the above four types of the nonconstructability.

Moreover, the continuous ***(dotted,dashed)*** marking of lines on fig. ***7*** and on fig. ***8*** designates accordingly the admissibility ***(impossibility)*** of the existence of predecessors c^{-1} for any configuration ***c**** in classical ***d–HS (d≥1).*** At that, if ***NCF (NCF-3) –*** *absolute* nonconstructability relative to the set $C(d,A)=C(d,A,\phi)\cup C(d,A,\infty)$, whereas ***NCF–1,*** and ***NCF–2*** will be nonconstructability relative to sets $C(d,A,\phi)$ and $C(d,A,\infty)$ accordingly. It is obvious, at constructability of a finite configuration $c^* = \langle c_b \rangle$, the *nonconstructability* of a block configuration c_b is impossible. Therefore, ***NCF, NCF–1, NCF–2*** and ***NCF–3*** cover four most interesting occasions of existence of nonconstructible configurations in classical ***HS-***models. The following basic result expressed by theorem ***10*** show the relations between the above four types of nonconstructability in classical ***d–HS.***

<u>Theorem 10</u>. An arbitrary classical d–HS (d≥1) has at least one type of nonconstructability NCF (and, possibly, NCF–3), NCF–1, NCF–2. The non-empty sets of NCF, NCF–1, NCF–2, and NCF-3 for a classical HS-model are infinite. A global transition function of any classical d–HS can possess the types of nonconstructability according to the table 2, presented below. If for a classical structure d–HS the set C(A,d,∞) is nonclosed relative to the mapping determined by its global transition function $\tau^{(n)}$, then the structure will possess the nonconstructability such as NCF–1 and/or NCF while converse proposition in general case is incorrect. At that, a classical structure d–HS not possessing NCF–1 and NCF (NCF-3) will possess NCF–2; an arbitrary classical structure d–HS (d≥1) that not possesses NCF–2 will possess NCF-1 and/or NCF. If a classical structure d–HS (d ≥ 1) possesses the nonconstructability such as NCF–1 without NCF, then the structure does not possess the nonconstructability such as NCF-2 too. Thus, there are classical d–HS for which an arbitrary finite configuration has at least 1 predecessor from the set C(A,d,∞) of all infinite configurations.

In particular, a classical structure can possess the nonconstructability such as ***NCF*** in case of closure of the set ***C(A,d,∞)*** relative to the above mapping $\tau^{(n)}$. Namely, an arbitrary classical structure ***d–HS (d ≥ 1)*** can possess any combination of the nonconstructabilities such as {***NCF–1, NCF***}. As a simple enough example three binary classical structures ***1–HS*** whose local transition functions are defined as follows:

$$\sigma_1^{(3)}(x,y,z)=\begin{cases}x, \text{ if } <xyz>\in\{100,101\}\\ z, \text{ otherwise}\end{cases};\qquad \sigma_3^{(3)}(x,y,z)=\begin{cases}1, \text{ if } <xyz>\in\{001,100\}\\ 0, \text{ otherwise}\end{cases}$$

$$\sigma_2^{(3)}(x,y,z)=x+y+z \pmod 2;\quad x,y,z\in B=\{0,1\}$$

can serves. Obviously, that easily proves the incorrectness of converse proposition; indeed, the local binary functions $\sigma_1^{(3)}$, $\sigma_2^{(3)}$ and $\sigma_3^{(3)}$ define classical structures which possess ***NCF*** without ***NCF–1***, possess ***NCF-1*** without ***NCF***, and possess both ***NCF*** and ***NCF-1*** accordingly. Whereas there are classical structures ***d-HS (d≥1)*** which do not possess both the nonconstructability such as ***NCF*** and ***NCF–1***, possessing the ***NCF–2***.

If ***NCF (NCF–3)*** is an ***absolutely*** nonconstructible configuration with respect to the set ***C(A,d,ϕ)∪C(A,d,∞)***, then configurations ***NCF–1*** and ***NCF–2*** are ***relatively*** nonconstructible configurations concerning the sets ***C(A,d,ϕ)***, and ***C(A,d,∞)*** accordingly. In the table **2** the mark **«+ (–)»** identifies the existence ***(absence)*** of appropriate type of *nonconstructible* configurations in classical ***d-HS***, defining the admissible combinations of types. The detailed discussion of the given questions from various standpoints can be found in our works [54–56,79,88,90,545,567,618].

Table 2

Admissible types of nonconstructability for the classical structures d–HS (d ≥ 1)				***Possibility of combinations***
NCF	***NCF–1***	***NCF–2***	***NCF–3***	
+	+	+	+	***Yes***
+	+	+	–	***Yes***
+	+	–	+	***Yes***
+	+	–	–	***Yes***
+	–	+	+	***Yes***
+	–	+	–	***Yes***
+	–	–	+	***Yes***
+	–	–	–	***Yes***
–	+	–	+	***No***
–	+	–	–	***Yes***
–	–	+	+	***No***
–	–	+	–	***Yes***
–	+	+	+	***No***
–	+	+	–	***No***
–	–	–	+	***No***
–	–	–	–	***No***

In particular, from the given table **2** follows, that classical ***d–HS*** have at least one of types of nonconstructible configurations, namely**:** ***NCF, NCF–1, NCF–2, NCF–3.*** In particular, ***128*** binary classical structures ***1–***

HS with Moore neighbourhood index concerning the nonconstructible configurations of the above types are differentiated as follows:

Type	*at least NCF*	*NCF–1 without NCF*	*only NCF–2*
Quantity	*113 ≈ 88.3%*	*10 ≈ 7.8%*	*5 ≈ 3.9%*

Proof of the theorem ***10*** along with a whole series of rather interesting examples of classical ***HS***-models can be found in works [3,5,9,53-56,88, 567,640]. In addition, the result represented below allows to consider the concept of ***closure (nonclosure)*** of the set $C(A,d,\infty)$ wide enough.

<u>Proposal 1</u>. For an arbitrary classical structure d–HS ($d \ge 1$) ***there are configurations*** $c^* \in C(A,d,\infty)$ ***that satisfy the relation*** $c^*\tau^{(n)} = c \in C(A,d,\phi)$ ***if and only if for the structure there are configurations*** $c_o^\infty \in C(A,d,\infty)$ ***that satisfy the relation*** $c_o^\infty\tau^{(n)} = $ ***, excepting case of trivial structure whose LTF*** $\sigma^{(n)}$ ***satisfies the following relation, namely:***

$$(\forall <x_1x_2 \dots x_n> \mid x_j \in A;\ j=1..n)(\sigma^{(n)}(x_1,x_2,\dots,x_n) = 0)$$

In addition, completely null configuration « » is referred by us to the set $C(A,d,\phi)$ ***of finite configurations defined in a state alphabet A.***

Hence, the existence for a non–trivial ***d***–dimensional mapping $\tau^{(n)}$ of such configurations $c^\infty \in C(A,d,\infty)$ that relation $c^\infty\tau^{(n)} = $ takes place is equivalent to the existence for the given *mapping* of configurations c_o^∞ such that $c_o^\infty\tau^{(n)} = c \in C(A,d,\phi)$. Thus, in more general case as ***closure (non–closure)*** of the set $C(A,d,\infty)$ of all infinite configurations relative to ***GTF*** $\tau^{(n)}$ of a classical structure ***d–HS*** ($d \ge 1$) along the whole length of the monography we will understand the ***existence (absence)*** in the set $C(A,d,\infty)$ of such configurations $c^\infty \in C(A,d,\infty)$ for which the relation in the form $c^\infty\tau^{(n)} = c \in C(A,d,\phi)$ takes place.

We a little bit more in details illustrate the *nonconstructability* concept of type ***NCF-3.*** Formally, a finite configuration $c^* = \ \ c_b \ \ \{c_b = x_1x_2 \dots x_p;\ x_1, x_p \in B \backslash \{0\};\ x_j \in A;\ j=2..p-1,\ B=\{0,1\}\}$, being the nonconstructible ***CF*** of type ***NCF–3,*** is defined by the following condition, namely:

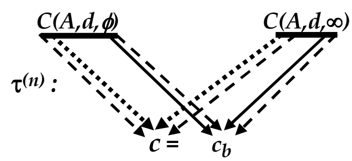
$$(\forall c \in C(A,d,\phi) \cup C(A,d,\infty))(c\tau^{(n)} \ne c^*)\ \&$$
$$(\exists c \in C(A,d,\phi) \cup C(A,d,\infty))(c_b \subset c\tau^{(n)})$$

Binary classical ***1-HS*** with neighbourhood index $X=\{0,1,2\}$, whose ***LTF*** $\sigma^{(n)}$ is defined by the formula of the following kind, is considered as a simple enough but not trivial example, namely:

$$\sigma^{(3)}(x,y,z) = \begin{cases} z, & if \;\; xyz \in \{000,001\} \\ y, & if \;\; xyz \in \{010,011\} \\ x+y+z+1 & (mod\ 2),\ otherwise \end{cases}$$

Immediate examination shows that the given binary ***1–HS*** possesses block–nonconstructible configuration ***c* = <0010100>*** [54,617]. On the other hand, the block configuration c_b***=<101> –*** a core of the previous configuration ***c* –*** is a constructible configuration, what easily follows from the existence for c_b of predecessor of the following simple kind, namely: c^{-1} ***= <01001>.*** Thus, in case of the block constructability of a configuration c_b, ***c* =*** c_b can be configuration–nonconstructible ***CF,*** i.e. to not have predecessors from the set ***C(A,d,ϕ)*** ∪ ***C(A,d,∞)*** of finite and infinite configurations. It is necessary to note, the both concepts of configuration nonconstructability and block nonconstructability are essentially various and are appreciably caused by classical type of ***HS-***models, which allows naturally to differentiate the set ***C(A,d).***

Graphically basic distinction between a *configuration*–nonconstructible configuration ***c=*** c_b and *block*–nonconstructible configuration c_b can be represented by the following diagram, namely:



where: ——— – existence of predecessors c^{-1}, and .... – absence of the predecessors c^{-1} for case of ***NCF–3;*** ------ – absence of predecessors c^{-1} for case of ***NCF.*** Of this diagram the difference of nonconstructability such as ***NCF–3*** from nonconstructability such as ***NCF*** is enough clear. In the first case the *nonconstructability*, called *configurational*, concerns the finite configurations, whereas in the second case we deal with the ***block*** nonconstructability.

Thus, it is possible to show [80] ***(<u>the generalized criterion</u>): A classical d–HS (d ≥1) possesses nonconstructability of type NCF and, probably, NCF–3 only if for the d–HS exist configurations c∈C(A,d,ϕ) which not have predecessors*** c^{-1} ***from the set C(A,d,ϕ)∪C(A,d,∞).*** The problem of existence of configurations of the given type for an arbitrary classical ***d–HS (d ≥ 1)*** is solvable for ***d = 1,*** and unsolvable for ***d ≥ 2.*** Our proof of this result is based on nonsolvability of known «*domino*» problem [54].

Hence, the nonconstructability such as ***NCF–3*** can be considered as a ***special*** subclass of the general nonconstructability such as ***NCF*** which in some cases represents quite certain interest both in the theoretical, and in the applied considerations of the classical ***HS–***models. First of all, it concerns the cases of research of ***HS–***models as formal parallel systems of processing of finite words in finite alphabets, and also at simulation on their base at a formal level of some processes, including processes of computing character. Results concerning the given type of nonconstructability present the quite certain interest as compound components of own apparatus of researches of dynamics of classical ***HS–***models and a series of their abstract appendices [5,54–56,80,90].

Certainly, the nonconstructability such as ***NCF–3*** can be considered as a special case of the ***NCF–***nonconstructability, which defines existence of nonconstructible configurations of a special kind interesting from many points of view. At that, concept ***NCF–3*** lays at the turn of ***block*** nonconstructability and *configurational* nonconstructability, belonging to both of them. Thus, if nonconstructability of two types ***NCF–1*** and ***NCF–2*** is caused by the differentiation of the set ***C(A,d)*** onto two non-overlapping subsets ***C(A,d,ϕ),*** and ***C(A,d,∞)*** of the finite and infinite configurations, then distinction in the set of ***NCF*** of a separate subset of ***NCF–3*** is caused by differentiation of the *absolute nonconstructability* according to the configurations kind, most natural to ***HS–***axiomatics of classical structures, which determines their global dynamics. Thus, configuration $c^* = \ c_b \ \{c_b = x_1x_2x_3 \ldots x_p;\ x_1, x_p \in A \backslash \{0\};\ x_j \in A;\ j=2..p-1\}$, being ***NCF–3,*** is defined by the following relation, namely:

$$(\forall c \in C(A,d))(c\tau^{(n)} \neq c^*) \ \&\ (\exists c \in C(A,d))(c_b \subset c' = c\tau^{(n)})$$

Essence of an arbitrary configuration ***c**** such as ***NCF–3*** the following rather simple diagram well enough illustrates, namely:

$$\tau^{(n)}: \quad \begin{array}{ccc} c \in C(A,d) & & c \in C(A,d) \\ \not\downarrow & & \downarrow \\ c^* = \ c_b & & c' \supset c_b \end{array}$$

Thus, via definitions of four types of nonconstructability ***(NCF, NCF-1, NCF–2, NCF–3)*** we cover the given fundamental concept as a whole. The *block* nonconstructability ***(NCF–type)*** determines the strongest and leading component of nonconstructability concept as a whole: ***A block configuration c_b is nonconstructible only if not exists a configuration $c \in C(A,d)$ such, that $c_b \subset c\tau^{(n)}$.*** Whereas other base nonconstructability types ***(NCF–1, NCF–2, NCF–3)*** have relative character and are defined only by distinctions of definitions of the sets of *block*, *finite*, and *infinite* configurations relative to global mappings $\tau^{(n)}$ of classical ***HS***-models. The nonconstructability concepts of ***NCF, NCF–3,*** and ***NCF–1, NCF–2*** are considered relative to the sets ***C(A,d)*** & ***C(A,d,ϕ)*** of finite & infinite configurations accordingly, being based on quite natural *differentiation* of the set ***C(A,d)*** of all possible configurations of a classical ***HS–***model.

Meantime, other interesting definitions of ***relative*** nonconstructability are possible too. In particular, ***U. Golze*** [232] has defined concepts of sets of *recursive* (C^r) and *rational* (C^q) configurations, which satisfy the relation $C(A,d,\phi) \subset C^q \subset C^r \subset C(A,d)$, and has investigated the kinds of the nonconstructability relative to the sets C^q and C^r for *classical* ***d–HS (d=1,2). U. Golze*** has shown, if in ***1–HS*** a configuration $c \in C(A,1,\phi)$ has a predecessor c^* then it necessarily has also a predecessor from the set C^q; there are configurations $c \in C(A,2,\phi)$, which have predecessors only from the set $C(A,2) \backslash C^r$. From the given results the non–equivalence of classical structures ***1–HS*** & ***d–HS*** ($d \geq 2$) relative to nonconstructability concept which had been entered and investigated by ***U. Golze*** follows.

Let's consider now a little bit more in details a question of existence of possible combinations of types of *nonconstructability* for classical ***1-HS***. First of all, on the basis of the theorem ***6*** [5,9] along with the aforesaid the bottom ***(****combinations* ***9..16*** *of nonconstructability types****)*** of the table **2** identifies opportunity of the specified combinations in case of absence of nonconstructability such as ***NCF*** for a structure ***1–HS***. The proof of the given part of the theorem is relatively simple, not demanding any special elucidations. Whereas for investigation of an admissibility of combinations ***1..8 (****table* **2*)*** certain concrete examples of simple enough classical structures ***1–HS*** are considered, then for them the presence of appropriate types of nonconstructability is established [10,88,90]. The given proof is enough easily generalized to the general ***d***–dimensional ($d \geq 2$) case of classical homogeneous structures. The detailed proof of the above assertions can be found in our works [54–56,79,88,90,536].

Of the theorem ***10***, in particular, follows, that an arbitrary ***d–HS*** ($d \geq 1$) does not possess property of ***absolute constructability***. Below, under designations ***NCF, NCF-1, NCF-2*** and ***NCF-3*** we shall understand both the concrete non-constructible configurations of appropriate type, and the sets of all such configurations relative to the given ***GTF*** $\tau^{(n)}$ and ***A***-alphabet of a classical ***d–HS*** ($d \geq 1$). In view of told, it is quite possible to represent in a sense the upper boundary of existence of types of the nonconstructability in classical ***HS***–models (*theorem* ***7*** [1,5]; [567,640]).

<u>*Theorem 11*</u>***. For an arbitrary classical structure d–HS (d ≥ 1) the next relations take place, namely:*** $NCF\text{-}3 \subset NCF \subset C(A,d,\phi)$, $NCF\text{-}1 \subset C(A,d,\phi)$ ***and*** $NCF \cup NCF\text{–}1 \cup NCF\text{–}2 \cup NCF\text{–}3 \subset C(A,d,\phi)$***. Let*** $G = C(A,d,\phi) \backslash \{\ \}$ ***then exist structures d–HS (d ≥ 1) for which relation*** $G \cup \{\ \} = NCF\text{–}2$ ***or G = NCF (primitive case) takes place excepting the case G = NCF–1.***

So, the result of the theorem ***11*** gives one more argument in favour of essential distinction between types of nonconstructability ***NCF,NCF-1, NCF-2*** and ***NCF-3.*** Relative to *nonconstructability* problem the question about quotas of classical ***HS***–models, possessing by non–constructible configurations of the different types represents undoubted interest. In work [274] ***E. Moore*** has put forward the hypothesis that the quota of ***d–HS (d ≥ 1)*** possessing ***NCF*** approach to ***1*** with growth of cardinality of ***A***-alphabet of the structures. Therefore in our books [1,5] have been presented asymptotic estimations of quotas of ***d***-*dimensional* structures possessing ***NCF*** and ***NCF–1*** in the absence of ***NCF*** for them. On basis of simple enough combinatorial approaches utilizing basic criteria of existence of the nonconstructability such as ***NCF*** and ***NCF–1*** together with some other considerations a series of relations has been received in this direction [10]. At that, it is necessary to have in mind, that here an obtaining of *optimal* lower bounds was not pursued but they can be useful enough at some quantitative analysises of classical structures.

<u>*Theorem 12*</u>***. Let N(a,n) is number of all classical structures d-HS with alphabet A={0,1,...,a-1} and neighbourhood template of size n whereas N1(a,n), N2(a,n), N3(a,n) and N4(a,n) is number of classical structures which not possess NCF, possess NCF–1 without NCF, possess NCF–2, not possess NCF–3 accordingly. Then the next relations take place:***

$$N(a,n) = a^{a^{n}-1}; \quad N1(a,n) > (a!)^{a^{n-1}} / a; \quad N1(a,n)/N(a,n) > \left(a!/a^{a}\right)^{a^{n-1}}$$

$$N2(a,n) \geq \left[a^{a-1} - (a-1)^{a-1}\right](a!)^{a^{n-1}} / a^{a}; \; N3(a,n) > (a-1)^{a^{n}}; \; N4(a,n) > (a-2)^{a^{n}}$$

Quota of classical structures d-HS possessing the nonconstructability such as NCF and/or NCF–1 is more than (e–1)/e, irrespective of size of the neighbourhood template of the structures (d≥1); at that, the quota approaches to 1 with growth of cardinality of the state alphabet. On the other hand, quota of classical structures d–HS (d≥1) which do not possess the nonconstructability such as NCF and NCF–1 less than 1/e, irrespective of size of the neighbourhood template of the structures; in that, for these structures all configurations are periodical. Number of 1–HS(a,n) possessing the NCF > $a^{a^{n}-n}$***, and of non–possessing >*** $(a!)^{a^{n-1}}$***.***

In particular, quantity of classical structures ***1–HS*** with state alphabet ***A***={***0,1, ..., a–1***} and the simplest neighbourhood index ***X***={***0,1***} that not possess the nonconstructability such as ***NCF–1*** is defined as:

$$N(a) = (a-1)^{a-1}(a^2-1)^{(a^2-a)/2}; \qquad \lim_{a\to\infty} N(a)/a^{a^2-1} = e^{-3/2}$$

while their quota concerning all structures of such type equals $e^{-3/2}$.

The proof of first two asymptotic relations corresponds to theorems ***8*** and ***9*** [5]. In the monography [1] and in a whole series of other works the systematic investigation of it and the questions connected to it had been initiated. So, ***E. Ikaunieks,*** using simple stochastic procedure has shown that «*almost all*» classical ***d–HS*** $(d \geq 1)$ possess ***NCF*** [276]. Using the concept of **γ**-configurations [2] introduced and considered by us in the following section, we managed to receive an *asymptotic* estimation of number of the classical structures ***d–HS*** not possessing ***NCF.*** So, the second part of the theorem ***12*** shows the basic result, valid for general ***d***–dimensional case $(d \geq 1)$. This result not only completely has closed the problem of ***E.F. Moore*** but also has shown the degree of generality of nonconstructability concept such as ***NCF.*** Absolutely other picture takes place for case of other types of nonconstructability. For example, with growth of cardinality of state alphabet ***A*** the quota of ***1–HS,*** that possess ***NCF–1*** enough quickly decreases. So, by degree of generality the concept of ***NCF*** seems to us the most representative relative to the types of nonconstructability ***NCF–1, NCF–2*** and ***NCF–3.***

At that, at finding of estimations for ***N3(a,n)*** and ***N4(a,n)*** the problem of their optimization has been not studied (*which in itself represents the certain interest for a series of the further researches*), and we only pursued purpose to show, that quantities of structures ***1–HS*** with the specified *nonconstructability* types are great enough. Meantime, despite absence of exact estimations of number of structures possessing those or other *nonconstructability* types, the received estimations allow to draw quite certain conclusions about degree of generality of the specified types of nonconstructability in classical ***1***-dimensional ***HS***-models and in ***d-HS*** $(d \geq 1)$ as a whole. The proof of the relation represented below is based on the following rather useful lemma [54–56,545,567,617,618,640].

<u>Lemma 1</u>. Number of the structures d–HS (d ≥ 1) without NCF (NCF-3) with alphabet A={***0,1, ..., a–1***} ***and neighbourhood template containing n elementary automata is less than the following value N, namely:***

$$N = \prod_{j=0}^{a-1} C^{a^{n-1}}_{a^{n-1}(a-j)} = \frac{1}{(a^{n-1}!)^a}\prod_{j=0}^{a-1}\frac{[a^{n-1}(a-j)]!}{[a^{n-1}(a-j-1]!} = \frac{(a^n)!}{(a^{n-1}!)^a}$$

Hence, because of simple enough combinatory considerations and the result of lemma ***1***, quota ***Δ(a,n)*** of homogeneous structures ***d-HS (d≥1)*** that do not possess the *nonconstructability* such as ***NCF*** and ***NCF–3*** can be represented by the following rather useful asymptotic relation [90].

<u>*Theorem 13*</u>***. Quota of homogeneous structures d–HS (d ≥ 1) with state alphabet A={0,1, ..., a–1} and with neighbourhood template containing n elementary automata (here their classical type is optional) without the nonconstructability such as NCF (NCF–3) can be defined by means of the following asymptotical relation, namely:***

$$\Delta(a,n) \approx \frac{1}{\sqrt{(2\pi)^{a-1} a^{a(n-1)-n}}}$$

Thus, the received estimation directly does not depend on dimension of a structure ***d–HS***, confirming the fact, that with growth of values ***a*** and ***n*** the quota of structures which are not possessing ***NCF (NCF–3)*** with which property of dynamics irreversibility of ***d–HS*** is associated not fully opportunely, becomes arbitrarily small, i.e. ***HS***–models with the above *feature* with growth ***a*** and ***n*** become more and more «*exotic*» on the general background of all abundance of structures ***d–HS (d ≥ 1)***. The technique of obtaining of other relations of the above theorem ***12*** is simple enough and can be found in [5,88,90]. Ibidem the discussion of various types of relations similar to the mentioned ones along with a whole series of interesting enough questions for the further research has been represented. The related estimations can be found in [536].

A whole series of interesting enough results concerning existence of ***NCF*** and ***NCF–1*** in classical structures ***d–HS (d ≥ 1)*** with state alphabet ***A={0,1, ..., a–1}*** and an arbitrary neighbourhood index ***X***, whose global transition functions satisfy the condition $(\forall c \in C(A, d, \phi)(|c\tau^{(n)}| > |c|)$ have been received, where $|c|$ is maximal diameter of a certain finite configuration ***c;*** i.e. structures of such type produce the configurations sequences strictly increasing in size from a finite configuration that is different from fully null configuration c_0 = « ». In particular, the next results can be mentioned in this connection [5,54–56,79,88,90]:

- ***quota of all configurations such as NCF-1 for a linear structure 1-HS with binary alphabet B and the simplest neighbourhood index X={0,1} equals 1/2 concerning the set of all finite binary configurations of the set*** $C(B,1,\phi) = \{1, 11, 1x_1x_2 \ldots x_n1 \mid x_j \in B;\ j = 1..n;\ n = 1..\infty\}$;
- ***each finite configuration containing odd number of states «1» in the above classical structure is configuration such as NCF–1;***
- ***there are structures d–HS (d ≥ 1) of the above type for which the set***

C(A,d,ϕ)\{ } can be fully generated only from a set J of configurations such as NCF, NCF–1 or NCF∪NCF–1; i.e. the next relation takes place:

$$\bigcup_j \left\{ c_j \tau^{(n)k} \mid k = 0,1,2,\ldots \right\} = C(A,1,\phi) \setminus \{\square\}; \quad c_j \tau^{(n)0} \equiv c_j,$$

where $\{c_j\}$ is a set J of finite configurations such as NCF and/or NCF-1, i.e. $J \subseteq NCF$, $J \subseteq NCF\text{-}1$ or $J \subseteq NCF \cup NCF\text{-}1$;

• ***any configuration of size less than neighbourhood template of d–HS (d ≥ 1) of such type is a configuration such as NCF–1 or NCF; i.e. these structures possess the nonconstructability such as NCF and/or NCF-1.***

The structures of the above sort present essential enough interest from theoretical and applied standpoints; their quota is more than $1/e^2 \approx 0.14$ irrespective of main parameters of classical ***HS***–models [54,79,88,90].

A set of the classical structures ***1–HS*** with binary alphabet ***B=***{***0,1***} and neighbourhood index ***X=***{***0,1,2***} has been considered as an example of distribution of structures ***1–HS*** relative to the *nonconstructability* types presented above. Obviously, quantity of such structures is equal ***128.***

For simplicity we shall enumerate the given structures by appropriate *discriminating* numbers; i.e. from ***0*** up to ***127.*** For example, a structure determined by ***LTF*** $\sigma^{(3)}(x_0, x_1, x_2) = \sum_j x_j \pmod 2$; $x_j \in B = \{0,1\}$, $j=0..2$ will have the discriminating number ***105*** (*see item* ***1.1***)***.*** On basis of the carried out analysis [90,567,617] we have shown, that among classical binary structures ***1–HS*** of the above type there are ***113*** structures that possess the ***NCF*** (*possibly, additionally* ***NCF–3*** *or/and* ***NCF–1*** *too*)***;*** their *discriminating* numbers are ***0..14, 16..23, 24..29, 31..44, 46..50, 52..59, 61.. 74, 76..84, 87..88, 91..100, 103..104, 107..119, 121..127.***

Structures of this subset are studied enough in detail. In particular, we studied the structure with discriminating number ***29,*** that represents a certain interpretating character and whose local transition function is defined by the following parallel substitutions as follows, namely:

"000" –> "0", "001" –> "0", "010" –> "0", "011" –> "1",
"100" –> "1", "101" –> "1", "110" –> "0", "111" –> "1"

It became clear that the structure possesses the single block ***NCF 1100*** of the *minimum* size ***4.*** Evidently, an arbitrary block configuration that contains the subconfiguration ***"1100"*** is nonconstructible too. The next fragment represents a program evaluation of ***NCF*** of the *minimum* size for the structure with *discriminating* number ***29*** by means of procedure **MinNCF** considered below. While procedure **NcfAll** allows to study number of nonconstructible block configurations of length ***n*** and their density *(ρ)* relative to all block configurations of the same length ***n,*** i.e. the call returns the list {***n,ρ***} whose the first element defines number of nonconstructible block configurations of length ***n*** whereas the second element defines their density relative to all block configurations of the same length ***n.*** The following fragment represents a source code of the procedure along with examples of its use for the mentioned ***1–HS.***

```
In[432]:= MinNCF[Ltf, 10]
Out[432]= "1100"

In[433]:= NcfAll[l_List, n_Integer] := Module[{a = Tuples[{0, 1}, n], b},
                                   b = Map[ListToString[#, ""] &, a];
                                      b = Map[{#, NcfQ[l, #, 1]} &, b];
                              b = Flatten[Select[b, #[[2]] == True &]];
                                        b = Sort[DeleteDuplicates[b]];
                            {Length[b] – 1, N[(Length[b] – 1)/2^n]}]

In[434]:= Ltf := {"000" –> "0", "001" –> "0", "010" –> "0", "011" –> "1",
                 "100" –> "1", "101" –> "1", "110" –> "0", "111" –> "1"};
In[435]:= NcfAll[Ltf, 10]
Out[435]= {424, 0.414063}
```

It is known that for a structure possessing ***NCF-1,*** there have to be the infinite nonzero configurations c^{∞} such that $c^{\infty}\tau = \square$, where **τ** – a global transition function of structure and □ – the infinite zero configuration. While for the above structure this condition is wrong as the following scheme illustrates enough obviously, namely:

↑	*1*	*1*	*0*	–	–	–	–
τ^{-1}: ...	*0*	*1*	*0*	–	–	–	–
	0	*0*	*1*	*0*	–	–	–
	0	*0*	*1*	*0*	–	–	–
□ = ...	*0*	*0*	*0*	*0*	....	....	....

So, the structure with discriminating number ***29*** not possesses ***NCF–1.***

While the structure ***1-HS*** with discriminating number ***120*** represents a certain interpretating and applied character and whose *local* transition function is defined by the following parallel substitutions as follows:

"000" –> "0", "001" –> "1", "010" –> "1", "011" –> "1",
"100" –> "1", "101" –> "0", "110" –> "0", "111" –> "0"

Dynamics of the given structure is characterized by chaotic behavior, generating complicated, in many respects casual configurations from simple enough initial configurations. Local transition function of this structure is used as a generator of pseudorandom numbers. The given local transition function was offered for use as a shifrator of sequences in cryptography. Meanwhile, ***M. Sipper*** and ***M. Tomassini*** showed, in the first case the function badly passes a test for criterion of a consent of Pearson in comparison with other pseudorandom sequences which were received by means of other cellular automata.

On the assumption of the criterion of the nonconstructability on basis of the mutually–erasable configurations, it is simple to be convinced that similar structure doesn't possess a nonconstructability of the ***NCF*** type. Furthermore, the given structure possesses ***NCF–1,*** inasmuch as for it exists at least one infinite nonzero configurations $c^{\infty}=\ldots 111111\ldots$ such that $c^{\infty}\tau=\square$, where τ – a global transition function of the structure and $\square$ – the infinite zero configuration.

The local transition function $\sigma^{(3)}$ of the structure is defined as follows:

$$\sigma^{(3)}(x,y,z)=\begin{cases} x+y+z \ \ (mod\,2), & if\ xyz\in\{000,001,100,101\} \\ x+y \ \ (mod\,2), & otherwise; \quad x,y,z\in\{0,1\} \end{cases}$$

Theoretical-experimental analysis of dynamics of finite configurations under the influence of global transition function defined by the given local function allows to reveal that the ***structure with discriminating number 120 possesses the property of essential self–reproducibility in the Moore sense of finite configurations.***

The detailed analysis of both constructive opportunities and dynamic features of the classical binary ***1-HS*** with Moore neighbourhood index ***X***={***0,1,2***} can be found in our works [641–643].

Structures of this subset can support complex enough dynamics. In a sense to critical structures of the given subset we fully can ascribe the structures with discriminating numbers ***0*** and ***127.*** Thus, for the *first* structure all finite configurations excluding configuration { } are ***NCF,*** whereas for the *second* structure exclusively configurations of the kind $\{1^k \mid k=3,4,5,\ldots\}$ are constructible configurations.

Three structures with discriminating numbers ***15, 51, 85*** do not possess ***NCF*** and ***NCF–1,*** generating identical ***(accurate to shift)*** sequences of configurations of type ***NCF–2*** and forming the second subset, which from the point of view of *dynamics* of special interest do not represent. The last subset contains ***12*** structures with discriminating numbers ***30, 45, 60, 75, 86, 89, 90, 101, 102, 105, 106, 120*** which do not possess the nonconstructability such as ***NCF.*** Besides, if structures with numbers

45 and ***101*** do not possess ***NCF*** and ***NCF–1,*** then others ten structures possess ***NCF–1*** in the absence for them ***NCF.*** The detailed analysis of structures of the above set relative to nonconstructability problem can be found in [567,617]. Of ***128*** structures of the considered set:

– 113 *possess* ***NCF*** *and, perhaps,* ***NCF–1, NCF–2, NCF–3;***
– 3 *with discriminating numbers* **15, 51** & **85** *do not possess* **NCF** & **NCF–1** *in the presence for them* **NCF–2;**
– 12 *do not possess* **NCF;** *at that, only two of them with numbers* **45** *and* **101** *in addition do not possess as well* **NCF–1;** *i.e. any finite configuration for the structures is* **NCF–2,** *more precisely, is the periodical configuration***.**

So, *quota* of the above structures possessing ***NCF (NCF-3)*** and perhaps ***NCF-1*** amounts approximately ***0.88,*** whereas only two structures with numbers ***45*** and ***101*** (*excepting the structures with numbers* **15,51,85** *with primitive dynamics*) do not possess the basic types of *nonconstructability* ***NCF*** and ***NCF–1,*** possessing ***NCF–2.*** At that, each sequence generated in structures with numbers ***45*** and ***101*** from a finite configuration ***c**,*** is periodical. At that, it is easy to show [54], the both structures ***45*** and ***101*** for each binary configuration $c=1x_1x_2x_3 \dots x_n1$ generate sequences $\{c\tau^{(3)t} \mid t > 0\}$ with the same period. Moreover, for two given structures the useful enough determinative relation takes place, namely:

$$(\forall c \equiv 1^k \mid k \geq 1)(\forall t \geq 1)(c\tau_{45}^{(3)t} \equiv (c\tau_{101}^{(3)t})^R), \;\; \text{where } x^R - \text{inversion of } x$$

Thus, only ***5*** structures from ***128*** with *discriminating* numbers ***15,41,45, 85*** and ***101*** possess property of full *reversibility* of finite configurations generated by them, where under ***reversibility*** is being understood an possibility of one–valued calculation of all chain of finite predecessors c^{-1} for any finite configuration; i.e. possibility of overall determination of its *prehistory* in the given structure. While, taking into consideration their generative possibilities they do not represent particular interest from point of view of model appendices. So, the greatest interest from such standpoint only binary structures ***1–HS*** represent which possess the nonconstructability such as ***NCF–1*** and / or ***NCF.***

In a great extent the aforesaid is fully generalized and to more general cases of *classical* structures ***d–HS.*** Thus, for obtaining enough complex *dynamics* we should pay the attention to the ***HS***-models possessing the types of the nonconstructability ***NCF*** and/or ***NCF–1.*** Thus, absence for classical ***HS***–models of *nonconstructability* such as ***NCF*** (*especially in the aggregate with type* ***NCF–1***) can serve as a certain kind of «*filter*» in case of selection of structures with complex enough dynamics of the finite configurations representing the certain applied interest [5,90,536,567].

Four types of non-constructible configurations introduced above have been considered concerning the sets ***C(A,d), C(A,d,ϕ), C(A,d,∞),*** basing on concepts of *block*, *finite* and *infinite* configurations, that characterize so-called *absolute* concept of *nonconstructability* in classical ***HS***–models. A series of interesting enough interpretations in *theoretical* and *applied* aspects of ***HS***-models has such concept, as *relative* nonconstructability when *constructability* is considered relative to a certain subset ***C**** of set ***C(A,d),*** for example, the most known cases ***C* ≡C(A,d,ϕ), C* ≡C(A,d,∞)*** or ***C*** $\subset$ ***C(A,d) –*** a finite nonempty or infinite set of configurations. In our opinion, it is necessary to attend to the *relative* nonconstructability in view of a whole series of interesting enough interpretations in both theoretical and applied aspects of dynamics of classical ***HS***–models.

So, ***U. Golze*** has researched the *nonconstructability* concept relative to the sets of recursive and rational configurations [232]. Let's introduce the more general concept of the *relative* nonconstructability in classical homogeneous structures regardless of a basic set of configurations.

<u>Definition 5</u>. A configuration c∈C(A,d) is nonconstructible relative to a set of configurations B⊂C(A,d) and GTF $\tau^{(n)}$ of a classical structure d–HS (d≥1) if and only if the relation (∀c*∈B)(c*$\tau^{(n)}$ ≠ c) takes place.

Meanwhile, the further specification of the *relative* nonconstructability is possible similarly to the considered above. The concept of ***relative*** nonconstructability in the more wide sense than it was done in case of classical ***HS***–models, has a series of interesting enough interpretations that stimulate its further research [5,54]. First of all, important enough question about interrelation of *absolute* and *relative nonconstructabilities* in classical ***HS***–models arises. The next result gives the partial answer to the given question interesting from many points of view [54–56].

<u>Theorem 14</u>. If a classical structure d–HS does not possess NCF, then it does not possess NCF–3 too; such structure with respect to a set B = C(A,d)\C possesses NCF-1 and/or NCF-3, and can possess NCF-2 and/ or NCF-3 and NCF–3, if some set C will be strict subset accordingly of sets C(A,d,ϕ), C(A,d,∞) & C∩C(A,d,ϕ)≠Ø, C∩C(A,d,∞)≠Ø. If a classical structure d–HS does not possess the nonconstructability such as NCF in the presence of the nonconstructability such as NCF-1 then for such structure the nonconstructible finite configurations relative to the set C(A,d,ϕ) exist. If a classical structure d–HS (d≥1) does not possess the nonconstructability such as NCF & NCF–1 then for the structure any finite configuration is nonconstructible relative to the set C(A,d,∞). If some classical structure d–HS possesses the nonconstructability such

as NCF, then for such structure nonconstructible finite configurations as concerning the set C(A,d,ϕ) and the set C(A,d,∞) will exist.

Of the theorem ***14*** follows that types of the *absolute nonconstructability* ***NCF, NCF-1, NCF-2*** and ***NCF-3*** generally speaking are not equivalent also and at a level of relative nonconstructability. At that, much more detailed research of problem of relative nonconstructability, including classes of polygenic, non-deterministic and asynchronous ***HS–***models, seems a rather interesting. In particular, absolutely different picture of *nonconstructability* phenomenon takes place in case of *non-deterministic* ***HS–***models. In view of definition of the class of non-deterministic ***HS–***models ***(****section* ***1.2)*** the following basic result takes place [5,55,88,90]**.**

Theorem 15**. *An arbitrary nondeterministic d–HS (d≥1) possesses NCF if and only if all GTF of allowable set G possess nonempty sets B_j of NCF and (∀k,j)($k \neq j \rightarrow B_k \cap B_j = Ø$). There are nondeterministic d–HS ($d \geq 1$) which possess NCF and for which any allowable GTF of their sets G possesses, in turn, the nonconstructability such as NCF.***

In view of result of the second part of the theorem ***15*** the condition of non-empty crossings of sets B_j is very essential. So, the results similar to the formulated in theorem ***15*** take place and for nonconstructability such as ***NCF–1*** for case of non-deterministic structures ***1–HS*** which do not possess ***NCF (NCF–3).*** At that, attempts of their generalization to case of higher dimensions and the nonconstructability such as ***NCF–2*** so far meet the certain impediments [5,54-56,79,88,90,536,567,617,640]**.**

At last, the *asynchronous* structures represent a rather interesting class of ***HS–***models for which in *different* areas of homogeneous space Z^d at each discrete moment ***t*** the different ***GTF*** of a certain allowable finite set ***G*** of global transition functions are acted. Such structures present a definite interest as from standpoint of formal models of information multiprocessing, and certain analogues of cosmological models of the Universe. Concerning the given class of models the *nonconstructability* problem and a series of other problems similar to case of classical and non–deterministic ***HS–***models are naturally formulated.

In works [5,54–56,88,90] only the most general questions connected to the given problematics have been discussed. In particular, it is shown that asynchronous ***HS–***models can possess the nonconstructability of types ***NCF*** and**/**or ***NCF–1*** even if functions ***GTF,*** which compose them, do not possess them.

In a series of interesting enough appendices the ***HS–***models with the variable neighbourhood index determined by the current state of an elementary automaton within of the certain maximal neighbourhood template, and also ***HS***–models with variable local transition function dependent on the current state of an elementary automaton represent the quite defined interest. In addition, the ***HS***–models combining both specified cases, i.e. having variables both neighbourhood index, and local transition function represent doubtless interest. Amount of such ***HS***–models is $N = \left(\sum_{k=1}^{n-1} \frac{\Gamma(n) * a^{a^{k+1}}}{\Gamma(k+1) * \Gamma(n-k)}\right)^a$ with repeat of resultant models.

The structures represent *subclasses* of the class of all ***HS***–models which have been researched by theoretical and computer methods. A series of rather interesting results concerning the above types of ***HS***–models the reader can find in [5,54–56,88,90]. So, at the made assumptions we can consider structures of the given classes in a sense in quality of the *structural compositions* of certain substructures, for example, with more simple neighbourhood indexes and/or local transition functions **σ.** At that, *substructures* composing the resultant structure can differ enough essentially with the resultant structure by own properties.

Let's consider a simple example of two structures of one of the above classes, whose neighbourhood indexes are determined by state of the central automaton of the maximal neighbourhood template, whereas local transition functions are linear and are defined by the following formulas, namely:

$$\sigma_1^{(3-x_o)}(x_o, x_1, x_2) = \begin{cases} x_o + x_1 + x_2 \ (mod\ 2), & if\ x_o = 0 \\ x_o + x_2 \ (mod\ 2), & otherwise \end{cases}$$

$$\sigma_2^{(3-x_o)}(x_o, x_1, x_2) = \begin{cases} x_o + x_1 + x_2 \ (mod\ 2), & if\ x_o = 0 \\ x_o + x_1 \ (mod\ 2), & otherwise \end{cases}$$

Obviously, ***LTF*** of the first resultant binary structure ***1–HS*** is defined by two linear ***LTF*** $\sigma_1^{(3)}$ and $\sigma_1^{(2)}$ with neighbourhood indexes ***X***={***0,1,2***} and ***X***={***0,2***} accordingly, whereas ***LTF*** of the *second* resultant structure is defined by two linear ***LTF*** $\sigma_2^{(3)}$ and $\sigma_2^{(2)}$ with neighbourhood indexes ***X*** = {***0,1,2***} and ***X*** = {***0,1***} accordingly. It is simple to show, that the first structure has discriminating number ***106*** and not possesses ***NCF*** while the second structure has discriminating number ***108*** and possesses the ***NCF*** already of the kind ***<101010>;*** in addition, for both structures the substructures composing them not possess the ***NCF.*** In a sense similar results in this direction can be found in our monographs [5,9,54–56,79, 88,90,567,618] along with other interesting enough associated results.

2.3. Criteria of existence in classical HS-models of the basic nonconstructability types; the related questions

The first ***criterion*** of ***nonconstructability*** in the classical ***HS***–models is connected to names of ***E. Moore*** and ***J. Myhill*** [5,123,128,131,274,275], and is based on concept of ***mutually erasable*** *configurations* ***(MEC).*** In slightly generalized kind which is equivalent to the initial the classical concept of ***MEC*** is introduced as follows.

<u>Definition 6</u>. Two various configurations $c_1,c_2 \in C(A,d,\phi)$ $(c_1 \neq c_2)$ form up a pair of MEC concerning GTF $\tau^{(n)}$ in a structure d–HS (d≥1) if and only if the following relation $c_1\tau^{(n)} = c_2\tau^{(n)}$ takes place.

It is easy to be convinced that this definition is equivalent to definition of ***MEC*** of ***E.F. Moore,*** but it is more convenient for definite theoretical *qualitative* investigation of dynamics of classical ***HS***–models. Using the concept of ***MEC, E.F. Moore*** and ***J. Myhill*** have received [10,274,275] a ***criterion*** of existence in structures ***d–HS*** (*not necessarily* ***classical*** *ones*) of non-constructible configurations such as ***NCF*** which by us has been generalized to a case of nonconstructability such as ***NCF–3,*** namely.

<u>Theorem 16</u>. An arbitrary classical structure d–HS (d≥1) possesses the nonconstructability such as NCF (and, perhaps, NCF–3) if and only if for its global transition function $\tau^{(n)}$ the pairs of MEC exist.

The given criterion of existence of ***NCF*** in classical ***d-HS (d≥1)*** remains fair also for case of *nonstable* structures for which the condition $\sigma^{(n)}(x, ..., x) = x$ is not carried out, i.e. for these structures a quiescent state «***x***» is missing. The given criterion supposes research of certain questions, connected with properties of the ***MEC;*** some of them are considered below. However, previously we need in the initial concept of the ***MEC*** strictly according to ***Moore–Myhill.*** For the greater obviousness and without loss of generality, the definition of ***MEC*** according to ***Moore–Myhill*** we shall give for case of structures ***1–HS*** [1,5,274,275,536,545].

<u>Definition 7</u>. Let W will be a certain block of m adjacent automata of a structure 1–HS and B will be a set of all neighbour automata for W according to neighbourhood index X={0,1, ..., n–1}. Let now CF(P) will be an arbitrary configuration of a finite P–block of automata of this structure. Then two block configurations $CF(B_1)\cup CF(W_1)\cup CF(B_2)$ & $CF(B_1)\cup CF(W_2)\cup CF(B_2)$ are called a pair of MEC for global function $\tau^{(n)}$ in the structure 1–HS if and only if the next relations take place:

$$[CF(B_1) \cup CF(W_1) \cup CF(B_2)]\tau^{(n)} \equiv [CF(B_1) \cup CF(W_2) \cup CF(B_2)]\tau^{(n)}$$

$$CF(W_1) \neq CF(W_2)$$

At the made assumptions the block W we name below as the internal block (IB) of a pair of MEC or in abbreviated form simply IB MEC.

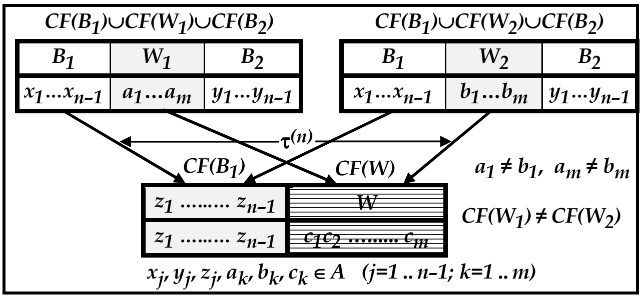


Fig. 9. Illustration of mapping of a pair of MEC by means of GTF $\tau^{(n)}$ of a structure 1–HS into the same block configuration $CF(W) \cup CF(B_1)$.

Essence of ***MEC*** for case of ***1–HS*** well illustrates fig. ***9.*** Of definition ***7*** it is simple to be convinced that the set of pairs of ***MEC*** is infinite, and their structure is of interest for theoretical and applied researches. At that, research of structure of ***MEC*** for dimensions $d \geq 2$ meets essential enough difficulties and is allowable only in the special cases, therefore the most interesting and essential results today are received in case of structures ***1–HS*** of different types [1,5,19,20,54–56,79,88,90,640–643].

In one's time relative to ***MEC*** by ***E. Moore*** a whole series of interesting enough questions has been formulated, whose solution has allowed to obtain a series of rather interesting results for ***1–***dimensional case. The following result illustrates variety of types of ***IB*** for ***MEC*** even in case of simple enough *binary* classical structures ***1-HS*** with neighbourhood index $X = \{-1,0,1\}$ [1,3,5,10,19,20,54–56,79,88,90,617,618,640–643].

<u>Theorem 17</u>. The binary classical structures 1–HS with neighbourhood index X = {-1,0,1} exist that have pairs of MEC with a simple IB of size L only of one of the following basic types, namely:

1) $L = \{p^+ \mid p \geq 1\}$; *2)* $L = \{1^+, 2^+, 3^+, p^- \mid p \geq 4\}$; *3)* $L = \{1^-, p^+ \mid p \geq 2\}$

4) $L = \{1^+, 2p^-, (2p+1)^+ \mid p \geq 1\}$; *5)* $L = \{1^-, 2^+, p^- \mid p \geq 3\}$; *6)* $L = \{1^+, p^- \mid p \geq 2\}$

where upper index {+ | –} defines {existence | absence} of pairs of MEC with simple IB (which does not contain other pairs of MEC).

For research of a whole series of questions of the nonconstructability problem and of the classical, and the unstable structures the following a rather simple lemma turns out useful enough.

<u>Lemma 2</u>. A structure d–HS (d≥1) will possess the pairs of MEC if and only if the structure possesses the pairs of MEC whose internal blocks have both the odd and the even sizes. The inside block of minimal size can contains even or odd number of elementary automata in case of a structure whose local transition function $\sigma^{(n)}$ depends on coordinates of the current elementary automata of the structure.

Without loss of the generality the validity of this lemma easily follows in case of using of the scheme for an example of structure ***1–HS*** with a state alphabet ***A*** = {***0,1, ..., a–1***} and neighbourhood index ***X*** = {***0,1,2***}.

A pair of MEC							*A pair of MEC*						
p	q	x_1	…..	x_n	c	d	p	q	x_1	…..	x_n	c	d
p	q	y_1	…..	y_n	c	d	p	q	y_1	…..	y_n	c	d
p'	q'	z_1	…..	z_n	c'	d'	p'	q'	z_1	…..	z_n		
$p,q,p',q',c,d,c',d',x_j,y_j,z_j \in A$; $j=1..n$; n – an odd integer													

Obviously, on basis of a pair of ***MEC*** with ***IB*** of odd size we can easily construct a pair of ***MEC*** with ***IB*** of even size, and vice versa. Thus, the lemma **2** concerns the classical and unstable structures, mainly. So, it is easy to show that the lemma **2** is incorrect for structures whose local transition function depends on coordinates of the current elementary automata. As example a structure ***1–HS*** with alphabet ***A*** = {***0,1, ..., a–1***} can serve with a variable neighbourhood index depending from {*odd* **|** *even*} of coordinate of the current elementary automaton. At that, local transition function of such structure is defined by the formula:

$$\sigma^{(3)}(x_j, x_{j+1}, x_{j+2}) = \begin{cases} \sigma^{(2)}(x_j, x_{j+2}), & \text{if } j \text{ is an even number} \\ x_j + x_{j+1} + x_{j+2} \pmod{a}, & \text{if } j \text{ is an odd number} \end{cases}$$

It is simple to show, that if extreme automata of the ***IB*** take positions {**<***even*, *odd***>**, **<***odd*, *even***>**, **<***odd*, *odd***>**}, the pairs of ***MEC*** with ***IB*** of such kind do not exist. On the other hand, if local transition subfunction $\sigma^{(n)}$ is defined by parallel substitutions {***00→0, 01→1,10→0, 11→0***}, then for structure there are pairs of ***MEC*** with ***IB*** of odd size ***n ≥ 3***, only.

ev	*od*	*ev*	*od*	*ev*	*od*	*ev*
1	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**1**	**1**	**0**	**0**
0	**0**	**0**	**0**	**0**		

An example of a pair of MEC; where 'ev' is an even position and 'od' is an odd position of an elementary automaton of the structure

A whole series of examples shows that with increase of state alphabet

A and neighbourhood index ***X*** of a classical ***1–HS*** the allowable types of ***IB*** of ***MEC*** expand [78,79]. Moreover, the structure of state alphabet ***A*** appreciably influences the nonconstructability concept in classical ***HS***–models. So, the *nonconstructability* problem in its general posing is connected not only to a differentiation of alphabet ***A***, but also with its cardinality. Classical ***HS***–models have an alphabet ***A = {0,1,2,3, ..., a–1}***, where «***0***» – a chosen quiescent state and «***a***» – some fixed integer. For example, if the set ***N*** of all non–negative integers is chosen as alphabet ***A*** of a structure ***1–HS*** then already relative to the structure ***1–HS≡***$<Z^1$***,*** ***A=N,*** $\tau^{(2)}$***,X***$>$ the *standard sense* of the nonconstructability concept such as ***NCF*** is being lost [1,5,19,20,640–643].

Meanwhile, if the abolition of *differentiation* of alphabet ***A*** in a context of defining of special *quiescent* state «***0***» uphold a criterion of existence of ***NCF***, abolishing all other types of *nonconstructability* ***(NCF–1,NCF–2*** & ***NCF–3)*** while its expansion onto *infinite* case results in infringement of the nonconstructability concept in such ***HS***–models. Thus, structure of alphabet ***A*** of *classical* ***HS***-models is rather essential for definition of the *nonconstructability* problem; some useful discussions of other cases confirming importance of structure of a state alphabet ***A***, can be found in our works [5,10,54–56,79,88,90,536,545,567,617,618,640–643].

As it was noted earlier, a lot of results linked with nonconstructability problem form an effective enough part of the basic means of research of dynamics of classical ***HS***–models, therefore estimations of various sort of ***IB*** of ***MEC*** along with other aspects of the problem represent a special interest [5,10]. In this connection concerning important enough question about the minimal size of simple ***IB*** of ***MEC*** which goes back to ***E.F. Moore***, one of answers the next basic theorem gives [19,20,22].

Theorem 18. For arbitrary integers a ≥ 3 & n ≥ 2 the classical structures 1–HS with state alphabet A={0,1, ..., a-1} and neighbourhood index X= {0,1, ..., n-1} exist which possess the MEC with simple internal block of minimal size L = n. The determination problem of the MEC of minimal size in an arbitrary d–HS (d ≥ 2) is algorithmically unsolvable.

In connection with study of the nonconstructability in classical ***d–HS***, we considered a whole series of problems of existence of ***MEC*** in the classical ***d–HS (d ≥ 1)*** enough in detail. The obtained results have both the qualitative and the numerical character as well [5,19,20,22]. In this connection the desire to receive estimations for minimal size of simple ***IB*** for ***MEC*** is quite natural. In case of the classical structures ***1–HS*** the following rather useful results take place [1,3,5,9,88,567]. Naturally, it is simple enough to give some examples of the classical structures that possess pairs of ***MEC*** with ***IB*** of minimal size ***1***, their quota is defined by the following theorem [545,567,617,618,640–643].

Theorem 19. Quota $\Delta(a, n)$ of classical structures 1–HS concerning all structures 1–HS with alphabet A = {0,1,2,3, … , a-1} and neighbourhood index X ={0,1,2,3,4, … , n-1}, which possess the pairs of MEC with IB of minimal size 1, satisfies the following relation $\Delta(a,n) > (2a^n - 1)/a^{2n}$.

An arbitrary classical structure ***d–HS (d≥1)*** with state alphabet ***A={0,1, 2,3,...,a-1}*** and neighbourhood index ***X={0,1,2,3,...,n-1}*** will possess the nonconstructible configurations such as ***NCF*** of minimal size ***1*** if and only if for local transition function $\sigma^{(n)}$ of the structure, the following rather obvious relation takes place, namely:

$$(\exists y \in A\backslash\{0\})(\forall <x_1,x_2, ..., x_n>)(\sigma^{(n)}(x_1,x_2, ..., x_n) \neq y); \quad x_j \in A \ (j=1..n)$$

Basing on this relation, we can easily make sure that quantity of such structures equals $\sum_{j=0}^{a-2} C_{a-1}^{j}(j+1)^{a^n-1}$ while their quota is $\sum_{j=0}^{a-2} C_{a-1}^{j}\left(\frac{j+1}{a}\right)^{a^n-1}$. At that, quota of classical structures that possess the nonconstructible configurations such as ***NCF*** of minimal size ***1*** is rather negligible both concerning all classical structures as a whole and the structures which possess the nonconstructability such as ***NCF.***

The method basing on the nonconstructability such as ***NCF*** & ***NCF–1*** is a very powerful tool in analysis of ***HS***–models. The effectiveness of such approach in a considerable degree depends on the quantitative knowledge of main correlations between ***γ–CF, MEC, NCF*** and ***NCF–1*** on the level of *local* transition functions and *global* transition functions as well. At present, exhaustive knowledge in this topic does not exist. We present the certain separate results in this direction. So, as a whole the following rather interesting estimations for the minimal size of ***IB*** of pairs of the ***MEC*** take place [3,5,9,54–56,79,88,90,545,567,617,640].

Theorem 20. If an arbitrary classical 1–HS possesses the pairs of MEC with simple IB of minimal size L, the relation $1 \leq L < a^{n-1}(a^{n-1}-1)+n-2$ ***takes place. Number G(a,n) of classical 1–HS with alphabet A={0,1,..., a–1} and neighbourhood index X={0,1,...,n–1} which possess only pairs MEC with IB of simple type (which not contains other pairs of MEC) in the form*** $\{<0^{n-1}|1|0^{n-1}> \mid <0^{n-1}|0|0^{n-1}>\}$ ***and*** $\{<0^{n-1}|10^p|0^{n-1}> \mid <0^{n-1}|0^p1|0^{n-1}>\}$ ***(where p=1 ..n; h^p – concatenation of p symbols 'h'), not less than*** $G(a,n) = (a!)^{a^{n-1}}/a^{n+1}$***; in addition, the quota of classical***

structures 1–HS of this type relative to all classical structures 1–HS of the same type is not less than $\left(a!/a^a\right)^{a^{n-1}}/a^n$.

Unfortunately, we can't receive a similar estimation for case of *classical* ***d-HS,*** and that is one of consequences of *algorithmic unsolvability* of the problem of existence in any classical ***d–HS (d≥2)*** of the *nonconstructible* configurations such as ***NCF.*** Indeed, otherwise this problem would be algorithmically solvable. Whereas, on the other hand, on the basis of the theorem ***19*** the algorithmic solvability of the problem of existence of pairs of ***MEC*** (*as well as* ***NCF)*** for the general case of ***1–***dimensional both ***classical*** and ***nonstable HS–***models is easily established [1,3]. By using an approach of ***E.F. Moore*** [274] and result of the above theorem ***19,*** it is possible to receive the upper limit for minimal size of ***NCF*** in classical ***1–HS*** [1]. However, such estimation seems a very rough and, practically, of little use in applications. On basis of appreciably other approach we have received considerably the best estimations for the minimal sizes of ***NCF (L)*** and ***NCF-1 (P)*** for case of classical ***1–HS*** [19].

Theorem 21. ***Minimal sizes L & P for NCF and NCF–1 for an arbitrary classical 1–HS are defined by relations*** $L \le (2^a-1)^{n-1}+1$ ***and*** $P \le (2^a-1)^{n-1}$. ***There are classical structures 1–HS with state alphabet A={0,1,…,a–1} along with neighbourhood index X={0,1,…,n–1} which possess NCF of the minimal size L = n + 1.***

The given result is not based at all on the concept of ***MEC*** and allows to determine constructively in an arbitrary ***1–HS*** not only existence of nonconstructability such as ***NCF*** and ***NCF–1*** but in many respects and their structure. Similar results concerning the *minimal* sizes of ***NCF*** for simple *binary* ***1-HS*** with a lot of *interpretations* of the *nonconstructability* concept have been represented by ***A. Adamatzky*** & ***A. Wuensche*** [269, 536]. At that, in [5,8,9,567,618] the well-founded refutation of incorrect doubt presented in [269] concerning estimations of the theorem ***21*** has been given. In view of told it would be rather interesting to establish concerning even the class of binary ***1–HS*** with a neighbourhood index ***X={0,1,2,3,...,n–1}*** the distribution of sizes of minimal blocks of ***NCF,*** is more exact: ***whether can form the given sizes a subset of set of integers*** $N^+ = \{0, 1, 2, 3, 4, ..., G\}$ $(G = 3^{n-1} + 1)$?

The carried out computer *analysis* of a series of rather interesting ***1–HS*** has allowed to hold the opinion, what the given assumption is cogent enough [90]. Let's represent now certain estimations for minimal sizes of the configurations such as ***NCF*** in case of classical structures ***1–HS.***

To the above questions the problem of existence in *classical* ***d–HS (d≥1)*** of so–called ***vanishing*** configurations which play a very essential part in researches of a lot of dynamic properties of such class of structures, including nonconstructability problem represents undoubted interest.

Definition 8. ***A configuration $c \in C(A, d, \phi)\backslash\{\ \}$ is called the vanishing configuration (VCF) for a classical structure d-HS ($d \geq 1$) if and only if relation $(\exists m>0)(c\tau^{(n)m}=\ )$ takes place where ' ' and $\tau^{(n)}$ – are the fully null configuration and global function of the structure accordingly.***

Obviously, a set of ***VCF*** in a classical ***HS***–model is empty or infinite. If in a classical ***HS***–model exist the ***VCF***, for the structure the set ***C(A,d,ϕ)*** is nonclosed relative to mapping induced by global transition function of the structure, ensuring presence for the structure of *nonconstructible* configurations such as ***NCF*** and, perhaps, ***NCF–1;*** while the converse statement, generally speaking, is not true, for example in case of *linear classical* structures. Problem of existence for classical ***HS***-models of the ***vanishing*** configurations presents the certain interest from standpoint of structures dynamics. The next theorem presents an useful result.

Theorem 22. ***If in a classical d–HS there are the VCF then the structure possesses nonconstructability such as NCF. The minimal length L(a,n) of VCF in a classical 1–HS with alphabet A={0,1, ..., a–1} and index of neighbourhood X={0,1, ..., n–1} is determined by the following relation $L(a,n) < a^n + n - 1$. There are classical 1–HS with minimal size of VCF no less than n–1. Exists a classical structure 1–HS with the simplest neighbourhood index X = {0, 1}, which possesses VCF of minimal size L(a,2)=a–1. Quota of classical structures 1–HS with alphabet A = {0,1, ...,a–1} and neighbourhood index X={0,1,...,n–1} which possess the VCF of minimal size 1 along with pairs of MEC with IB of minimal size 1 relative to structures of the same kind is not less than $(2a^n - 1)/a^{2n}$. If the set C(A,d,∞) is nonclosed concerning global mapping $\tau^{(n)}$, then for d–HS ($d \geq 1$) with global transition function $\tau^{(n)}$ which not possesses the NCF (NCF–1) the nonconstructability such as NCF-1 (NCF) exists.***

Let's consider an interesting enough example of binary ***1–HS*** with the neighbourhood index ***X = {0,1,2}*** whose ***LTF*** $\sigma^{(3)}$ is defined as follows:

$$\sigma^{(3)}(x_0,x_1,x_2) = \begin{cases} 0, & if\ <x_0x_1x_2> = <000> \\ 1, & if\ <x_0x_1x_2> = <111> \\ x_0+x_1+x_2+1 & (mod\ 2),\ otherwise \end{cases} \quad ; x_0,x_1,x_2 \in A = \{0,1\}$$

The carried out analysis shows that the given structure possesses the block configuration ***c=1001*** that is simultaneously both ***NCF*** and ***VCF,*** forming the pair of ***MEC*** with ***IB*** of minimal size ***1*** together with null configuration ***c*= .*** In view of told it is possible to show that ***quota of non–constructible finite configurations such as NCF which contain c= 1001 as subconfigurations, relative to all configurations of such set as C(B,1,ϕ) = {1, 11, 1x_1x_2 ... x_n1 | x_j∈B; j=1..n; n=1..∞} not less than 1/8.***

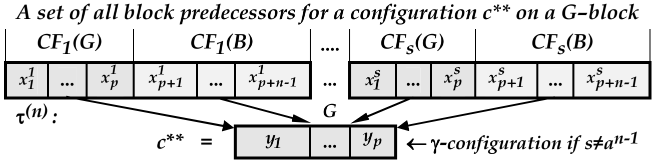
Meanwhile, detailed researches of the nonconstructability problem in the *classical* ***HS***–models have suggested to us an idea not only to better understanding of insufficient efficiency of approaches on the basis of concept of ***MEC***, but also have allowed to introduce the concept of so–called **γ–***configurations* ***(γ–CF)*** which appeared fruitful enough [37]**.** Our concept of ***γ–CF*** is slightly *distinct* from the concept of ***k-****balanced global transition functions* introduced by ***A. Maruoka*** and ***M. Kimura*** at ***1976*** irrespective of us, however both concepts are completely equivalent.

The concept have been introduced by us with the purpose of research of the nonconstructability problem in classical ***HS***–models while by ***A. Maruoka*** and ***M. Kimura*** for research of parallel global mappings $\tau^{(n)}$: ***C(A,d) → C(A,d)*** determined by global functions $\tau^{(n)}$ in classical ***d–HS (d≥1).*** We shall introduce now actually concept of *block* **γ***-configurations* ***(****or for brevity simply* ***γ–CF).***

<u>Definition 9</u>. Let G will be an arbitrary block of elementary automata in a certain structure d–HS, B will be a block of elementary automata neighboring the all automata of block G according to neighbourhood index of the structure and CF(P) will be configuration of an arbitrary block P of elementary automata (d ≥ 1). We shall say that a structure d–HS (d ≥ 1) possesses γ–configuration on the block G if and only if at least for one CF(G) exist s ≠ a^{n-1} predecessors where a, n is cardinality of state alphabet of d–HS and automata set of block B accordingly.

The following rather evident diagram well illustrates essence of above definition of **γ–**configurations for case of an arbitrary structure ***1–HS*** with state alphabet ***A={0,1,...,a-1}***, neighbourhood index ***X={0,1,...,n–1}*** and global transition function $\tau^{(n)}$.

A set of all block predecessors for a configuration c** on a G–block



On basis of the concept **γ–*CF*** it is possible to receive a new criterion of existence of ***NCF*** in ***HS***-models [3,37,45,53-56] that presents significant enough interest for a lot of researches in the ***HS***–problematics, above all, of dynamic aspects of ***HS***–models. In view of definition ***9*** the next theorem has been proved [37].

<u>Theorem 23</u>. A structure d–HS (d≥1) possesses the nonconstructability such as NCF (perhaps, and NCF–3) if and only if in the structure exist γ–configurations; in addition, this assertion concerns both nonstable, and classical homogeneous structures.

Theorem **23** gives another criterion of existence of the *nonconstructible* configurations ***(NCF)*** in the ***HS***-models of both classical and nonstable ones, and it is more convenient for a lot series of theoretical researches in ***HS***–models. The criterion not depends on the concept of ***erasability (MEC)*** in the ***HS***–models. However, it is necessary to note that in spite of entire equivalence of both criteria of existence of *nonconstructability* such as ***NCF (Moore–Myhill* & *Aladjev–Maruoka–Kimura)*** a series of rather essential distinctions exist at their concrete use [3,5,37,53-56,88].

The criterion has allowed us to obtain more acceptable estimations for some numerical characteristics of ***d-HS (d ≥ 1).*** Let's give an estimation of the minimal size of ***NCF*** in classical structures ***d–HS (d = 1, 2),*** using the criterion ***(**theorem* **23)** of the configurations nonconstructability on basis of the introduced concept of **γ**-configurations [3,5,37,54,79,88,90].

<u>Theorem 24</u>. If in a structure 2–HS with Moore's neighbourhood index and state alphabet A = {0,1,...,a-1} exist γ–CF on blocks of predecessors of size PxP, then for such structure exist NCF of size LxL, namely:

$$L = 2(P+2)\left]\frac{2\ln a}{4(P+1)\ln a - \ln s}\left(P+3+\sqrt{P^2+\frac{\ln s}{\ln a}}\right)\right[- 2$$

For case of structures 1–HS with the above neighbourhood index and state alphabet A the similar estimation assumes the following form:

$$L = \left]\frac{(n-1)\ln a}{(n-1)\ln a - \ln s}\right[(P+n-1) - n + 1$$

where n – size of neighbourhood template of the structure, the value s corresponds to the definition 9 whereas a is cardinality of alphabet A of the structure and]Z[denotes an integer ≥ Z.

In addition, the above theorem is true for both classical and nonstable structures. Using results of the theorem **24,** it is possible to make clear rather essential contrast of results concerning use of concepts of ***MEC*** and **γ–*CF.*** In particular, an interesting enough example of application of the given approach for estimation of the minimal sizes of ***NCF*** can be illustrated on known game «***Life***» [5,172,239,409]. It is simple to be convinced, that the given game is nothing else than a binary classical ***2-HS*** with Moore's neighbourhood index [3,5,37]. Investigations of the given game ***'Life'*** are carried out by a whole series of mathematicians, programmers along with amateurs, as a rule, on the basis of methods of computer modelling. Here, for today, multitude of both interesting and simply amusing results published in different editions have been obtained [536,640]. This classical binary ***2–HS*** is one of most famous.

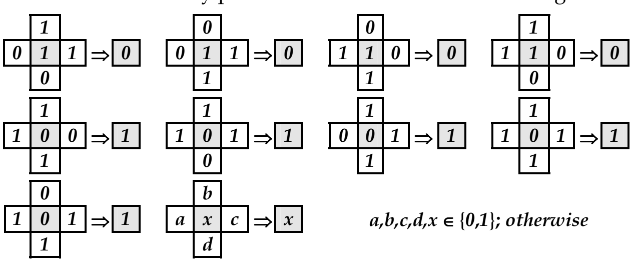
So, ***A.R. Smith,*** researching the binary ***2–HS,*** appropriate to the game «***Life***», has shown [131], that this structure possesses ***NCF*** of minimal size ***LxL*** ($L=10^{10}$). For an obtaining of the given estimation ***A.R. Smith*** has used the approach on the basis of concept of ***MEC,*** meanwhile, as on the basis of concept of ***γ-CF*** we could essentially improve the given estimation, reducing it to quite foreseeable size, namely: ***49x49.*** It does by rather real the obtaining of kind of ***NCF*** in such structure using the possibilities of modern computers [3,96,114]. A number of others ***Life–***like ***2–HS*** with Moore's neighbourhood index exists among which it is possible to note, for example, binary ***2–HS Seeds,*** initially investigated by ***B. Silverman,*** and whose local transition function is defined as:

$$\sigma^{(9)}(x_o, x_1, ..., x_8) = \begin{cases} 1, & \text{if } x_o = 0 \ \& \sum_{j=1}^{j=8} x_j = 2 \\ 0, & \text{otherwise} \end{cases}$$

where as a x_j the generalized coordinates of elementary automata and their states associated with them of the alphabet ***A***={***0,1***} are presented. Under the *generalized* coordinates of automata making neighbourhood template of a ***(i, j)***–automaton are understood coordinates {***(i,j), (i+1,j), (i+1,j+1),(i,j+1),(i-1,j+1),(i-1,j),(i-1,j-1),(i,j-1),(i+1,j-1)***}. Below, current states of elementary automata are associated with their coordinates. In particular, the central automaton x_o of a neighbourhood template can have a state ***0*** or ***1.*** For receipt of the estimation of *minimal* size of ***NCF*** it is expedient to use the concept of ***γ–CF*** that allows to obtain a rather reasonable estimation, namely: ***18x18.*** It does by rather real the receipt of a sort of ***NCF*** in such structure using the facilities of computers. At that, for the receipt of the above results the theorem ***24*** has been used, whereas theorem ***25*** allows to obtain both lower and upper bounds of the ***NCF*** of minimal size. In particular, the lower bound of the ***NCF*** of minimal size for the above two cellular automata equals ***10,*** whereas the upper bound is essentially top–heavy, that is conditioned the fact that real quantity of disproportion in maps of block configurations in such automata undef influence of the global transition functions was not taken into account. So, using the theorems ***24, 25,*** it is possible to obtain very essential contrast of results concerning the application of concepts of ***NCF*** and **γ–*CF*** as the above examples of estimation of the ***NCF*** of minimum sizes allow to illustrate.

Thus, the ***Aladyev–Kimura–Maruoka*** **γ–*CF*** concept and the criterion of nonconstructability based on it allow to investigate quite effectively a number of *quantitative* aspects of dynamics of classical ***HS***–models, whereas the ***MEC*** concept in a number of cases is more acceptable for their *qualitative* researches. So, in many respects both concepts a rather well supplement each other. In a number of cases instead of optimum or asymptotic estimates of the ***NCF*** sizes in classical ***HS***–models there are sufficiently simple formulas, as functions of key parameters of the structure. In the given context these formulas represented by theorem **25** were repeatedly used at researches of the ***NCF*** [5,37,640]. The above examples sufficiently well illustrate aforesaid.

At that, similarly we also researched universal classic binary ***2–HS*** of ***E.R. Banks*** [9,132], which today is *minimal* by complexity; the classical binary ***2-HS*** has Neumann's neighbourhood index and local transition function $\boldsymbol{\sigma}^{(5)}$ defined by parallel substitutions of the following kind:



It is possible to show that the given classical binary ***2–HS*** is suitable as an *environment* of realization of computing circuits of any complexity. Now, on the basis of the theorem ***24*** we received an estimation of size ***14x14*** of such ***NCF;*** i.e. in universal ***HS***-model of ***E.R. Banks*** exist ***NCF*** already on square blocks of size ***14x14.*** In this connexion the following rather interesting hypothesis arises, namely:

<u>Hypothesis</u>. The universal classical structures of minimal complexity {dimension*(size of neighbourhood template)*(alphabet cardinality)} possess the nonconstructability such as NCF and/or NCF–1.

For today, all minimal *universal* ***HS***-models known to us conform with the given hypothesis [5,8,9,54-56,79,88,90,131,132,158,536,545,567,640].

Thus, the concept of **γ–***configurations* along with the *nonconstructability* criterion basing on them allow to investigate enough effectively a lot of quantitative aspects of dynamics of ***HS***–models (*of both classical, and nonstable ones*) whereas the concept of ***MEC*** in whole series of cases is more acceptable for their qualitative researches. In addition, in many respects both concepts well supplement each other.

In a whole series of cases instead of *optimum* or *asymptotic* estimations of sizes of ***NCF*** in classical ***HS***–models the simple enough formulas in functional form from general parameters are entirely sufficient. In the given context the similar formulas repeatedly were used in researches of these problems. In this respect the result of theorem ***24*** assumes the form which in case of need is rather convenient for a finding of certain numerical estimations for series of problems connected to research of the nonconstructability such as ***NCF*** [5,37,54–56,79,88,90,618,640–643].

Theorem 25. If in a structure 2–HS with Moore's neighbourhood index and state alphabet A={0,1, ..., a-1} exist γ–CF on blocks of predecessors of size PxP, then for such structure exist NCF of size LxL, namely:

$$\left]\frac{2P^2+5P+4}{P+1}\right[\le L \le 8(2P^2+9P+10)\,]\,ln\,a\,[\,a^{4P+4}-2$$

For case of structures 1–HS with the above neighbourhood index and state alphabet A the similar estimation assumes the following form:

$$P \le L \le 2(P+n-1)(n-1)a^{n-1}\,]ln\,a[$$

where]M[is an integer not less than M.

The theorem gives estimations for minimal sizes of ***NCF*** in the *classical* ***HS***–models in the form of a function from their main parameters: size of predecessors of **γ–*CF*,** neighbourhood template, and cardinality of a alphabet ***A***. The results of theorems ***24–25*** can be useful enough both for the theoretical and for numerical researches in the ***HS*** problems, as well. Furthermore, the results can be enough easily extended onto ***HS***-models with arbitrary neighbourhood indexes and higher dimension. They seem useful enough at researches of certain aspects of dynamics of the classical ***HS***–models [3,5,54–56,79,88,90,545,640–643].

Both for deeper comprehension of the *nonconstructability* concept and for creation on its base of effective apparatus of research of dynamics of the ***HS***–models the determination of various interrelations between various characteristics of ***MEC***, **γ–*CF***, ***NCF***, ***NCF–1***, ***NCF–2*** and ***NCF–3*** both quantitative and qualitative ones is extremely desirable. Now the full picture in this question is absent, excepting a series of the separate results represented below [3–5,8,9,54–56,88,90,567]. In particular, from the theoretical point of view certain interest represents ascertainment of dependences between the sizes ***(S)*** of the *minimal* blocks containing ***MEC, γ–CF, NCF*** and ***NCF–1*** in classical ***HS***–models. So, for example, for ***γ–CF*** and ***NCF*** the following relation takes place: ***S(γ–CF) ≤ S(NCF)*** [3,5,37] while for case of ***MEC*** and ***NCF*** the picture is essentially more complex, namely, the next rather important result takes place [5,88,90].

Theorem 26. For each integer n ≥ 2 the classical structures 1–HS with neighbourhood index X = {0,1, ..., n–1} exist, for which minimal blocks of the MEC and NCF satisfy the relations of kind S(MEC)/S(NCF) = n or S(MEC)/S(NCF)=1/(n+1) accordingly where minimal S(γ–CF)=1. For each integer n ≥ 2 the classical 1–HS exist that do not possess the NCF and NCF–3, but have NCF–1 of minimal size L ≥ n–1. For each integer n ≥ 3 a classical binary 1–HS exists, which possesses γ–CF of minimal size L = n along with simple IB of MEC of minimal size 1.

Determination of an upper limit for the minimal sizes of ***IB*** of pairs of ***MEC,*** of sizes of ***γ–CF*** or nonconstructible configuration of a required type ***(NCF, NCF–1, NCF–2*** *and* ***NCF–3)*** is important enough question. Among received results, in particular, for case of classical structures ***1–HS*** the following rather useful theorem can be noted [1,5,640–643].

Theorem 27. For any integer n ≥ 2 there are classical binary structures 1–HS with global transition function $\tau^{(n)}$ which possess the following properties simultaneously, namely:

- ***they possess γ–CF of the minimal length L = n;***
- ***difference of minimal sizes of γ–CF and of IB of MEC equals n–1;***
- ***they do not possess the nonconstructability such as NCF–1;***
- ***for each integer k ≥ 1 their global transition functions $\tau^{(n)k}$ possess the γ–CF of minimal size n;***
- ***there are integers $t_1=t_1(n)$ & $t_2=t_2(n)$ for which block configurations in the form of $c_b = 0^{p_1 \ge t_1}10^{p_2 \ge t_2}$ are nonconstructible such as NCF in such structures, where $t_1 < t_2$ – growing functions from variable n;***
- ***configurations in the form of $c_p = 1^p$ (p≥n-1) are passive in similar structures, i.e. $c_p\tau^{(n)} = c_p$.***

Theorem ***27*** is enough easily generalized to case of higher dimensions by means of a special embedding of ***1–HS*** that satisfies the conditions of the theorem, into a classical ***d–HS (d ≥ 2).*** Of the given theorem and other certain results in the given direction follows [1,3] that, generally, it is impossible to receive satisfactory quantitative relations between minimal sizes of ***γ–CF*** and ***IB*** of ***MEC*** for classical ***HS***–models.

The given situation serves as one of principal causes of difficulties in quantitative researches of the nonconstructability problem in classical ***HS***–models of general type. Moreover, from theorem ***27*** and that fact that the minimal size of ***γ–CF*** no more than minimal size of ***NCF*** easily follows, that classical ***HS***–models with any *predetermined* minimal size of the nonconstructible configurations such as ***NCF*** exist.

Meantime, the question of revealing of relations between *minimal sizes* of ***NCF, IB*** of ***MEC*** and ***γ–CF*** seems interesting enough. At that, above we marked, that minimal size of ***γ-CF*** is not more than minimal size of ***NCF,*** i.e. ***min S(γ–CF)≤min S(NCF).*** On the other hand, we have shown [79,88,90] that the relation ***min S(IB MEC) ≤ min S(NCF)*** is valid. So, in context of the question about relations between minimal sizes of ***γ–CF, IB*** of ***MEC*** and ***NCF*** it is possible to formulate the next useful result.

<u>Theorem 28</u>. If an arbitrary classical structure d–HS (d ≥ 1) possesses the nonconstructability such as NCF then for minimal sizes of NCF, γ-CF and IB of MEC the following relation takes place:

$L^{ncf}_{min} \geq L^{ibmec}_{min}$ {≤|≥} L^{γ}_{min}; where L^{ncf}_{min}, L^{ibmec}_{min}, and L^{γ}_{min} are minimal sizes of the block configurations NCF, IB of MEC, and γ-CF accordingly.

Number N of structures d–HS (d ≥ 1) with alphabet A={0,1, ..., a–1} and neighbourhood index X={0,1, ..., n–1}, which possess the configurations NCF and γ–CF of minimal size 1, is determined by the next formula:

$$N(a,n) = \sum_{j=1}^{a-1} (-1)^{a+j+1} C_a^j \, j^{a^n}$$

The given result have a series of appendices at researches of questions of nonconstructability in classical ***d–HS (d ≥ 1),*** whereas the following result represents a certain interest for case of classical structures ***1–HS.***

<u>Theorem 29</u>. Quantity T(a, n) of all classical structures 1–HS with an alphabet A={0,1,2, ..., a–1} and neighbourhood index X={0,1,2, ..., n–1}, which simultaneously possess the γ–CF and NCF of minimal sizes one along with pairs of MEC with IB of simplest type (which not contain other pairs of MEC) in the form of {<0^{n-1} | 1 | 0^{n-1}>, <0^{n-1} | 0 | 0^{n-1}>} & {<0^{n-1} | 10^p | 0^{n-1}>, <0^{n-1} | 0^p1 | 0^{n-1}>} (p = 1..n; h^p is concatenation of p symbols «h»), satisfies the following relation, namely:

$$T(a,n) \geq \begin{cases} 1 & , \; if \;\; a = 2 \\ \sum_{j=1}^{a-2} (-1)^{a+j} C_{a-1}^{j} \, j^{a^n - n - 1} & , \; if \;\; a \geq 3 \end{cases}$$

At that, it is necessary to mark, the result represented in theorem ***29*** is naturally generalized to dimensionality ***d > 1.*** The results represented above not only more deeply uncover the nonconstructability problem in classical structures, in particular in ***1–HS*** but also to a certain extent expand opportunities of the apparatus of research of dynamics of the given type of the structures which is based on the nonconstructability concept. So, on basis of the theorem ***28*** follows that it is rather difficult to speak about preferability of that or other criterion as a whole ***(****even under the condition of equivalence of both criteria****),*** though the criterion on basis of ***γ–CF*** in a whole series of cases seems more preferable. At that, namely criterion on basis of ***γ–CF*** determines for finite ***HS–***models the existence of the nonconstructability such as ***NCF.***

For analysis of binary classical structures ***1–HS*** with index ***X* = {*–1, 0, 1*}** for the purposes of classification of their in context of possessing the minimal sizes of ***NCF*** and ***γ–CF*** the *Maple* procedure ***T1_HSb*** has been created. The source code of ***T1_HSb*** realized in ***Maple*** with use of our library [545,637] is presented below [97–118].

```
T1_HSb:= proc() local a, b, c, t, k, j, v, h, p, pr, HS, Q, R, T, Z, V, d, x, o;
  HS := [op({seq(k, k = 0 .. 127)} minus {15,30,45,51,60,75,85,86,89,90,
                                          101,102,105,106,120})];
  for k to 9 do Z[k], Q[k] := {}, {} end do;
  pr := proc(t::posint) local a, b, k, j;
       assign('b' = {cat(seq("0", k = 1 .. t))}, 'a' = {});
       for k to 2^t – 1 do c := cat("", convert(k, 'binary'));
       b := {op(b), cat(seq("0", j=1 .. t – length(c)), c)} end do end proc;
  for v in HS do x := cat("", convert(v, 'binary'));
       x := cat(seq("0", k = 1 .. 8 – length(x)), x);
     for k to 8 do o := cat("", convert(k – 1, 'binary'));
       T[cat(seq("0", j = 1 .. 3 – length(o)), o)] := x[k]   end do;
     for h to 100 do assign('b' = pr(h + 2), 'a' = {});
       for k in b do a:={op(a), cat(seq(T[k[j .. j + 2]], j=1 .. length(k) – 2))}
       end do;
 if nops(a)<2^h then Z[h]:={v,op(Z[h])}; break end if end do end do;
  for v in HS do x := cat("", convert(v, 'binary'));
     x := cat(seq("0", k = 1 .. 8 – length(x)), x);
     for k to 8 do o := cat("", convert(k – 1, 'binary'));
```

```
        T[cat(seq("0", j = 1 .. 3 – length(o)), o)] :=  x[k]  end do;
      for h to 100 do  assign('b' = pr(h + 2), 'R' = table([]));
        for k in b do  a := cat(seq(T[k[j .. j + 2]], j = 1 .. length(k) – 2));
          R[a] := [op(R[a]), 1]  end do;
        if nops(map(op, [entries(R)])) = 1 or
          nops({op(map(op, [seq(map(nops, d), d=entries(R))]))}) <> 1
        then Q[h] := {v, op(Q[h])};  break  end if end do end do;
  op(map(deltab, [Z, Q], 1, {})), eval(Z), eval(Q)
end proc:   # Maple 8, for other versions an adaptation can be needed
```

The procedure call ***T1_HSb()*** returns ***2*** tables whose indices determine *minimal* sizes of ***NCF*** and ***γ–CF*** accordingly whereas entries determine discriminating numbers of ***1–HS*** which correspond to them, namely**:**

1={0}, 2={2,4,8,12,16,24,32,34,48,64,66,68}, 3={1,3,6,10,11,14,17,18,19,20, 28,36,40,42,46,47,50,55,56,63,70,72,76,80,81,84,96,98,112,116,117,119,126, 127}, 4 = {7,9,13,21,26,27,29,31,33,35,38,39,44,49,52,53,58,59,65,69,71,74, 78,79,82,83,87,88,92,93,100,111,114,115,123,125}, 5 = {5,23,25,41,43,54,61, 62,67,77,94,95,97,103,107,108,110,113,118,121,122,124}, 6={57,73,99,109}, 8 = {22,104}, 9 = {37,91}   ⇐⇐⇐⇐   ***NCF***

1 = {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,24,25,26,28,31, 32,33,34,35,36,37,38,40,41,42,44,47,48,49,50,52,55,56,59,61,62,63,64,65,66, 67,68,69,70,72,73,74,76,79,80,81,82,84,87,88,91,93,94,95,96,97,98,100,103, 104,107,109,110,111,112,115,117,118,119,121,122,123,124,125,126,127},
2 = {23,27,29,39,46,53,54,57,58,71,77,78,83,92,99,108,114,116},
3 = {43,113}   ⇐⇐⇐⇐   ***γ–CF***

Thus, among all binary structures ***1–HS*** with neighborhood index ***X* = {*-1,0,1*}** that possess nonconstructability such as ***NCF*** with *minimal* size ***9*** only two structures with numbers ***37*** and ***91*** exist. In addition, ***1–HS*** with number ***104*** possesses ***γ–CF*** with minimal size ***1*** and pair of ***MEC*** with ***IB*** of minimal size ***1***, for example {***<11|0|11>, <11|1|11>***} along with nonconstructible configurations such as ***NCF*** of minimal size ***8*****.**

We have similar situation for structure with number ***37*** that possesses ***γ–CF*** with minimal size ***1***, and pairs of ***MEC*** with ***IB*** of minimal size ***2***, for example, ***<00|00|00>*** and ***<00|11|00>*****.** In the following table the columns ***1-HS, NCF, γ–CF*** and ***IBMEC*** define numbers of binary ***1-HS***, minimal sizes of ***NCF***, ***γ–CF*** and internal blocks of ***MEC*** accordingly.

1–HS	*NCF*	*γ–CF*	*IBMEC*
37	*9*	*1*	*2*
91	*9*	*1*	*2*

22	8	1	1
104	8	1	1
57	6	2	2
73	6	1	2
99	6	2	2
109	6	1	2
23	5	2	1
43	5	3	1
108	5	2	2
113	5	3	1
7	4	1	1
100	4	1	1
125	4	1	1

Thus, from this table follows, that for classical binary ***1–HS*** with index $X = \{-1, 0, 1\}$ the following possibilities take place, namely:

$$min_{\gamma-CF} > min_{IBMEC} \quad (23, 43, 113)$$
$$min_{\gamma-CF} < min_{IBMEC} \quad (37, 73, 91, 109)$$
$$min_{\gamma-CF} = min_{IBMEC} \quad (7, 22, 57, 99, 100, 108, 125)$$

One more interesting enough example in the given context is familiar game «***Life***» [3,5,150,168,172,409,429,536,567,617,618]. It is simple to be convinced; binary ***2–HS*** with Neumann–Moore's neighborhood index that is equivalent to the structure with local transition function ***(LTF)***

$$\begin{matrix} z_6\, z_5\, z_4 \\ z_7\, z_o\, z_3 \\ z_8\, z_1\, z_2 \end{matrix} \rightarrow z'_o = \begin{cases} 1, \left[\begin{array}{l} if \;\; z_o = 0 \;\&\; \sum_{j=1}^{j=8} z_j = 3 \\ if \;\; z_o = 1 \;\&\; \sum_{j=1}^{j=8} z_j = \{2 \mid 3\} \end{array} \right. \\ 0, \;\; otherwise \end{cases}$$

possesses the ***γ–CF*** of minimal size ***1*** along with pairs of ***MEC*** with ***IB*** of minimal size ***1***. On the other hand, on basis of the concept of ***γ–CF*** we have received an estimation for minimal size of ***NCF*** in the above structure, namely no more than ***49x49*** [96,114]. Now, using resources of the modern personal computers, we can receive the more precise estimation. For analysis of the above ***2–HS*** for the purposes of receipt of minimal size of ***NCF*** a special procedure ***MinNCFlife(n)*** in ***Maple 9*** has been created [545]. The procedure call ***MinNCFlife(n)*** returns a set of binary representations of discovered ***NCF*** of size ***(n–1)x(n–1)*** along with output of the appropriate warning; otherwise, the procedure call returns the empty set ***S*** with output of appropriate warning. A series of computer experiments allows to receive the estimation no less than ***20x20***, i.e. for the above structure the minimal size of ***NCF*** satisfies the following relation $20 \le min_{NCF} < 49$. So, the more powerful computers will allow to receive both ***NCF*** of minimal size in the above structure, and their ourselves. Thus, the above estimations can serve as a *sensible* argument in favour of impossibility to give priority to ***γ–CF*** or ***MEC*** as the *basal cause* of possession of the nonconstructability such as ***NCF*** by a structure. In the final analysis, we in full measure can consider ***γ–CF*** and ***MEC*** as the equivalent reasons of presence of configurations such as ***NCF*** in homogeneous structures ***d–HS (d ≥ 1).***

Meanwhile, these reasons and some others quite convincingly explain the fact that exactly on basis of concept of ***MEC*** is impossible to obtain satisfactory estimations of minimal sizes of ***NCF*** and a series of other rather important numerical characteristics for ***d–HS (d≥1).*** As a whole the nonconstructability phenomenon is seemingly caused by a certain type of asymmetry of functions ***LTF (GTF)*** of the structures [54–56]. In connection with the aforesaid, we with full ground can now ascertain that the criterion of ***Moore–Myhill*** (*theorem* **16**) of existence of ***NCF*** in classical and nonstable ***HS***–models in many respects essentially yields to the equivalent criterion of ***Aladjev–Maruoka–Kimura*** (*theorem* **23**), which bases on the above concept of ***γ–CF.***

For example, on the basis of criterion of ***Aladjev-Maruoka-Kimura*** we can obtain the more acceptable estimations for number of *classical* and *nonstable* structures possessing the ***NCF.*** At the heart of such approach the calculation of number of the structures possessing ***γ–CF*** on blocks of one elementary automaton lays. The following theorem gives some of such estimations which on the basis of an experimental testing have shown quite satisfactory accordance.

<u>Theorem 30</u>. Number n(a, n) and quota q(a, n) of classical & nonstable structures d–HS (d ≥ 1) with neighborhood index X = {0,1,2, ..., n–1} and a state alphabet A={0,1, ..., a–1} which possess the nonconstructability such as NCF satisfy the following relations, namely:

$$n(a,n) \geq a^{a^n} - \frac{\Gamma\left(a^n+1\right)}{\Gamma\left(a^{n-1}+1\right)^a} \qquad q(a,n) \geq 1 - \frac{\Gamma\left(a^n+1\right)}{\Gamma\left(a^{n-1}+1\right)^a a^{a^n}}$$

where Γ is the Gamma function. For case of the simplest binary 1–HS with neighborhood index X = {0, 1} the strict equalities exist. Number of 1–HS with alphabet A={0,1,2,… ,a–1}, neighbourhood index X={0,1} and symmetrical LTF without NCF equals at least BarnesG(a+2).

Table of values ***q(a, n)*** which have been obtained on basis of theorem ***30*** for ***a*=2 .. 6** and ***n*=2 .. 10** enough evidently illustrates quick enough converging of quota of the structures possessing ***NCF*** to **1** already for small enough values of integers ***n*** and ***a*.**

a/n	2	3	4	5	6	7	8	9	10
2	0.625	0.7266	0.8036	0.8600	0.9007	0.9296	0.9502	0.9648	0.9751
3	0.9146	0.9701	0.9899	0.9966	0.9989	0.9996	0.9999	1	1
4	0.9853	0.9980	0.9998	1	1	1	1	1	1
5	0.9979	0.9999	1	1	1	1	1	1	1
6	0.9997	1							

Thus, with growth of values ***n*** and ***a*** both the classical and non–stable homogeneous structures not possessing ***NCF*** become more and more exotic objects of research regardless of their dimensionality.

Of the concept of **γ–*CF*** and a lot of other reasons follows [54]**,** a certain asymmetry of ***GTF*** lays at the heart of the nonconstructability concept since the concept of **γ–*CF*** is nothing else than ***asymmetry*** in mappings of configurations on finite blocks of elementary automata by means of global transition functions $\tau^{(n)}$**.** At presence of the detailed picture of such *asymmetry* it would be possible to essentially advance in research of both the nonconstructability problem, and in rather numerous and interesting contiguous questions concerning dynamics of classical and nonstable ***HS***–models.

We pass to discussion of other *three* nonconstructability types ***(NCF–1, NCF-2*** *and* ***NCF-3)*** in classical ***HS***-models which according to theorem ***10*** can combine in wide enough bounds. Having considered criteria of existence of ***NCF,*** we shall characterize the *current state* of the question for three other types of nonconstructability in classical ***HS***–models. Of definitions ***1, 3, 4, 23*** and theorem ***10*** it is simple to be convinced, that an arbitrary finite configuration ***c∈C(A,d,ϕ)*** can't has more than ***1*** type of *nonconstructability* simultaneously. Thus, the pair–wise crossings of sets ***NCF, NCF-1, NCF-2*** for global function $\tau^{(n)}$ of a classical ***d–HS*** are empty. The following theorem represents a *criterion* of existence of the nonconstructability such as ***NCF–1*** and ***NCF–2*** in classical ***d–HS (d ≥ 1)*** that not possess the nonconstructability such as ***NCF*** and ***NCF–3.***

<u>Theorem 31</u>. Let's speak, that the set C(A,d,∞) is closed concerning the global transition function $\tau^{(n)}$ ***in a classical d–HS if the next relation (∀c*∈ C(A,d,∞))(c****$\tau^{(n)}$***∈ C(A,d,∞)) is carried out (d ≥ 1); otherwise, the set C(A,d,∞) will be non-closed. A classical d-HS (d ≥ 1) which not has NCF (NCF-3), possesses NCF–1*** {***NCF–2***} ***if and only if the set C(A,d,∞) is non–closed*** {***closed***} ***relative to mapping induced by GTF*** $\tau^{(n)}$ ***of the structure. At that, a classical structure d–HS (d ≥ 1) will possess the nonconstructability such as NCF and/or NCF-1 if the set C(A,d,∞) will be non–closed concerning mapping induced by GTF of the structure. If***

for some classical structure d–HS ($d \geq 1$) the set C(A,d,∞) is non-closed concerning mapping induced by global transition function $\tau^{(n)}$ of the structure along with absence of the nonconstructability such as NCF (NCF–1), then the structure will possess the nonconstructability such as NCF-1 (NCF). If for a classical structure d-HS ($d \geq 1$) the set C(A,d,∞) is closed relative to mapping defined by global transition function $\tau^{(n)}$ the structure does not possess the nonconstructability such as NCF–1, irrespective of the presence of the nonconstructability such as NCF.

Below, it will be shown that quota δ of classical structures ***d–HS ($d \geq 1$)*** for which the set ***C(A,d,∞)*** is non–closed concerning a mapping $\tau^{(n)}$ is more than ***(e–1)/e*** on condition that ***n>2*** or/and ***a>2,*** i.e. ***δ > 0.632.*** Thus, a *quota* of classical structures that possess the nonconstructability such as ***NCF*** and/or ***NCF-1*** more than ***(e-1)/e*** irrespective of parameters ***d, n.*** Then quota of classical structures ***d–HS ($d \geq 1$)*** that do not possess the nonconstructability such as ***NCF–1*** and/or ***NCF*** less than ***1/e,*** i.e. less than ***0.37.*** In addition, it was shown that a quota of classical structures ***d–HS ($d \geq 1$)*** which do not possess the nonconstructability such as ***NCF*** and/or ***NCF–1*** tends to ***1*** with growth of cardinality of a state alphabet of structures [88,90]. Furthermore, in the assumption that completely null configuration $c_o =$ was ascribed by us to the set ***C(A,d,ϕ)*** of finite configurations, it is possible slightly differently to formulate theorem ***31*** which represents essentially more convenient criterion of existence of ***NCF–1*** in classical structures not possessing the nonconstructability such as the ***NCF*** as well as ***NCF–3*** [3,5,8,9,11,54–56,79,88,90,536,567].

<u>*Theorem 32*</u>***. Existence for an arbitrary classical structure d–HS ($d \geq 1$) of the configurations c'∈C(A,d,∞) such that relation c'$\tau^{(n)}$=c∈C(A,d,ϕ) takes place is necessarily but not sufficiently for existence in the d-HS of the nonconstructability such as NCF–1. For a classical structure d–HS ($d \geq 1$) which does not possess the nonconstructability such as NCF there is the nonconstructability such as NCF–1 only then when for its global transition function $\tau^{(n)}$ exist such configurations c**∈C(A,d,∞) that the relations c**$\tau^{(n)}$ = c∈C(A,d,ϕ) take place.***

At that, the first part of the theorem is proved by existence of classical structures for which sets ***C(A,d,∞)*** are non-closed relative to mappings induced by their global transition functions $\tau^{(n)}$, for which each finite configuration, different from ***NCF,*** has predecessors from set ***C(A, d, ϕ)*** and the set ***C(A,d,∞),*** i.e. such structures do not possess the ***NCF-1.*** For instance, classical structure ***1–HS*** with simplest neighbourhood index

X=$\{0,1\}$, a state alphabet ***A***=$\{0,1,2\}$ along with local transition function $\sigma^{(2)}$ which is determined by the following formula:

$$\sigma^{(2)}(x,y) = \begin{cases} 0, & if\ y = 0 \\ x + y\ (mod\ 2), & if\ y = 1 \\ x\ (mod\ 2) + y - x, & if\ y = 2 \end{cases} ; \qquad x, y \in A = \{0,1,2\}$$

can be considered as a simple example proving the above assertion.

The given theorem gives an answer to a series of questions, raised in our monographs [1,3] and in other works. At that, the theorem can be used for generalization of certain results about the nonconstructability problem in classical structures ***d–HS*** $(d\geq 1)$. Our result of researches of classical structures ***d–HS*** $(d\geq 1)$ allows to formulate the following

Proposal 2. If a classical structure d–HS $(d \geq 1)$ ***possesses the NCF–1 without the nonconstructability NCF then an arbitrary configuration*** $c \in C(A, d, \phi)$ ***has at least one predecessor from the set*** $C(A, d, \infty)$***, i.e. in such structure the set*** $C(A,d,\phi)$ ***can be generated from a set*** $G \subseteq NCF\text{–}1$***, i.e. the dynamics of an arbitrary finite configuration and the structure as a whole are irreversible in our comprehension.***

At that, the further research of the given problematics has allowed us to introduce a new concept of the ***MEC*** as some base of the *generalized* criterion of the nonconstructability in classical structures ***d–HS*** $(d\geq 1)$ [5,8,9,54–56,79,88,90,545,567,617,618,640–643], namely.

Definition 10. Two configurations $c_1, c_2 \in C(A,d)$ $(c_1 \neq c_2)$ ***form a pair of MEC-1 for global transition function*** $\tau^{(n)}$ ***of a classical structure d-HS*** $(d\geq 1)$ ***if and only if for these configurations the following relation is valid, namely:*** $c_1\tau^{(n)} = c_2\tau^{(n)} = c^{\#} \in C(A,d,\phi)$.

At that, the pairs of ***MEC–1*** analogously to pairs of ***MEC*** can be made up by configurations such as ***NCF–1*** and/or ***NCF***, i.e. {***NCF–1,NCF–1***}, {***NCF,NCF***}, {***NCF,NCF–1***}. As distinct from the internal block of ***MEC*** for ***MEC–1*** a certain analogue in the form of in a sense '*absorption node*' $c^{\#}$ is defined whose size presents the certain interest in a whole series of numerical researches of the nonconstructability problem caused by presence of pairs of ***MEC–1***. The expediency of definition of the given concept is caused by the fact, that pairs of ***MEC–1*** permit also infinite configurations. In particular, it was shown that minimal sizes of $|c^{\#}|$ and configurations such as ***NCF*** and ***NCF–1*** can be equal. In addition, if absorption node $c^{\#}$ can be a nonconstructible configuration such as ***NCF–1***, then $c^{\#}$ cannot be as a nonconstructible configuration such as

NCF. At the heart of the given property the definition ***10*** and principal difference between the *nonconstructability* such as ***NCF*** and ***NCF–1*** lay**.**

Meanwhile, between the concepts of ***MEC*** and ***MEC-1*** a series of other differences takes place. In particular, presence for a classical structure ***d–HS (d ≥ 1)*** of pairs of ***MEC*** entails presence for the structure of pairs of ***MEC–1*** that are formed by infinite structures on basis of ***MEC;*** but on the other hand, presence for a classical structure of pairs of ***MEC–1*** that are *structurally* distinct from ***MEC*** not necessarily entails presence for the classical structure of pairs of ***MEC.*** This circumstance is caused by the fact that presence for an arbitrary classical structure ***d–HS (d≥1)*** of pairs of ***MEC*** is one of **2** criteria of existence for the structure of the nonconstructability such as ***NCF*** whereas existence for some classical structure of pairs of the ***MEC–1*** not necessarily causes presence for the structure of nonconstructability such as ***NCF.***

The above concept of the ***MEC–1*** of mutual erasability of a pair of two different configurations in classical structures ***d–HS (d ≥ 1)*** is linked by the intimate way with the general nonconstructability problem in ***HS–*** models what the following rather important result testifies [56,90,567]**.**

<u>Theorem 33</u>. An arbitrary classical structure d–HS (d ≥ 1) possesses at least the nonconstructability such as NCF (perhaps, NCF–3) or NCF–1 if and only if for the structure exist pairs of the MEC–1. Furthermore, if for a structure d–HS (d ≥ 1) pairs of MEC–1 are absent, the structure possesses NCF–2; at that, existence of the nonconstructability such as NCF–2 can entirely combine with presence of the MEC–1.

Classical structure ***1–HS*** with simplest neighbourhood index ***X =*** {***0, 1***}***,*** a state alphabet ***A=***{***0,1,2***} along with local transition function $\sigma^{(2)}$ that is determined by the following formula**:**

$$\sigma^{(2)}(x,y) = \begin{cases} 0, & if\ y = 0 \\ x^o + y, & if\ y = 1; \\ 1, & if\ y = 2 \end{cases} \qquad x, y \in A = \{0, 1, 2\}$$

can be considered as a simple example proving the second part of the second assertion of the above theorem. In this structure there are pairs of ***MEC–1*** in their general comprehension, more precisely ***MEC,*** along with existence for the structure of the nonconstructability such as ***NCF*** and ***NCF–2*** in the absence of the nonconstructability such as ***NCF–1.***

The result of the theorem ***33*** is essential generalization of well–known criterion of ***Moore-Myhill (****theorem* ***16)*** and also of criterion of ***Aladjev-Kimura–Maruoka (****theorem* **23*)*** that is equivalent to the first, extending them onto other nonconstructability types too. As concept of ***NCF–3*** is direct consequence of nonconstructability differentiation such as ***NCF*** onto block and configurational *nonconstructability*, then determination of criterion that a finite configuration is ***NCF–3*** seems rather useful. In this respect a criterion is represented by the following theorem [5,8,9].

<u>Theorem 34</u>. An arbitrary configuration $c = c_b$ ***in a classical 1–HS is nonconstructible configuration such as NCF–3 if and only if the block configuration*** c_b ***in the 1–HS is constructible, but block configuration*** $c^*_b = 0^p c_b 0^p$ ($p \geq a^n + n - 1$) ***in the 1–HS is nonconstructible such as NCF.***

The result of theorem ***34*** represents in a sense the constructive test for a belonging of any configuration $c \in C(A,\phi)$ to the type ***NCF–3*** by being thus a rather effective tool in a lot of theoretical researches concerning dynamics of classical ***HS***–models. Meanwhile, this nonconstructability type is not ascribed by us to the basic. The given solution is based on the circumstance caused by the following result [9,54–56,80,567,640].

<u>Theorem 35</u>. Existence in a classical d-HS (d≥1) of nonconstructability such as NCF-3 causes existence in it of the nonconstructability of type NCF also while the converse assertion is generally false. In a classical structure d–HS (d ≥ 1) exist NCF (NCF-3) if and only if in it exist finite configurations not having predecessors from set $C(A,d,\infty) \cup C(A,d,\phi)$.

Meanwhile, in a whole series of cases it will be convenient enough for us to group both *nonconstructability* types by common concept «***NCF***», separately not discriminating thus peculiar type ***NCF-3*** which in some cases appears useful enough. The following theorem ***36*** summarizes a number of useful enough interrelations between ***MEC, γ–CF, NCF*** and ***NCF–1*** [54–56,79,88,90,545,567,617,618,640–643].

<u>Theorem 36</u>. Let $\tau^{(n)} = \tau^{(m)}\tau^{(p)}$ ***is a decomposition of some function*** $\tau^{(n)}$ ***into two global transition functions*** $\tau^{(m)}$, $\tau^{(p)}$ ***of the same dimension 1 and which are determined in the same state alphabet A. If a GTF*** $\tau^{(m)}$ ***not has MEC, and*** $\tau^{(p)}$ ***has a set D of γ–CF then GTF*** $\tau^{(n)}$ ***has the same set D of γ–CF. If some GTF*** $\tau^{(m)}$ ***not possesses MEC and*** $\tau^{(p)}$ ***has pairs of MEC then the GTF*** $\tau^{(n)}$ ***has the same set of NCF as the function*** $\tau^{(p)}$. ***There are transition functions*** $\tau^{(n)}$ ***having MEC with the limited size of minimal simple IB of MEC and NCF of an arbitrary given minimal size. There are functions GTF*** $\tau^{(n)}$ ***for which any pair of MEC contains the nonconstructible configurations such as NCF.***

Let a global function $\tau^{(n)}$ ***has a set M of pairs of MEC. Then functions*** $\tau^{(n)m}$ ($m > 1$) ***have the set M of MEC identical with function*** $\tau^{(n)}$ ***if and***

only if at least only one configuration of each pair of MEC from M is NCF for GTF $\tau^{(n)}$; otherwise, $(\forall k\geq1)(M_k \subset M_{k+1})$, where M_k are sets of all pairs of MEC for global transition functions $\tau^{(n)k}$ ($k \geq 1$). Function $\tau^{(n)}$ possesses a NCF if and only if at least one of functions $\{\tau^{(m)}, \tau^{(p)}\}$ possesses NCF. If at least one function from pair $\{\tau^{(m)}, \tau^{(p)}\}$ possesses a NCF–1, then their composition $\tau^{(n)}=\tau^{(m)}\,\tau^{(p)}$ ($n=m+p-1$) will possess NCF and/or NCF–1. Last 2 assertions are valid for case of dimensions $d \geq 1$ and any finite state alphabet A of the homogeneous structures.

For example, in view of the presented results follows, that in classical ***HS***-models there are global transition functions having various sets of ***MEC*** along with identical sets of ***NCF.***

We shall consider the set ***G*** of the binary classical structures ***1-HS*** with neighbourhood index ***X***={***0,1,2***} as a rather simple example concerning distribution of the above nonconstructability types. It is obvious that structures quantity of the given set equals ***128*** while its each structure is uniquely identified by an appropriate *discriminating* number whose principle of calculation has been described earlier.

Firstly, according to the nonconstructability criterion on basis of ***γ–CF*** (*theorem* **23**) we can simply make sure, that all ***1–HS*** with numbers:

0..14, 16..29, 31..39, 40..42, 44, 46..50, 52, 54..56, 59, 61..74, 76, 77, 79..84, 87, 88, 91, 93..98, 100, 103, 104, 107, 109..112, 115, 117..119, 121..127

possess ***NCF*** and, possibly, ***NCF–1*** (*we relinquish confirmation of it to the reader*). The of such structures is ***93.*** A whole series of structures of the given subset can support a rather complex dynamics whose features we here disregard. The following subset is formed by the structures possessing pairs of ***MEC,*** and consequently ***NCF.*** Quantity of similar structures is ***11,*** their numbers are ***43, 53, 57, 58, 68, 92, 99, 108, 113, 114,*** and ***116.*** In particular, structures with numbers ***53, 99, 113*** not possess ***NCF–1,*** while structures with numbers ***92, 108, 114*** moreover possess ***NCF–1.*** Three structures with numbers ***15, 51, 85*** not possess ***NCF*** and ***NCF–1,*** generating identical sequences of configurations (*within shift*) and from standpoint of dynamics of especial interest do not represent. At last, the structures which not possess the ***NCF*** owing to absence for them the ***MEC*** belong to the last subset. *Discriminating* numbers of the structures with certain useful comments are represented below:

30 – configurations of the kind { $(1110)^k11$ | $k=0,1,2, ...$} are ***NCF–1*** in the structure;
45, 101 do not possess ***NCF–1*** but any finite configuration is ***periodic;***

60 **–** configurations of the kind { 1^{2k+1} | ***k=0,1,2, ...***} are ***NCF–1*** in the structure**;**
75, 102, 105, 106, 120 possess ***NCF–1*** at least of the simplest kind ***c = 1;***
86, 90 possess ***NCF–1*** of the simple kind ***c= 11 ;***
89 **–** configurations of the kind **<** $1x_1x_2x_3 .... x_n111$ **>** are ***NCF–1*** in the structure**;** $x_j \in A=\{0,1\}$, ***j=1..n.***

Thus, from total ***128*** binary structures of the considered set ***G:***
– 113 possess ***NCF*** and, possibly, ***NCF–1;***
– 3 structures with numbers ***15, 51, 85*** do not possess ***NCF*** & ***NCF–1;*** in addition**,** they are not of any interest from standpoint of dynamics**;**
– 12 do not possess ***NCF;*** in addition, only two of them with numbers ***45*** and ***101*** do not possess ***NCF–1*** in addition**.**

For the more detailed research of binary classical structures ***1–HS*** with neighbourhood index ***X=***{***0, 1, 2***} that possess ***NCF*** nonconstructability, the following three procedures can be a rather useful.

```
In[2251]:= NcfQ[Ltf_List, S_String, t_/; IntegerQ[t] &&
           MemberQ[Range[0, 9], t]] := Module[{a, b, c = "", d = {}, j, p,
                                   n = StringLength[Part[Ltf[[1]], 1]], k = 1},
                        a = Tuples[Range[0, t], p = StringLength[S] + n – 1];
                  b = Map[Map[ToString, #] &, a]; b = Map[StringJoin, b];
                     For[k, k <= Length[b], k++, For[j = p – n + 1, j >= 1, j––,
               c = StringReplace[StringTake[b[[k]], {j, j + n – 1}], Ltf] <> c];
                                 d = DeleteDuplicates[AppendTo[d, c]]; c = ""];
                                                If[! MemberQ[d, S], True, False]]
In[2252]:= L118 := {"000" –> "0", "001" –> "1", "010" –> "1", "011" –> "1",
                     "100" –> "0", "101" –> "1", "110" –> "1", "111" –> "0"};
In[2253]:= NcfQ[L118, "01010", 1]
Out[2253]= True
In[2254]:= MinNCF[Ltf_List, n_Integer] := Module[{a, k = 1},
                         a = Map[StringJoin, Map[Map[ToString, #] &,
                     Flatten[Map[Tuples[{0, 1}, #] &, Range[1, n]], 1]]];
                                                  For[k, k <= Length[a], k++,
                    If[NcfQ[Ltf, a[[k]], 1], Return[a[[k]]], Continue[]]]]
In[2255]:= MinNCF[L118, 10]
Out[2255]= "01010"
```

```
In[5]:= NfToLtf[m_Integer, n_Integer] := Module[{b, c, d = {},
        a = IntegerDigits[n, 2]}, If[n >= 2^2^m, Defer[NfToLtf[m, n]],
            b = Map[ToString, Join[Table[0, {2^m – Length[a]}], a]];
                      c = Map[StringJoin, Map[Map[ToString, #] &,
                  Flatten[Map[Tuples[{0, 1}, #] &, Range[m, m]], 1]]];
          Do[AppendTo[d, Rule[c[[k]], b[[k]]]], {k, 1, Length[b]}]; d]]

In[6]:= NfToLtf[3, 118]
Out[6]= {"000" –> "0", "001" –> "1", "010" –> "1", "011" –> "1",
         "100" –> "0", "101" –> "1", "110" –> "1", "111" –> "0"}
In[7]:= MinNCF[NfToLtf[3, 118], 10]
Out[7]= "01010"
In[8]:= b = Map[{#, StringLength[MinNCF[NfToLtf[3, #], 10]]} &, H];
In[9]:= b = Sort[b, #1[[2]] >= #2[[2]] &]; Gather[b, #1[[2]] == #2[[2]] &]
Out[9]= {{{37, 9}, {91, 9}}, {{22, 8}, {104, 8}}, ..., {{0, 1}}}
```

where ***H*** **–** list of all binary structures ***1–HS*** with neighbourhood index ***X***=**{*0,1,2*}** that possess the ***NCF*** nonconstructability. The procedure call **NcfQ[*Ltf, S, t*]** returns ***True,*** if a finite block configuration ***S*** is the ***NCF*** in a classical structure ***1–HS*** with local transition function, defined by the list of parallel substitutions ***Ltf,*** and in alphabet ***A*=**{***0,1,…, t***} ***(t ≤ 9);*** otherwise the procedure call returns ***False.***

Whereas the procedure call **MinNCF[*Ltf, n*]** returns the minimal block with ***NCF*** for a classical binary structure ***1–HS*** with the local transition function, defined by the list of parallel substitutions ***Ltf;*** in addition, ***n*** defines a predictable size of minimal ***NCF;*** furthermore, it is supposed that this structure possesses the ***NCF*** nonconstructability. At last, the procedure ***NfToLtf*** has an auxilary character**;** its call **NfToLtf[*p, n*]** ***(p –*** *length of neighbourhood template*, ***n –*** *an integer* **< 2^2^*m*)** returns the list of parallel substitutions defining a local transition function for binary ***1–HS*** with a discriminating number ***n.*** Note that procedure **NfToLtf** along with use for the problem considered here can be a rather useful in a number of other problems of experimental study of binary ***1–HS.***

MinNCF	*LTF with the discriminating numbers*
9	***37, 91***
8	***22, 104***
6	***57, 73, 99, 109***
5	***5, 23, 25, 41, 54, 61, 62, 67, 77, 94, 95, 97, 103, 107, 108, 110, 113, 118, 121, 122, 124***
4	***7, 9, 13, 21, 26, 27, 29, 31, 33, 35, 38, 39, 44, 49, 52, 53,***

	58, 59, 65, 69, 71, 74, 79, 82, 83, 87, 88, 92, 93, 100, 111, 114, 115, 123, 125
3	***1, 3, 6, 10, 11, 14, 17, 18, 19, 20, 28, 36, 40, 42, 46, 47, 50, 55, 56, 63, 70, 72, 76, 80, 81, 84, 96, 98, 112, 116, 117, 119, 126, 127***
2	***2, 4, 8, 12, 16, 24, 32, 34, 48, 64, 66, 68***
1	***0***

Using the above ***3*** procedures, implemented in system ***Mathematica,*** we researched the problem of minimum size of ***NCF*** in the structures with the discriminating numbers from diapason ***0..127.*** The obtained result reflects the above **2–**column table whose the *first* column defines minimum size of ***NCF,*** whereas the *second* defines the discriminating numbers of the binary structures ***1–HS*** with such ***NCF.***

At that, any sequence generated from an initial finite configuration in structures ***15,45,51,85*** and ***101*** is periodic**;** length of its period depends on length of an initial configuration and its kind. In particular, for the configurations ***c(k)*** the lengths of sequences *periods* generated by them are defined by the following simple formula**:**

$$c(k) = 1^k \equiv \underset{\leftarrow\, k\, \rightarrow}{11 \ldots 11} \quad (k \geq 1); \qquad p = 2^{[\log_2 (k+1)]};$$

decimal representation $1^k \equiv 2^k - 1$, *where* $[x]$ – *an integer is no more than x*

Hence, a quota of structures possessing ***NCF*** and, probably, ***NCF–1*** is ***≈ 0.88*** while only ***2*** structures with numbers ***45*** and ***101*** do not possess the basic types ***NCF*** and ***NCF–1*** of nonconstructability**.** Furthermore in this direction the following rather interesting result has been proved.

Proposal 3. For an arbitrary classical structure d–HS (d ≥ 1) any finite configuration of the set C(A,d,ϕ) is periodic if and only if the structure not possesses the nonconstructability such as NCF and NCF-1. Thus if a classical structure does not possess the nonconstructability such as NCF and NCF–1 then all its finite configurations will be periodic and the structure can't possess the property of universal computability.

So, only ***5*** structures from ***128*** with numbers ***15, 45, 51, 85, 101*** possess ***full reversibility*** of finite configurations generated by them**;** where an possibility of calculation of all chain of finite *predecessors* for any finite configuration ***c ∈ C(A,1,ϕ)*** is understood as this concept, i.e. possibility to unambiguously determine its *prehistory* in the structure. Thus, they do not represent particular interest in view of their limited generative possibilities from standpoint of modeling applications. So, from such standpoint the structures of the set ***G*** that possess the types ***NCF*** or/

and ***NCF–1*** of nonconstructability can represent the greatest interest only. At that, it substantially is generalized to the more general cases of classical ***HS–***models. Hence, we should pay the attention to models ***1–HS*** that possess the nonconstructability such as ***NCF*** and/or ***NCF–1*** for providing their complex enough dynamics.

Concept of types ***NCF*** & ***NCF-1*** of *nonconstructability* is enough closely connected to types of *dynamics* (*state graphs*) of classical ***d–HS (d≥1).*** In particular, the following result takes place, namely:

If a classical d–HS (d ≥ 1) not possess the nonconstructability such as NCF and NCF–1, then for any configuration $c_j \in C(A,d,\phi)$ the d–HS will generate the configurations sequences (state graphs) of only one of the following three kinds, namely:

(a) all configurations sequences $\Theta_j=\{c_j\tau^{(n)k} \mid k\geq 0; j=1..\infty\}$ are periodical in such structure;

(b) $\ldots \to c_{j-k} \to \ldots \to c_{j-2} \to c_{j-1} \to c_j \to c_{j+1} \to c_{j+2} \to \ldots \to c_{j+k} \to \ldots$;

(c) the structure has configurations sequences Θ_j of types (a) and (b).

Absence of nonconstructability of type NCF for classical HS-models is necessary condition, but not sufficiently for guarantee of reversibility of their dynamics relative to all finite configurations. If an arbitrary classical d–HS (d≥1) possesses the nonconstructability such as NCF–1 without NCF, then for arbitrary configuration $c_j \in C(A, d, \phi)$ the given structure generates the configurations sequence (state graph) of only one of the following two kinds, namely:

(a) $c_j \to c_{j+1} \to \ldots \to c_{j+p} \to \ldots \quad \bigcup_{j=1}^{\infty}\{c_j\tau^{(n)k} \mid k\geq 0\} = C(A,d,\phi); \quad c_j - NCF\text{-}1$

(b) the structure has configurations sequences $\{c_j\tau^{(n)k} \mid k\geq 0; j = 1..\infty\}$ of type (a) along with periodical sequences.

So, for case of the above binary ***1–HS*** among all structures possessing ***NCF–1*** without ***NCF*** the structures with numbers ***75,89,90,102,105*** and ***106*** belong to the type ***(a)*** whereas only three structures with numbers ***30, 60*** and ***86*** belong to the type ***(b).*** At that, this fact is rather obvious, not demanding any special explanations. Now, we can easily be made sure in correctness of the following result [54]:

For existence in an arbitrary d–HS (d ≥1) of the nonconstructability of type NCF–1 without NCF is enough that for each finite configuration $c \in C(A, d, \phi)$ the d–HS generates as a whole the ascending sequence of configurations concerning the sizes of configurations, i.e.

$$(\forall c \in C(A,d,\phi))(\exists\{j_1>j_2>...>j_q>...\})\left(k<p \to \left|c\tau^{(n)j_p}\right| > \left|c\tau^{(n)j_k}\right|\right) \&$$
$$(\neg\exists(k<p))\left(\left|c\tau^{(n)k}\right| > \left|c\tau^{(n)p}\right|\right); \quad j_q - \textit{ascending sequence of integers;}$$
$$|h| - \textit{size of a configuration } h \ (q=1,2,...\infty)$$

If for some d–dimensional global transition function $\tau^{(n)}$, defined in a finite alphabet A, the following relation $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$ takes place where $|c^*|$ – maximal diameter of a configuration c^* then the existence problem of the nonconstructability such as NCF and/or NCF–1 for the GTF $\tau^{(n)}$ is algorithmically solvable.

Furthermore, if a classical structure d–HS ($d \geq 1$) does not possess the nonconstructability such as NCF and for structure the above relation takes place then the structure possesses the nonconstructability such as NCF–1, the set C(A,d,∞) is non–closed concerning mapping induced by GTF $\tau^{(n)}$ of the structure, and the set $C(A,d,\phi)\backslash\{\ \}$ can be generated by means of only of configurations such as NCF-1; in addition, the set NCF–1 is minimal generative set for the set $C(A,d,\phi)\backslash\{\ \}$, i.e. the next relation takes place, namely:

$$\bigcup_j <c_j>\left[\tau^{(n)}\right] = C(A,d,\phi)\backslash\{\square\}; \ (\forall i,j)(i \neq j \to <c_i>\left[\tau^{(n)}\right] \cap <c_j>\left[\tau^{(n)}\right] = \varnothing$$
$$\textit{where } c_j \textit{ are configurations forming the set NCF–1}$$

Thus, for classical structures of the above sort a finite set of generators such as ***NCF–1*** for the set ***$C(A,d,\phi)\backslash\{\ \}$*** does not exist. It comparatively easily follows of obvious enough second relation represented above.

On the other hand, ***if a classical structure d-HS ($d \geq 1$) does not possess the nonconstructability such as NCF-1 and for the structure the above relation takes place the structure possesses the nonconstructability of type NCF and the set $C(A,d,\phi)\backslash\{\ \}$ can be generated by means of only of configurations from a minimal set $K \subseteq NCF$; whereas for a structure possessing the nonconstructability such as NCF and NCF-1 and in the presence of the above relation the set $C(A,d,\phi)\backslash\{\ \}$ can be generated by means of only of configurations from a minimal set $Kr \subseteq NCF \cup NCF–1$. In addition, case of strict inclusion is conditioned by the fact that the configurations such as NCF and/or NCF–1 can form pairs of MEC–1.***

Among classical structures satisfying the relation $(\forall c \in C(A,d,\phi))(|c^*| < |c^*\tau^{(n)}|)$ along with absence of the nonconstructability such as NCF makes sense to seek structures that possess the property of universal reproducibility in Moore's sense of finite configurations, e.g. the class of linear classical structures along with few other types of structures possessing the same universal property which are considered below.

On basis of theorem ***31*** estimations of number ***N(a,n)*** and quota **δ*(a,n)*** of the classical structures ***d–HS (d≥1)*** with a state alphabet ***A***={***0,1,2,…, a–1***} and neighbourhood index ***X*** = {***0,1,2, …, n–1***} that will possess the nonconstructability such as ***NCF (NCF–3)*** and/or ***NCF–1*** are expressed by the following relations, namely:

$$N(a,n) > a^{a^n-a}\left[\sum_{j=0}^{a-1} C_{a-1}^{j}(a-1)^{a-j-1} - (a-1)^{a-1}\right] = a^{a^n-a}\left[a^{a-1} - (a-1)^{a-1}\right]$$

$$\delta(a) > N(a,n)/a^{a^n-1} = \left[1-(1-1/a)^{a-1}\right]; \quad \lim_{a\to\infty}\delta(a) > 1-1/e; \quad \delta 1(a) < 1/e$$

where [x] – a least integer ≥ x. While quota **δ*(a)*** of such structures with respect to all classical structures ***d–HS*** is defined by the above relation regardless of parameters ***d*** and ***n*** of a structure. Hence, quota **δ*1(a)*** of classical structures ***d–HS (d ≥ 1)*** not possessing the nonconstructability neither ***NCF*** nor ***NCF–1*** less than ***1/e***. At that, **δ*(a)*** and **δ*1(a)*** is enough appreciably decreased and overstated accordingly with growth of the cardinality of ***A*** depending on our numerous experimental researches.

Obviously, for existence in an arbitrary classical structure d-HS (d≥1) of the nonconstructability such as NCF-1 the presence for the classical structure of such infinite configurations $c^{\infty} \in C(A,d,\infty)$ ***that*** $c^{\infty}\tau^{(n)} =$ ***is necessary but not sufficient condition.***

On basis of analysis of four basic types of the nonconstructability and computer simulation, it is possible to establish the fact, the dynamics of configurations $c_o \in C(A,d,\phi)$ in classical ***HS***–models is characterized by transition graphs of the following kind, namely:

1. ***<u>An infinite non–periodic sequence</u> of finite configurations from the set*** $C(A,d,\phi)$: $c_o \to c_1 \to \ldots c_j \to \ldots$ for which there are **2** possibilities:

(a) defines algorithm of the structural organization of components of configurations depending on an initial configuration $c_o \in C(A,d,\phi)$ and a moment of time ***t;*** binary classical structures ***1–HS*** considered above with numbers ***105*** and ***111*** can serve as such examples.

(b) such algorithm is absent or is complex enough for determination.

2. ***<u>Sequence of a pure cycle</u>;*** in addition, the *passive configurations* ***(PCF)*** $c_o \in C(A,d,\phi)$ determined by the relation $c_o\tau^{(n)} = c_o$ ***(****accurate to shift****)*** fall under the given definition too:

$$c_o \to c_1 \to c_2 \to \ldots c_j \quad (c_j \to c_o)$$

There are ***HS***–models in which each configuration c_o generates a pure cycle whose period is at least **2** and depends as on kind of $c_o \in C(A,d,\phi)$

and its size. The binary structures considered above with numbers ***45***, ***101*** quite can serve as similar examples.

3. ***<u>Sequence of a mixed cycle</u>***; it is characterized by presence in it of the certain configuration c_j generating some pure cycle, namely:

$$c_o \to c_1 \to c_2 \to .... c_j \to .... \to c_{j+p} \to$$

The binary structures ***1–HS*** with numbers ***44*** (*configurations* ***101*** *and* ***10101***) and ***66*** (*configuration* ***1011***) can serve as examples. At that, it is simple to be convinced, the ***HS***–models whose dynamics includes transition graphs such as ***(3)*** possess nonconstructability such as ***NCF***.

Dynamics of the *classical* ***HS***–models does not possess *transition graphs* of other type; at that, dynamics can correspond both to the sets of the specified graphs, and exceptionally to one of them. For example, the dynamics of ***1–HS*** with number ***105*** is being described by transition graphs exceptionally only such as ***(1.a)***, whereas ***1–HS*** with numbers ***45*** and ***101*** by transition graphs only such as ***(2)***. A ***NCF*** is absolutely nonconstructible configuration with respect to set $C(A,d,\phi) \cup C(A,d,\infty)$, however ***NCF–1*** and ***NCF–2*** are nonconstructible configurations with respect to the sets $C(A,d,\phi)$ and $C(A,d,\infty)$ accordingly.

The above arguments have been considered by us as ***one*** of possible approaches to *classification* of dynamics of classical ***HS***–models [29,30] in context of discussion of an interesting enough work of ***A.W. Burks*** [314]. Later on, systematizing ***HS***–models concerning their behaviour, ***S. Wolfram*** has determined into them ***four*** classes in many respects similarly to our approach. Meanwhile, the classification carries purely phenomenological character and not gives any recommendations for designing on its base the required rules of behaviour of ***HS***–models, it only characterizes possible types of dynamics as a whole. So, there are also other *phenomenological* criteria of *classification* of rules of dynamics of classical ***HS***–models on which we not dwell here by a whole series of reasons, referring the interested reader to the appropriate editions presented in the expanded bibliographies [536,640]. Meantime, in any case *phenomenological* criteria only externally qualitatively characterize the dynamics of ***HS***–models, not allowing to use them as a toolkit for direct programming of ***HS***–models with the required dynamics.

While graphs of ***predecessors*** for configurations $c_o \in C(A,d,\phi)$ belong to the following basic types, namely:

1. ***<u>An infinite non–periodic sequence</u> of configurations from*** $C(A,d,\phi)$:

$$\ldots \leftarrow c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o$$

The above binary ***1–HS*** with number ***39*** (*configuration* $\{1^k \mid k = 1..n\}$) can serve as such example.

2. ***<u>A finite non–periodic sequence</u> of configurations from set C(A,d,φ):***

$$c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o$$

This case takes place, if a configuration c_{-j} is non–constructible such as ***NCF.*** For instance, for the above-mentioned ***1–HS*** with number ***39,*** one of the possible predecessors sequences for configuration $c_o = 1$ looks like $c_{-3} = 101 \rightarrow 111 \rightarrow 11 \rightarrow 1 = c_o$ where $c_1 = 101$ is ***NCF.***

3. <u>A sequence of pure cycle;</u> at the same time, the *passive* configurations $c \in C(A, d, \phi)$ (*accurate to shift*) fall under this definition also, namely:

$$c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o \quad (c_o \rightarrow c_{-j})$$

The above binary structures ***1–HS*** with numbers ***45*** and ***101*** can serve as similar simple examples.

4. <u>A sequence of the mixed type;</u> it is characterized by existence in it of a configuration c_{-j} for which predecessors can be from the set ***C(A,d,φ)*** or from the set ***C(A, d,∞),*** or from both sets simultaneously, namely:

$$\leftarrow \ldots c_{-j-p} \leftarrow \ldots \leftarrow c_{-j-1} \qquad c_r \in C(A,d,\phi);\ r=0..-j-p$$

$$c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o$$

$$\leftarrow \ldots b_{-j-k} \leftarrow \ldots \leftarrow b_{-j-1} \qquad b_q \in C(A,d,\infty);\ q=-j-1..-j-k$$

The above binary structures ***1–HS*** along with other types of ***HS–****models* [54–56,90] can serve as examples. At that, it is possible to be convinced that graphs of predecessors such as ***(4)*** starting with configuration c_{-j} admit also the subgraphs such as ***(1) – (3).*** Meantime, the following an essential enough result has been received [54–56,79,88,90,640–643].

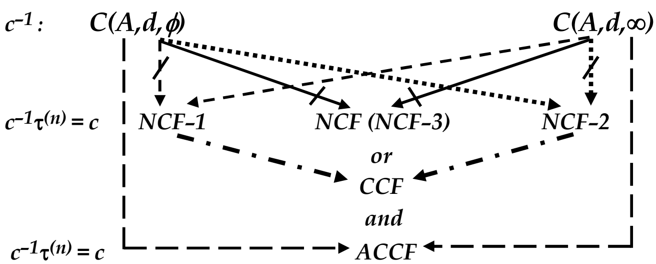
<u>Theorem 37</u>. The problem of classification of classical structures d–HS (d≥2) according to their types of transitions graphs of states, i.e. finite configurations, is algorithmically unsolvable in general case.

Our *classification* of dynamics of classical structures along with similar classification of ***S. Wolfram*** have mainly phenomenological character and do practically not play any classifying part. They both have been received on the basis of experimentation with simple enough types of classical ***HS–***models and rather speculative experiments. Meanwhile, each classification which pretends to this name should provide some algorithm either direct or indirect which allows to ascribe any *classical* ***HS***–model to this or that type. However, both mentioned *classifications* do not allow to do it, because, for example, in view of unsolvability of the existence problem of *nonconstructability* such as ***NCF*** in ***HS***-models of dimensionality $d \geq 2$ we cannot differentiate an arbitrary ***HS***–model already concerning type **4.** Impossibility of similar classification have been proved also by ***K. Culik*** and ***S. Yu*** [555] on basis of one *unsolvable* problem, namely.

Proposal 4. The existence problem for any structure d–HS (d ≥ 2) of all finite configurations as vanishing ones is algorithmically unsolvable.

Among other ways of classification of ***HS***–models it is possible to note approach of ***C. Langton*** on base of ***λ***–parametrization which measures quota of *non-zero* values of ***LTF*** together with approaches of ***N. Israeli, J. Dubacq, H. Goldenfeld,*** which have offered parametrization of ***LTF*** on basis of known complexity concept of ***A. Kolmogorov*** [5,88,90,536]**,** and also a rather interesting stochastic approach of ***A.V. Lebedev*** [569]**.** Now, there are some other approaches to classification of ***HS***–models.

Along with differentiation of the nonconstructability concept relative to classical ***d-HS (d≥1)*** the question of differentiation of *constructability* of the finite configurations represents indubitable interest also. So, an arbitrary configuration $c \in C(A,d,\phi)$ in classical ***d–HS (d≥1)*** is called the *constructible* configuration ***(CCF)*** if it has predecessors c^{-1} from the set $C(A,d,\phi)$ or the set $C(A,d,\infty)$, i.e. $c^{-1}\tau^{(n)} = c$. Obviously, a constructible configuration can't be as ***NCF (NCF–3),*** but it can be ***NCF–1*** or ***NCF–2.***



On the other hand, in classical ***d–HS (d ≥ 1)*** a configuration $c \in C(A,d,\phi)$ is called the ***absolutely constructible*** configuration ***(ACCF)*** if and only if it has predecessors as from set $C(A,d,\phi)$ and from set $C(A,d,\infty)$**.** Thus, obviously, an ***absolutely*** constructible configuration ***c*** can't be as ***NCF (NCF–3)*** but the configuration ***c*** can't be as ***NCF–1*** or ***NCF–2*** as well.

The diagram presented above enough visually illustrates interrelation of all four nonconstructability types ***(NCF, NCF–1, NCF–2, NCF–3)*** and constructability ***(CCF, ACCF)*** in the classical structures ***d–HS (d ≥ 1).***

On basis of concept of the *absolutely constructible configurations* and the theorem ***31*** we can prove the following rather interesting result [90].

<u>*Theorem 38*</u>. ***For a classical structure d–HS (d≥1) the set C(A,d,ϕ) can't consist only of the absolutely constructible configurations as well as finite configurations such as NCF–1 while the set C(A,d,ϕ) can consist of the constructible configurations only of type NCF–2, for example.***

In the given context a rather interesting question about the *maximal set* of absolutely constructible finite configurations arises. Thus, detailed enough discussion of the given question can be found in [114,567,617]. The following result [567,617] gives an answer to the given question.

<u>*Theorem 39*</u>. ***For each classical d–HS (d≥1) one of three relations takes place, namely: CCF ⊆ C(A,d,ϕ), CCF ≡ ACCF, ACCF ⊂ C(A,d,ϕ), where CCF and ACCF are the sets of all finite constructible and absolutely constructible configurations accordingly; in addition, the following relations take place: (∃d–HS)(C(A,d,ϕ) ≡ NCF–2) & (∀d–HS)(C(A,d,ϕ) ⊃ NCF–1 & C(A,d,ϕ) ⊃ ACCF).***

The above ***1–HS*** with neighborhood index ***X=***{***0,1,2***} and *discriminating* number ***116*** can be considered as an interesting enough example. So, a block configuration ***c* = <010>*** is ***NCF*** of minimal size in the structure; therefore any configuration containing this block configuration ***c**** will be in such ***1–HS*** as ***NCF*** while others are absolutely constructible. The detailed analysis of such structure allows to formulate one interesting enough result, namely:

There is a structure 1–HS without NCF–1 for which the configurations such as NCF compose «almost all» set C(A,1,ϕ), while the remaining configurations of the set are absolutely constructible configurations, i.e. they have predecessors from the set C(A,1,ϕ) and the set C(A,1,∞).

Above all, the above example well enough illustrates the fact that non-closure of the set ***C(A, d, ∞)*** concerning mapping defined by a classical structure ***d–HS (d ≥ 1)*** is the requirement, but not sufficient condition for presence of the nonconstructability such as ***NCF–1*** in the structure. Thus, on the one hand, for any classical ***d–HS (d ≥ 1)*** the set ***C(A,d,ϕ)*** of finite configurations cannot consist only from absolutely constructible configurations while, on the other hand, there are classical structures ***d–HS*** for which all constructible configurations of the set ***C(A,d,ϕ)*** are as well absolutely constructible. So, the determinative attribute of the concept of *classical* ***HS***–models allows to *differentiate* naturally not only the nonconstructability but *constructability* of finite configurations too. And in it one more essential feature of the classical ***HS***–models. Along with this feature the *classical* structures ***d–HS (d≥1)*** are of interest, first of all, from the *applied* standpoint since they have such special state as a «*quiescent*» which causes a lot of quite natural interpretations and of this state itself, and a whole series of *dynamic* properties of the *classical* structures determined by its availability.

As it was already marked, for existence in ***d-HS*** of nonconstructability such as ***NCF–1*** for the structure necessarily existence at least of ***1*** such infinite configuration $c \in C(A, d, \infty)$ that $c\tau^{(n)} =$, or $c\tau^{(n)} = c' \in C(A, d, \phi)$ in the more general case. However, here it is necessary to note one rather essential aspect. Among all infinite configurations relative to classical structures ***1–HS*** it is expedient to distinguish configurations of **2** basic types**:** ***(1)*** *infinite configurations into both sides* (c^∞) and **(2)** *configurations infinite only to the left* (c_-^∞) or *to the right* (c_+^∞)**.**

Let for a classical structure ***1-HS*** without ***NCF*** there are configurations of the kind $c_+^\infty = \square x_1 x_2 \dots x_n \dots x_j \infty \; (x_1, x_j \in A \backslash \{0\})$ such, that the following relation $c_+^\infty \tau^{(n)} = \square \in C(A,1,\phi)$ takes place. But then it is simple to prove existence in the structure at least of two different finite configurations $c_p = \square x_1 x_2 \dots x_n \dots x_p \square$ and $c_k = \square x_1 \dots x_k \square \; (p \neq k; \; x_1, x_p, x_k \in A \backslash \{0\})$ made up on basis of such configuration c_+^∞ that $c_p \tau^{(n)} = c_k \tau^{(n)} = c \in C(A,1,\phi)$ takes place (*in a specific case the relation* $c_p \tau^{(n)} = c_k \tau^{(n)} = \square$ *may be also*), where ' ' belongs (*according to earlier marked agreement about structure of the set of all finite configurations*) to the finite configurations. Hence, the ***1–HS*** will possess the pairs of ***MEC*** and therefore by the nonconstructability such as ***NCF***, contradicting the assumption. Analogous situation takes place and for the infinite configurations c_-^∞ too. Thus, for the classical structures ***1–HS*** at absence of nonconstructability such as ***NCF*** only c^∞ configurations infinite into both sides can exist, which satisfy the next relation $c^\infty \tau^{(n)} = \square \in C(A,1,\phi)$. While the fact of configurations existence of type c_-^∞ or c_+^∞ for which $c_+^\infty \tau^{(n)} = \square \in C(A,1,\phi)$ or $c_-^\infty \tau^{(n)} = \square \in C(A,1,\phi)$ for a classical ***1–HS*** provides existence of the nonconstructability such as ***NCF*** for such ***1–HS***. In the given context the following rather useful result can be formulated, namely.

<u>Theorem 40</u>. A classical structure 1–HS possesses nonconstructability such as NCF if for the structure exist such configurations $c^* \in C(A, 1, \phi) \cup c_+^{\infty} \cup c_-^{\infty}$ ***that the following relation takes place, namely:*** $c^*\tau^{(n)} =$ **.**

Thus, concerning the nonconstructability concept, the set ***C(A, 1, ∞)*** of all infinite configurations of classical structures ***1–HS*** is differentiated. In the certain cases the given circumstance is essential enough. At that in a whole series of cases the importance of the given differentiation is not less significant than differentiation of ***C(A, 1)*** into the sets ***C(A, 1, ϕ)*** and ***C(A,1,∞).*** Proof of a series of interesting enough results relative to classical structures above all dealing with the set ***C(A,d,∞) (d≥1)*** of the infinite configurations, enough essentially uses the following entirely obvious lemma.

<u>Lemma 3</u>. For any integer d≥1 and alphabet A={0,1,...,a-1} an arbitrary configuration in the alphabet A of d-dimensional hypercube with edge of size L defined by the following simple formula

$$L = n \left]\sqrt[d]{a^{n^d}+1}\right[, \quad where\ ]x[\ - an\ integer\ greater\ than\ x$$

will contain at least 2 identical subconfigurations on d–dimensional hypercubes with edge of size n. Whereas for a singular 1–dimensional case and an integer $n \geq 1$ ***exists such integer*** $m=n(a^n+1)$ ***that each tuple*** $P_j=<x_1x_2 \ldots x_m>$ ***will contain at least two identical disjoint subtuples*** $<y_1y_2y_3 \ldots y_n>$ $(x_k, y_p \in A;\ k=1..m;\ p=1..n;\ n < m)$ ***of configurations.***

Particularly, the given result has been used at the proof of theorem ***40.*** Along with the considered questions, there is a whole series of other more special questions of global dynamics of the classical ***HS–***models connected to the *nonconstructability* problem. For example, a question about influence of types of ***LTF*** $\sigma^{(n)}$ onto existence of nonconstructible configurations in the classical ***HS–***models seems rather interesting. So, researches of classical structures ***1–HS*** determined by symmetric ***LTF,*** have shown [1,3,37]**,** that the structures determined by such ***LTF*** will possess configurations $c \in C(A, 1, \phi)$ such as ***NCF (NCF–1)*** if and only if they will possess configurations c^R, inverse to them, as ***NCF (NCF–1)*** too. Hence, *symmetry* of ***LTF*** $\sigma^{(n)}$ in a classical structure ***1-HS*** expands, generally speaking, the sets of ***NCF*** and ***NCF-1*** whereas for *asymmetric* ***GTF*** $\tau^{(n)}$ both configurations $c \in C(A, 1, \phi)$ and c^R can be ***NCF*** or ***NCF–1*** separately. Moreover, at more general posing the «***symmetry***» of ***LTF*** in classical ***HS–***models can be considered and concerning the separate *subclasses****.*** A whole series of other special questions of dynamics of the classical ***HS***–models connected to the nonconstructability problem the reader can find in our works [54–56,79,88,90,640–643].

The *reversibility* concept considered above of *classical* ***HS***–models plays rather important role in theoretical and applied aspects, especially in case of use of the ***HS*** as models of *spatially–distributed dynamic* systems from which physical systems represent a *special* interest. Thus, we can imagine a ***HS***-model as an infinite automaton for a processing of some input words [*configurations from the set* $C(A, d, \phi)$] into output words of the same infinite set $C(A, d, \phi)$. At that, each output of such automaton becomes its next input word. Thus, we can consider the classical ***HS***–models as *infinite autonomous automata* whose description and research of dynamics allow to successfully use both the language of *diagrams* of states and the graphic language of transitions. Thus, the approach on basis of *state graphs* is effective enough facility of dynamics research of the *classical* ***HS***-models what by us was mentioned earlier. In addition, the given *graphic* approach admits a whole series of modifications and interpretations responding specificity of the researched problems.

Widely enough the given graph approach to dynamics research of the classical ***HS***-models is used in works [158,160,177,178,268,281,314], for example. In the given terms, functioning of a classical ***HS***–model can be determined by a states graph, where the current configuration of a model as a state of an infinite automaton is being understood. In turn, states graph of such infinite automaton consist of subgraphs of certain elementary types, namely:

(a) $\tau^{(n)}: c_o \rightarrow c_1 \rightarrow c_2 \rightarrow c_3 \rightarrow \dots c_j \rightarrow \dots \rightarrow c_k \rightarrow \dots \rightarrow c_p \rightarrow$ (with c_p returning to c_j)

(b) $\tau^{(n)}: c_o \rightarrow c_1 \rightarrow c_2 \rightarrow \dots c_j \rightarrow \dots \rightarrow c_k \rightarrow \dots \rightarrow c_p \dots \rightarrow$

(c) $\tau^{(n)}: \dots \rightarrow c_{-2} \rightarrow c_{-1} \rightarrow c_o \rightarrow c_1 \rightarrow c_2 \rightarrow \dots \rightarrow c_k \rightarrow \dots \rightarrow c_p \rightarrow \dots$

$$c_k \in C(A, d, \phi)\ (k = -\infty\,..+\infty)$$

As a result of a work on the questions stated by ***A.W. Burks*** [128,314] and in connexion with researches of the reversibility problem for ***HS***–models we had investigated the *state graphs* of the classical ***HS***-models in their connection with the nonconstructability problem. Here a basic result can be formulated as follows.

<u>Theorem 41</u>. If a classical structure d–HS (d ≥ 1) does not possess NCF (NCF–3) and NCF–1, then already concerning input / output alphabet C(A,d,ϕ) state graph of the d–HS can contain only subgraphs of types (a; for j=p=0) and/or (c); in other cases the combinations of subgraphs of types (a..c) in wide enough ranges are permitted.

A whole series of results of researches concerning state graphs of the classical ***HS***–models represent special interest in case of consideration of such class of parallel dynamic systems as infinite automata in their traditional understanding [54–56,158,281,389,390,536,640]. A little bit more in details the given question was discussed earlier in section **2.2.**

Meanwhile, a question concerning the *nonconstructability* problem for certain special subclasses of the classical structures ***d–HS*** $(d \geq 1)$ seems rather interesting [617]. In particular, it is possible to define one such subclass of structures ***d–HS*** $\equiv \langle Z^d, A, \tau^{(n)}, X_z \rangle$ in which neighbourhood index is variable and is determined by an internal state of the current elementary ***z***–automaton, i.e. $X_z = F(s[z])$. Obviously, that the set X_z of all allowable neighbourhood indexes of the similar ***HS***–model is finite and for state alphabet $A = \{0, 1, ..., a-1\}$ its cardinality equals ***a***. Whereas amount ***N*** of all such ***HS***–models is defined by the following formula:

$$N = a^{\sum_{j=0}^{a-1} a^{[X_j]-1} - 1}$$

where $[Xj]$ – quantity of elementary automata forming a neighborhood template corresponding to neighborhood index X_j $(j = 0..a-1)$. It is easy to be sure the introduced structure ***d-HS*** $(d \geq 1)$ with a variable neighbourhood index X_z and a state alphabet ***A*** of the current elementary automaton is strictly equivalent to an appropriate classical structure ***d-HS*** with the same state alphabet. So, the ***HS***-models which are defined in above way compose a *subclass* of the class of all classical ***HS***–models. In the models of the given kind the parallel substitutions determining them local transition functions are differentiated by the first symbol of their left parts, i.e. in a sense fibering of local functions concerning states of the current elementary automata takes place.

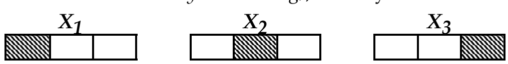
Thus, depending on the current state the elementary automaton of the structure defines one's own neighbours from which it is necessary to receive the information for definition of the next state; i.e. *behaviour* of an elementary ***z***–automaton of a ***HS***–model of similar type carries in a sense «*intellectual*» character. At that, ***HS***-models of the above type are characterized by an *individual* choice of a neighbourhood index by the elementary ***z***-automaton, expanding a diapason of a rather interesting problems effectively represented on their base [54–56,79,88,90,640].

Meantime, and from theoretical standpoint such ***HS***–models represent the quite certain interest. In particular, linear local transition functions appropriate to a variable neighbourhood index X_z do not possess the nonconstructability such as ***NCF*** at existence for them ***NCF-1;*** and vice versa, a resultant ***HS***–model basing on such neighbourhood index X_z does not possess the nonconstructability such as ***NCF–1,*** having ***NCF.*** As a supporting example we shall consider the binary ***1***–model whose neighbourhood index X_z is defined as $X_z=\{X_o = \{0,1\}, X_1 = \{0,1,2\}\}$ and the appropriate *partial* local transition functions are defined as follows $\sigma_0^{(2)}(x,y)=x+y \ (mod\ 2),\ \sigma_1^{(3)}(x,y,z)=x+y+z \ (mod\ 2);\ x,y,z\in\{0,1\}$. Using the *nonconstructability* concept and related concepts, considered below obviously, these local functions are linear, and the appropriate global transition functions do not possess the *nonconstructability* such as ***NCF*** at existence such as ***NCF–1.*** On the other hand, it is easy to make sure, for the resultant ***1***–model already the pair {***<01|01|01>, <01|10|01>***} of mutually–erasable configurations exists, i.e. this model possess the nonconstructability such as ***NCF.*** Furthermore, it is easy to make sure, that the set ***C(B,1,∞)*** of all ***1***–dimensional *infinite* binary configurations is closed relative to the mapping defined by global transition function of the above binary ***1***–model with variable neighbourhood index X_z; i. e. the model does not possess the nonconstructability such as ***NCF–1.*** Hence, in a context of results of chapter **3** it is possible to ascertain, for such ***1***–model, contrary to linearity of partial local transition functions defining the resultant global function of the ***1***–model, the property of the universal reproducibility in Moore's sense of finite configurations will not have a place. At that, in context of results of chapter **2** in such ***1***–model, contrary to linearity of partial local transition functions, the *nonconstructible* configurations such as ***NCF*** will exist without ***NCF–1.***

Furthermore, the above binary ***1***–model can generate parallel formula language $L(\tau_3)$ *(ch. 5);* so, from axiom $c_o=1011$ the formula sequence of configurations is generated $\{c_{2k-1}=(10)^{k+1}1,\ c_{2k}=(10)^{k+1}11\}\ (k=1,2,\ldots)$, and from axiom $c_o=1101$ the same formula sequence of configurations is generated. Along with applied aspects the ***HS***–models of the above kind is characterized by a series of other rather interesting theoretical properties [88,90]. As the further expansion of ***HS***-models of the given kind it is possible to consider also a case when a neighbourhood index X_z of their elementary automata depends on history onto some depth of their previous states [5,15,79,88,90,518,536,545,567,640–643].

Meanwhile, a class of the structures basing on the concept of classical structures, but whose neighbourhood index is defined by the current configuration of neighbourhood template of some constant size seems rather interesting, defining one more kind of structures different from classical and interesting ones from certain standpoints. So, in this case within of some *fixed* neighbourhood template depending on its *current* configuration, a neighbourhood index is determined on whose basis a calculation of the next state of its central automaton will be made. But, at similar definition of structures quite probably appearance of many–valuedness at calculations of states of elementary automata with time.

As a simple example we shall consider ***1-***dimensional binary structure whose neighbourhood template has size ***3*** within which three indexes of neighbourhood are applied, namely**:** X_1={***0,1,2***}, X_2={***-1,0,1***} and X_3= {***-2,-1,0***}**.** At that, a system of parallel substitutions defining ***LTF*** of the structure with its affixment to the above neighbourhood indexes have the following kind ***(****central automata****,*** *appropriate for the specified indexes of neighbourhood are marked by the shading****)*,** namely**:**



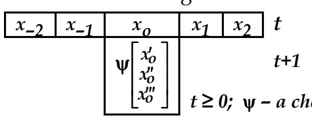
X_1	X_2	X_3
000 → 0	***110 → 1***	***011 → 0***
001 → 1	***111 → 0***	***100 → 1***
010 → 1		***101 → 1***

Thereby, within a geometrically aninted neighbourhood template the formative elementary automata are endowed by a little more complex function of commutation relative to the choice of a central automaton, i.e. the automaton whose state changes at the next moment depending on the calculated neighbourhood index. Unlike the classical structures the using to the current configuration of global transition function of a structure determined thus can be the most conveniently programmed by a method presented in [567,640]**.** At that, for a single–valued choice of the following state it is necessary to define a certain choice function which solves this problem. In particular, to that end the logic function ***XOR*** can be used as a choice function.

It is simple to make sure that this class of structures ***(***𝕽***–****class)* expands the classical structures already on a level of generative opportunities concerning a fixed neighbourhood index within some neighbourhood template [3,88]**.** Relative to structures of the **𝕽–**class it has been shown that criterion of existence of the nonconstructability such as ***NCF*** that is based on the ***MEC*** in classical structures is correct and for structures of the **𝕽–**class, namely the following result takes place [90,640–643]**:**

A structure of the 𝕽-class will possess the nonconstructability such as NCF if and only if for the structure there are pairs of the MEC in their classical conception.

And that is not some unexpected result since it is simple to show, that as a whole **𝔑**-class is a subclass of the class of all classical structures of the same dimensionality with same state alphabet but with increase of neighbourhood template. For example, an arbitrary structure ***1–HS*** of the **𝔑**–class with a state alphabet ***A*** and a neighbourhood template of size ***n*** is modeled strictly real time by means of an appropriate *classical* structure ***1-HS*** with neighbourhood index ***X***={***-(n-1), ..., -1,0,1, ..., (n-1)***} and the state alphabet ***A*.** On the above example the following scheme enough evidently illustrates a principle of definition of local transition function of the modeling structure ***1–HS*,** namely**:**



t ≥ 0; ψ – a choice function of the next state

Furthermore, this result can be spread to a similar kind of structures with a variable neighbourhood index, not restricting oneself by fixity of neighbourhood template. More circumstantially with questions of appendices of structures of the **𝔑**–class, and also of a whole series of their generalizations along with a series of their interesting properties and appendices it is possible to familiarize oneself in [77,88,90,567]**.**

To the above question the existence of the nonconstructability such as ***NCF*** for asynchronous structures directly adjoins too. In particular, it appears that the criterion of the nonconstructability such as ***NCF*,** that is based on the concept of mutual–erasable configurations, extends on wide enough class of asynchronous structures [54]**.** As an example we will present **2** binary *asynchronous* structures. Above all, let's consider a rather simple example of asynchronous structure ***1-HS*** whose global transition function $\tau^{(3)}$ are determined by means of the following local binary transition function $\sigma^{(3)}$, i.e. we deal with *asynchronous* structure with variable neighbourhood index and local transition function $\sigma^{(3)}$ that are determined by position {***even | odd***} of the current elementary automaton of the structure, namely**:**

$$\sigma^{(3)}(x_j, x_{j+1}, x_{j+2}) = \begin{cases} x_j \oplus x_{j+1} \oplus x_{j+2}, & \text{if } j \text{ is an even number} \\ x_j \oplus x_{j+1}, & \text{if } j \text{ is an odd number} \end{cases}$$

$$x_j, x_{j+1}, x_{j+2} \in B = \{0,1\}; \quad j = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$\text{where } \oplus \text{ is addition operation modulo } 2$$

Obviously, the local transition subfunctions composing the *generalized* local transition $\sigma^{(3)}$ of such asynchronous structure do not possess the nonconstructability such as ***NCF*.** At that, these subfunctions possess the nonconstructability such as ***NCF–1*** and possess the property of the universal reproducibility in the Moore's sense of finite configurations. Furthermore, as application of local transition subfunctions $\sigma^{(3)}$ in the current moment causes ambiguity at calculation of following states of automata of the structure, then in this case the principle of ***full*** *identity* of configurations of intersecting parts of neighbourhood templates is used. A simple scheme defining the process of calculation of pairs of *mutually–erasable* configurations *(depending on initial {even | odd} position of elementary automaton)* shows, that for the structure the pairs of ***MEC*** are absent**;** meanwhile all block configurations are constructible in the structure, i.e. such structure does not possess the nonconstructability such as ***NCF.*** On the other hand, the following scheme of generating of all possible configurations of an elementary automata {***i, i + 1***} ***(i = 0, ±1, ±2, ±3, ±4,...)*** of an asynchronous structure ***1–HS,*** whose generalised global transition function is defined by local transition function $\sigma^{(2)}$**:**

$$\sigma^{(2)}(x_j, x_{j+1}) = \begin{cases} \sigma_1^{(2)}(x_j, x_{j+1}), & \text{if } j \text{ is an even number} \\ \sigma_2^{(2)}(x_j, x_{j+1}), & \text{if } j \text{ is an odd number} \end{cases}$$

$$\sigma_1^{(2)} = \begin{bmatrix} 00 \to 0 \\ 01 \to 1 \\ 10 \to 1 \\ 11 \to 1 \end{bmatrix} \qquad \sigma_2^{(2)} = \begin{bmatrix} 00 \to 0 \\ 01 \to 1 \\ 10 \to 1 \\ 11 \to 0 \end{bmatrix}$$

$$x_j, x_{j+1} \in B = \{0,1\}; \quad j = 0, \pm 1, \pm 2, \ldots$$

shows that one of the subfunction, composing the generalised local transition function *(more precisely the subfunction* $\sigma_1^{(2)}$ ***)*** of the structure possesses the nonconstructability such as ***NCF.*** Whereas from the next scheme *(where «**ev**» and «**od**» denotes even and odd positions of automaton accordingly)* easily follows, that an arbitrary block configuration in the structure is constructible, i.e. the given structure does not possess the nonconstructability such as ***NCF.***

ev	***od***	***ev***		***ev***	***od***
0	***0***	***0***	⇒	***0***	***0***
0	***0***	***1***	⇒	***0***	***1***
0 ***1***	***1*** ***0***	***1*** ***0***	⇒	***1***	***0***
0 ***1***	***1*** ***1***	***0*** ***0***	⇒	***1***	***1***

od	***ev***	***od***		***od***	***ev***
0	***0***	***0***	⇒	***0***	***0***
0 ***1*** ***1***	***0*** ***1*** ***1***	***1*** ***0*** ***1***	⇒	***0***	***1***
1	***0***	***0***	⇒	***1***	***0***
0 ***1*** ***0***	***1*** ***0*** ***1***	***0*** ***1*** ***1***	⇒	***1***	***1***

In addition, this structure does possess the pairs of mutually–erasable configurations, for example, already of the following kind, namely:

ev	*od*	*ev*	*od*
1	***0***	***0***	***1***
1	***1***	***1***	***1***
1	***0***	***1***	

Carried out the detailed enough analysis of asynchronous structures of this type along with a whole series of some other types of structures of the given class has allowed to us to formulate the following a rather interesting result, namely:

There is a wide enough class of asynchronous structures for which the criterion of existence of the nonconstructability such as NCF, basing on concept of pairs of mutually–erasable configurations analogously to case of classical and unstable structures is invalid, as a whole.

Of this result follows, that such important enough concepts as ***MEC* &** ***NCF*** are inherent, mainly, in classical and unstable structures, concept of ***NCF–1*** is inherent to classical structures only, whereas for case of a wide enough class of asynchronous structures these concepts lose own primary meaning. So, relative to interrelation of the ***NCF*** and mutual erasability the class of *asynchronous* structures essentially differs from classical and unstable structures, generally speaking.

Of the above results it is possible to confidently conclude that concept of nonconstructability in the classical ***HS***–models has been researched enough in detail, therefore today the given section of the ***HS*** theory is one of most advanced. Meantime, the problematics contains a number of open questions and rather perspective directions of research. First, it concerns the absence of satisfactory criteria of existence for classical structures of combinations of various types of the nonconstructability according to table **2** in its complete volume. The results represented in the given section cover only some part **(***though enough considerable part***)** of the given question, whereas the more detailed information relative to the nonconstructability problem can be found in literature quoted in the present section and in the extensive enough bibliography [536]**.** The further research of the problematics has allowed us to introduce a new concept of the ***MEC*** as a some basis of the generalized criterion of the nonconstructability in classical structures ***d–HS (d≥1)*** [5,8,9,54–56]**.** Finally, the results obtained from researches of the nonconstructibility problem, can be considered as a sufficiently effective tool for studying of the dynamic properties of the classical homogeneous structures.

2.4. The nonconstructability problem for structures on splitting and finite homogeneous structures

Along with classical ***HS***–models the large enough applied interest the so–called finite ***HS***–models represent as well which consist of any but finite number of elementary *z*–automata. The given class of structures from the theoretical standpoint is most intensively investigated by the Japanese mathematicians [135,185,230], and also by a number of other researchers [1,3,9,138,147,148,158,159,161,166,170,171,173,175–179,205, 267]. Our results in the given direction are presented in monographies [1,3–5,9,54–56,90]. Researches in this direction are perspective enough, having in mind numerous applied aspects of class of *finite* ***HS***-models, first of all at their use as the *parallel* discrete models of various objects.

In the previous sections of the chapter the *nonconstructability* problem has been considered concerning the infinite classical ***HS***-models, but it takes place and for finite ***HS***–models, however with essential enough differences on which accent is being done in the present section. The given problematics is presented more in details, first of all, in works of the Japanese school [135,230] on finite ***HS***–models and in a lot of other works [156,171,175,184–187,240], but here we for the first time shall try to carry out comparison concerning the *nonconstructability* problem of *infinite* and *finite* ***HS***–models. First of all, a finite ***HS***–model represents the finite automaton with a certain *specific* internal organization which does it as a rather convenient model in a lot of interesting appendices. A finite ***HS***–model is similar to some finite automaton without inputs which processes internal states (*global configurations*) under influence of a global transition function at discrete moments ***t***, and its output at a moment ***t > 0*** corresponds to its internal state at same moment ***t***. As a matter of fact, each finite ***HS***–model is one of examples of the above automata of Moore with own *specific internal* organization. In addition concerning the nonconstructability problem some results for the finite ***HS***–models are presented below, here we shall represent only a result which is directly connected to the *general* nonconstructability problem in classical structures having infinite number of elementary automata.

<u>Theorem 42</u>. If a GTF $\tau^{(n)}$ causes the existence of NCF, NCF–3 and/or NCF–1 then there is wide enough class of finite closed structures d-HS with GTF $\tau^{(n)}$ that possess the NCF_f, and vice versa. Among set of all NCF_f direct and indirect analogues of the nonconstructability such as NCF, NCF–3 and NCF–1 are directly established.

The result of this theorem is a certain spreading of the results received by us earlier to case of the nonconstructability such as ***NCF–3*** [3,55,56] however, generally speaking, this theorem has not place for the finite structures**;** above all it concerns the nonconstructability such as ***NCF-1*** since it is directly linked with existence of infinite predecessors.

Number ***N*** of global configurations of a finite ***HS***–model is finite and equals to $N=a^m$, where ***m*** – number of elementary automata of Moore which compose the ***HS***–model, and ***a*** – cardinality of their alphabet ***A***. Hence, each global configuration ***(CF)*** of a finite ***HS***–model is a certain mapping ***CF:*** $Z_m^d \Rightarrow A$; Z_m^d is a finite connected block of ***m*** elementary automata of a homogeneous space Z^d similarly to the classical infinite ***HS***-models. At that, in consequence of finiteness of such ***HS***–model an uncertainty in the field of its *boundary* elementary automata **(***according to its neighbourhood index***)** arises at application to them of ***LTF*** $\sigma^{(n)}$, by demanding of specifying of appropriate boundary conditions **(***a block of boundary automata together with its configuration***)**. Specifically, a finite ***(mxn)***–rectangular ***2–HS*** with Neumann–Moore neighbourhood index requires determination of configuration of elementary automata into one layer surrounding body of the ***2–HS (fig. 10)***.

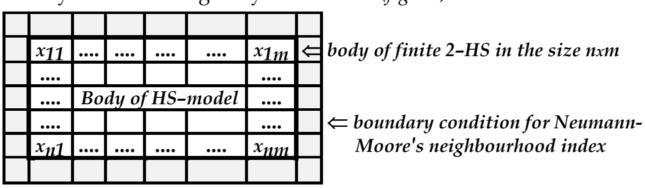


Fig. 10. An organization of a finite 2–HS with stiff border.

The configuration of boundary automata can be both the *constant*, and the *variable*, simulating a certain interaction of a finite ***HS***–model with an environment. In particular, one of ways of modelling in structures is creation for them of finite blocks simulating work of those or other devices, including communication channels between them. So, ***J. von Neumann*** and a lot of his followers have acted in this way at research of the first cellular models – of prototypes of modern ***HS*** [124,125,128, 132,536]. Above described by us a way of determination of boundary conditions we shall name ***stiff***. Convolution of a finite homogeneous environment, achieved by a «***gluing***» of its opposite borders, is second way of determination of boundary conditions. At that, obviously, this approach can be illustrated by example of finite ***1-HS***, whose left edge incorporates (*is glued*) with right edge, namely:

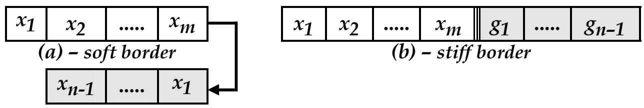


Fig. 11. Determination of soft and stiff boundary conditions for case of the finite structures 1–HS.

In case of organization of *soft boundary conditions* (*fig.* ***11, a***) connection of elementary automata of a ***HS***–model is being not interrupted and behind its ***m***–th automaton the first follows at once, i.e. cyclic scheme of communication of elementary automata of the model is organized. While in case of stiff boundary conditions its boundary g_k–automata (*in quantity determined by its neighbourhood index* ***X***) join on the right (*at the left*) from the extreme automata of the model (*fig.* ***11, b***). If in case of soft border its configuration is variable, then in case of rigid border it can be both by the permanent configuration, and the variable.

In view of told, we can define without loss of generality a finite ***d–HS*** ($d \geq 1$) as the ordered six $HS_G^m \equiv \langle Z^d, A, \tau^{(n)}, X, m, G\rangle$, for which the first four components are defined similarly to case of classical infinite ***HS***–model; ***m*** – the size of edge of ***d***-dimensional hypercube of elementary automata of space Z^d (*body of the model*), ***G*** – boundary conditions (*way of determination of boundary automata and their configurations*). At that, a configuration of all elementary automata composing its body will be understood as the *global* state of HS_G^m–model. A set ***C(A,d,m)*** of global states of such finite model is named the ***complete*** if the given model at the initial moment $t = 0$ admits any possible global body configuration determined in a state alphabet ***A*** as an initial condition, irrespective of boundary conditions of a HS_G^m–model. Obviously, the set ***C(A,d,m)*** of global states of a HS_G^m-model is finite and its cardinality equals $N=a^m$.

If a HS_G^m–model does not possess the nonconstructible configurations ***(NCF)*** then, using all configurations from the set ***C(A,d,m)*** as the *initial* configurations, at the following moment we can receive the full set of configurations; i.e. the mapping $\tau^{(n)}: C(A,d,m) \Rightarrow C(A,d,m)$ takes place, namely: global transition function of the model maps the set ***C(A,d,m)*** onto itself. Fig. ***12 (a)*** evidently enough illustrates the told. While, if at the made assumptions certain global configurations (*states*) of a model remain unattainable, then they are named the ***nonconstructible (NCF);***

in addition, the following mapping ***τ$^{(n)}$: C(A,d,m) ⇒ C⊂C(A,d,m)*** takes place, i.e. global transition function of such HS_G^m–model maps the set ***C(A,d,m)*** into itself ***(**fig.* ***12, b).***

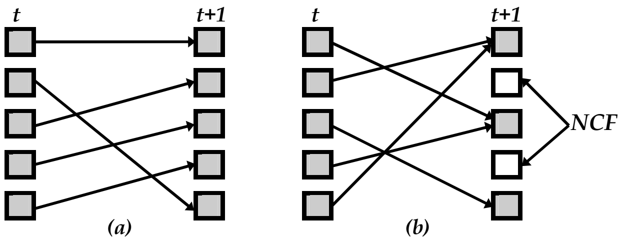


Fig. 12. Illustration of the nonconstructability existence such as NCF for the finite HS–models.

Below, we will show that the given condition, generally speaking, and determines a nonconstructability criterion for finite HS_G^m–models:

A finite HS_G^m–model possesses the non-constructible configurations if and only if for such model the mapping τ$^{(n)}$: C(A,d,m) ⇒ C* ⊂C(A,d,m) takes place, where τ$^{(n)}$ is a global transition function of the model.

Thus, the criterion has rather general character that fully corresponds to our general concept of ***mutual erasability*** **(***definition* **6).** In addition, the essential distinction in dynamics of infinite and finite classical ***HS***–models become apparent already at such level of their possibilities as existence for them of universal configurations, considered below. So, if the infinite ***HS***-models do not admit configurations of such type, the finite models can have *single* universal configuration, and *all* universal configurations. One of criteria of existence of nonconstructability such as ***NCF*** for infinite ***HS***–models consists in the existence for them of the pairs of ***MEC*** in ***Moore-Myhill*** sense **(***definition* **7).** But for case of finite ***HS***–models the given criterion has not place generally speaking, being based only on our most general concept of erasability. Therefore, here it is necessary to use other certain approaches for investigations of the nonconstructability. As a whole, it is necessary to mark, that the given nonconstructability questions along with questions of reversibility of dynamics of the finite ***HS***–models are not so and simple [54,88,90,536].

Let's consider two classes ***AG ≡ <Z,A,τ$^{(n)}$,X,m,G1>*** and ***VS ≡ <Z,A,τ$^{(n)}$, X,m,G2>*** of the finite models, where ***G1*** and ***G2*** represent *soft* and *stiff* boundary conditions of models accordingly ***(**fig.* ***11), A* = {*0,1,2, ..., a–1*},**

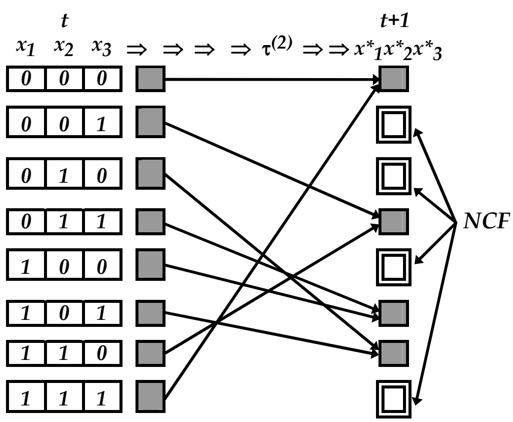
X **= {*0,1,2,3, ..., n–1*}** and ***GTF*** $\tau^{(n)}$ for models of both classes are defined by linear local functions $\sigma^{(n)}(x_1,x_2,...,x_n) = \sum_k x_k \ (mod\ a)$, $x_k \in A \ (k=1..n)$.

In consideration of the definition of soft and stiff boundary conditions for a finite ***1–HS*** by length ***m,*** and also linearity of ***LTF*** $\sigma^{(n)}$ for models of both classes, a state x_k^{t+1} of an automaton x_k of models from classes ***AG*** and ***VS*** in the moment ***t+1*** $(t \geq 0;\ k=1..m)$ is calculated according to the following simple formulas ***(10–11)*** accordingly**:**

$$\begin{cases} x_k^{t+1} = \sum_{j=0}^{n-1} x_{k+j}^t \ (mod\ a); \quad 1 \leq k \leq m-n+1 \\ x_k^{t+1} = \sum_{j=k}^{m} x_j^t + \sum_{j=1}^{n-m+k-1} x_j^t \ (mod\ a); \quad m-n+1 < k \leq m \end{cases} \quad (k=1..m) \qquad (10)$$

$$\begin{cases} x_k^{t+1} = \sum_{j=0}^{n-1} x_{k+j}^t \ (mod\ a); \quad 1 \leq k \leq m-n+1 \\ x_k^{t+1} = \sum_{j=k}^{m} x_j^t + \sum_{j=1}^{n-m+k-1} g_j^t \ (mod\ a); \quad m-n+1 < k \leq m \end{cases} \quad (k=1..m) \qquad (11)$$

where formulas ***(10)*** and ***(11)*** are linked with HS_G^m–models of the first and second class accordingly**,** that for convenience are denoted simply as ***AG≡<a,n,m>*** and ***VS≡<a,n,m>*** accordingly**.** Then the direct testing confirms existence for model ***AG(2,3,2)*** of *four* ***NCF*** what the following scheme evidently enough illustrates.

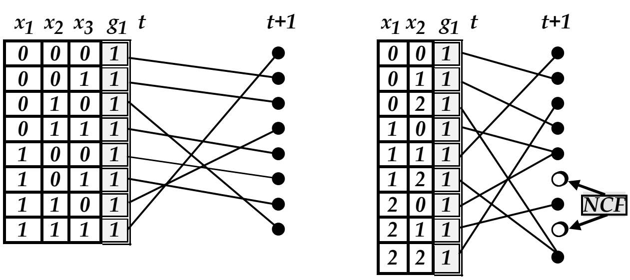


Thus, it is possible to show, that in general case for models ***AG(2,m,2)*** exists exactly $N = 2^{m-1}$ $(m \geq 2)$ of ***NCF*** and with growth of value ***m*** the quota of ***NCF*** for such models tends to $q = 1/2$. The similar situation is valid and for models ***AG(2,m,2)*** if instead of an elementary automaton x_1 the automaton x_{m-1} is used as their soft border ***G1;*** i.e. to carry out their counting out in the order, opposite to accepted at convolution of *finite* ***HS-***model ***(****fig.* ***11.a).*** However, already for models ***AG(3,m,n)*** the situation is completely different, namely**:** if for model ***AG(3,n,n)*** exists $N=a^n-a$ of ***NCF,*** then already for model ***AG(3,3,2)*** the ***NCF*** are absent. Direct check establishes the absence of ***NCF*** both for model ***AG(2,4,2),*** and model ***AG(2,5,3).*** So, in the class of the ***AG*** models, whose linear ***LTF*** do not support existence of pairs of ***MEC*** the criterion of ***Moore–Myhill*** of existence in the models of ***NCF*** not takes place, namely**:** ***In the absence of pairs of the MEC in Moore–Myhill's sense, the models of this class can both possess and not possess the nonconstructability such as NCF.***

Let's consider now a case of finite HS_G^m–models with stiff boundary conditions ***G2*** and linear local transition functions of the above kind**;** i.e. class of models ***VS≡<Z,A,τ***$^{(n)}$***,X,m,G2>.*** Let's analyse the dynamics of two simple structures ***VS1≡<3,2,2>*** and ***VS2≡<2,3,2>*** with identical boundary conditions ***G2*** ***(****only one automaton is in state* ***«1»)*** as concrete examples of models of such type.

In addition, if for the first structure the ***LTF*** $\sigma 1^{(2)}(x,y) = x+y \ (mod\ 2)$ is used, then for the second structure local transition function $\sigma 2^{(2)}(x,y)$ is used, which is defined by following rather simple formula, namely**:**

$$\sigma 2^{(2)}(x,y) = \begin{cases} y, \ if \quad x=0 \\ x+y \ (mod\ 2), \ if \quad xy \in \{10, 11, 22\} \quad x,y \in A = \{0,1,2\} \\ x+y \ (mod\ 2)+1, \ \ otherwise \end{cases}$$



It is easy to be convinced that local transition functions $\sigma 1^{(2)}$ and $\sigma 2^{(2)}$ completely exclude the existence for the corresponding structures of pairs of ***MEC*.** However, that does not guarantee absence for the finite structures of the nonconstructability such as the ***NCF*.** The schemes of transitions, represented above, of global states for the finite structures are transparent enough and do not demand any special elucidations.

So, for case of finite models with *stiff* boundary conditions the absence of pairs of ***MEC*** does not guarantee the absence of nonconstructability too. Thus, in general case of finite HS_G^m–models the existence of ***MEC*** can be as sufficient condition but not necessary condition for existence for them of the nonconstructability such as ***NCF*.**

Moreover, the nonconstructability problem for the finite ***HS***–models is very closely connected to type of boundary conditions. For example, if models ***AG(2,m,2)*** with soft boundary conditions ***G1*** possess the ***NCF*,** then the appropriate models ***VS(2,m,2)*** with stiff boundary conditions ***G2*** do not possess the nonconstructability property. At that, the model ***VS(2,2,2)*** with boundary condition ***g1 = 1*** has all global configurations as *universal configurations* ***(UCF)*,** while with boundary condition ***g1 = 0*** the model possesses neither ***NCF*** nor ***UCF*.** Moreover, it is possible to show, that validity of the statement about existence of $N = 2^{m-1}$ ***NCF*** for models ***AG(2,m,2)*** remains in force and for models ***VS(2,m,2)*** if the boundary conditions of some periodic types and of some other types are given as their stiff boundary conditions ***G2*.** It is shown that finite structures with soft borders whose boundary conditions are variable can have one or all global configurations as universal configurations**;** in addition, in the first case each configuration different from the sole universal configuration generates all global configurations excluding the universal one. Many very interesting properties of finite structures were received by us on basis of computer modeling with use of some programming languages, and packages *Maple* and *Mathematica***.** Many fragments for that can be gathered of our programs simulating certain dynamic aspects of classical ***HS***–models [6,15,54–56,65,67,74,85,87,118, 545]**,** and also from other interesting works [166–168,236,243,280,536]**.**

For a *reversible* finite ***HS***–model its injective global mapping should be as well bijective**.** Meanwhile, if global mapping of a finite ***HS***–model is injective, it does not entail obligatory reversibility of its dynamics. So, dynamics of the finite ***HS***-models is *reversible***,** if their global mappings are bijective. In general case, it is enough non-simply to determine the reversibility of the finite ***HS***-models and more in detail it is possible to familiarize oneself with the given questions of dynamics of the finite ***HS***–models in a series of interesting enough works of ***M. Harao*** and ***S. Noguchi*** [536]. In addition, in case of linearity of global mappings $\tau^{(n)}$ the *reversibility* problem becomes much more accessible. Discussion of properties of linear or additive ***HS***–models can be found in interesting works of ***O. Martin*** and ***K. Morita,*** and of a series of other researchers [5,536]. Consequently, concerning the nonconstructability problem the finite and infinite ***HS***–models define essentially different classes of the parallel dynamic cellular systems, stimulating the further research in the given direction which seems to us rather interesting from a whole series of standpoints, above all, the applied character.

Meanwhile, with growth of the size of finite ***HS***–models the classical criteria of existence of non–constructible configurations such as ***NCF*** start to play the more and more increasing part, by basing on concepts of ***MEC*** *(in sense of* ***Moore–Myhill*** *and our)* and ***γ–CF.*** In particular, in case of sizes exceeding the minimal sizes of ***NCF*** to the ***HS***-models the classical nonconstructability criteria such as ***NCF*** are quite applicable.

The *homogeneous structure on splitting* ***(HSoS)*** is defined as the ordered tuple of five base component ***HSoS*** $\equiv$ $<Z^d,A,m,\Psi^{(h)},\Xi>$ where the first two components Z^d and ***A*** are similar to case of classical ***HS***–models; ***m*** – size of an edge of a ***d***–dimensional *hypercube* into which the space Z^d of the structure is broken; $\Psi^{(h)}$ – local block function of transition ***(LBF;*** $h=m^d$***)***; Ξ – rules of switching of blocks of space Z^d of structure. Functioning of the structures ***d–HSoS*** is simple enough and has been considered enough in detail in section ***1.2.*** In the same place, a certain comparative analysis of both types of models ***(HS*** *and* ***HSoS)*** has been carried out. At present, structures such as ***HSoS*** find wide application, above all, for a lot of interesting tasks of physical modeling, by having of *software* and *hardware* on the above ***CAM***–machines which are based on computing ***HS***–models [165,376,394,430,536,545,618,640–643].

Consequently, research of the nonconstructability problem for case of ***HSoS***–models seems interesting enough. Meanwhile, from standpoint of the nonconstructability problem between classical ***HS*** and ***HSoS*** the certain differences exist. Above all, for ***HSoS***–models the classification of the nonconstructability, similarly to case of the classical ***HS***–models ***(NCF, NCF–1, NCF–2*** *and* ***NCF–3)*** is not valid. Indeed, according to the definition ***4*** a finite configuration ***c**** is ***NCF-1*** in a classical ***d–HS*** $(d\geq 1)$ if and only if for it there are predecessors only from the set $C(A,d,\infty)$ of the infinite configurations. That implies nonclosure of the set ***C(A,d,∞)*** relative to a global transformation $\tau^{(h)}$ of classical ***HS***–model.

On the other hand, from definition of a ***HSoS***–model directly follows, the non-closure of the set ***C(A,d,∞)*** concerning a global transformation $\tau^{(h)}$ determines the need for existence of substitutions of the following kind $(\exists x_j \neq 0)(x_1 x_2 x_3 \ldots x_h \Rightarrow 000 \ldots 0)$ among parallel block substitutions determining ***LBF*** $\Psi^{(h)}$ of the model. Hence, such mapping $\Psi^{(h)}: A^h \Rightarrow A^h$ will not be by *biunique* mapping, therefore the ***HSoS***–model should possess the nonconstructability such as ***NCF,*** i.e. the most *general* type of nonconstructability in ***HS***–models. At that, the existence in a ***HSoS***–model of configurations such as ***NCF–1*** with need entails the existence in it of the ***NCF*** also, i.e. that is a *sufficient* condition of existence in the ***HSoS***–models of nonconstructability such as ***NCF.***

Generally speaking, existence for an arbitrary ***HSoS***–model of *mapping* of two different states of by means of ***LBF*** $\Psi^{(h)}$ into the same state we entirely can consider as the existence for such model of a pair of ***MEC.*** Consequently, the criterion of existence of nonconstructability such as ***NCF*** for ***HSoS***–models can be formulated as follows, namely**:**

An arbitrary d–HSoS possesses the nonconstructability such as NCF if and only if for the d–HSoS the pairs of MEC in the general sense exist, i.e. in concordance with definition 6.

It is simple to be convinced that quantity of ***HSoS*** ≡ $<Z^d,A,m,\Psi^{(h)},\Xi>$, which not possess the pairs of ***MEC*** and hence of ***NCF,*** equals $\left(a^{m^d}\right)!$ whereas their quota concerning all structures of the given type equals $\Delta = \left(a^{m^d}\right)! \Big/ a^{m^d a^{m^d}}$, i.e. enough quickly approaches to *zero* already for small enough values ***a, m, d.*** Thus, and in the class of structures ***HSoS*** the structures possessing the reversibility property to a certain extent are «***exotic***»***.*** On the other hand, absence for some ***HSoS***–model of the nonconstructability of type ***NCF*** entails also closure of the set ***C(A,d,∞)*** relative to global transformation $\tau^{(h)}$ of the model, and hence, absence for it of ***NCF–1.*** While for the *classical* ***HS***–models the given statements generally speaking are incorrect. At that, it is simple enough to make sure, that the following result takes place, namely**:**

The closure problem of the set C(A,d,∞) (d ≥ 1) concerning a global $\tau^{(h)}$ ***transformation determined by local block function*** $\Psi^{(h)}$ ***of a structure d–HSoS is algorithmically solvable while a set of NCF for the d–HSoS***

is recursive. Nonclosure of the set C(A,d,∞) concerning mapping which is determined by local block function $\Psi^{(h)}$ of an arbitrary structure d–HSoS causes the presence for the structure of the nonconstructability such as NCF while the opposite assertion, generally speaking, is false. Consequently, for ***HSoS***–models the existence of ***NCF–1*** without ***NCF*** is impossible. On the other hand, for a classical ***HS***–model the types of nonconstructability ***NCF*** and ***NCF–1*** are not equivalent, in the absence in it of ***NCF*** the model can possess a ***NCF–1***. At that, nonclosure of the set ***C(A,d,∞)*** concerning the global transformation $\tau^{(n)}$ of an arbitrary classical ***HS***–model is a criterion of existence in the model of ***NCF–1*** in case of absence in it of ***NCF*** (*theorem* **29**). And if the existence problem of nonconstructability such as ***NCF*** for *general* case of classical ***d–HS*** ***(d ≥ 2)*** is algorithmically unsolvable, then in a class of ***d–HSoS*** ***(d ≥ 1)*** the given problem is algorithmically solvable, and a constructive solution algorithm is reduced to ascertainment of existence/absence of mutual unambiguity of a mapping $\Psi^{(h)}: A^h \Rightarrow A^h$.

The one–to–oneness of local mappings $\Psi^{(h)}: A^h \Rightarrow A^h$ of ***HSoS***–models is *criterion* of absence of ***NCF*** for them; at that, the *nonconstructability* is determined at once on blocks of size ***(mx...xm)*** of elementary automata of the models while a set of all ***NCF*** for any ***HSoS***–model is recursive. It is shown that in general case ***the nonconstructability property such as NCF relative to mutual modeling of structures d–HS and d–HSoS is not invariant*** [9,88,260,536]. In addition, a modeling of an irreversible structure by appropriate reversible structure is quite allowable.

Of the arguments represented above follows, being based on the same nonconstructability definition such as ***NCF***, we receive that its causal–investigatory bases for the classical ***HS***–models and ***HSoS***–models are essentially different. The given *difference* underlies serious distinctions of a lot of fundamental dynamic properties of the classical ***HS***–models and ***HSoS***–models, and causes essentially large demand of the second for problems of modeling of processes and phenomena which need in the reversibility property of their dynamics [5,54,79,88,90,536,567,640].

Once more it is expedient to note that in comparison with the classical structures one of fundamental criteria of the nonconstructability such as ***NCF*** for finite structures and ***HSoS*** is based on the general concept of mutually erasable configurations (*definition* **6**), instead of mutually erasable configurations in sense of ***Moore–Myhill*** (*definition* **7**). Indeed, for classical structures our concept of mutual erasability is based on a pair of different finite configurations that by global transition function of a structure is mapped into the same finite configuration what in the full measure corresponds to absence of one–oneness for mappings of finite configurations of body of a finite structure by means of its local transition function and block of fragmentation of space a ***HSoS*** by its local block function of transition.

Properties of the parallel mappings such as surjectivity and injectivity which are determined by global functions of transition $\tau^{(n)}$ of the ***HS***–models have the most *direct* attitude to the *nonconstructability* problem and they play fundamental role at studying of dynamic properties of ***HS***–models. A series of researchers has worked in this direction, and a lot of interesting enough results has been received [1,3,5,9,19,20,25,28–30,53,54–56,68,131,268-277,279,311,317,396]. In particular, within of the given researches ***G.A. Hedlund*** [127] has studied the given theme for a ***1***–dimensional case of dynamic shift systems both in combinatory and topological aspects. ***M. Nasu*** has investigated the further combinatory aspects of local mappings determined by surjective global mappings along with local mappings determined by injective global mappings in ***1–HS*** [311]. ***A. Maruoka*** and ***M. Kimura*** have researched four new properties of parallel mappings in ***HS***–models: ***strong R–surjctivity*** and ***weak R–surjctivity*** along with ***strong R–injectivity*** and ***weak R–injectivity*** [522]. Of them the first two concepts are not equivalent to concepts known earlier, filling interspace between the bijectivity and the surjectivity. On the other hand, it has been proved that other two concepts are equivalent to the surjectivity. At that, these concepts are characterized by the strengthened balanced conditions introduced in work [271] which are equivalent to our concept of the ***γ–CF*** [5,37,640].

Basic results of ***Maruoka–Kimura*** in the given direction can be easily generalized to classical ***HS***–models and with an arbitrary dimension, and with neighbourhood indexes of the more general types [41]. With some other interesting results concerning the *surjectivity* and *injectivity* of global parallel mappings in the ***HS***–models the reader can acquaint oneself in the works presented in the extended bibliography [536,640].

By completing by the present section a discussion of the basic results concerning the general nonconstructability problem in classical ***d–HS*** ***(d≥1)***, we shall dwell on features of the problem in connexion with the reversibility problem of dynamics of the ***HS***–models which represents important enough both the theoretical, and the applied interest [54-56, 88,536]. At that, the ***reversibility*** is understood by us as unambiguity of reverse dynamics of finite configurations in the classical structures.

2.5. The reversibility problem of dynamics of the classical homogeneous structures *(HS–models)*

Reversibility of ***HS***–models is one of the major properties, first of all, from the point of view of the theory of calculations and simulation of multifarious physical processes which is a closely enough connected to presence for the models of nonconstructability such as ***NCF***, first of all. Meantime, here, perhaps, certain remarks of the general character which concern the reversibility problem as a whole which, in turn, is closely connected to the *nonconstructability* problem for ***HS***–models as a whole and for the classical ***HS***–models in particular would be quite pertinent. So, at a formal level the *reversibility* problem of an arbitrary function ***F*** from ***n*** variables $\{x_1, x_2, ..., x_n\}$ is reduced to a question of an opportunity of one–valued restoration for it of a tuple $<x_1, x_2, x_3, ..., x_n>$ according to known kind of function ***F*** and its value $F(x_1, x_2, x_3, ..., x_n)$ on the sought tuple.

Naturally, on ***n*** inputs and ***(n–k)*** $\{k=1..n-1\}$ outputs of some algorithm provided that they belong to the same finite alphabet, to achieve such type of reversibility impossibly. Therefore, along with a result $F(x_1, x_2, ..., x_n)$ we should have ***(n–1)*** values of tuple $<x_1, x_2, ..., x_n>$ for *restoration* of missing value x_j; $j \in \{1, 2, 3, ..., n\}$, i.e. we should have some additional information, allowing on basis of kind of the function ***F*** and its value on the tuple, to restore all sought tuple. In principle, a few other ways for receiving of such additional information can be used. So, the next scheme enough evidently illustrates the given aspect.

$$\overbrace{\begin{bmatrix} x_1, \\ x_2, \\ ..., \\ x_n \end{bmatrix}}^{t=0} \;\Bigg|\; \rightarrow \underbrace{\overbrace{\underbrace{F\left(\begin{bmatrix} x_1, \\ x_2, \\ ..., \\ x_n \end{bmatrix}\right)}_{irreversibility} \;,\; \bigcup\{x_{k_j} \mid j = 1..n-1\};\; k_j \in \{1, 2, ..., n\}}^{t=1}}_{reversibility}$$

As a concrete example we shall consider logic ***XOR***–function defined as follows, namely:

a	*b*	*a XOR b*
0	*0*	*0*
0	*1*	*1*
1	*0*	*1*
1	*1*	*0*

Obviously, the given function is irreversible, because, for example, the value «*1*» for the function we can receive on two various tuples ***<0, 1>*** and ***<1, 0>.*** However, if on an output we give in addition also one of values, for example, «***a***»**,** then we will receive the following relations between inputs and outputs of the function, namely**:**

a b		***a r***	***a b***		***a r***	***a b***		***a r***	***a b***		***a r***
0 0	→	***0 0***	***0 1***	→	***0 1***	***1 0***	→	***1 1***	***1 1***	→	***1 0***

where ***a*** and ***r –*** values of ***a*–**input and result ***a XOR b*** accordingly. Of the above relations obviously, that one–to–one dependence between **2** inputs and **2** outputs takes place, providing possibility of one–valued restoration of ***b*–**input in the form of value ***a XOR r.*** A *Boolean* function with ***n*** outputs and ***n*** inputs is called the *reversible* if it maps any input tuple of values into a single output tuple. In view of told, it is simple to notice, that any of standard logic functions ***(****excepting unary function* ***NOT)*** is non–reversible**;** i.e. by its result a tuple of source variables is not defined uniquely**.** Meanwhile, it is simple to be convinced, that for the given type of functions it is possible to provide the reversibility by a combination of their results and value of one of inputs. So, in quality of an quite natural condition of providing of reversibility for a system is support of equal quantity of inputs and outputs.

We can achieve this requirement in the various ways, and now large enough amount of various reversible logic gates is specifically offered. So, logic reversible gates ***NOT, CNOT, SWAP,*** of ***Toffoli*** and ***Fredkin*** are used most widely. As a matter of fact, in any logic gate at which the amount of inputs exceeds amount of outputs, a loss of information will be inevitable to happen as it is impossible to determine the states of inputs on the basis of states of its outputs.

The reversibility problem of calculations and creation of the reversible computers is especially urgent today, when serious works on creation of new architecture of computer equipment with orientation onto the manufactures using ***nanotechnology*** have been begun**.** So**,** calculations which are carried out on modern computers are being done by means of the irreversible operations erasing the information.

Meanwhile, already in ***1973 C. Bennet*** has shown that at calculations it is possible to do without erasing of information and *irreversible* logical elements. Later, validity of the given position has been shown on a lot of computational models. In particular, we note here the logic gate of ***E. Fredkin*** which has three input lines and three output lines [3,536]**.** It does not lose the information since a state of inputs can be determined always by state of outputs. ***E. Fredkin*** has shown that any logic device necessary for operation of a computer can be created in the form of a corresponding *combination* of such reversible gates. Of a series of other interesting reversible computing models it is possible to note «*billiard*» computer of ***Fredkin–Toffoli*** along with certain others [536,545,640].

The above *reversible* computing models are based, mainly, on classical dynamics and electronics, however a series of researchers has offered also some other models of reversible computers basing on principles of quantum mechanics. Inherently, particles in such models should be located so that *quantum mechanics* rules controlling their interaction in accuracy were similar to rules predicting values of signals on outputs of reversible logic gates. The given models do not scatter energy and submit to laws of quantum mechanics. More in detail the reader can familiarize oneself with reversible models of similar type in the list of original sources [536,545,618,640–643].

The aforesaid relates, mainly, to the information processing. However computer should not only process the information, but also remember it. So, the interrelation between storage and information processing in the best way, perhaps, can be described by means of Turing machine, that in computing *attitude* can model any modern computer as well as solve any problem. ***C. Bennet*** has proved possibility of construction of reversible Turing machine, i.e. such machine which does not lose any information and for the given reason during own work can consume any preset small quantity of energy. However not all Turing machines are reversible, however it is quite possible to construct the reversible Turing machine capable to execute any given calculation [3,88,90,536]. At that, for creation of *reversible* computing models the approaches on biomolecular and chemical bases are offered. So, enzymatic reactions which are well known in genetics are reversible. Thus, it is shown that a hypothetical enzymatic Turing machine can perform computations with arbitrary small expense of energy. With an interesting model of such reversible Turing machine, and also with interesting discussions from the physical point of view relative to reversible calculations and *reversible* computers the reader can familiarize oneself in the extended bibliography [536]. Here, above all, we recommend to pay the special attention on in many respects pioneer ideas and works of researchers such as ***R. Landauer, C. Bennett, E. Fredkin, T. Toffoli, M. Margolus, K. Morita*** along with certain others.

Meanwhile, in connexion with application of ***HS***-models as the formal and perspective prototypes of computational models, the questions of dynamics reversibility of the given models also lay in a channel of the marked researches. Problems of dynamics reversibility of ***HS***–models play extremely important part, first of all, from point of view of their use in quality of simulation environment for various phenomena and processes, first of all, of physical character, and also in quality of some prototypes of perspective computational devices supporting *reversible* calculations and supposing use nanotechnology. In the given attitude one of the major aspects of researches on classical ***HS***–models falls on the reversibility problem of their dynamics.

At present a series of interesting classes of the ***HS***–models possessing the general property of reversibility among which it is possible to note the above ***HSoS***–models for the first time introduced by ***N. Margolus*** [388,536] and widely used by him in collaboration with ***T. Toffoli*** for modeling of certain reversible processes [150–152,273,376,430,536,640] along with the reversible ***HS***–models specially designed by ***T. Toffoli*** and researched from point of view of *computational* and *constructional* universality [187,268,318,536]**.** A series of works has been devoted to various questions of reversibility of ***HS***–models of various types and classes [536]**.** So, ***D. Richardson*** [305] has proved, that a classical ***HS***–model is reversible if and only if its global mapping defined by ***GTF,*** is injective. However the topological approach used for that does not give a constructive algorithm for a providing of immediate inversion. From the theoretic-automaton point of view the approach to the given problem can be found at ***K. Culik*** [536]**.** ***S. Amoroso, Y. Patt*** [270] and ***V. Aladjev*** [19,20,88] have proved existence of the effective procedure that decides the *reversibility* problem for classical ***1–HS.*** At that, on the other hand, ***J. Kari*** [277] and ***V.Z. Aladjev*** [88] on the basis of various approaches have proved unsolvability of the reversibility problem for ***d-HS*** ***(d≥2).*** At that, it is necessary to note that receiving of *constructive* algorithm of determination of reversibility of a classical ***d–HS (d ≥ 2)*** is a difficult enough problem even under the condition of its existence.

Meanwhile, for whole series of cases of linear and some special types of ***d–HS (d ≥ 2)*** such constructive algorithms exist. So, ***G. Manzini*** and ***L. Margara*** have deduced a formula for reversibility of linear classical ***d–HS; T. Sato*** has given an example of algorithm for determination of reversibility in case of one special class of ***d–HS*** whereas ***K. Sutner*** has given an example of algorithm providing determination of existence of *reversibility* and *surjectivity* of global mapping for linear ***d–HS (d ≥ 2)*** at quadratic time [536]**.** ***K. Morita*** has proved existence of a reversible

1–HS which model an arbitrary *1–HS*, including and irreversible ones, whereas ***J. Dubacq*** has proved an opportunity of simulation of Turing machines by means of reversible *1–HS* [536]. ***T. Toffoli*** has proved an opportunity of modelling of an arbitrary *d–HS* by means of reversible *(d+1)–HS*, having proved thus computing universality of reversible *d–HS (d≥2)* [268], whereas ***K. Morita*** and others have proved computing universality of *reversible* *1–HS* [79,321,322]. ***T. Toffoli*** and ***N. Margolus*** have presented a quite interesting review about reversible *HS*–models [273]. More in detail with the reversibility problem for *HS*–models the reader can acquaint oneself in the extended bibliography [536,545].

Thus, even within such general concept as *W*–modelling, in general it is impossible to model an arbitrary classical *d–HS (d≥1)* by means of a *reversible* structure of the same dimension. The given result serves as a rather serious argument in favor of the assumption of impossibility of such modeling. However beyond frames of finiteness of the alphabet of internal states, there is a possibility of modeling of the classical *d-HS* possessing the nonconstructability of *NCF*-type by means of structures of the same dimension but without this property. Naturally, structures with infinite alphabets of elementary automata can't be considered as the classical cellular automata however they a certain extent allow to estimate those limits outside of which such modelling is possible. So, without violation of community, we will consider a case of *1–HS* with alphabet $A = \{0,1,\ldots, a-1\}$ and neighbourhood index $X=\{0,1,\ldots,n-1\}$. It is obvious, if for the local transition function of such *1–HS* the following relation is carried out:

$$(\forall < x_1, x_2, \ldots x_{n-1} >)\left(x_o \ne x_o^* \to \sigma^{(n)}(x_o, x_1, \ldots x_{n-1}) \ne \sigma^{(n)}(x_o^*, x_1, \ldots x_{n-1})\right)$$
$$x_j, x_o^* \in A = \{0,1,2,\ldots,a-1\}; \qquad j = 0..n-1 \qquad (\psi)$$

then the given structure doesn't possess of pairs of *MEC* and *NCF*. We structure internal states of the elementary automaton of the simulated *1–HS** at two levels $<b/p>$, where $b\in A$, $p\in B=\{k*a+1 \mid k=1,2,\ldots\}$. At that, the set *B* is ordered in ascending order and its elements are numbered, since unit. At the made assumptions the local transition function of the simulated *1–HS** is defined as follows, namely:

$$\sigma_1^{(n)}: \begin{bmatrix} x_1x_2x_3\ldots x_n \\ y_1y_2y_3\ldots y_n \end{bmatrix} \to \begin{bmatrix} \sigma^{(n)}(x_1, x_2, x_3, \ldots, x_n) \\ \varphi(y_1 + x_1) \end{bmatrix}; \quad x_j \in A,\ y_j \in B,\ j = 1..n$$

where $\varphi(m)$ calculates an element of *B* with order number *m*, whereas $\sigma^{(n)}$ defines *LTF* of the simulated binary *1–HS*. Due to this definition can be seen, the *top* level of state of configuration $<c_o/\ldots(a+1)\ldots>$ of the modeling ***1–HS**** completely simulates dynamics of the structure ***1–HS*** for an arbitrary finite configuration $co \in C(A, 1)$. Now, proceeding from the made assumptions we will estimate correctness of the relation ***(ψ)*** relative to the modeling structure ***1–HS****.

$$\sigma_1^{(n)}: \begin{bmatrix} x_1x_2x_3...x_n \\ y_1y_2y_3...y_n \end{bmatrix} \rightarrow \begin{bmatrix} x_1^* \\ y_1^* = \varphi(x_1+y_1) \end{bmatrix} \equiv S_1^*$$

$$\sigma_1^{(n)}: \begin{bmatrix} x_1^1x_2x_3...x_n \\ y_1^1y_2y_3...y_n \end{bmatrix} \rightarrow \begin{bmatrix} x_1^{1*} \\ y_1^{1*} = \varphi(x_1^1+y_1^1) \end{bmatrix} \equiv S_1^1$$

$$x_1, x_1^1, x_1^*, x_1^{1*} \in A = \{0, 1, 2, ..., a-1\}$$

$$y_1, y_1^1, y_1^*, y_1^{1*} \in B = \{k*a+1 \mid k = 1, 2, 3, ...\}$$

Meanwhile, as the relation $\left\langle x_1 / y_1 \right\rangle \neq \left\langle x_1^1 / y_1^1 \right\rangle$ has to take place then to us it is enough to limit oneself to consideration of only three of four cases:

(1) $(x_1 = x_1^1) \& (y_1 \neq y_1^1)$ **–** it is obvious, in this case the following relation takes place, namely:

$$y_1^* = \varphi(x_1 + y_1) \neq y_1^{1*} = \varphi(x_1^1 + y_1^1)$$

i.e. the states $\langle S_1^*, S_1^1 \rangle$ of automata will be different $(S_1^* \neq S_1^1)$;

(2) $(x_1 \neq x_1^1) \& (y_1 = y_1^1)$ **–** in this case on the basis of the above function ***φ(m)*** a rather easily can be proved the following relation, namely:

$$y_1^* = \varphi(x_1 + y_1) \neq y_1^{1*} = \varphi(x_1^1 + y_1^1),$$

i.e. and in this case the states $\langle S_1^*, S_1^1 \rangle$ of the corresponding automata of the modelling structure ***1–HS**** will be different $(S_1^* \neq S_1^1)$;

(3) $(x_1 \neq x_1^1) \& (y_1 \neq y_1^1)$ **–** at last, in the given case we will proceed from the fact that the difference between two neighbour members of the set ***B*** is equal ***a***. Therefore, at simultaneous performance of two following conditions, namely:

$$(x_1 \neq x_1^1) \& (x_1, x_1^1 \in A) \text{ and } (y_1 \neq y_1^1) \& (y_1, y_1^1 \in B)$$

from the definition of the above function ***φ(m)*** also a rather easily to obtain that the following relation takes place, namely:

$$y_1^* = \varphi(x_1 + y_1) \neq y_1^{1*} = \varphi(x_1^1 + y_1^1)$$

Consequently, and in the given case the states $\langle S_1^*, S_1^1 \rangle$ of automata will be different $(S_1^* \neq S_1^1)$. Thus, the above condition ***(ψ)*** of absence for the modelling structure ***1–HS**** of pairs of ***MEC,*** and consequently of ***NCF,*** is entirely proved. In this connexion the following interesting enough proposal can be formulated, namely:

On condition of usage of the infinite alphabet of internal states of the elementary automaton exists d–HS (d ≥ 1) with such alphabet without nonconstructability of type NCF which in strictly real time simulates an arbitrary classical structure of the same dimensionality.

On account of importance of study of nonconstructability of NCF type on the basis of concept of pairs of *mutually–erasable* configurations, the questions of spread of classical concept of the cellular automata which simulate the classical cellular automata and not possess ***NCF*** represent undoubted interest. In this connexion we researched a whole series of extensions of classical cellular automata [618,635,638,641,644].

However, the question of modelling of an arbitrary classical ***d–HS*** by means of *reversible* ***d–HS*** still remains not up to the end the developed. So, our results concerning ***WM–*** and ***W–***modelling of a classical ***d–HS*** ***(d ≥ 2)*** definitely speak in favour of a rather essential complexity of the proof of fact itself of such modelling under the condition of operation with ***HS***-models at formal level and on basis of the criterion of ***Moore-Myhill*** of the nonconstructability existence ***(****theorem* ***16)*** [54,79,88,90].

In the most widespread understanding a reversible ***HS***-model is being understood as the ***HS***–model that not loses the information with time, namely: ***At each moment t > 0 the HS–model is completely reversible.*** Meanwhile, in general case for determination of similar reversible ***HS***-model of a special difficulty does not exist. With that end in view, it is quite enough to define local transition function $\sigma^{(n)}$ of a ***HS***–model as follows, namely:

$$z(t+1) = F(NT_z(t)) \# z(t-1) \qquad (12)$$

where $z(t)$ is state of elementary z-automaton of the model in moment $t \geq 0$, $NT_z(t)$ is a configuration of neighbourhood template with central z–automaton in moment t, F – a mapping $NT_z(t) \to A$, # – some binary operation and A – alphabet of the ***HS***-model. In the presented solution the number of inputs of the logic gate which represents an elementary automaton of the ***HS***–model equals to number of outputs, i.e. two; the following rather simple scheme well illustrates that, namely:

$$\begin{matrix} \ldots \to \\ \ldots \to \end{matrix} \begin{bmatrix} \sigma_{t-1}(NT_z) \\ z(t) \end{bmatrix} \begin{matrix} \to \\ \to \end{matrix} \begin{bmatrix} \sigma_t(NT_z) \\ z(t+1) \end{bmatrix} \begin{matrix} \to \\ \to \end{matrix} \begin{bmatrix} \sigma_{t+1}(NT_z) \\ z(t+2) \end{bmatrix} \begin{matrix} \to \ldots \\ \to \ldots \end{matrix}$$

Thus, we have an opportunity of one–valued restoring of a state of an elementary z–automaton of the model in the moment $t-1$ on basis of its state in the current moment $t+1$ and its value $\sigma_t(NT_z)$; i.e. its state in the moment t, namely:

$$z(t–1) = z(t+1)\ \#^{-1}\ \sigma(NT_Z(t))$$

where $\#^{-1}$ **–** an inverse function to the function # with values in ***A*****.** So, class of ***HS*****–**models, offered above we can entirely define as structures with memory, and their elementary automaton in the certain degree is similar to the above logic gate of ***E. Fredkin*** in which the states of the top level of the above scheme successfully carry out a role of a control channel, i.e. $\sigma_p(NT_Z)$ ***(p=t+k, k=±0, ±1, ±2, ±3,...).*** Thus, local transition function $\sigma^{(n)}$ can be arbitrary, providing an opportunity of definition of reversible ***HS*****–**models with wide enough set of local functions.

The binary structure ***1–HS*** with *discriminating* number ***122 (****alphabet* ***B=*** {***0,1***} *and neighbourhood index* **X=**{***-1,0,1***}***)*** is considered as an example of the above type of structures. It is simple to make sure**;** the structure ***1–HS*** which is defined thus, is *irreversible***,** because according to criterion on basis of ***γ–CF (****theorem* ***23)*** in the structure ***NCF*** exist. In addition, in such structure ***NCF–1*** already of simple kind ***c*** **=** ***11*** exist too.

Let's define now a new structure which is constructed on basis of the above structure ***1-HS*** and whose operating is defined by the following equations for rules of transition of its elementary automata. We need now only in proof that a structure ***1–HS,*** determined thus, should not possess the nonconstructability such as ***NCF,*** i.e. in traditional posing such structure is reversible, for example, a structure with function**:**

$$z(t+1) = \sigma^{(3)}(NT_Z(t))\ \ XOR\ \ z(t–1)$$

In many researches of ***HS*****–**models the set ***A*** **=** {***0,1,2, ..., a–1***}**,** formative a finite commutative ring relative to operations of multiplication and addition *modulo* ***a,*** is considered as an alphabet of internal states of an elementary automaton. For our purposes the operation of subtraction modulo ***a,*** which is inverse to addition operation in the given ring, is of interest**;** this operation for two sets ***B=***{***0,1***} and ***A=***{***0,1,2***} is defined by operating tables of the following kind, namely**:**

–	0	1
0	0	1
1	1	0

–	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

In general case of set ***A=***{***0,1,2, ..., a-1***} the table of *subtraction* operation modulo ***a*** is also simply derived. It is simple to be convinced, that the structure determined thus will be *reversible* what the following simple scheme well enough illustrates, namely**:**

$z(t-1)$	← 0 ←	← 1 ←
$\sigma^{(3)}(NT(t))$	← 0/1 →	← 0/1 →
$z(t+1)$	→ 0/1 →	→ 1/0 →

of which it is simple to draw a conclusion, that according to equation of functioning of an elementary ***z***–automaton of the structure on basis of information about states of the ***z***–automaton at moments ***t + 1*** and ***t*** it is possible to determine unambiguously its state in the moment ***t – 1,*** i.e. in such sense the above structure is reversible.

Let's present a history of the given structure during its three first steps for configuration c_{t-1}= ***111111*** ***(****an initial state****)*** in moment ***t-1*** and for initial configuration c_t = ***1000010*** , namely:

t–1	***... 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 ...***
t	***... 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 ...***
t+1	***... 0 0 0 0 1 0 0 1 1 0 0 1 0 0 0 0 ...***
t+2	***... 0 0 0 1 1 0 1 1 1 1 0 1 1 0 0 0 ...***
t+3	***... 0 0 1 1 0 0 1 1 1 1 0 0 1 1 0 0 ...***

It is natural that the above reversible structure is modeled strictly real time by means of a classical ***1–HS*** with an alphabet ***A*** of the structured states and with neighbourhood index ***X = {–1,0,1},*** however such ***1–HS*** will not be reversible. Indeed, such modeling structure is represented by both the structured alphabet ***A*** of states of its elementary automata and neighbourhood index ***X={–1,0,1}*** with the local transition function determined by parallel substitutions of the following kind, namely:

$$\begin{matrix} t-1 \\ t \end{matrix}\begin{bmatrix} S_{j-1} \\ S^*_{j-1} \end{bmatrix}\begin{bmatrix} S_j \\ S^*_j \end{bmatrix}\begin{bmatrix} S_{j+1} \\ S^*_{j+1} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma^{(3)}(S^*_{j-1}, S^*_j, S^*_{j+1}) \\ \sigma^{(3)}(S^*_{j-1}, S^*_j, S^*_{j+1}) \ XOR \ S_j \end{bmatrix}$$

$$S_j, S^*_j \in \{0,1\}; \ j = 0, \pm1, \pm2, \dots; \ t = 0,1,2,\dots$$

It is simple to show, the simulating structure ***1-HS*** is *classical* structure with neighbourhood index ***X={–1,0,1}*** and the *structured* alphabet ***A*** of cardinality ***4;*** at that, its first level of states defines a configuration of the modelled structure in moment ***t,*** while the *second* ***-*** a configuration in moment ***t–1 (t=1,2,...).*** At that it is supposed that in moment ***t=0*** the second level of states of this modelling structure determines a certain initial condition ***(****an initial configuration****)*** while the first level defines an initial configuration of the modelled structure immediately.

For proof of *irreversibility* of the modelling structure it is quite enough to specify for it a pair of ***MEC,*** what implies existence for the structure of the nonconstructability such as ***NCF*** and, consequently, absence of the reversibility property of its dynamics. On account of the aforesaid it is simple to be convinced of validity of existence for the modelling structure already pairs of ***MEC*** of the following kind with ***IB*** of length **3,** what stipulates the existence of the nonconstructability such as ***NCF*** for the structure, hence, and absence for it of dynamics reversibility.

$$\sigma^{(3)}: \left\langle \begin{array}{c|ccc|c} 0\,0 & 0 & 0 & 0 & 1\,0 \\ 0\,1 & 0 & 0 & 1 & 1\,0 \end{array} \right\rangle \to \left\langle \begin{array}{c|c|c} 1 & 1\,1\,1 & 1 \\ 1 & 1\,1\,1 & 0 \end{array} \right\rangle$$

$$\sigma^{(3)}: \left\langle \begin{array}{c|ccc|c} 0\,0 & 0 & 0 & 0 & 1\,0 \\ 0\,1 & 1 & 0 & 0 & 1\,0 \end{array} \right\rangle \to \left\langle \begin{array}{c|c|c} 1 & 1\,1\,1 & 1 \\ 1 & 1\,1\,1 & 0 \end{array} \right\rangle$$

In addition, a whole series of other operations can be quite used as the *operation* **# *(12)*** analogously to the aforesaid. So, the above example of the reversible ***HS***–model is only one of possible examples of such type whereas a receiving of reversible ***HS***–models for concrete appendices demands occasionally of complex enough research work. Now, there is a whole series of other interesting enough examples of definition of reversible ***HS***–models [536]**.** We also had represented a few reversible ***HS***–models, distinct from classical ones, with application to biology of development [5]**.** Meanwhile, the reversibility problem for the *classical* ***HS***–models is more many–sided and is considered below. As a whole, the problem of reversibility of ***HS***–models is not so unambiguous.

Below we shall use the more strong reversibility concept that is being understood as a possibility of *one–valued* restoration of all dynamics of a ***HS***–model at any moment**;** i.e. such ***reversibility,*** when it is possible to determine precisely at each moment t for each finite configuration in the ***HS***–model of its single predecessor at the previous moment $t-1$**.**

Considering the *classical* ***HS***–models as converters of both *finite***,** above all, and *infinite* configurations we quite can identify the configurations processed by such converters with their inner states. In case of similar interpretation a ***HS***–converter admits, as a rule, several inputs ***(states)*** for a receiving of a single output. Thus, the ***HS***–converter as a certain infinite automaton with n inputs and one output does not provide the reversibility property of in case of $n > 1$**.** From such point of view we and shall consider further the *reversibility* problem for the *classical* ***HS***–models. Reversibility of ***HS***–models of such type can be considered as the *global* reversibility, when on the basis of a configuration $c^* \in C(A,d)$ in moment $t \geq 0$ we can unambiguously determine for the ***HS***–model a single predecessor $c^{*-1} \in C(A,d)$, using only configuration c^* itself and local transition function or global transition function $\tau^{(n)}$ of the model, i.e. a mapping $\tau^{(n)}: C(A,d) \to C(A,d)$ should be *bijective* mapping. Thus, concerning the class of structures d–***HS*** $(d \geq 1)$ we can define naturally enough the reversibility of their dynamics as existence for an arbitrary configuration $c \in C(A,d)$ of a single predecessor c^{-1} from the set $C(A,d)$, i.e. the following relation should be carried out, namely:

$$(\forall c^* \in C(A,d))(E!\, c^{-1} \in C(A,d,\phi) \cup C(A,d,\infty))(c^{-1}\tau^{(n)} = c^*)$$

The following definition summarizes the aforesaid.

__Definition 11__. ***Dynamics of an arbitrary d-HS≡<Z^d,A,$\tau^{(n)}$,X> is called reversible if and only if for each its configuration $c^* \in C(A,d)$ there is a single predecessor c^{*-1} from the set C(A,d) such that $c^{*-1}\tau^{(n)}=c^*$; if not, the dynamics of such structure is called irreversible.***

Graphically the given definition can be illustrated as follows:

(a) $\dots \leftarrow c_{-j} \leftarrow \dots \leftarrow c_{-3} \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_0 \in C(A,d)$ – *reversible dynamics*

$\dots \leftarrow c^{\infty}_{-(j+1)}$

(b) $\leftarrow c_{-j} \leftarrow \dots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_0 \in C(A,d)$ – *irreversible dynamics*

$\dots \leftarrow c^{f}_{-(j+1)}$

(c) $c_{-j} \leftarrow \dots \leftarrow c_{-3} \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_0 \in C(A,d)$ – *irreversible dynamics*

Thus, in the state graph such as ***(a)*** a configuration c_0 has an infinite prehistory, for example, in case of its periodicity, whereas for the state graph such as ***(b)*** amount of predecessors on a step $-(j+1)$ necessarily not less than *two* and they can belong to the set $C(A,d,\phi)$ and/or the set $C(A,d,\infty)$. At last, the state graph such as ***(c)*** is finite, ending by certain configuration c_{-j} such as ***NCF***.

Thus, *irreversibility* of dynamics of a structure ***d–HS*** $(d \ge 1)$ is naturally defined or by absence of predecessors at all for a certain configuration $c^* \in C(A,d)$ or by existence of more than one predecessor c^{*-1} from the set $C(A,d)$ for a configuration c^*. Meanwhile, for case of classical ***d–HS*** $(d \ge 1)$ the irreversibility can be ***absolute*** and ***relative***.

Above all, in view of the nonconstructability definition such as ***NCF-1***, conditioned by presence for classical ***HS***–models of the determinative condition $\sigma^{(n)}(x,x,\dots,x) = x$, where $x \in A$ – a quiescent state, specifically, $x \equiv 0$. This condition allows to differentiate quite naturally the set of all possible configurations into two nonoverlapping sets – $C(d,A,\phi)$ and $C(d,A,\infty)$ of finite and infinite configurations accordingly; so, if the set $C(d,A,\phi)$ is closed concerning $\tau^{(n)}$, i.e. $(\forall c^* \in C(d,A,\phi))(c^*\tau^{(n)} \in C(d,A,\phi))$, the set $C(d,A,\infty)$ can be *nonclosed*; i.e. $(\exists c^{\infty} \in C(d,A,\infty))(c^{\infty}\tau^{(n)} \in C(d,A,\phi))$, that urgently causes the necessity of specification of interpretation of the ***reversibility*** concept for classical ***HS***–models, i.e. those ***HS*** which satisfy the above determinative condition. Existence in a ***HS***–model of nonconstructability such as ***NCF-1*** in the absence even of ***NCF*** leads to *irreversibility* of such model**,** more precisely of its dynamics, generally.

The given definition can be considered as an especially formal because the uniqueness of the infinite predecessor has the disputable enough interpretation, in our opinion. Therefore, the concept of *formal* and *real* dynamical reversibility are entered below. Actually, from the formal standpoint we can consider any admissible possibilities whereas from applied standpoints the instant transition from infinite configuration into finite one and vice versa, in our opinion do not admit transparent enough interpretations. However, various interpretations of transition from an infinite state to a finite, and vice versa are here possible. So, in mathematics it is possible to discover many of similar interpretations, for example, $\Sigma_k 1/p^k = 1,\ k = 1 .. \infty$, etc. While in backward dynamics of some finite configuration it is possible to associate its *instant* transition into an infinite configuration with some kind of *singularity***.** So, in case of classical structures the instant transition of an infinite configuration to a finite configuration is possible, whereas the instant transition of a finite configuration to an infinite configuration is impossible. Thus, in structures of such type a singularity can arise for backward dynamics of finite configurations only. On the basis of results of research of the nonconstructability problem in classical structures can be shown [54]**:**

There are no classical structures d–HS (d ≥ 1) for which in the absence of the nonconstructability such as NCF and in case of presence of the nonconstructability such as NCF–1 each finite configuration such as NCF–1 has one predecessor from the set C(A,d,∞) whereas other finite configurations not possess such predecessors, i.e. a classical structure without NCF in the presence of NCF–1 possesses finite configurations having at least two predecessors from the set C(A,d,ϕ)∪C(A,d,∞).

Thus, even on the assumption of absence of the nonconstructability of type ***NCF***, but in the presence of the nonconstructability such as ***NCF–1*** the dynamics of classical structures can be considered irreversible as a whole that according to proposal **2** conforms to our comprehension of dynamics irreversibility in classical homogeneous structures.

In addition, if to consider presence for a finite configuration of single predecessor from the set ***C(A,d,∞)*** as a singular point of its backward dynamics which is not breaking its reversibility, then and in this case the dynamics of a classical structure which does not possess ***NCF***, will be irreversible as a whole. Hence, the dynamics of a classical structure possessing the nonconstructability such as ***NCF*** and/or ***NCF–1*** is as a whole *irreversible* as we see it even under the assumption of possibility of reversible dynamics of certain finite configurations.

Absolute irreversibility for a classical ***d-HS*** $(d \geq 1)$ takes place if for the structure at least one configuration $c^* \in C(A,d)$ exists, which possesses several predecessors c^{*-1} from the set $C(A,d,\phi) \cup C(A,d,\infty)$ or at all has no predecessors. It is obvious if a structure ***d–HS*** $(d \geq 1)$ possesses the nonconstructability such as ***NCF*** then it is *absolutely irreversible*. On the other hand, absolutely other situation takes place in case of absence of the nonconstructability such as ***NCF*** for *classical* structures ***d-HS*** $(d \geq 1)$. For the given type of structures ***d–HS***, the nonconstructability such as ***NCF–1*** which is characterized by existence of the finite configurations having predecessors only from the set $C(A,d,\infty)$ is defined concerning finite configurations. During researches of nonconstructability of such type, an interesting class of ***d–HS*** $(d \geq 1)$ has been discovered for which the interesting enough following offer takes place, namely [79,88,90].

Theorem 43. There are classical structures d–HS $(d \geq 1)$ ***not possessing the nonconstructability such as NCF for which a finite configuration having single predecessor from the set*** $C(A,d,\phi)$ ***will has predecessors also from the set*** $C(A,d,\infty)$***; in addition, each finite configuration such as NCF–1 has at least two predecessors from the set*** $C(A,d,\infty)$***, i.e. any finite configuration has at least one predecessor from the set*** $C(A,d,\infty)$***. There are classical structures 1–HS with a state alphabet*** $A=\{0,1,2, ..., a-1\}$ ***which not possess the NCF in the presence of nonconstructability such as NCF–1 and whose configurations such as NCF–1 have*** $N=a^{n-1}$ ***while any other finite configuration has N-1 predecessors from the set*** $C(A,d,\infty)$***, where n is size of neighbourhood template. If some classical structure d–HS*** $(d \geq 1)$ ***with a state alphabet*** $A=\{0,1,2, ..., a-1\}$ ***does not possess the nonconstructability such as NCF, and for it there are at least two such different configurations*** $c^\infty, b^\infty \in C(A,d,\infty)$ ***that*** $c^\infty\tau^{(n)} = b^\infty\tau^{(n)} =$ ***, then for such structure finite configurations such as NCF–1 can possess more than two predecessors from the set*** $C(A,d,\infty)$ ***which pairwise differ between themselves by the infinite number of states.***

As a simple example it is possible to present a classical structure ***1–HS***

$$\sigma^{(2)}(x,y) = \begin{cases} x, & \text{if } y = 0 \\ x+y \pmod 2), & \text{if } <xy> \in \{11,22\}; \quad x,y \in A = \{0,1,2\} \\ y, & \text{otherwise} \end{cases}$$

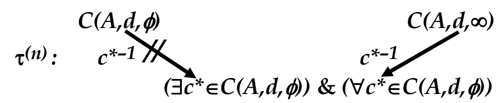
with alphabet $A = \{0,1,2\}$, neighbourhood index $X = \{0,1\}$ and ***GTF*** $\sigma^{(2)}$, determined by means of the above mentioned simple formula.

The following rather transparent schema confirms correctness of the above mentioned assertion for case of dimension $d = 1$, namely:

$$\begin{array}{r|l|l} c^{-1} \quad =\ldots 00 & 1\ x_1 \ldots x_n \square =\ldots 00 & 2\ x_1 \ldots x_n \square \\ \{c^{-1} | c^{-2}\}\tau^{(2)} =\ldots 01 & x'_o\, x'_1 \ldots x'_n \square =\ldots 02 & x''_o\, x''_1 \ldots x''_n \square \\ c^{-2} \quad =\ldots 22 & 1\ x_1 \ldots x_n \square =\ldots 11 & 2\ x_1 \ldots x_n \square \end{array}$$

$$x_j, x'_j, x'_o, x''_j, x''_o \in \{0,1,2\};\ j = 1..n;\ c^{-1}, c^{-2} - predecessors$$

Schematically the given type of *irreversibility* of dynamics of a classical structure ***d–HS*** $(d \geq 1)$ concerning some configuration $c^* \in C(A,d,\phi)$ can be submitted as follows, namely:



As an example, we shall consider a binary ***1–HS*** with neighbourhood index $X=\{0,1,2\}$ and local transition function defined by the formula:

$$\sigma^{(3)}(x,y,z) = y + z \pmod 2, \qquad x,y,z \in B = \{0,1\}$$

As shown, the given classical binary ***1–HS*** does not possess ***NCF.*** On the other hand, may be shown that for the structure ***NCF–1*** already of the simplest kind $c^* = \quad 1$ exist where « » – a chain of quiescent states «***0***», infinite to the left *(right)*. At that, during definition of all possible predecessors for the given configuration it is possible to show that the configuration not only is ***NCF–1,*** i.e. has no predecessors from the set $C(B,1,\phi)$; in addition, in the set $C(d,A,\infty)$ of infinite configurations, this configuration has two various predecessors, namely $c_1\tau^{(3)} = c_2\tau^{(3)} = c^*$ $\{c_1 = \ldots 1111\ \ ,\ c_2 = \ \ 1111\ldots;\ c_1 \neq c_2\}$, not allowing to uniquely restore a predecessor for the configuration c^*, i.e. unambiguously to restore its prehistory. Consequently, dynamics of the above classical ***HS***–model is ***irreversible.*** So, we can formulate important enough result relative to reversibility of classical ***HS***–models [54–56,640–643], namely.

Theorem 44. Absence for an arbitrary classical structure d–HS (d ≥ 1) of the nonconstructability such as NCF is the necessary condition but not sufficient for reversibility of its dynamics of finite configurations.

This result demands the more precise interpretation of the *reversibility* concept for case of classical ***HS***–models. In our understanding, under the reversibility of dynamics of a classical ***HS***–model is understood an opportunity of one–valued restoration of a single predecessor for an arbitrary finite configuration ***c****, in particular, for case of a structure ***1–HS*** of a configuration ***c***** of the kind ***c**=*** $x_1x_2 \ldots x_p$; $x_1, x_p \in A \backslash \{0\}$ on basis of analysis of its local transition function $\sigma^{(n)}$. Such ***reversibility*** concept is naturally enough at consideration of dynamics of the finite configurations in classical structures. So, in view of above concept, the absence in some classical ***HS***–model of *nonconstructability* such as ***NCF*** does not provide the *reversibility* of its dynamics that with all evidence and follows of the above example of classical binary structure ***1–HS.***

Thus, among classical ***d–HS*** which not possess the nonconstructability such as ***NCF*** but possess the nonconstructability such as ***NCF–1 (d≥1),*** the *irreversible* structures exist according to a quite natural *reversibility* definition represented above. Moreover, the fulfilled analysis confirm that all classical binary structures ***1-HS*** with neighbourhood index ***X=*** {***0,1,2***} that not possess the nonconstructability such as ***NCF,*** however possess ***NCF-1*** (*i.e. structures with discriminating numbers* ***30,60,75,86,89, 90,102,105,106*** *and* ***120;*** *section* ***2.2)*** are *irreversible* ones in context of the above definition ***11.*** So, structures with discriminating numbers ***30,60, 75*** and ***120*** have finite configurations possessing a predecessor from the set ***C(B,1,φ)*** and a predecessor from the ***C(B,1,∞),*** while structures with discriminating numbers ***86,89,90,102,105*** and ***106*** have the finite configurations possessing *two predecessors* from the set ***C(B,1,∞).*** Thus, existence in classical structures ***d–HS (d ≥ 1)*** of the nonconstructability such as ***NCF–1,*** not looking even on absence of the nonconstructability such as ***NCF,*** can provoke the irreversibility of dynamics of the given class of structures as a whole. In this connection an interesting enough question arises: ***Whether can guarantee the global irreversibility of a classical structure d–HS (d ≥ 1) the existence for such structure of the nonconstructability such as NCF–1 in the absence of the NCF?*** Taking into account our comprehension of irreversibility of dynamics of finite configurations in classical structures along with our results relative to allowable predecessors for finite configurations the positive answer to the given question arises all by itself; see some considerations below.

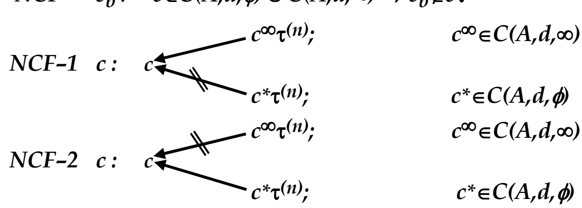
Meanwhile, taking into consideration a whole series of reasons below we shall introduce two more reversibility concepts of dynamics of the classical structures – concepts of ***formal*** and ***real*** reversibility. At that, if the concept of formal reversibility fully coincides with reversibility concept defined by the definition ***11,*** the concept of real reversibility is appreciably distinctive. So, under the real reversibility is understood the reversibility relative to the finite configurations only; i.e. existence for an arbitrary finite configuration c^* of such sole configuration $c^{\#}$ of the set ***C(A,d,ϕ)*** only that the relation $c^{\#}\tau^{(n)}=c^*$ takes place. As distinct from the definition ***11,*** the definition ***13*** below gives another approach to the reversibility concept of dynamics, allowing to look at important enough concept from different points of view.

In addition, below, this question will be considered in more detail in the network of the given comprehension of the *reversibility* concept. In case of the classical ***HS***–models the condition $\tau\tau^{(n)} =$ in the course of this monography ***will not considered as openness condition of the set C(A,d,∞) relative to GTF*** $\tau^{(n)}$ ***of the structures because above we have agreed to ascribe the fully null configuration « » to the set C(A, d, ϕ).*** At that, obviously, if a certain ***d–HS*** $(d \geq 1)$ possesses ***NCF–1,*** then the following relation $(\exists c' \in C(A,d,\infty))(c'\tau^{(n)} =\ )$ will take place. Moreover, absence for classical structures of the nonconstructability such as ***NCF*** and ***NCF–1*** at all does not guarantee the dynamics *reversibility* of their configurations. In this connexion, it is possible to show [54,88,90,640], that the interesting enough following offer takes place, namely.

Theorem 45. There are classical structures d–HS which do not possess the nonconstructability such as NCF and NCF–1, but for which exist infinite configurations having at least two various predecessors from the set C(A,d,∞) (d ≥ 1) of infinite configurations.

Thus, the aforesaid naturally stipulates expediency of introduction for classical structures ***d–HS*** $(d \geq 1)$ of the concept of ***relative reversibility (irreversibility)*** of their dynamics. Taking into account importance of reversibility concept for research of dynamics of classical ***HS***–models, we once again will address to three basic nonconstructability types in the models of such type, namely: ***NCF, NCF–1*** and ***NCF–2.*** For three basic nonconstructability types in classical structures ***d–HS*** $(d \geq 1)$, the following rather evident graphic interpretations can be represented:

NCF $c_b: \quad c \in C(A,d,\phi) \cup C(A,d,\infty) \rightarrow c_b \notin c\tau^{(n)}$



where c_b is a block configuration, i.e. configuration of states of a finite block of elementary automata of an arbitrary ***HS***–model***;*** ***C(A,d,ϕ)*** and ***C(A,d,∞)*** are sets of all *finite* and *infinite* ***d***–dimensional configurations of the ***HS***–model in alphabet ***A***=**{*0,1, ..., a–1*}** accordingly***;*** $\tau^{(n)}$ is global transition function of the ***HS***–model***;*** at last, {***c,c****} and c^{∞} are *finite* and *infinite* configurations of the ***HS***–model, which have finite and infinite number of states distinct from a «***quiescent***» state «***0***» accordingly. At that, among the represented types of nonconstructability, two types of nonconstructability ***NCF*** and ***NCF–1*** we shall consider as basic types. The following result represents one rather important relation between *basic* types of the nonconstructability [54-56,88]**.** At that, it is necessary to have in mind, that completely null configuration ***c=*** by a lot of the important reasons is included into the set ***C(A,d,ϕ).*** For this reason, for an arbitrary classical structure the set ***C(A,d,ϕ)*** can't consist completely of finite configurations such as ***NCF.***

<u>Theorem 46</u>. If for an arbitrary classical structure d–HS (d ≥ 1) the set C(A,d,∞) is nonclosed concerning global mapping induced by a global transition function $\tau^{(n)}$ ***of the structure, then such classical structure will possess the nonconstructability such as NCF, NCF–1 or by both types of the nonconstructability concurrently. A classical structure d–HS (d ≥ 1) not possessing NCF (perhaps, NCF–3) and NCF–1 possesses the nonconstructability such as NCF–2. On the other hand, a classical structure d–HS (d ≥ 1) which does not possess the NCF–2 will possess the nonconstructability such as NCF or/and NCF–1.***

Existence of the quiescent state «***0***» differentiating the set ***C(A, d)*** into two nonoverlapping subsets ***C(A,d,ϕ)*** and ***C(A,d,∞)*** along with **2** basic types of the nonconstructability ***NCF*** and ***NCF–1*** allows to determine enough naturally three types of dynamics ***irreversibility*** for classical structures ***d–HS (d ≥ 1)*** what can be formulated as follows.

<u>Definition 12</u>. We shall name an arbitrary configuration c**∈C(A,d,ϕ) for a classical structure d–HS (d≥1) by configuration with irreversible dynamics of the first type, of the second type and the third type if the configuration c** has zero or at least a pair of predecessors only from the set C(A,d,ϕ), at least 1 predecessor only from the set C(A,d,∞), and predecessors from the set C(A,d,ϕ) and the set C(A,d,∞), accordingly.

Above all, on basis of the given definition and results concerning the nonconstructability, it is simple to make sure, for an arbitrary classical structure ***d–HS (d≥1)*** the set ***C(A,d,ϕ)*** cannot consists of the irreversible configurations only of the ***second*** or only ***third*** irreversibility type, but the set can consists of irreversible configurations only of the ***first*** type of irreversibility. In the latter case, the existence of classical structures ***d–HS (d≥1)*** for which each finite configuration of the set ***C(A,d,ϕ)\{ }*** is or ***NCF,*** or has not less than two predecessors from the set ***C(A,d,ϕ)*** ***(****i.e. its predecessors are pairs of the* ***MEC)*** has been proved. Indeed, such classical structures are characterized by simple enough dynamics of finite configurations. Meanwhile, an arbitrary classical structure ***d–HS*** admits various combinations of the above three types of irreversibility in wide enough limits. In addition, in view of above definition ***12*** the following interesting enough result takes place [54–56,88,618,640]**.**

Theorem 47. Among classical structures d–HS (d≥1) with an arbitrary state alphabet A = {0,1,2, ..., a–1} there are such structures as:

1. Structures possessing the nonconstructability such as NCF without NCF–1 can have irreversible finite configurations of the 1st type only;

2. Structures without the nonconstructability such as NCF for which mappings $\tau^{(n)}$:C(A,d,ϕ)→C(A,d,ϕ) are bijective whereas mappings $\tau^{(n)}$: C(A,d,∞) → C(A,d,∞) are not bijective; at that, structures that possess the nonconstructability such as NCF cannot have bijective mappings such as and $\tau^{(n)}$: C(A,d,ϕ) → C(A,d,ϕ), and $\tau^{(n)}$: C(A,d,∞) → C(A,d,∞);

3. Structures possessing the nonconstructability such as NCF can have all finite configurations excluding NCF as irreversible configurations of the third type only;

4. Structures not possessing the nonconstructability such as NCF, but at existence of NCF–1 can have arbitrary finite configurations as the irreversible configurations of the second or third type; in addition any finite configuration different from NCF–1 is irreversible configuration of the third type, i.e. the configuration is absolute constructible, while each NCF–1 is irreversible configuration of the second type;

5. Structures, whose each finite configuration from the set C(A, d, ϕ) is irreversible, having the type 2 or 3; at that, each configuration such as NCF–1 has more than one predecessor from the C(A,d,∞), whereas the others have sole predecessor from the C(A,d,ϕ) along with single from C(A,d,∞). There are such classical structures that: (1) set C(A,d,ϕ) can be generated only from finite configurations while the set C(A,d,∞) is generated only from infinite configurations, and (2) the set C(A,d,ϕ) is generated from infinite configurations; in addition, its infinite subset NCF–1 is generated only from the infinite configurations, whereas the set C(A,d,ϕ)\NCF–1 is generated both from the finite, and the infinite

configurations. For an arbitrary classical structure d-HS (d ≥1) the set C(A,d,ϕ) cannot be generated only from the infinite configurations;

6. For an arbitrary state alphabet A={0,1,2, ..., a–1} (a ≥ 3) there are the classical structures d–HS (d ≥ 1) in the absence of nonconstructability such as NCF and for which an arbitrary finite configuration possesses (a–1) different predecessors from the set C(A,d,∞) if the configuration is different from a configuration such as NCF–1; otherwise, it will has a few different predecessors from the set C(A,d,∞).

If for an arbitrary classical structure d-HS (d≥1) with global function $\tau^{(n)}$ ***the set C(A,d,∞) is non–closed concerning the mapping defined by GTF*** $\tau^{(n)}$***, the mapping*** $\tau^{(n)}$***: C(A,d,ϕ) → C(A,d,ϕ) will not be bijective. A mapping*** $\tau^{(n)}$***: C(A,d,ϕ) → C(A,d,ϕ) for an arbitrary classical structure d–HS (d ≥ 1) with GTF*** $\tau^{(n)}$ ***will be bijective if and only if the structure not possess the nonconstructability such as NCF in the presence of the closed set C(A,d,∞) concerning the mapping defined by the GTF*** $\tau^{(n)}$***.***

So, the classical binary structure ***1–HS*** with connected neighbourhood template of the size three and with *discriminating* number ***113*** does not possess the nonconstructability of type ***NCF-1*** at presence in it of ***NCF.*** For this structure each irreversible finite configuration possesses pairs of predecessors only from the set ***C(A,d,ϕ),*** i.e. the irreversibility of the first type takes place. It is simple to make sure in validity and of other assertions ***2–5*** of the given theorem**;** in addition, the second part of the theorem follows of the above results on nonconstructability [54-56,88]**.**

Thus, if the first type defines the dynamics reversibility relative to the set of block configurations, and in wider understanding of the finite configurations concerning the set ***C(A,d,ϕ)∪C(A,d,∞),*** then the second type relative to the set ***C(A,d,ϕ).*** At that, if a certain classical ***HS-***model possesses the reversibility of the first type, then for it the reversibility of the second type can be absent. So, if for a certain classical ***HS-***model each finite configuration is periodic, the reversibility of the first type takes place along with absence of the second type, because each finite configuration does not possess predecessors from the set ***C(A,d,∞).*** It takes place only if a structure does not possess the nonconstructability of types ***NCF*** and ***NCF–1,*** i.e. its dynamics is rather simple. At that, at existence of irreversibility of the second type, the finite configurations can have more one predecessor from the set ***C(A,d,∞).***

So**,** for the above structure with number ***60*** in the absence of ***NCF*** in it**,** i.e. in the presence of reversible dynamics of the *first* type, an arbitrary configuration of the kind $\{1^{2k+1} \mid k=0,1,2,\ldots\}$ is ***NCF–1*** in the structure; in addition, a pair of various infinite configurations are predecessors c^{-1} for these configurations. Furthermore, from standpoint of general sense the *second* type of irreversibility deserves the essentially smaller attention, because both transition of an *infinite* configuration into *finite* configuration and vice versa is enough problematically interpreted. In this context rather natural desire to associate dynamics of the classical structures with *trajectories* of dynamic systems plays essential part too.

In principle, in compliance with the definition ***11*** as a reversible finite configuration a configuration having only single predecessor from the set ***C(A,d,ϕ)*** and, perhaps, predecessors of the set ***C(A,d,∞)*** should be regarded. This assumption is caused by the aforesaid. In this context the following interesting enough result takes place, namely:

The set C(A,d,ϕ) for an arbitrary classical structure d–HS (d ≥ 1) will consist only of reversible configurations if and only if such structure does not possess the nonconstructability such as NCF and NCF–1; at that, all finite configurations in the structure will generate periodical sequences of configurations only.

In addition, if to consider the ***HS***–models with the above reversibility (*i.e. to deal with full reversibility*) then from point of view of modelling appendices they not represent any especial interest by reason of their dynamics and insignificant quota, how it was repeatedly noted above. Consequently, from such standpoint only ***HS***–models that possess the nonconstructability such as ***NCF,*** above all, and/or ***NCF–1*** can present the greatest interest. So, the ***HS***–models with irreversibility only of the second type possess dynamics of a limited enough complexity. While, the binary classical structures ***1–HS*** considered above, possessing the *full* reversibility have simple enough dynamics of finite configurations whose applied interpretation of special interest does not represent.

It is necessary to mark, that it is enough expedient to consider also the dynamics reversibility concerning the set ***C(A,d,ϕ);*** i.e. when existence for a finite configuration of a single predecessor from the set ***C(A,d,∞)*** does not provide the *reversibility* of classical structure ***d-HS (d ≥ 1).*** The given assumption is well founded because *infinite* configurations from the standpoint of a lot of applied aspects not have enough satisfactory interpretation, and from the theoretical standpoint a case of potential infinity is more interesting. But in certain cases infinite configurations can appear useful enough at research of the certain theoretical aspects [567,640]. Roughly speaking, a ***reversible*** classical structure ***d-HS (d≥1)*** never forgets the history of any finite configuration concerning the set ***C(A,d,ϕ)***. If from the abstract standpoint the transition for a step from an infinite configuration into a finite configuration is quite allowable whereas at use of the classical structures as a modelling environment the interpretation of the given opportunity is problematic enough. In this case such possibility should be considered only as accompanying the base modeling process on basis of dynamics of *finite configurations*. At that, the classical structures of such type represent especial interest for modelling of a series of processes investigated by modern physics [536]. In connection with that, the reversibility concept of dynamics of *finite* configurations for classical ***HS***–models can be defined as follows.

Definition 13. An arbitrary classical structure d-HS (d ≥ 1) which does not possess the nonconstructability such as NCF–1 and NCF is called real reversible classical structure relative to all finite configurations if not, the irreversible in the presence of possible formal reversibility.

That allows to us to formulate the following result about interrelation of the nonconstructability and dynamics reversibility of the structures concerning finite configurations, i.e. from point of view of trajectories of finite configurations in the classical structures, namely [54–56,88]:

A classical structure d-HS (d ≥1) for that the set C(A,d,∞) is nonclosed concerning global mapping determined by its GTF $\tau^{(n)}$ is irreversible relative to dynamics of finite configurations. There is not a classical structure d–HS (d ≥ 1) for which each finite configuration could has a sole predecessor from the set C(A,d,ϕ) in case of nonclosure of the set C(A,d,∞) concerning a mapping defined by global transition function $\tau^{(n)}$ of such classical structure.

In addition, on basis of theorem **47** the following result follows:

There are classical structures d–HS (d≥1) for which any configuration c*∈C(A,d,ϕ) has at least one predecessor from the set C(A,d,∞). There are not the classical structures d–HS (d ≥ 1) which do not possess the nonconstructability such as NCF under the condition of the presence of nonconstructability such as NCF–1, and for which a configuration c∈C(A,d,ϕ) has only a single predecessor from the set C(A,d,ϕ) or from the set C(A,d,∞) of all infinite configurations.

However it is enough easy to make sure, that for an arbitrary classical structure ***d–HS (d ≥ 1)*** all finite configurations can't have predecessors solely from the set ***C(A,d,∞)*** of all infinite configurations. The second assertion directly follows from the above axiom that the entirely null configuration ***c*=' '**, i.e. the configuration consisting only of quiescent states, is ascribed by us to the set ***C(A,d,ϕ)*** of all finite configurations**;** furthermore, this assertion can be enough easily received on basis of out results on the nonconstructability such as ***NCF–1*** [79,88]**.** Having defined dynamical reversibility of classical structures concerning the finite configurations ***(****definition* **13)** which is quite natural in view of its study for the purpose of presence for an arbitrary finite configuration of its quite defined unique trajectory reversible in time, we generally speaking should have in view such essential enough circumstance.

Thus, an arbitrary configuration ***c∈C(A,d,ϕ)*** has reversible dynamics if for it the following relation takes place, namely**:**

$$(\forall t\geq 0 \mid t, t-1, \dots, 1)(E!c^{-t}\in C(A,d,\phi))(c^{-t}\tau^{(n)} = c^{-t+1}) \qquad c^{0}\equiv c$$

Consequently, taking into account the aforesaid there are no classical structures which possess the nonconstructability such as ***NCF*** in the presence of the nonconstructability such as ***NCF-1*** and simultaneously possess as a whole *reversible* dynamics of finite configurations**;** at that, some infinite subsets of the set ***C(A,d,ϕ)*** can quite contain dynamically reversible configurations.

Hence, only classical structures ***d–HS (d ≥ 1)*** which do not possess the nonconstructability such as ***NCF*** and ***NCF–1*** will possess the property of dynamical reversibility ***(****in sense of definition* **13)** as a whole, but such structures represent a relatively simple periodical dynamics of infinite and finite configurations that excludes, in particular, presence in such structures of such important properties as *universal* computability and reproducibility in the Moore's sense of finite configurations.

In view of differentiating of the set ***C(A, d)*** of all configurations of the classical structures into two nonoverlapping subsets, the question of reversibility of finite configurations concerning each of these subsets separately is quite natural. In this connection the following result can be formulated [88]**.** We shall say that an arbitrary finite configuration is ***reversible*** concerning the set ***C(A,d,ϕ)*** or ***C(A,d,∞)*** if it has definitely *one* predecessor from the appropriate set**;** otherwise, the configuration will be ***irreversible*** concerning the appropriate set.

Theorem 48. An arbitrary finite configuration in a classical structure d–HS (d ≥ 1) is reversible concerning the set C(A,d,ϕ) if and only if the structure not possesses the nonconstructability of both type NCF and NCF–1. At once all finite configurations in a classical structure d–HS (d ≥ 1) cannot be reversible concerning the sets C(A,d,ϕ) and C(A,d,∞)

simultaneously, i.e. an arbitrary finite configuration in the structure cannot possess only by one predecessor from of each set C(A,d,ϕ) and C(A,d,∞); at that, there are classical structures d–HS (d ≥ 1) for which each finite configuration, except NCF–1, possesses single predecessor from the set C(A,d,ϕ) along with one or two predecessors from the set C(A,d,∞). In addition, there are classical structures d–HS (d ≥1) whose a finite configuration from the set C(A,d,ϕ) possesses sole predecessor from the set C(A,d,ϕ) or will be nonconstructible configuration of type NCF–1 with at least one predecessor from the set C(A,d,∞). There are classical structures d–HS (d ≥ 1) in which depending on availability or lack of the nonconstructability such as NCF or NCF–1 the set C(A,d,ϕ) can be generated only from the set of nonconstructible configurations such as NCF-1, NCF or NCF∪NCF-1. In addition there is not classical structure d–HS (d ≥ 1) for which a finite configuration will possess the predecessors from the sets C(A,d,∞) and C(A,d,ϕ) simultaneously. If an arbitrary classical d–HS (d ≥1) possesses the nonconstructability such as NCF–1 in the absence of the nonconstructability such as NCF, then for two arbitrary different configurations c_1 ***and*** c_2 ***such as NCF–1 the following relation*** $\{c_1\tau^{(n)p}\}\cap\{c_2\tau^{(n)p}\} = \varnothing$, $p = 0 \,..\, \infty$ ***takes place.***

As seen from the above, the reversibility problem of the classical ***d–HS*** is directly based on the above concept of predecessors. Meanwhile, the theoretical study of the reversibility of dynamics of classical structures is quite difficult (*however, as well many important problems in this class of parallel dynamical systems*), therefore the use of computer simulation for this purpose is extremely effective. Likewise as for other problems, for experimental study of classical homogeneous structures we have used the corresponding procedures programmed in the software of systems of computer mathematics of ***Maple*** and ***Mathematica*** [635,637,638].

Thus, for experimental study of the reverse dynamics of classical ***1-HS*** in certain cases the procedure **Predecessors[*Ltf, Co, n*]** programmed in system ***Mathematica*** turned out a rather useful; the formal arguments of the procedure are defined as follows, namely:

Ltf – the list defining local transition function of a structure;
Co – an initial configuration;
n – the size of a neighbourhood template.

At that, the call **Predecessors[*Ltf, Co, n*]** returns the predecessors set of a configuration ***Co*** for the given conditions. In the following fragment the source code of the ***Predecessors*** procedure along with examples of its application for definition of predecessors of ***1–HS*** is presented.

```
In[3242]:= Predecessors[Ltf_/; ListQ[Ltf], Co_/; StringQ[Co],
                              n_/; IntegerQ[n]]:= Module[{L, a, b, c,
                              h = {}, i, j, k, d = StringLength[Co]},
        a = Gather[Ltf, StringTake[#1, –1] == StringTake[#2, –1] &];
      For[k=1, k <= Length[a], k++, L[StringTake[First[a[[k]]], –1]] =
                     Map2[StringDrop, Map[ToString1, a[[k]]], {–1}]];
                                            b = L[StringTake[Co, 1]];
      For[k = 2, k <= d, k++, c = L[StringTake[Co, {k, k}]];
      For[i = 1, i <= Length[b], i++, For[j = 1, j <= Length[c], j++,
                     If[StringEnd[b[[i]], StringTake[c[[j]], n – 1]],
           h = Append[h, b[[i]] <> StringTake[c[[j]], –1]], Null]]];
                     b = h; h = {}]; If[Length[b] != (n – 1)^Length[a],
 Print["Structure possesses the nonconstructability of NCF–type"], Null]; b]

In[3243]:= Ltf = {"0000", "0011", "0101", "0110", "1000", "1010", "1100", "1111"}
Out[3243]= {"0000", "0011", "0101", "0110", "1000", "1010", "1100", "1111"}
In[3244]:= Predecessors[Ltf, "11001110", 3]
           Structure possesses the nonconstructability of NCF–type
Out[3244]= {"0010111110"}
```

In particular, the return by the ***Predecessors*** procedure of the *empty* set speaks with certainty about availability in the corresponding structure of nonconstructability of ***NCF*** type and allows to define at least one of kinds of nonconstructible configurations [635]. And what is more, the ***Predecessors*** procedure in some cases allows to determine availability in the studied structure of the nonconstructability of ***NCF*** type on the basis of number of predecessors of a block configuration ***Co.*** Thus, the previous fragment represents a certain analog of ***Maple***-procedure of the same name in system ***Mathematica.*** Algorithm of the analog isn't based on table structures, and parallel substitutions of ***x1x2 ... xn → x'1*** that define a local transition function of structure, are defined not by a table, but by the list ***Ltf*** of strings of the format "***x1x2 ... xnx'1***". At that, **2** arguments ***Co*** and ***n*** are identical to the arguments of the procedure prototype of the same name from ***Maple*** [632,641,644].

It is known that along with theoretical research of classical ***HS***-models their experimental study by means of computer simulation is enough widely used. With this purpose a rather large collection of software of various appointments and complexity was created. A whole series of means of this type can be found in [536]. In particular, in context of the general problem of researches of the nonconstructability problem the question of *predecessors* generation for an arbitrary block configuration ***Co*** arises. The assessment of number of these predecessors allows on the basis of ***Aladyev–Maruoka–Kimura*** criterion in a whole series of cases to resolve an existence issue for structure of nonconstructability of ***NCF*** type. A number of the ***Mathematica***–procedures solving this problem for the case of classical ***1–H*** with the arbitrary alphabet ***W*** of internal states and neighbourhood index ***X*** were programmed for this purpose. The above procedure ***Predecessors –*** one of their number. For the experimental investigation of predecessors of block configurations in the classical ***1–H*** in addition to the presented procedure, the certain interest the procedure **TestOnNCF[*Ltf, n, m*]** essentially basing on the previous procedure represents. The procedure call **TestOnNCF[*Ltf, n, p*]** for a classical ***1–HS*** with the neighbourhood template of size ***n*** and the local transition function ***Ltf*** returns the first of block configurations of size ***p*** being **γ**–configuration, and prints about it the corresponding message. In case of absence of **γ**-configurations of size ***p***, the procedure call ***TestOnNCF*** returns *Null* and prints the corresponding message. In large, considering the more large reactivity of ***Mathematica*** relative to ***Maple***, it would seem quite natural to use ***Mathematica*** for modelling of dynamics and research of a number of properties of homogeneous structures, however it not entirely so [632,634,635,638,641,644].

Thus, full reversibility of the finite configurations concerning the set $C(A,d,\phi) \cup C(A,d,\infty)$ demands absence of the nonconstructability such as ***NCF*** and ***NCF–1***, sharply limiting the number of structures of such type; in addition, relative to dynamics such structures seem to us of a rather little interest. Indeed, in a lot of examined cases the *dynamics* of classical homogeneous structures of such type turns out rather simple and predictable since such structures for any *finite initial* configuration generate *periodic* sequences, not allowing to simulate complex enough processes, algorithms and phenomena. So, if to consider dynamics of classical structures as abstract algebraic systems of parallel processing of the finite words defined in finite alphabets, the completely *reversible* classical structures demonstrate rather simple dynamics.

Thus, on account of definition ***13*** of reversibility of dynamics of finite configurations in classical structures, what seems to us quite natural, we are compelled to be limited to a subclass of structures whose finite configurations are periodic, defining simple enough dynamics of such type of structures. So, the ***direct*** modeling of complex enough objects, phenomena and processes by means of algebraic systems such as the classical structures demands use of the classical structures possessing the nonconstructability such as ***NCF*** or/and ***NCF–1***. Certainly, at the certain assumptions it is possible to simulate the irreversible classical structures simulating those or other processes by *reversible* structures, for example, from the standpoint of absence for them ***NCF***, however a mediated process of modelling of the first will be far enough from the ***direct*** modeling. The *reversibility* problem of dynamics for the classical structures, in addition, will be discussed a little below.

Having finished a discussion of the questions relating to the problem of nonconstructability in ***HS***-models, we turn to algorithmic aspects of ***HS***-problematics, i.e. to questions of algorithmic solvability of those or other its problems. This theme is being represented enough important for the ***HS***–problematics from many rather interesting standpoints. In the first place that have the most immediate attitude toward research of dynamics of classical homogeneous structures as a whole.

2.6. Algorithmical aspects of the nonconstructability problem and some connected questions of dynamics of the classical homogeneous structures *(HS–models)*

Algorithmic *solvability* of the nonconstructability problem is one of *key* questions of the mathematical theory of ***HS***–models and a series of its important appendices, especially by way of use of classical structures in quality of both *conceptual* and *practical* models of *spatially–distributed* dynamic systems from which the real physical systems represent the greatest interest [3,5,54–56,88,536,567,640]. In the general formulation, the *solvability* of the *nonconstructability* problem is reduced to the next question: ***Whether there is an algorithm for definition of, whether an arbitrary classical HS–model will possess the nonconstructability of types NCF, NCF–1, NCF–2 and NCF–3?*** In similar general formulation the problem remains open up till now, however there are answers to a whole series of more particular, but not less important questions that represent significant independent interest [1,3,5,8,9,88,90,536,567,640].

The fullest decision of this problem is received for case of the classical structures ***1–HS***. First of all, concerning an arbitrary *block configuration* or finite configuration the following basic result having a series of the important enough appendices takes place [54–56,88,90,640], namely.

Theorem 49. ***Concerning an arbitrary finite configuration as well as a block configuration the problem of determination of its type (NCF–1, NCF, NCF–2, NCF–3, constructible) for an arbitrary classical structure 1–HS is algorithmically solvable.***

The approaches used at proof of the given theorem allow not only to constructively determine type of an arbitrary block configuration and finite configuration [55], but also to establish structure of a set of their direct predecessors what in a series of cases is important enough. For the general ***d*-**dimensional case ***(d≥1)*** the question of determination for a concrete block configuration of its type *(constructible, **NCF** or **NCF–3**)* is algorithmically solvable but it nothing does speak about solvability of the problem as a whole, i.e. existence of the nonconstructability of type ***NCF (NCF–3)*** for an arbitrary structure ***d–HS (d ≥ 2).***

It is well-known that with *transition* from dimension ***d=1*** to dimension ***d=2*** the research of many questions of dynamics of classical structures ***d–HS*** enough appreciably becomes complicated, and many of solvable problems for ***1–***dimensional case become *unsolvable* for ***d–***dimensional case ***(d≥2).*** In particular, in work [158] algorithmic unsolvability of the closure problem of a set of all finite configurations, distinct from fully null configuration « » and a series of other problems of dynamics of the classical structures ***d–HS (d ≥ 2)*** has been proved. In works [1,3-5,8-12,54–56,90,131,147,160,184–187,232,240,241,277,306] a whole series of interesting enough results on algorithmic unsolvability of problems of dynamics of the classical ***HS–***models is represented. Below, this theme in a certain sense will be continued.

One of known approaches to the solution of the solvability problem of existence in classical ***HS–***models of this or that *nonconstructability* type consists in determination of an upper limit for the *minimal* sizes of the internal block of pairs of ***MEC,*** of sizes of ***γ–CF*** or a nonconstructible configuration of a required type ***(NCF, NCF–1, NCF–2** and **NCF–3).*** In case of classical ***1-HS*** we acted thus and in this direction a series of the results representing a certain independent interest has been received [5,8,9,37,53-57,73,88,90,536,567,640]. This question plays an important part for estimation of the minimal size of ***γ–CF*** both at research of the nonconstructability problem generally, and at studying a whole series of dynamic properties of classical ***HS–***models.

In context of research of solvability of the *nonconstructability* problem we and a lot of other authors have studied a question of interrelation of minimal sizes of ***NCF*** and of ***IB*** of ***MEC*** in the classical ***HS–***models [5,54–56,88,131]. Meanwhile, contrary to the undertaken efforts in this direction, the satisfactory solution has not been received. However, a series of the essential results received in this direction has allowed to formulate the following interesting enough statement.

Proposal 5. In general case of the classical structures d–HS (d ≥ 2) it is impossible to receive any rather satisfactory quantitative estimation for minimal size of NCF as a function from minimal size of IB of the MEC, and vice versa.

The given proposal has allowed to clear a whole series of the principal questions existing up to it [5,53–57,88]. However, concerning one type of classical structures we have interesting enough result rather useful in a whole series of applications of theoretical nature [54–56,79,88,90].

Theorem 50. If for global transition function $\tau^{(n)}$ of a structure d–HS (d ≥ 1) the following relation $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$ takes place, where $|GS|$ is size of maximal diameter of a finite configuration GS, then the structure will possess the nonconstructability such as NCF and/or NCF–1. It is easy to make sure that quota of such structures is asymptotically more than e^{-2}. If a classical structure d–HS (d ≥ 1) of the above type possesses the nonconstructability only such as NCF–1 {NCF} then its set NCF–1 {NCF} generates all set $C(A,d,\phi)\backslash\{\ \}$; at that, the detection problem of NCF–1 {NCF} of minimal size is solvable.

On basis of a series of results concerning the *decomposition* problem of *global transition functions* ***(GTF)*** in the classical ***HS***–models considered a little bit below, in some cases it is possible to reduce the solution of the existence problem of nonconstructability such as ***NCF, NCF–3*** and ***NCF–1*** to the solution of similar problems for essentially more simple functions ***GTF*** $\tau^{(n)}$ of the same dimension and alphabet. In a number of cases similar approach considerably simplifies solution of the given problem, however generally speaking, its direct using of any especial result does not give. However, basing on this approach, the following interesting enough result has initially been received [5,55,56,88,90].

Definition 14. An algorithm determining the existence of pairs of MEC for classical HS–models, we shall name «essentially constructive» if it for any global function $\tau^{(n)}$ not only gives the answer «NO/YES» upon the question about existence of MEC, but also in case of the positive answer determines all types of pairs of MEC existing for a HS–model.

Constructive algorithms represent special interest above all there, where the researcher collides with necessity of real model realization of that or other character. But as ***HS***–models represent the most considerable interest from the standpoint of their opportunities in the constructive attitude, the given definition is being represented a quite pertinent. In light of definition ***14*** along with results of research of the *decomposition* problem of ***GTF*** $\tau^{(n)}$ in classical structures ***d–HS (d ≥ 2),*** it was possible to throw light and on the given interesting question [37,640–643].

Theorem 51. Any essentially constructive algorithm which solves the existence problem of pairs of MEC for classical structures d–HS (d ≥ 2) is absent in general case.

The further development of the proof technique of the given theorem has allowed also to prove algorithmical unsolvability of the existence problem of the *nonconstructability* such as ***NCF-3*** for *classical* structures ***d–HS (d ≥ 2);*** namely, the following result having a whole series of the important enough applications as an apparatus of investigation in this direction has been proved [54–56,79,88,90,640–643].

Theorem 52. The existence problem for an arbitrary classical structure d–HS (d ≥ 2) of nonconstructability such as NCF–3 is algorithmically unsolvable. The existence problem for an arbitrary classical structure d–HS (d ≥ 2) of pairs of the MEC–1 is algorithmically unsolvable too.

But according to the aforesaid ***(****theorem* ***11)*** for each ***HS***–model the set of all configurations ***NCF–3*** is a strict subset of the set of ***NCF,*** then an enough simple modification allows to prove and unsolvability of the existence problem of ***NCF*** in classical structures ***d–HS (d ≥ 2);*** that has been earlier proved also by ***J. Kari*** [90,277] on basis of other approach. Thus, in general case for classical structures ***d–HS (d ≥ 2)*** the existence problems of the *nonconstructability* such as ***NCF–3*** and ***NCF,*** and ***γ–CF,*** and pairs of ***MEC*** & ***MEC–1*** are algorithmically unsolvable. Thus, the unsolvability of the general nonconstructability problem such as ***NCF*** and ***NCF–3*** for classical structures ***d–HS (d ≥ 2)*** presumes development of certain partial methods of the solution of the specified problems for ***HS***–models of the certain types and classes. It can have a lot of rather important theoretical and applied outcomes. The essential operational experience with the classical ***HS***–models of dimension ***d=2*** shows that despite algorithmic unsolvability of the determination problem of the existence for them of the nonconstructability such as ***NCF*** and ***NCF–3*** in case of concrete structures ***2–HS*** we always received solution in the form of certain constructive algorithms. Thus here only about absence of unified decision algorithm concerning class of all structures ***d–HS (d ≥ 2)*** makes sense to speak, whereas in each concrete case the given problem, in our opinion, as a rule, has a constructive solution of some efficiency defined by concrete specificity of ***LTF*** of a ***2–HS.*** So, one of approaches to the solution of similar problem is a method ***(****in a series of cases, invariant concerning the nonconstructability property****)*** of modelling of one classical structure ***d-HS (d≥2)*** by means of some other structure of the same class and dimension but with the simplest neighbourhood index **(***section* ***1.1),*** for example, for a case of ***2–HS*** the given index has the kind ***X*** **=**{***(0,0),(0,1),(1,0)***}**.** The increase of cardinality of an alphabet ***A*** of a modeling structure in a series of cases is essentially balanced by significant simplification of procedure of analysis of ***LTF*** $\sigma^{(n)}$ with the object of to reveal the existence in the modelled structure ***d–HS (d*** **≥** ***2)*** of the nonconstructability phenomenon. Meanwhile, in view of series of the reasons such approach should be applied circumspectly enough [5,640]**.** Completely other picture takes place for case of the structures ***1–HS*** and, above all, of nonconstructability such as ***NCF–1*** and ***NCF–2*** in them. So, according to theorems ***20*** and ***21*** the existence problem of pairs of ***MEC,*** and ***NCF*** for classical structures ***d–HS (d*** **=** ***1)*** is solvable. The following result proves solvability of the problem for generalized case of ***MEC–1*** for dimension ***d*** **=** ***1*** along with unsolvability for ***d*** **≥** ***2.***

Theorem 53. The existence problem of pairs of MEC–1 for an arbitrary classical structure d–HS is algorithmically solvable for case d = 1 and algorithmically unsolvable for case d ≥ 2.

The proof of theorem ***53*** can serve as a constructive test of existence of ***MEC–1*** for each classical structure ***1–HS.*** At the same time, the given test supposes rather simple program realizations, allowing to use for the solution of the problem the advanced computer facilities in each concrete case [1,5,54-56,88,90,536,567,640]**.** On basis of theorem ***53*** and the approach to its proof it is possible to receive a series of interesting enough results concerning the solvability of those or other aspects of the nonconstructability problem for an arbitrary structure ***1–HS.*** Thus, from the results presented here and also from works [1,2,3,5,7–9,27,30, 43,53-56,277,536,567] concerning solvability of the basic aspects of the nonconstructability problem in the classical ***HS–***models it is easy to be convinced, that their decision essentially depends on dimension of the models. If in case of classical structures ***1–HS,*** mainly, the algorithmic solvability takes place while already for ***2–****dimensional* case very much many important questions in this direction remain unsolvable in spite of relative simplicity of the given type of classical ***HS–***models [536]**.**

The following main theorem establishes full *solvability* of the existence problem of the nonconstructability of all types for case of the classical ***1–***dimensional ***HS–***models [5,8,9,54–56,79,88,90,618,640–643]**.**

Theorem 54. The existence problem of the nonconstructability such as NCF, NCF–1, NCF–2, and NCF–3 for an arbitrary classical structure

1–HS is algorithmically solvable. The existence problem of admissible combinations of types of nonconstructability according to the table 2 for an arbitrary classical structure 1–HS is algorithmically solvable, while for case of classical structures d–HS ($d \geq 2$) a complete solution of the given question remains open until now.

In the same direction the following result represents a certain interest, above all, from theoretical standpoint [54–56,79,88,90,545,567,640].

<u>Theorem 55</u>. The existence problem of the nonconstructability such as NCF-1 in classical structures d-HS not possessing nonconstructability such as NCF is solvable for case $d = 1$ and unsolvable for case $d \geq 2$.

So, the proof of the theorem for case $d \geq 2$ is based on the algorithmical unsolvability of the «***domino***» problem [260] that has been considered enough in detail in [640] and on definition ***4*** establishing enough close connection between existence of the *nonconstructability* such as ***NCF–1*** and closure of the set ***C(A,d,∞)***. Here, concerning the solvability of the determination problem of non–closure of the set ***C(A,d,∞)*** of all *infinite* configurations for an arbitrary *classical* structure ***d–HS*** ($d \geq 1$) with state alphabet ***A*** the following important enough result takes place [88,90].

<u>Theorem 56</u>. The determination problem of closure of the set C(A,d,∞) of all infinite configurations concerning mapping induced by a global transition function $\tau^{(n)}$ of an arbitrary classical structure d–HS ($d \geq 1$) is algorithmically solvable for case $d = 1$ and unsolvable for case $d \geq 2$.

Proof of this theorem is based on unsolvability of the above «***domino***» problem whose brief sketch is represented in section ***2.8*** [617]. At that, in more general posing the represented arguments allow to formulate the following interesting enough result [54–56,617,640], namely.

<u>Theorem 57</u>. For a classical structure d–HS ($d \geq 2$) with an alphabet A and global function $\tau^{(n)}$, the existence problem of such configurations c* that the relation $c^*\tau^{(n)} = c^{\infty}_r$ takes place is unsolvable (where c^{∞}_r is an infinite configuration consisting only from r–states; $r \in A$).

Substantially, under the ***nonclosure (closure)*** of the set ***C(A,d,∞)*** of the infinite configurations concerning global parallel mapping induced by ***GTF*** $\tau^{(n)}$ of a classical structure ***d–HS*** ($d \geq 1$) we understand ***existence (absence)*** in the set ***C(A,d,∞)*** of such configurations $c^{\infty} \in C(A,d,\infty)$ that relation $c^{\infty}\tau^{(n)} =$ takes place, where – completely *null* configuration which owing to a whole series of rather essential reasons is attributed by us to the set ***C(A, d, ϕ)*** of all finite configurations. Of our results in this direction an interesting enough offer follows [54–56,79,88,90,640].

Theorem 58. The existence problem of such infinite configurations c_o^∞ that the relation $c_o^\infty\tau^{(n)} = c \in C(A,d,\phi)$ takes place concerning a global parallel mapping induced by GTF $\tau^{(n)}$ of a classical structure d–HS is algorithmically solvable for case $d = 1$ and unsolvable for case $d \geq 2$.

At that, it is shown that the following interesting result takes place.

Theorem 59. If for an arbitrary classical structure d–HS ($d \geq 1$) the set $C(A,d,\infty)$ is nonclosed concerning a global mapping induced by global function $\tau^{(n)}$, then the structure will possess the nonconstructability such as NCF or NCF-1, or by the mentioned nonconstructability types simultaneously.

Hence, for the decision of the solvability problem of existence of the nonconstructability such as ***NCF*** and ***NCF–1*** for any classical structure ***d–HS ($d \geq 1$),*** the question of nonclosure of the set ***$C(A,d,\infty)$*** concerning a global mapping induced by the global transition function $\tau^{(n)}$ of the structure plays an essential enough part. On the assumption of rather transparent definitions of the nonconstructability such as ***NCF–1*** and ***NCF–2*** and also of the aforesaid arguments, already it is rather simple to conclude that in the absence of *nonconstructability* such as ***NCF*** and ***NCF–1*** for a classical structure ***d–HS ($d \geq 1$)*** for it the *nonconstructability* such as ***NCF–2*** should exist. The above results concerning the problem of nonconstructability once again confirm basic distinctions between the considered nonconstructability types in classical ***HS***–models, and strong influence of dimensionality of the ***HS***–models upon the results connected to them. Obviously, inasmuch as the existence problems of nonconstructability such as ***NCF*** & ***NCF–1*** are unsolvable concerning classical structures ***d–HS ($d \geq 2$),*** the problem of revealing of *dynamical reversibility* for classical structures is unsolvable as a whole too.

In most general posing from the standpoint of researches of analogies between formal models on basis of classical structures ***d–HS*** and real physical processes and phenomena, embedded in them, it would be extremely interesting to make clear more in detail not only influence of dimensionality upon their global properties, but also the problem of nonconstructability directly connected to dynamical reversibility of classical structures ***d–HS ($d \geq 1$).*** In works [5,8,9,53-56,88,90,536,567] the influence of key parameters of classical ***HS***–models upon questions of investigation of their dynamic properties is considered with sufficient degree of completeness. This question seems to us a rather important.

Research of deep properties of parallel global mappings $\tau^{(n)}$***: C(A,d) →*** ***C(A,d),*** induced by ***GTF*** $\tau^{(n)}$ of the classical ***HS***–models, has the direct attitude to the *nonconstructability* problem and plays the fundamental part in research of dynamic properties of such ***HS***–models. Properties of parallel mappings such as *injectivity* and *surjectivity* have *immediate* connection with the nonconstructability problem and were studied by a lot of researchers whereas the review of their results can be found in our monographs [3,5] and also in works [90,183–187,270,272,310,311].

According to the theorem ***31*** the necessary and sufficient condition of the nonconstructability existence such as ***NCF-1*** in a classical structure ***d–HS (d ≥ 1)*** not possessing ***NCF*** consists in nonclosure of set ***C(A,d,∞)*** concerning its parallel mapping $\tau^{(n)}$***: C(A,d,∞) →C(A,d,∞).*** Whereas the existence of ***NCF*** (***NCF-3)*** in a classical structure ***d–HS (d ≥1)*** is directly connected to ambiguity of its mapping $\tau^{(n)}$***: C(A,d,ϕ) → C(A,d,ϕ).*** In the *general* case of nonconstructability it is rather interesting to investigate interrelation between existence in a classical ***HS***-model of ***NCF, NCF–3*** and/or ***NCF–1*** along with properties of parallel mapping $\tau^{(n)}$***:C(A,d) → C(A,d).*** With this purpose we shall determine influence of existence in a classical structure ***1–HS*** of ***NCF, NCF–3*** and/or ***NCF–1*** upon mutual unambiguity of its parallel mapping $\tau^{(n)}$, and vice versa. Whereas for case of classical structures ***d–HS (d≥1)*** the following result takes place.

<u>***Theorem 60***</u>***. If the set C(A,d,∞) of infinite configurations of a classical structure d–HS is nonclosed relative to d–dimensional transformation*** $\tau^{(n)}$***, then mapping*** $\tau^{(n)}$ ***: C(A) → C(A) appropriate to it will not be one–to–one, where C(A) = C(A,d,∞) ∪ C(A,d,ϕ). If a classical structure d–HS does not possess nonconstructability such as NCF then*** $\tau^{(n)}$***:C(A,d,ϕ)→ C(A,d,ϕ) is biunivocal mapping, while*** $\tau^{(n)}$***: C(A,d,∞) → C(A,d,∞) can be not biunivocal mapping. If the set C(A,d,∞) of a classical structure d–HS (d ≥ 1) is nonclosed concerning d–dimensional transformation*** $\tau^{(n)}$***, then a mapping*** $\tau^{(n)}$ ***: C(A,d,ϕ) → C(A,d,ϕ) can't be by bijective mapping while the contrary assertion is false, generally speaking.***

On basis of theorems ***31*** and ***60*** it is possible to show, that at existence of the nonconstructability such as ***NCF (NCF–3)*** and / or ***NCF–1*** for an arbitrary *classical* structure ***1–HS*** the global parallel mapping $\tau^{(n)}$***: C(A) → C(A)*** is not biunique, whereas the converse proposition is generally speaking false. Consequently, fact of many–valuedness of a mapping $\tau^{(n)}$***: C(A) → C(A)*** does not entail existence in a classical structure ***1–HS*** of the nonconstructability such as ***NCF, NCF–3*** and ***NCF–1.*** Moreover, if ambiguity of a mapping $\tau^{(n)}$: *C(A,ϕ)→C(A,ϕ)* gives rise to occurrence of the nonconstructability such as ***NCF*** (*perhaps*, ***NCF–3***) the ambiguity of a mapping $\tau^{(n)}$: ***C(A,∞) → C(A,∞)*** is not connected immediately with the nonconstructability in *classical* structures ***1–HS***. Being based on the theorem ***60*** and some other results [54], it is enough simple to receive a decision of the solvability problem of existence of one–to–one global mapping $\tau^{(n)}$: ***C(A) → C(A)*** for any classical ***1–HS***. In this direction we have received a series of rather interesting results [1,3–5,7,15,47,54–56, 68,75,567] which are expressed by the following basic theorem having a series of interesting enough appendices [536,545,567,617,640–643].

<u>*Theorem 61*</u>. ***Problems of determination of mutual one–valuedness of parallel mappings for case of classical structures 1–HS, namely:***

$$\tau^{(n)}: C(A,\phi) \to C(A,\phi),\ \ \tau^{(n)}: C(A,\infty) \to C(A,\infty)\ \ \text{and}\ \ \tau^{(n)}: C(A) \to C(A)$$

are algorithmically solvable. The existence problem of reverse parallel mapping τ_n^{-1} ***for a parallel global mapping*** $\tau^{(n)}$***: C(A)→C(A) generated by a classical structure 1–HS is algorithmically solvable too.***

In addition, the second part of this theorem presents non–constructive proof of solvability of the existence problem of reverse global function τ_n^{-1} for ***GTF*** $\tau^{(n)}$ of an arbitrary classical structure ***1–HS***. Thereupon, it would be interesting enough to receive also a constructive decision of this problem what will allow to receive the reverse local function $\sigma^{(n)}$ on basis of concrete kind of ***LTF*** $\sigma^{(n)}$ of a classical structure under the condition of its existence. On the other hand, the *solvability* problem of mutual one–valuedness of global mappings $\tau^{(n)}$: ***C(A,d) → C(A,d)*** for general case of classical structures ***d–HS*** ($d \geq 2$) the following principal theorem decides [5,54–56,79,88,90,618,640–643].

<u>*Theorem 62*</u>. ***Problems of determination of mutual one–valuedness of global mappings*** $\tau^{(n)}$***: C(A,d,ϕ) → C(A,d,ϕ) and*** $\tau^{(n)}$***: C(A,d) → C(A,d) for general case of the classical structures d–HS*** ($d \geq 2$) ***are unsolvable.***

Proof of the given assertion directly follows of result of the theorem ***52*** and consequences ensuing from it which determine unsolvability of the existence problem of the nonconstructability such as ***NCF (NCF–3)*** for an arbitrary classical structure ***d–HS*** ($d \geq 2$). Of the theorem ***62*** also follows, that ***the definition problem of existence of the reverse parallel mappings*** τ_n^{-1} ***for parallel mappings*** $\tau^{(n)}$***: C(A,d) → C(A,d) induced by the classical structures d–HS*** ($d \geq 2$) ***is unsolvable.*** Using results on the nonconstructability such as ***NCF*** along with property of *compactness* of the topological product, ***D. Richardson*** has proved [305] that a global parallel mapping $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$ is biunique only if a mapping $\tau_n^{-1}: C(A,d) \rightarrow C(A,d)$ is being defined by global function of a certain ***d–HS (d≥1).*** In this respect the given result plays rather important part in theoretical researches of dynamical properties of ***d–HS (d ≥ 1);*** on basis of it and theorem ***61,*** in particular, the above assertion easily follows.

It is known that an arbitrary classical structure ***d–HS (d ≥ 1)*** possesses the nonconstructability such as ***NCF*** (*and, perhaps,* ***NCF–3)*** if and only if for the structure there are configurations $c \in C(A,d,\phi)$ which have not predecessors c^{-1} from the set $C(A,d) = C(A,d,\phi) \cup C(A,d,\infty)$ [80]. So, this question directly adjoins to *reversibility* question of dynamics of ***d–HS.*** In context of result of the solution of the given problem it is necessary to note the important result of ***T. Yaku*** [224,317] consisting in that that the determination problem of predecessors c^{*-1} for any configuration $c \in C(A,d)$ for a structure ***d–HS (d ≥ 2)*** is algorithmically unsolvable. The given result is very weighty argument that the determination problem of mutual one-valuedness of the global mappings $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$ should be algorithmically unsolvable for general case of the classical structures ***d–HS (d≥2).*** While for classical structures ***1–HS*** the problem of determination of direct predecessors c^{-1} for any finite configuration $c \in C(A,d,\phi)$ is algorithmically solvable and the constructive proof of it is based on the following important result representing independent interest. This question has quite determinate meaning for elaboration of different reversible computational and physical ***HS***–models [536].

<u>***Theorem 63***</u>***. The determination problem of predecessors*** c^{-1} ***and their types for an arbitrary finite configuration*** $c^{**} \in C(A,d,\phi)$ ***in a classical structure d–HS is solvable for case d = 1 and unsolvable for case d ≥ 2.***

Proof of the theorem can be found in works [55,90,224,536]; thus, our proof of this problem is based on *nonsolvability* of the known «*domino*» problem. However already for case of the classical structures ***1–HS*** the definition problem of «***related***» attitudes for **2** arbitrary configurations {***c, c****} is unsolvable, i.e. the following question is unsolvable, namely: ***Whether has a place the relation*** $(\forall c,c^* \in C(A,1,\phi))(c^* \in \langle c \rangle[\tau^{(n)}])$ ***for an arbitrary classical structure 1–HS?***

It is well–known that resolvability problems are being investigated by means of constructive and non–constructive methods. While from the standpoint of applied aspects of ***HS***–models the greatest interest just constructive methods represent. Our research concerning the criteria and resolvability of a lot of aspects of the nonconstructability problem in general case of classical structures ***d–HS (d≥1)*** confirm a rather high level of complexity of the given problematics. A series of separate and most special results in the given direction can be found, in particular, in works [3,5,7,15,47,54–56,68,75,79,88,90,306,536,545,567,617,618,640].

Of the presented results relative to *solvability* of the *nonconstructability* problem for the classical ***HS***–models it is simple to make sure, that its solution essentially depends on dimensionality of the models. So, if in the ***1***–dimensional case all basic aspects of the nonconstructability are solvable with existence of constructive decision algorithms whereas in the **2**–dimensional case some questions of existence of the *basic* aspects of the *nonconstructability* are unsolvable. In this connexion the problem of influence of values of key parameters of ***HS***–models (*neighbourhood index*, *dimensionality*, *state alphabet*, *local transition function*) on research of their dynamic properties represents indubitable interest. A series of the solvability problems of more subtle properties of dynamics of the classical ***HS***–models on basis of the theory of creative and productive sets of finite configurations has been considered in [5,54–56,88,90,536].

Meanwhile, the acuteness of algorithmic unsolvability of the problem of nonconstructability such as ***NCF*** and ***NCF–3*** for general case of ***HS***–models is reduced also by that fact, that with growth of cardinality of an alphabet ***A*** and/or size of the neighbourhood template the share of the ***HS***–models possessing ***NCF*** and, perhaps, ***NCF–3,*** greatly swiftly aspires to ***1*** (*theorem* **12**), i.e. «***almost all***» complex enough ***HS***–models possess the nonconstructability such as ***NCF.*** To some extent a similar picture takes place for case of the nonconstructability such as ***NCF–2*** and ***NCF–1*** relative to the solvability problem of their existence in the general case of classical structures ***d–HS (d ≥2)*** [5,8,9,53,88,90,536,567].

The results represented in the section solve as a whole the solvability problem of existence for the classical ***HS***–models of non–constructible configurations of various types whereas with *separate* questions of the given problems it is possible to familiarize oneself more thoroughly in a lot of the works containing additional primary sources [1,3,5,8,20,22, 30,37,53-57,70,72,88,90,135,158,160,166,184-187,190,232,269,277,307,310, 314,322,536,567,617]. The latest decades, the special attention is given to the questions connected to reversibility of ***HS***–models, in quality of some base of representation of spatially–distributed dynamic systems which, in turn, is closely linked to the *nonconstructability* problem such as ***NCF*** [273]**.** At that, a little bit in more details questions of *dynamical reversibility* of classical ***HS***–models we have considered in section **2.5.**

We consider the further research on the nonconstructability problem in the classical ***HS***–models are being represented rather interesting by two principal causes, namely**:**

(1) the nonconstructability is one of fundamental concepts in research of dynamics of classical and of some other types of HS–models;
(2) the base results received on the nonconstructability problem allow not only to detail this fundamental concept, but also to form effective enough apparatus of investigation of dynamics of HS–models of both classic types and of some other important enough types.

On given place a discussion of the questions related to the property of *nonconstructability* for classical ***HS***–models is finished**;** meantime, from this directly follows, excepting a few fundamental questions the given problematics by present time has received full enough resolution. But its fundamentality for mathematical ***HS***–problematics along with a lot of the open questions connected to it continue to attract attention to it enough many researchers [88,90,536,567,617]**.** The results presented in chapter **2** solve the nonconstructability problem in classical structures ***d–HS*** as a whole. The more special results in this direction are enough numerous and allow to successfully research the nonconstructability problem in detail. Along with that the results in this direction allow to form rather effective methods for research of dynamics of the classical structures ***d–HS (d≥1).*** A lot of the similar results are discussed below**;** many interesting enough results in this direction can be found in our monographs and in references represented here and in [536,545,640]**.**

The given problematics composes a rather essential part of the general theory of dynamic properties of classical ***d-HS (d≥1)*** therefore we paid such close attention to it. Furthermore, the problematics play a rather essential part in formation of general apparatus of research of ***d–HS.***

Once more it is necessary to note that the results which are presented here bear descriptive character inasmuch as their proofs are lengthy enough and too technical to be included in this book. The interested reader has opportunity to familiarize oneself with them in references cited in the appropriate places. In addition, in order to maintain a self-contained presentation–the definitions of the main concepts are given, with the exception of a minimum of simplest mathematical concepts. Such approach allows to get acquainted conceptually here with given material, without spreading out, at times, on voluminous proofs.

Chapter 3. Extremal constructive opportunities of the classical homogeneous structures

It is known that any *Turing machine* ***(TM)*** can be modeled by means of a classical structure ***1–HS,*** proving the ability of *universal computability* for ***1–HS,*** i.e. ability to calculate any recursive function or to realize an arbitrary computational algorithm, or information processing [1,5,536, 567,640]**.** However, the given universality, generally speaking, needs the use of some additional states for an elementary automaton of such structure. Namely, for calculation of a certain function determined in a finite alphabet ***A,*** a ***HS–***model can claim some expansion of alphabet ***A,*** for example, only upon one symbol. Below we shall show that such minimal expansion is sufficient for possibility of calculation by means of ***HS-***model of an arbitrary *recursive* function, including generation of an arbitrary finite configuration from the given initial configuration.

Apparently, the given opportunity is inherent any enough in complex system, i.e. within internal axiomatics of the system the solution of all problems inherent in it is impossible**;** for opportunity of the decision of non–solvable problems is demanded an expansion of axiomatics of system ***(****specifically of state alphabet of a system****).*** Formally this axiom has rigorous proofs for a whole series of formal theories.

Axiomatics of classical ***HS–***models is defined by their key parameters, namely**:** dimensionality ***d*** of homogeneous space $\boldsymbol{Z^d}$**;** state alphabet ***A*** of an elementary ***z–***automaton, neighbourhood index ***X,*** along with a local transition function $\boldsymbol{\sigma^{(n)}}$**.** Within of the given base axiomatics the question of constructive opportunities of classical ***HS–***models presents special interest, namely**:** ***As far as powerful opportunities of classical HS-models (within of their base axiomatics) concerning generation by them of the finite configurations?*** On the assumption of own interests and tastes, many researchers differently define the *maximal* generative ***(****dynamic****)*** opportunities of ***HS–***models within their base axiomatics.

Meanwhile, for today we have no any unique concept of the maximal *generative* opportunities of classical ***HS–***models, and it has appreciably subjective character [1,3–5,8,9,88,90]**.** In particular, as contrasted to the nonconstructability the essential enough interest represents definition of the general properties reflecting the maximal constructive means of ***HS–***models relative to generating by them of finite configurations. We shall consider two most known approaches**:** on basis of ***universal*** and ***self–reproducing*** finite configurations [1,3–5,8,9,54–56,88,90,536,545]**.**

3.1. Universal finite configurations in the classical homogeneous structures

In the well–known monograph ***S. Ulam*** has formulated an interesting problem about existence of simple universal matrix system [220]. Its positive solution would give an example of simple generating formal system which could be researched quite effectively by means of well–known mathematical methods. In view of this problem we introduce the necessary concepts and definitions used further.

Definition 15*. *A square matrix U(n, a) of order n with elements from a set A={0,1, ..., a–1} is named the universal matrix relative to a class of all matrixes of order m < n if for any matrix B(k, a) (k ≤ m) there is such integer p > 0 that matrix B will be principal minor of a matrix $U^p(n,a)$.

In terms of the definition the following theorem resolves the existence problem of the ***universal*** self–reproducing matrix system [1,5,9,16,88].

Theorem 64*. *There is such integer $n_0 > 0$ ***that universal matrixes U(n,a) cannot exist for an arbitrary integers*** $n \geq n_0$ ***and a ≥ 2.***

Of result of the theorem follows, that the universal generating matrix systems of enough high order not exist. While for infinite matrixes the given question still remains open, i.e. in the initial posing of ***S. Ulam*** the existence problem of the universal reproducing matrix system still waits for own decision. Moreover, a series of questions of existence of similar matrix systems over the field ***A*** also remain open. Besides the above mentioned applied interest, work in this direction will present significant interest also for the theory of infinite matrixes and, above all, by the used apparatus which is developed still enough faintly.

As an interesting applied aspect of the given problem it is possible to specify, in particular, the use of classical structures ***d–HS*** for modeling of logic deductive systems in pure mathematics. In the given case the configurations from the set ***C(A,d,ϕ)*** are associated with *offers* of logic calculus, the initial configurations of a structure with *axioms* and ***GTF*** with *derivation* rules of a system. Then a sequence of global transition functions, applied to an initial configuration *(axiom)*, represents proof *(conclusion)* in the given deductive model. The problems of *deducibility* and *completeness* are basic in the given systems. These both problems are directly linked with the existence problems for *classical* ***HS***–models of ***NCF*** and ***UCF*** accordingly. Using of classical structures ***d–HS (d ≥ 1)*** for modeling of developing systems of cellular nature can be noted as the second applied aspect of the existence problem of the ***UCF.***

The existence problem of ***UCF*** for the classical ***HS***–models which was formulated by ***S. Ulam*** for case of regular lattices is closely connected to the ***completeness*** problem of ***H. Yamada*** and ***S. Amoroso*** for case of *polygenic* ***HS***–models [5,73,244,245,248,256]. The given problem can be formulated as follows: ***Whether exists a finite configuration (finite set of configurations) for the given classical structure d–HS of which the set C(A,d,ϕ) can be generated by means of GTF*** $\tau^{(n)}$ ***of the structure?*** In other words, the question is reduced to permissibility of the following determinative relation $\cup_k <c_k>[\tau^{(n)}] = C(A,d,\phi)$ ***(k=1 .. p).*** According to the told, the finite configurations c_k which satisfy the given condition are named the ***universal*** finite *configurations* (*shortly* ***UCF).***

For case of finite ***HS***-models the existence problem of ***UCF*** has *positive* solution, namely: ***There are finite structures d–HS (d ≥ 1) which have one or all configurations as the UCF.*** So, a series of examples of such structures can be found in the monographs [1,5]. However, absolutely other picture takes place in case of investigation of the *infinite* classical ***HS***–models [5,54,88,90,567,640]. Here we need an essential definition.

Definition 16. ***A set of finite configurations*** $c_k \in C(A,d,\phi)$ ***composes for global transition function*** $\tau^{(n)}$ ***of a classical structure d–HS (d ≥ 1) the set of universal configurations (UCF), if the following determinative relation*** $\cup_k <c_k>[\tau^{(n)}] = C(A,d,\phi)$ ***(k=1 ..p) takes place.***

In some works the question relative to so-called «*minimal*» ***HS***–models (*for example, in the Anglo–lingual literature in* ***S. Wolfram's*** *suggestion in book* [407]) defined as follows: ***Starting with a finite configuration h, a minimal cellular automaton is one which provides generating from h of the sequence whose elements in the aggregate will contain all finite block configurations*** was being enough actively discussed; i.e. in the above terminology for such classical structures the following relation should be carried out, namely:

$$(\exists c_o \in C(A,d,\phi))(\forall c^* \in C(A,d,\phi))(\exists t \geq 1)(c^* \subseteq c_o\tau^{(n)t}) \qquad (13)$$

First of all, in our opinion, the term «***minimal***» insufficiently correctly reflects essence of the question since it, above all, concerns complexity of a ***HS***–model itself which as a rule is determined by such parameters as: *dimensionality*, *size of neighbourhood template* and also *cardinality* of a state alphabet of an elementary automaton of a $d\text{–}HS \equiv <Z^d,A,\tau^{(n)},X>$, i.e. value ***SL = dxnx#(A),*** where ***#(A)*** – cardinality of an alphabet ***A.*** At that, many considerations about possibility of existence of similar ***HS–***models has been suggested; concrete examples of models of such type which are based only on empirical results were presented. However, as far as we know, strict result meanwhile is absent. Below, the given question will be a little bit detailed. We shall consider this problem in a little bit other posing, namely: ***Whether some HS–model can generate all set of finite configurations, since some initial finite configuration?*** In other words, in the above terminology for such classical ***HS–***models the following relation should be carried out, namely:

$$(\exists c_o \in C(A,d,\phi))\left(\bigcup_{j=0}^{\infty}\left\{c_o, c_{j+1} = c_j\tau^{(n)}\right\} = C(A,d,\phi)\right) \qquad (14)$$

A rather simple proof of impossibility of the above relation ***(14),*** using results on the nonconstructability in classical structures for case of the classical structures ***d–HS (d≥1)*** can be found, for example, in [567,640]. Moreover, using results concerning the *nonconstructability* such as ***NCF*** and ***NCF–1,*** it is possible to show [1,5,54], that the given problem even in more general formulating has the negative solution for the classical ***HS–***models; about that the following basic result having a lot of rather interesting appendices along with many theoretical aspects testifies [5, 90,536,567]. The result is enough widely used in many considerations.

<u>Theorem 65</u>. For an arbitrary classical structure d–HS (d ≥ 1) a finite set of universal finite configurations is inadmissible.

A simple enough proof basing on a series of well–known properties of nonconstructability such as ***NCF*** and ***NCF–1*** for the *classical* structures ***d–HS (d≥1)*** of the given theorem can be found, for example, in [640]. In addition, the approach used at proof of the given theorem and basing on the considered properties of nonconstructability such as ***NCF*** and ***NCF–1*** can be useful and in a lot of other cases. Thus, even essentially stronger result expressed by the following theorem, that presents and independent interest, takes place under certain conditions [5,54,56,88].

<u>Theorem 66</u>. If an arbitrary classical structure d–HS (d ≥ 1) possesses the nonconstructability such as NCF in the presence for the structure of a set G of configurations such as NCF–1 then for the structure there is not a finite set of configurations $c_j \in C(A,d,\phi)$ (j=1 .. p), for which the following determinative relation takes place, namely:

$$\bigcup_j \langle c_j \rangle\left[\tau^{(n)}\right] = C(A,d,\phi)\setminus G; \qquad c_j \in C(A,d,\phi) \quad (j = 1..p)$$

The proof of the given theorem basing on the nonconstructability such as ***NCF*** and ***NCF–1*** along with some other rather simple prerequisites is transparent enough and can be found in our works [54–56,79,88,90].

Moreover, from the given result directly follows, what in certain cases a narrowing of the set ***C(A,d,ϕ)*** of all finite configurations up to the set of only constructive configurations, which it is necessary to generate, does not lead to the positive decision of the existence problem of ***UCF*** for classical ***HS***–models. In addition, on basis of one rather interesting *algebraic* approach with application of results on the *nonconstructability* it is possible to prove essentially more general and strong result [9,15, 90,91], which gives answers to some questions, raised in our previous works [1,4,22,88,54–56,76], and also forms a rather essential part of the apparatus of research of dynamics of the classical ***HS***–models. At that, an analogue of the given result exists also and for ***HSoS***–models [9,15].

<u>Theorem 67</u>. If an arbitrary classical structure d–HS ($d \geq 1$) with state alphabet A={0,1,2, ..., a-1} (a – a prime number) and global transition function $\tau^{(n)}$ possesses a set M of NCF (perhaps, NCF–3) and/or NCF–1 then there are not such finite sets of global transition functions $\tau^{(n_j)}$ and finite configurations $c_j \in C(A,d,\phi)$ given in the same state alphabet A that the following two relations will take place, namely:

$$1)\bigcup_j < c_j > \left[\tau^{(n_j)}\right] = C(A,d,\phi)\setminus M; \quad 2)\bigcup_j < c_j > \left[\tau^{(n_j)}\right] = M \quad (d \leq 1;\, j = 1..p)$$

At that, for alphabet A (a – composite number) the formulation of the result only with the second relation takes place; the given statement takes place in case of prime number a and the nonconstructability of type NCF–2 also; i.e. global transition function $\tau^{(n)}$ will possess a set M of the nonconstructible configurations such as NCF–2.

Along with others [5,71–73,75,88,90], from the given result interesting enough consequence follows, namely: ***Two sets of the configurations C(A,d,ϕ)\GSAK*** and ***GSAK (GSAK –*** *a set of* ***NCF****, perhaps, and* ***NCF–3;*** *and also* ***NCF–1*** *or* ***NCF–2*** *in case of an alphabet* ***A*** *for prime* ***a) cannot be generated by means of finite sets $c_j \in C(A,d,\phi)$ and GTF $\tau^{(n_j)}$, that are defined in the same alphabet, regardless to an initial GTF $\tau^{(n)}$ relative to which the nonconstructability of the specified types is considered.***

Moreover, from result of the theorem ***66*** follows, that the classical ***HS***–models are not finite–axiomatized parallel formal systems even under condition of elimination from the set ***C(A,d,ϕ)*** of the nonconstructive finite configurations. Thus, each set of nonconstructive configurations

(NCF, NCF-1, NCF-2, NCF-3) concerning the completeness problem in classical structures ***d–HS (d≥1)*** possesses the same immunity as the set ***C(A,d,ϕ)*** of all finite configurations. This result allows to understand essentially more deeply the essence of the *nonconstructability* problem in classical ***HS***–models. At that, it turned out that exception of each of allowable four types of the nonconstructability does not exert serious influence on the maximal constructive opportunities, for example, in a context of existence for the classical ***HS***–models of a set of ***UCF.*** At the same time, use of an expanded alphabet of classical ***HS***–models allows to successfully and simply enough generate only from a single initial configuration the set ***C(A,d,ϕ)*** of all finite configurations [54,545,567].

Consequently, even exception out of a set ***C(A,d,ϕ)*** of nonconstructible configurations of any types concerning any ***GTF*** leaves in it complex enough finite configurations for generating of them even by means of a finite set of ***GTF*** $\tau^{(n_j)}$. This is direct step on a way of definition of the ***complexity*** concept of finite configurations in classical ***HS***–models. In addition, existence of a finite set of ***UCF*** and for some separate global transition function $\tau^{(n)}$, and for their finite set $\tau^{(n_j)}$ is impossible, i.e. in the *expanded* understanding the existence problem of ***UCF*** for classical ***HS***–models also has the negative decision.

In the expanded understanding, the given problem directly precedes the complexity problem of finite configurations while in initial posing speaks about impossibility of generation by means of global transition function of a classical ***HS***–model of all set of finite configurations from an arbitrary finite set of initial finite configurations, establishing a top unattainable limit of its generative opportunities. In this connection a number of more weakened approaches to determination of maximal generative opportunities of classical ***HS***–models is considered. Among them an approach on basis of *universal reproducibility* in Moore's sense of the finite configurations occupies especial place; it represents along with academic interest also a certain applied interest, in particular, in biology of development and in a lot of research problems concerning the questions of *reliability* and *restoration* in the various ***HS***–models of technical character, including questions of self–reproduction of robots [536]. The given question will be considered in the following section; here we shall consider the existence problem for classical ***HS***–models of universal finite configurations at reduction in requirement to their definition, namely:

Whether exist classical homogeneous structures d–HS (d ≥ 1) and such

initial finite configurations for them, that the determinative relation (13) takes place?

In such posing we require not generating from an initial configuration $c_o \in C(A,d,\phi)$ of some sequence containing all finite configurations, but only of presence in the generated finite configurations of entrances of all finite block configurations, what is essentially weaker condition in view of the concept of classical structures. So, it is simple to make sure that at existence of a structure ***d–HS (d≥1)*** with the specified property all configurations, generated from a certain initial finite configuration ***h*** and which contain all finite *block* configurations will be in the Moore sense as self–reproducing configurations. While for such structure the nonconstructability such as ***NCF (NCF–3)*** will be absent at presence of nonconstructability such as ***NCF–1.***

At the same time in this structure any finite configuration ***c'*** can't have predecessors from sets ***C(A,d,ϕ)*** and ***C(A,d,∞)*** simultaneously, i.e. each finite configuration ***c*** will ***NCF–1*** or *absolutely* constructive [88,90]. The general solution of the given problem is not known to us, however in [640] a series of arguments in favour of its positive decision for case of a *binary* classical ***1–HS*** with neighbourhood index of Neumann-Moore is given. The fulfilled analysis of the given class of ***1–HS*** for detection of the specified property of generating of all block configurations has shown [567,640], that among the investigated structures it is possible to single out only ***10*** structures with discriminating numbers ***30,60,75, 86,89,90,102,105,106,120*** which possess the nonconstructability such as ***NCF–1*** in the absence of the nonconstructability such as ***NCF*** (***NCF–3)*** and which are quite suitable as candidates for the required property. However, the subsequent the detailed enough theoretical analysis and computer modeling have shown that structures with the numbers ***30, 60,75,86,89,90,102,105*** cannot serve as structures with the mentioned property of universality. Thus, structures ***1–HS*** with numbers ***106*** and ***120*** remain whose more detailed analysis have been executed.

For computer researches of dynamic properties of classical structures ***1–HS*** by us a whole series of means in various programming systems has been created; one of them programmed in ***CAS Maple*** [5,13,15,88, 90,118] is presented below. As show numerous computer experiments carried out by means of a simple procedure ***HS_GS*** in ***CAS Maple*** the given structures not only possess the universal reproducibility in the Moore's sense of finite configurations but also any finite configuration distinct from zero generates a sequence of configurations which in the aggregate contain all block configurations in the binary state alphabet.

```
> HS_GS := proc(Co::string, C::string, n::posint, T::table)
local a, d, h, k, j, m;
  assign(h=cat(seq("0", k=1..n–1)), d="", m=0), assign(a = cat(h,C,h));
  do  m := m + 1;  if search(a, Co) then return m  else
  for k to length(a) – n+1 do d:=cat(d,T[cat(a[k+j]$(j=0..n–1))]) end do;
        assign('a' = cat(h, d, h), 'd' = "")  end if
   end do
end proc:   # Maple 8, for other versions an adaptation can be needed
> LTF:= table(["000"=`0`, "001"=`1`, "010"=`1`, "011"=`0`, "100"=`1`,
  "101"=`0`, "110"=`1`, "111"=`0`]);
> [seq(HS_GS(cat("1", convert(k,binary),"1"), "1", 3, LTF),k=900..1050)];
  [54, 64, 32, 92, 81, 92, 61, 56, 121, 48, 43, 16, 64, 156, 86, 53, 50, 103, 32,
  46, 43, 46, 46, 64, 67, 44, 107, 88, 49, 72, 185, 116, 202, 68, 52, 69, 90, 50,
  125, 130, 184, 97, 106, 66, 150, 121, 26, 50, 43, 70, 128]
```

The source code of this procedure whose sense is transparent enough based on the information, that can be found, for example, in [536,545]. So, computer experiments with *classical* structures ***1–HS*** with numbers ***106*** and ***120*** together with certain theoretical considerations based on dynamic properties of such structures that are caused by presence in them of the nonconstructability such as ***NCF–1*** in the absence of ***NCF*** enable us to formulate interesting enough following assumption [88]:

For the binary classical structures 1–HS with numbers 106 & 120, any finite configuration which is different from the zero configuration and is self–reproducing configuration in Moore's sense generates a certain sequence of configurations that in the aggregate will contain all finite block configurations given in the binary state alphabet.

In any case along with problem of *self–reproducing finite* configurations in the *Moore* sense which belong to a so–called problem of the *maximal generative* possibilities of classical homogeneous structures, the various facilitated generative opportunities of the structures represent a rather considerable interest. And in this sense the discovery of the structures supporting not only of self–reproduction of finite configurations, but also simply of the set of all finite subconfigurations from a fixed finite initial configuration represents quite certain interest. This question we researched as theoretically and experimentally. In particular, with the help of experiments with a rather simple procedure **SubConf** a whole series of interesting enough results had been obtained. The procedure call **SubConf[*Ltf, Cf, p, w*]** on the basis of the local transition function which is defined by the list ***Ltf*** of parallel substitutions along with the finite configuration ***Cf,*** number of the demanded copies ***p*** and a finite configuration ***w*** returns **2–**element list whose the first element defines number of steps used by the structure ***1–HS(a, n)*** with local transition function ***Ltf*** for generation ***p*** copies of configuration ***Cf*** from an initial configuration ***w*** whereas the second element defines number of really obtained copies of configuration ***Cf.*** On that ground was revealed one a rather interesting class of structures ***1–HS(a, n)*** a few of which from an arbitrary finite initial binary configuration ***w*** generates the whole set of the binary finite configurations as ***subconfigurations.*** Moreover, any binary finite configuration ***Cf*** in such structure is self-reproducing in the *Moore* sense. The following fragment represents the source code of the procedure ***SubConf*** with typical examples of its application.

```
In[3212]:= SubConf[Ltf_List, Cf_String, p_Integer, h_String] :=
    Module[{n = StringLength[Part[Ltf[[1]], 1]], a, b, c = "", k, d = 0, t},
          a = StringJoin[Map[ToString, NestList[Sin, 0, n – 2]]];
                                        b = a <> h <> a;  Label[AVZ];
                          For[k = StringLength[b] – n + 1, k >= 1, k––,
          c = StringReplace[StringTake[b, {k, k + n – 1}], Ltf] <> c];
                                        b = a <> c <> a; c = ""; d = d + 1;
                  If[t = StringCount[b, Cf]; t >= p, {d, t}, Goto[AVZ]]]
In[3213]:= Lf := {"0000" –> "0", "0001" –> "1", "0010" –> "1", "0011" –> "0",
               "0100" –> "1", "0101" –> "0", "0110" –> "0", "0111" –> "1",
               "1000" –> "1", "1001" –> "0", "1010" –> "0", "1011" –> "1",
               "1100" –> "0", "1101" –> "1", "1110" –> "0", "1111" –> "1"};
In[3214]:= SubConf[Lf, "10100110111", 7, "10100110111"]
Out[3214]= {1555, 8}
In[3215]:= SubConf[Lf, "10100110111", 7, "1"]
Out[3215]= {1055, 7}
In[3216]:= Ltf := {"00000" –> "0", "00001" –> "1", "00010" –> "0",
            "00011" –> "1", "00100" –> "0", "00101" –> "1", "00110" –> "1",
            "00111" –> "0", "01000" –> "0", "01001" –> "1", "01010" –> "1",
            "01011" –> "0", "01100" –> "1", "01101" –> "0", "01110" –> "1",
            "01111" –> "0", "10000" –> "1", "10001" –> "0", "10010" –> "1",
            "10011" –> "0", "10100" –> "1", "10101" –> "0", "10110" –> "0",
            "10111" –> "1", "11000" –> "1", "11001" –> "0", "11010" –> "0",
            "11011" –> "1", "11100" –> "0", "11101" –> "1", "11110" –> "0",
            "11111" –> "1"};
```

```
In[3217]:= SubConf[Ltf, "10100110111", 7, "10100110111"]
Out[3217]= {752, 7}
In[3218]:= Ltf90 := {"000" –> "0", "001" –> "1", "010" –> "0", "011" –> "1",
            "100" –> "1", "101" –> "0", "110" –> "1", "111" –> "0"};
In[3219]:= SubConf2[Ltf120, "1010011011101", 25, "1010011011101"]
Out[3219]= {248, 32}
```

Let ***G*** is a set of classical binary structures ***1–HS(2, n) (n ≥ 3)*** whose the global transition functions are defined by local transition functions by the following formulae, namely:

$$\begin{cases} \sigma^{(n)}(0,0,\ldots,0) = 0\,; \qquad \sigma^{(n)}(1,0,\ldots,0) = 1 \\ (\forall x_n \neq x_n^*)(\sigma^{(n)}(x_1,x_2,\ldots,x_{n-1},x_n) \neq \sigma^{(n)}(x_1,x_2,\ldots,x_{n-1},x_n^*))\ , \textit{otherwise} \\ \qquad x_n^*, x_k \in \{0,1\};\quad k = 1..n \end{cases}$$

It is obviously, that quantity of such binary structures equals $2^{2^{n-1}-2}$. Numerous experiments with the above procedure allow to formulate the following interesting enough assumption, namely:

Among all linear structures of the set G there are structures which by the generative possibilities can be characterized as follows, namely:

1. Structures for which an arbitrary finite binary configuration is self-reproducing in the Moore sense;

2. Structures which generate from an arbitrary finite configuration the sequence of configurations containing any beforehand given number of copies of an arbitrary binary finite configuration.

The first component of the assumption has been proved a rather long ago while the correctness of the second has substantially presumable character basing on positive results of numerous experiments with the above procedure ***SubConf.*** So, in case if the second component of this assumption is valid then we have structures possessing the generative property that is more strong then self–productivity in the *Moore* sense. At that, term «*assumption*» for the above assertion is caused by its final part because according to the following section its first part is true. So, in case of determination of classical structures of such type we receive a rather interesting class of the ***HS***–models possessing simultaneously the property of universal reproducibility in Moore's sense along with property of universal generative opportunities of block configurations from any finite initial configuration. In our opinion, researches in the given direction are being represented interesting enough. At that, on this way we would receive possibility to better make clear the essence of complexity of finite configurations in the ***HS***–axiomatics. The given question is discussed in chapter ***4*** of the present monograph in detail.

3.2. Self–reproduction of finite configurations in the classical homogeneous structures

If the existence problem of ***UCF*** characterizes *generative* opportunities of the classical ***HS***–models concerning the set of finite configurations as a whole, the universal *reproducibility* combines the given possibility with certain structural-dynamic aspect of generating of configurations sequences by means of the ***HS***–models. The essence of the universal reproducibility is that any finite configuration in a classical ***HS***–model is self–reproducing configuration in Moore's sense. The above class of the ***HS***–models possessing the property of universal reproducibility is interesting from many standpoints, and a lot of attention is devoted to the given question of global dynamics of ***HS***–models. In this direction a lot of such researchers as ***A. Waksman, T. Vinograd, A.R. Smith, T. Yaku, S. Amorozo, T. Ostrand, A. Fredkin, G. Cooper, P. Anderson, S. Ulam, V.Z. Aladjev,*** etc. have received rather interesting results [1,3-5, 12,15,54-56,88,90,131,185-187,221-225,294,298,299,536,567,618]. At that, our results in the given direction allow to establish a number of rather interesting interrelationships between the nonconstructability and the universal reproducibility in the classical ***HS***–models, and also to solve a series of especially mathematical problems. For the further we shall define the concept of «*reproducibility of configurations in **Moore**'s sense*».

Definition 17. We shall speak, a configuration $c \in C(A,d,\phi)$ contains m copies (accurate to shift and rotation of space Z^d) of the certain block configuration c_b if exists m nonoverlapping areas of the space Z^d each of which contains at least one copy of the configuration c_b. We shall speak, a configuration $c \in C(A,d,\phi)$ is self–reproducing configuration in Moore's sense in a classical structure d–HS ($d \geq 1$) if for any previously given integer $m > 0$ there is such integer $t > 0$, that configuration $c\tau^{(n)t}$ will contain not less m copies of the initial configuration c.

Along with attempts of formalization of self–reproduction process at the most abstract level the self-reproducing configurations in a certain measure can characterize constructive possibilities of the classical ***HS***–models and in the given attitude they are to a certain extent are *reverse* to nonconstructible configurations such as ***NCF, NCF-1, NCF-2, NCF-3.*** It is shown that classical homogeneous structures ***d–HS*** can have sets of rather complex finite self-reproducing configurations as at absence, and at presence in them of the nonconstructability of the above types

[1,3–5]. The discovered class ***L*** of linear classical structures possessing the property of universal reproducibility of the finite configurations is the most interesting in this respect.

A classical structure ***d–HS (d ≥ 1)*** is called the *linear* structure if its ***LTF*** $\sigma^{(n)}$ is defined by the formula of the following kind, namely:

$$\sigma^{(n)}(x_1, \dots, x_n) = \sum_{1}^{n} b_k x_k \pmod a \quad x_k, b_k \in A = \{0, 1, \dots, a-1\};\ (k = 1..n),\ a - prime$$

At the certain assumptions thanks to works of the above researchers the following interesting enough result takes place [5,54–56,88,90,536].

<u>Theorem 68</u>. For a classical linear structure d–HS (d ≥ 1) an arbitrary configuration c∈C(A,d,ϕ) is self–reproducing configuration in Moore's sense, i.e. the structure possesses property of universal reproducibility of finite configurations.

Using result of the theorem ***150*** [5], we receive the possibility not only for essential simplification of proof of the given result, but in a certain degree for characterization of the whole class of the given type of such structures further named as a class of ***linear classical structures d–HS (d ≥ 1).*** Below we believe that for structures of such linear class a state alphabet ***A=***{***0,1, ..., a–1***} satisfies the condition $a=p^k$ where ***p –*** a prime number and ***k –*** an arbitrary positive integer.

<u>Theorem 69</u>. A classical homogeneous structure d-HS (d≥1) with local transition function $\sigma^{(n)}$, defined by the following general formula

$$\sigma^{(n)}(x_1, x_2, \dots, x_n) = \Big(\sum_{k=1}^{n} b_k x_k\Big)^{a^m} \pmod a$$

there is at least a pair of such different integers j, p ∈ 1..n, that $b_j, b_p \neq 0$

possesses the property of universal reproducibility in Moore's sense of the finite configurations where $a = p^k$ (p – a prime number; m and k are positive integers), $b_j, x_j \in A = \{0,1, \dots, a-1\}$; j = 1 .. n.

Thus, the class ***L*** of linear classical ***HS***–models from the standpoint of their dynamics can be characterized by availability of the property of universal reproducibility of finite configurations. So, in this connexion a rather interesting question arises: ***Whether there are other classes of the d–HS (d ≥ 1) that possess the property of universal reproducibility and how they can be characterized formally?***

At that, the class of linear ***HS***–models possessing property of *universal reproducibility* of finite configurations till now was considered relative to the connected neighbourhood index ***X =*** {***0,1,2, ..., n–1***}. Meanwhile, the given property is extended to a case of the general neighbourhood index $X^*=\{j_1,j_2, ..., j_p\}$ $(0=j_1 < j_2 < ... < j_p = n-1)$, including disconnected indexes provided, that in them not less than two variables in each of dimensions are the leading variables, i.e. for structures ***1–HS*** their ***LTF*** $\sigma^{(n)}$ have the following formula representation, namely:

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = \sum_{j=j_1}^{j=j_p} a_j x_j \pmod{a}; \quad a_j, x_j \in A; \quad a = p^k$$
$$(0 = j_1 < j_2 < ... < j_h = n-1;\ 2 \le h \le n); \quad p - a\ prime;\ k - an\ integer \qquad (15)$$

where $A=\{0,1, ..., a-1\}$ – alphabet of the ***1–HS*** under condition $a=p^k$, p – a prime, k – a positive integer while neighbourhood template has size n [5,9,54–56,88]. It is simple to make sure that for each neighbourhood template of size n exists $2^{n-2}-1$ different disconnected neighbourhood templates. In addition, the minimal neighbourhood index has the next kind $X = \{0, n-1\}$. For example, for case of binary classical structures ***1–HS***, two linear structures with neighbourhood indices $X_3 = \{0,1,2\}$ and $X_2 = \{0,2\}$ possess the possibility of universal *reproducibility* in Moore's sense of finite configurations. As a whole, among all classical binary structures ***1–HS*** with maximal neighbourhood index $X = \{0,1,2, ..., n-1\}$ exists 2^n-n-1 *linear* classical structures possessing property of *universal reproducibility* in Moore's sense of finite configurations. Consequently, in this context quite pertinently to consider in a sense the ***generalized*** class of the linear classical structures characterized by such important dynamic property as the universal reproducibility in Moore's sense of finite configurations whose ***LTF*** are defined by the above relations of theorem ***68*** and ***(15)***. In [536,545,640] source codes of some procedures along with numerous examples of their use for analysis of dynamics of self–reproducing configurations in the classical structures ***1–HS*** are represented. For comprehension of the main principles of realization and functioning of these procedures of acquaintance with computer algebra system ***Maple*** of releases not below than ***6*** within the scope of our books is quite sufficiently [97–118].

During computer experimentation a whole series of rather interesting results relative to self–reproducing finite configurations in the class of linear classical ***HS***–models has been received, with some of them it is possible to familiarize oneself in [3,88,567,640]. So, the results received in the given direction allow us to speak, the property of universal or essential reproducibility in Moore's sense of the finite configurations, apparently, is inherent also in linear structures with an alphabet ***A =***

{***0,1,2, ..., a–1***}, where ***a*** **–** a positive integer, which can't be represented in the form of $a=p^k$ where ***p*** **–** a prime number **&** ***k*** **–** a positive integer. However, as against the above class of linear structures the process of generating of required quantity of copies of initial *finite* configurations in the linear structures of the given type demands considerably larger quantity of steps of ***GTF*** $\tau^{(n)}$ on the assumption of essential decrease of density of quantity of copies during generating. At that, size of an initial configuration and also its kind exert a rather essential influence on speed of generating. Thus, exists a whole series of other interesting enough results in this direction which seem to us interesting enough for the further researches [3,5,9,54–56,79,88,90,536,567,617,640–643].

In particular, a number of software tools for experimental study of the linear classical structures of dimensionalities ***1*** and ***2*** has been created; To a large extent, such tools have been focused on investigation of the *self-reproduction* phenomenon of finite configurations when cardinality of *alphabet* of internal states of an elementary automaton of structure is expressed by an arbitrary integer. As an example of one of these tools, the procedure **Selfreprod**, whose source code the following fragment represents, can serve, namely:

```
In[2230]:= SelfReprod[c_String, n_Integer, p_Integer, m_Integer] :=
                                                   Module[{a, b = "0", k,
          h = StringJoin[Map[ToString, NestList[Sin, 0, n – 2]]]},
                                      a = h <> c <> h; d = 1; Label[AVZ];
                             For[k = StringLength[a] – n + 1, k >= 1, k––,
                 b = ToString[Mod[Total[ToExpression[Characters[
                                  StringTake[a, {k, k + n – 1}]]]], p]] <> b];
                                      a = "0" <> b; t = StringCount[a, c];
                                         If[t >= m, d, d++; Goto[AVZ]]]
In[2231]:= SelfReprod["13302", 3, 4, 3]
Out[2231]= 15
In[2232]:= {d, t}
Out[2232]= {15, 10}
```

A successful procedure call **SelfReprod[*c, n, p, m*]**, implemented in the environment of system ***Mathematica***, returns the number of iterations of the linear global function $\tau^{(n)}$, defined by a local function $\sigma^{(n)}$

$$\sigma^{(n)}(x_1,\dots,x_n)=\sum_{1}^{n} x_k \pmod p \quad x_k \in A=\{0,1,\dots,p-1\};\ (k=1..n),\ p - \text{an arbitrary integer} \quad (Kr)$$

that was required to generate ***m*** of copies of an initial configuration ***c***.

In case of a rather long run of the procedure, it can be interrupted, by monitoring through the list {***d, t***} the reality of obtaining the required number of copies of the initial finite configuration ***c.***

For experimentally theoretic study of the self–reproductivity in Moore sense the below tools had been used. The call ***ToLTF*[*A, n, F, p*]** returns the list of parallel substitutions defining a local transition function of ***1–HS*** with alphabet ***A,*** template size ***n*** and ***p*=Length[*A*];** at that, ***F*** is a name of the function defined below. While the call ***SubConf1*[*Ltf,Cf,p*]** provides monitoring of generation from finite configuration ***Cf*** of its ***p*** copies through the returned list {***d, t***} where ***d*** – number of iterations of ***GTF*** $\tau^{(n)}$ defined by a local function ***Ltf,*** and ***t*** – real number of copies of a configuration ***Cf.*** The fragment presents source codes of the given tools along with the more typical examples of their applications.

```
In[850]:= F[x_List, n_Integer] := Mod[Total[x], n]

In[851]:= ToLTF[A_List, n_Integer, F_Symbol, p_Integer] :=
                    Module[{a = Tuples[A, n], b = {}, k = 1},
                          For[k, k <= Length[a], k++,
     b = AppendTo[b, Rule[StringJoin[Map[ToString, a[[k]]]],
                             ToString[F[a[[k]], p]]]]]; b]

In[852]:= SubConf1[Ltf_List, Cf_String, p_Integer] :=
  Module[{n = StringLength[Part[Ltf[[1]], 1]], a, b, c = "", k, d = 0, t},
       a = StringJoin[Map[ToString, NestList[Sin, 0, n – 2]]];
                           b = a <> Cf <> a;  Label[AVZ];
                For[k = StringLength[b] – n + 1, k >= 1, k––,
     c = StringReplace[StringTake[b, {k, k + n – 1}], Ltf] <> c];
                           b = a <> c <> a; c = ""; d = d + 1;
          If[t = StringCount[b, Cf]; t >= p, {d, t}, Goto[AVZ]]]

In[853]:= SubConf1[ToLTF[{0, 1, 2, 3, 4, 5}, 2, G, 6], "12304532", 4]
Out[853]= {720, 4}
In[854]:= SubConf1[ToLTF[{0, 1, 2}, 3, G, 3], "122211222101", 16]
Out[854]= {351, 27}
In[855]:= SubConf1[ToLTF[{0, 1, 2, 3}, 3, G, 4], "10213222333012", 6]
Out[855]= {224, 8}
In[856]:= ToLTF[{0, 1, 2, 3}, 2, G, 4]
Out[856]= {"00" –> "0", "01" –> "1", "02" –> "2", "03" –> "3", "10" –> "1",
 "11" –> "2", "12" –> "3", "13" –> "0", "20" –> "2", "21" –> "3", "22" –> "0",
 "23" –> "1", "30" –> "3", "31" –> "0", "32" –> "1", "33" –> "2"}
```

Numerous experiments with the procedures ***SelfReprod*** and ***SubConf1*** allowed to study a whole series of the classical ***1***-dimensional ***HS*** with different both alphabet ***A*** of internal states of elementary automata of structures and neighbourhood template size ***n***, allowing to formulate the following a rather interesting assumption, namely:

There is a rather wide set of classical structures 1–HS(a,n) with local transition functions of the form (Kr) where p is integers different from earlier considered types of integers that possess the property of rather essential self-reproducibility in Moore's sense of finite configurations.

Note, in many cases the above software allow to receive the structure of configurations containing copies of self–reproducing configuration. Other interesting properties of the generalized class of linear classical ***HS***–models characterized by property of universal reproducibility in Moore's sense of finite configurations can be found in [54–56,536,640]. Meanwhile, it would be rather desirable to receive some *determinative* characteristics of the *generalized* linear classical ***HS***–models as a whole. In the given direction quite definite interest represents the following basic result, used and in other purposes [88,90,567]. In this respect we have received the general characteristic of the given class of structures which is closely connected with the *nonconstructability* problem in the classical ***HS***–models [54–56,75,618,640–643].

Theorem 70. Existence of the nonconstructability such as NCF–1 in the absence of the nonconstructability such as NCF (NCF–3) in a classical structure d–HS (d ≥ 1) is the necessary condition but not sufficient for possession of the structure of property of universal reproducibility in the Moore's sense of finite configurations.

The given result represents a certain kind of test (*a necessary condition*) for verification of classical ***HS***–models for possession by the property of *universal reproducibility* and also illustrates rather prominent aspect of interrelation between *maximal* constructing and nonconstructability in classical ***HS***–models. Thus, we obtain a certain kind of «***filter***» for selection of structures for the purpose of the candidates for possession of the property of *universal reproducibility* according to ***E. Moore***. Thus, among the structures of such type it is necessary to search structures with property of universal reproducibility as well as with property of high degree of *reproducibility* in Moore's sense of finite configurations.

For example, among the above *binary classical* ***1–HS*** only the structures with discriminating numbers ***30,45,60,75,86,89,90,101,102,105,106,120*** possess the nonconstructability such as ***NCF–1*** without ***NCF***, however only the structures with numbers ***30,45,60,75,86,89,90, 101,102,105,106, 120*** possess the property of full *(universal)* or essential reproducibility of finite configurations according to the following table ***3*** [15,54,88,90].

Table 3

	NCF	*NCF–1*	*Increase*	*Periodic*	*Self–reproduction*
30	–	+	–	+	*No*
45	–	–	–	+	*No*
60	–	+	+	–	*Yes*
75	–	+	+	–	*No*
86	–	+	–	+	*No*
89	–	+	+	–	*No*
90	–	+	+	–	*Yes*
101	–	–	–	+	*No*
102	–	+	+	–	*Yes*
105	–	+	+	–	*Yes*
106	–	+	+	–	*Yes*
120	–	+	+	–	*Yes*

Proof of the *universal reproducibility* in Moore's sense of structures with numbers ***60,90,102*** and ***105*** is based on the fact, that they are the linear classical structures with connected and disconnected neighbourhood indexes; their ***LTF*** $\sigma^{(3)}$ are defined by the following formulas, namely:

$$\sigma^{(3)}_{60}(x,y,z) = x+y \pmod 2; \qquad \sigma^{(3)}_{90}(x,y,z) = x+z \pmod 2; \; x,y,z \in \{0,1\}$$
$$\sigma^{(3)}_{102}(x,y,z) = y+z \pmod 2; \qquad \sigma^{(3)}_{105}(x,y,z) = x+y+z \pmod 2$$

whereas the proof for structures with numbers ***106*** and ***120*** is based on the kind of their ***LTF*** $\sigma^{(3)}$ defined by the following formulas:

$$\sigma^{(3)}_{106}(x,y,z) = \begin{cases} y+z \pmod 2, & \text{if } y=1 \\ x+y+z \pmod 2, & \text{otherwise} \end{cases}$$
$$\sigma^{(3)}_{120}(x,y,z) = \begin{cases} x+y \pmod 2, & \text{if } y=1 \\ x+y+z \pmod 2, & \text{otherwise} \end{cases}$$

or in equivalent form:

$$\sigma^{(3)}_{106}(x,y,z) = (1-y)x+y+z \pmod 2 \qquad \sigma^{(3)}_{120}(x,y,z) = x+y+(1-y)z \pmod 2$$

Thus, last two functions are different from linear functions whose ***LTF*** $\sigma^{(n)}$ are defined in the form of ***(15).*** Furthermore, the above structures with numbers ***106*** and ***120*** possess not only the possibility of *universal* reproducibility in the Moore's sense of finite configurations but, most probably, along with that also for them a finite configuration distinct from zero generates a sequence of configurations that in the aggregate contain all ***block*** configurations in the binary alphabet, i.e. possess the property of universality relative to the binary block configurations. At that, if for generation of ***w*** copies of an arbitrary finite configuration ***h*** structures with discriminating numbers **106** and **120** demand ***p*** and ***q*** steps, then for the inverse configuration h^R these structures demand ***q*** and ***p*** steps accordingly.

It is easy to make sure that for any state alphabet $A=\{0,1,2,\ldots,a-1\}$ the global transition functions $\tau^{(n)}$ $(n \ge 2)$ constitute the noncommutative subset $T(a)$ concerning the operation of composition, i.e.

$$(\forall\tau^{(n)})(\forall\tau^{(p)})(\tau^{(n)}\tau^{(p)}\in T(a)\ \&\ (\exists\tau^{(n)},\tau^{(p)})(\tau^{(n)}\neq\tau^{(p)}\rightarrow\tau^{(n)}\tau^{(p)}\neq\tau^{(p)}\tau^{(n)}))$$

At that, the subset $T(a)$ does not possess a finite system of generators. The detailed enough consideration of the decomposition problem of global transition functions in the classical ***HS***–models is presented in chapter ***7,*** whereas here we use this approach for creation of *nonlinear* binary classical structures ***1–HS*** possessing the property of universal reproducibility in the Moore's sense. We illustrate the above approach on basis of simple enough example. Let $\tau^{(2)}$, $\tau^{(3)}{}_{106}$, $\tau^{(3)}{}_{120}$ are global transition functions whose local functions are determined as follows:

$$\sigma^{(2)}(x,y)=x+y\ \ (mod\ 2);\quad (1)\,\tau^{(2)}\tau^{(3)}_{106},\ (2)\,\tau^{(2)}\tau^{(3)}_{120},\ (3)\,\tau^{(3)}_{106}\tau^{(2)},\ (4)\,\tau^{(3)}_{120}\tau^{(2)}$$

$$\sigma^{(3)}_{106}(x,y,z)=(1-y)x+y+z\ \ (mod\ 2)\qquad \sigma^{(3)}_{120}(x,y,z)=x+y+(1-y)z\ \ (mod\ 2)$$

In the same place four compositions of global transition functions that were subjected to analysis are represented. It is easy to make sure that these compositions present four different nonlinear global transition functions $\tau^{(4)}$ according to the above numeration which are presented by the following local transition functions $\sigma^{(4)}$, namely:

$$\sigma^{(4)}_1(x,y,z,h)=\begin{cases}x+y+z+h+1\ \ (mod\ 2), & if\ \ <xyzh>\in\{0010,0011,1100,1101\}\\ x+y+z+h\ \ (mod\ 2), & otherwise\end{cases}$$

$$\sigma^{(4)}_2(x,y,z,h)=\begin{cases}x+y+z+h+1\ \ (mod\ 2), & if\ \ <xyzh>\in\{0011,0100,1011,1100\}\\ x+y+z+h\ \ (mod\ 2), & otherwise\end{cases}$$

$$\sigma^{(4)}_3(x,y,z,h)=\begin{cases}x+y+z+h\ \ (mod\ 2), & if\ \ <xyzh>\in\{0010,0011,1000,1001\}\\ z+h\ \ (mod\ 2), & otherwise\end{cases}$$

$$\sigma^{(4)}_4(x,y,z,h)=\begin{cases}x+h\ \ (mod\ 2), & if\ \ <xyzh>\in\{0001,0100,1001,1100\}\\ x+y\ \ (mod\ 2), & otherwise\end{cases}$$

Experimental–theoretical researches of ***1***–dimensional classical binary structures, whose global transition functions $\tau^{(n)}$ $(n \ge 4)$ are formed by means of the above method allowed to formulate the following rather interesting assertion, namely:

For each integer n ≥ 3 there are at least four nonlinear binary classical structures 1–HS with neighbourhood index X={0,1,...,n–1} that possess the property of universal reproducibility in the Moore's sense of finite

configurations among which two binary classical structures possess the universality property of finite block configurations.

At that, if the *first* part of the assertion has strict theoretical basing [88] then the *second* part is based on results of computer research of global transition structures $\tau^{(3)}_{106}\tau^{(2)}$, $\tau^{(3)}_{120}\tau^{(2)}$, i.e. bears especially experimental character. Meantime, the made numerous computer experiments with the above last two structures allow to say about a rather high level of certainty of the second part of the given assertion [88,90]. However, in this direction a theoretical confirmation would be very desirable.

At that, for the above classical structures with discriminating numbers ***75*** and ***89*** the following quite interesting regularity has been detected:

For each finite configuration c_o the structures generate configurations sequences $<c_o>[\tau_{75}{}^{(3)}]$ and $<c_o>[\tau_{89}{}^{(3)}]$ accordingly each of which will contain configurations subsequences of the following kind, namely:

$$75:\ \left\{pr(c_o)\,0^{2t-|pr(c_o)|}\,c_o \,\middle|\, t = 2^k;\ k = k(c_o) + j;\ j = 0,1,2,\dots\right\}$$

$$89:\ \left\{c_o\ 0^{2t-|sf(c_o)|}\,sf(c_o) \,\middle|\, t = 2^k;\ k = k(c_o) + j;\ j = 0,1,2,\dots\right\}$$

where $k(c_o)$ is a positive integer depending from a finite configuration c_o, whereas $pr(c_o)$ and $sf(c_o)$ – prefix and suffix of the configuration c_o accordingly that depend from this configuration, and t is step number of the sequence generating.

Furthermore, for structures with discriminating numbers ***60,75,89,90, 102,105,106*** & ***120*** the relation $(\forall c\in C(A,d,\phi))(|c|<|c\tau^{(n)}|)$ takes place (*column «**Increase**» in table* **3**), where $|S|$ is size of maximal diameter (*in* **1**–*dimensional case a length*) of a finite configuration ***S*** that according to theorem ***50*** provides these structures with nonconstructability such as ***NCF–1*** in the absence of the nonconstructability such as ***NCF*** and a generating of all set ***C(A,d,ϕ)*** from configurations such as ***NCF–1*** only.

In addition, for such structures rather interesting regularities relative to quantities of copies of configurations generated by them depending on number of steps of generating have been discovered. In particular, for structures with discriminating numbers ***90*** and ***102*** an interesting regularity has been found: ***For «almost all» finite configurations c*, if m copies of a configuration c* are generated by structure with number 90 during t steps, the same number of the configuration c* is generated by structure with number 102 during 2*t–1 steps with the exception of configurations from the set*** $\{1^{2k-1}\,|\,k=1,2,3,4,\dots\}$.

Above we noted a series of interesting enough properties of classical structures ***d–HS (d ≥ 1)*** with an arbitrary neighbourhood index ***X*** and state alphabet ***A***={***0,1,3, ..., a–1***}, whose global transition functions $\tau^{(n)}$ satisfy the following condition $(\forall c \in C(A,d,\phi)(|c\tau^{(n)}| > |c|)$, where $|c|$ is maximal diameter of a finite configuration ***c***, i.e. structures of such type produce the configurations sequences strictly increasing in size from an arbitrary finite configuration ***h*** that is different from fully null configuration c_0= . These structures constitute a special subclass *(let's denote as* ***GS–****class)* of all classical structures of the same state alphabet and dimensionality. As it was noted earlier, structures of this subclass possess at least the nonconstructability such as ***NCF–1*** or ***NCF.***

On the other hand, for today namely among structures of the ***GS–***class the structures possessing the opportunity of *universal reproducibility* in Moore's sense of finite configurations have been found. In particular, well–known linear classical structures also belong to the ***GS–***class. It is quite perhaps that one of tests of existence for classical structures ***d–HS (d≥1)*** of the property of *universal reproducibility* in Moore's sense of finite configurations can be formulated as follows, namely:

The structures possessing the property of universal reproducibility in Moore's sense, it is expedient to search among the classical structures which in absence for them of nonconstructability such as NCF satisfy the relation $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$ ***where*** $|h|$ ***is size of maximal diameter of a finite configuration h; thus, sought structures belong to the above GS–class whose GTF form a noncommutative subset GS in regard to the composition operation. The given structures possess the nonconstructability such as NCF–1, and for them the set C(A,d,ϕ) can be generated only by nonconstructible configurations such as NCF–1.***

Thus, from the above ***8*** binary classical structures satisfying the above test, i.e. structures of the ***GS–***class, ***6*** structures possess the property of universal reproducibility in Moore's sense of finite configurations. So, the above test allows to reveal ***75%*** of classical binary structures ***1–HS*** with neighbourhood index ***X*** = {***0,1,2***}. It would be desirable to narrow the above test by means of exception out of the above ***GS–***class of the structures not having any prospects in this respect.

Having determined on basis of experimental–theoretical researches of classical structures whose global functions $\tau^{(n)}$ satisfy the next relation $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$, where $|c|$ is size of maximal diameter of a finite configuration $c \in C(A,d,\phi)$ a test for the purpose of possibility of possessing by an classical structure of universal reproducibility in the Moore's sense of finite configurations, we are interested in a receiving of more concrete recommendations for search of such structures in the *GS*–class. Inasmuch as the *GS*–class forms noncommutative subset of global transition functions $\tau^{(n)}$ concerning operation of composition the search of such structures on basis of this operation quite naturally arises. It was shown that composition of global functions $\tau^{(n)}=\tau^{(p)}\tau^{(m)}$ from the *GS*–class gives a global function $\tau^{(n)}$ which will possess the property of universal reproducibility of configurations in the Moore's sense [15,79,88]. However, an extension of this technique is allowable, namely: ***Composition of global transition functions*** $\tau^{(n)}=\tau^{(p)}\tau^{(h)}$ ***from the GS–class when only one function*** $\{\tau^{(p)} \mid \tau^{(h)}\}$ ***possesses the property of universal reproducibility can give a new global transition function*** $\tau^{(n)}$ ***which also will possess the property of universal reproducibility in Moore's sense of finite configurations.*** For example, ***6*** compositions in the form $\tau^{(3)}_{75}\tau^{(2)}\neq\tau^{(2)}\tau^{(3)}_{75}$, $\tau^{(3)}_{89}\tau^{(3)}_{105}\neq\tau^{(3)}_{105}\tau^{(3)}_{89}$ and $\tau^{(3)}_{75}\tau^{(3)}_{105}\neq\tau^{(3)}_{105}\tau^{(3)}_{75}$ can serve as an illustration where global transition functions $\tau^{(3)}_{75}$ and $\tau^{(3)}_{89}$ do not possess the property of universal reproducibility in the Moore's sense whereas $\tau^{(3)}_{105}$ possesses the property. Furthermore, all compositions of such kind that have been examined by us have possessed by property of universal reproducibility in the Moore's sense.

Meantime, as a result of numerous computer experiments with global transition functions from the class ***GS***, we not succeeded in finding a pair of global transition functions whose composition would possess the property of universal reproducibility in Moore's sense. Thus, most likely, the following assertion takes place, namely:

Only compositions $\tau^{(n)} = \tau^{(p)}\tau^{(h)}$ ***of global functions from the class GS where at least 1 function*** $\{\tau^{(p)},\tau^{(h)}\}$ ***possesses the property of universal reproducibility in the Moore's sense can give as a result functions*** $\tau^{(n)}$ ***which will possess the above property of universal reproducibility.***

Simple enough examples represented above serve only for illustration of the said, whereas with the detailed experimental–theoretical aspect of this question the interested reader can familiarize oneself in [79,88].

Moreover, the result of theorem ***68*** allows to receive a decision of the following rather important question linked as well with constructive opportunities of the classical ***HS***–models, namely: ***Whether a classical HS–model can double an arbitrary finite configuration defined in the same alphabet A?*** Occupying oneself with questions of searching of a appropriate mathematical apparatus which will be isomorphic to the developing biological organization, we have suggested in this quality the parallel τ_n–grammar and ***A***–algorithms, and have carried out their analysis in a context of biological interpretations [3–5,33,46,54–56,88].

At that, the ***Rozen's*** logical paradox connected to the phenomenon of *self–reproduction* in formal developing systems was being investigated. The essence of this paradox consists in the condition, that the models of self–reproduction should include both the system of reproduction, and a certain specific environment. So, the known ***Markov*** algorithms determined in a certain alphabet ***A*** cannot double an arbitrary finite word in the same alphabet. It is quite reasonable to suppose, that the given result takes place both for τ_n, and for ***A***–algorithms. Concerning the ***A***-algorithms we have shown [3,5] that the problem of doubling of words is being solved by introduction of one additional symbol $b \notin A$. In addition, more detailed researches in this direction have allowed to formulate the important enough hypothesis representing indubitable theoretical interest from many standpoints.

Let ***P*** will be a production over a finite word ***s*** in some finite alphabet ***A*** which processes the given word into a new word ***s**** according to a certain algorithm using only the alphabet ***A***. Schema ***R*** presents some finite set of productions P_k ***(k = 1 .. n)*** together with algorithm of their application to any word ***s*** in the alphabet ***A***. Then we shall call ***F(R,A)*** a ***formal system*** in the alphabet ***A*** with scheme ***R***. In this terminology our hypothesis assumes the following kind, namely.

Hypothesis 1. There is not a formal system F(R, A) that can double an arbitrary finite word in an arbitrary finite alphabet A.

The given hypothesis remains open for today, and seems to us that its decision is rather complex, meanwhile, for case of classical ***HS***–models the given problem has the negative solution, namely.

Theorem 71. There is not a classical structure d–HS (d ≥ 1) with a state alphabet A and an arbitrary neighbourhood index which doubles each d–dimensional finite configuration defined in the same alphabet A.

The given result is immediate consequence of the more general result of the theorem ***70*** that sets a certain kind of restriction on the *universal* reproducibility of finite configurations in the classical ***HS***–models. Of results, represented below, it is possible to make sure in opportunity of the solution of the given problem for classical structures ***1–HS*** with an arbitrary finite alphabet ***A***, that is expanded only onto one symbol.

The given proof is not constructive and the reader is recommended as an useful enough exercise to determine a classical structure ***1–HS*** with an arbitrary state alphabet $A^* = A \cup \{\alpha\}$ $(\alpha \notin A)$ that doubles an arbitrary finite configuration determined in the state alphabet A.

It is well known that a series of properties intrinsic to global transition functions $\tau^{(n_j)}$ *(for some or for all)* is being inherited also by a transition function $\tau^{(n)}$ that can be presented in the form of a composition of the following general kind, namely [54–56,79,88,90,617,640–643]:

$$\tau^{(n)} = \tau^{(n_1)}\tau^{(n_2)} \ldots \tau^{(n_j)} \ldots \tau^{(n_p)}; \quad n = \sum_{j=1}^{p} n_j - (p-1); \quad 2 \le n_j < n; \quad j = 1..p \qquad (16)$$

and vice versa. So, if a ***GTF*** $\tau^{(n)}$ possesses the nonconstructability such as ***NCF,*** then at least one from functions $\tau^{(n_j)}$ that compose the above decomposition ***(16)*** will possess this property. With some other useful properties of similar type it is possible to familiarize oneself in book, whereas others can be found, for example, in [88,90]. Thus, we receive a rather natural mechanism of constructing the more complex global transition functions from less complex functions whose compositions will inherit the necessary properties inherent to some or all functions $\tau^{(n_j)}$, which form the composition of $\tau^{(n)}$ ***(16).*** In this context we shall consider the question of generating on base of the above composition approach of classical structures possessing the property of universal reproducibility in the Moore's sense of the finite configurations, that is essentially distinct from the classical example of linear structures.

By way of illustration we shall represent a rather simple example. For a composition *two* binary classical structures ***1–HS*** are chosen, namely: with neighbourhood index $X_2 = \{0,1\}$ and ***LTF*** $\sigma^{(2)}$ defined by formula $\sigma^{(2)}(x_o,x_1)=x_o+x_1 \ (mod\ 2)$ and with neighbourhood index $X_3=\{0,2\}$ and ***LTF*** $\sigma^{(3)}$ defined by formula $\sigma^{(3)}(x_o,x_1,x_3)=x_o+x_2 \ (mod\ 2)$. It is simple to make sure that composition $\tau^{(4)} = \tau^{(3)}\tau^{(2)}$ whose ***GTF*** are defined by above local transition functions gives as a result a new more complex global transition function $\tau^{(4)}$ whose local function $\sigma^{(4)}$ is determined by the next formula, namely: $\sigma^{(4)}(x_o,x_1,x_2,x_3)=x_o+x_1+x_2+x_3 \ (mod\ 2)$.

It is shown that ***1–HS*** with the given global function $\tau^{(4)}$ possesses the nonconstructability such as ***NCF–1*** in the absence of ***NCF.*** At that, the given structure possesses the property of universal reproducibility in Moore's sense [54,55,88,90,640]. Results received in the given direction allow to formulate the following rather interesting assertion.

<u>Theorem 72</u>. ***A classical structure d–HS (d ≥ 1) with global transition function $\tau^{(n)}$ given in alphabet A={0,1,2, ..., a–1} possesses the property of universal reproducibility in Moore's sense of finite configurations, if all global transition functions $\tau^{(n_j)}$, which constitute a composition of GTF $\tau^{(n)}$ (16) also possess the property of universal reproducibility.***

In particular, at constructing **1**–dimensional global functions $\tau^{(n+m-1)}$ with the property of universal *reproducibility* on basis of a composition of more simple global transition functions $\tau^{(n)}$ and $\tau^{(m)}$ that have the same property, the next relation $\tau^{(n+m-1)} = \tau^{(n)}\tau^{(m)} = \tau^{(m)}\tau^{(n)}$ seems a rather useful. The following rather transparent relations *(without loss of generality for **1**–dimensional case)* lay in basis of proof of this assertion:

$$\sigma_1^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sum_{t=1}^{m}\left(\sum_{j=t}^{t+n-1} x_j \pmod a\right) \pmod a$$

$$\sigma_2^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sum_{t=1}^{n}\left(\sum_{j=t}^{t+m-1} x_j \pmod a\right) \pmod a$$

$$(\forall < x_1 x_2 ... x_{n+m-1} >)(\sigma_1^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sigma_2^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}))$$

$$x_j \in A = \{0, 1, ..., a-1\}; \quad a - a \text{ positive integer}; \quad j = 1..n+m-1$$

In addition, the global transition functions $\tau^{(n)}$ and $\tau^{(m)}$ can be either linear functions, or their compositions. At the same time, it is possible to show that as a result of a composition of linear global functions $\tau^{(n)}$ and $\tau^{(m)}$ we again receive a linear ***GTF*** $\tau^{(n+m-1)}$, i.e. similar *linear **GTF*** constitute a closed subset relative to operation of composition of ***GTF*** in the form ***(16)*** whose elements will possess the property of universal reproducibility in the Moore's sense of finite configurations.

Meantime, among all global transition functions of the ***GS***–class there are functions that are distinct from ***linear*** ones, but which possess the property of universal reproducibility in Moore's sense. The following interesting enough result having a series of appendices takes place in the given direction. Moreover, the given result was generalized to the general case of classical structures ***d–HS (d ≥ 1)*** [54–56,79,88,90,640].

<u>Theorem 73</u>. ***A set containing all linear classical structures including classical structures whose global transition functions are made up as a composition (16) of linear global transition functions possesses the opportunity of universal reproducibility in Moore's sense of the finite configurations; at that, in the composition (16) the global transition functions providing for shifts of configurations along the coordinates axes can be used additionally (in some cases results of compositions***

can coincide with appropriate linear classical structures having the disconnected neighbourhood templates). Furthermore, there are also nonlinear classical structures possessing the opportunity of universal self–reproducibility in the Moore's sense of the finite configurations.

The essence of this result is any composition of linear global transition functions which possess the property of universal *reproducibility* again gives a global transition function possessing the same property; thus, the linear structures ***(15)*** constitute a subset concerning operation of *composition* ***(16)*** and the property of universal reproducibility; at that, evidently ***linearity*** is not so obligatory requirement. The more special questions of dynamics of linear classical structures are considered in a lot of works [5,9,10,54–56,140,160,283,284]. A lot of interesting enough properties of linear classical ***HS***–models has been investigated by the Japanese mathematicians on basis of *algebraic* methods using concepts of additive groups and commutative rings together with concepts and methods of dynamic systems [124,536]. In particular, a ***L***–class of *linear* structures was investigated by means of linear algebra in work [140].

By considering the universal reproducibility as a *maximal* constructive possibility of the classical ***HS***–models on generating by them of finite configurations enough interestingly to discover not only new classes of structures with the given property, but also to investigate classes of structures that possess the given property in a considerable extent. So, we have determined a class ***GW*** of classical structures with connected neighbourhood template of size ***n*** along with non–binary alphabet for which any continuous finite configuration ***(****inside without states* ***«0»)*** of the size $m \geq n$ is the self–reproducing in Moore's sense [54]. One more class ***LG*** of structures whose local transition functions $\sigma^{(n)}$ have been received on basis of ***LTF*** of structures of the classes ***L*** and ***GW*** possess a rather high degree of reproducibility of finite configurations. Rather interesting classes of the structures possessing a rather high degree of *reproducibility* of finite configurations, it is possible to receive on basis of compositions of finite number of ***GTF*** $\tau^{(n)}$ from the specified classes ***L*** and ***LG.*** Computer analysis has shown [5,9,54–56] that structures of the given types possess rather considerable reproducing properties of the finite configurations of quite definite types.

One more class of the ***HS***–models possessing a rather high degree of reproducibility of finite configurations, it is possible to determine on basis of a special algebraic system introduced by us for a polynomial representation of ***a***–valued logic functions [3,5,7]. Research of a whole series of classes of discrete *parallel dynamic systems* **(DPDS)**, including ***HS***–models, is very closely linked with research of properties of their ***LTF*** $\sigma^{(n)}$, which represent ***a***–valued logic functions ***(a–VLF)***. Among various approaches to research of similar functions the special place occupies the algebraic approach, when each ***a–VLF*** can be represented by a polynom of maximal degree ***n(a–1)*** over a field ***A*** modulo ***a***, and vice versa, where ***a–VLF*** is any mapping $R^{(n)}: A^n \rightarrow A$. Meanwhile, in case of composite number ***a*** far not each ***a–VLF*** can be represented in such polynomial form, or rather «***almost all***» functions have no such polynomial representation. Since alphabet ***A*** in a classical ***HS***–model can be arbitrary then the problem of spreading of algebraic method of research of ***LTF*** $\sigma^{(n)}$ onto the general case of the alphabet ***A*** arises. In this connection arises an interesting and important from many points of view a question: ***Whether it is possible to determine some algebraic system which would admit polynomial representation of each a–VLF in alphabet A for composite integer a analogously to case of prime a?***

With this purpose we have defined an algebraic system ***(AS)*** in which «*almost all*» ***a–VLF*** have polynomial presentation for case of composite module ***a*** [84]. The offered ***AS*** is defined as follows. A finite alphabet $A_a=\{0,1,2, ..., a-1\}$ of the system is being chosen, and on it usual binary operation of addition modulo ***a*** is defined. Simultaneously on A_a the binary operation of **#–***product* is defined according to the *multiplication* table of the following kind ***(****table* **4)**.

Table 4 (#–multiplication table)

#	0	1	2	3	4	5	....	a–6	a–5	a–4	a–3	a–2	a–1
0	0	0	0	0	0	0	....	0	0	0	0	0	0
1	0	1	2	3	4	5	....	a–4	a–3	a–2	a–1	0	a–1
2	0	2	3	4	5	6	....	a–3	a–2	a–1	0	a–1	1
3	0	3	4	5	6	7	....	a–2	a–1	0	a–1	1	2
4	0	4	5	5	7	8	....	a–1	0	a–1	1	2	3
5	0	5		7	8	9	....	0	a–1	1	2	3	4
6	0	6	7	8	9	10	....	a–1	1	2	3	4	5
....	....	....	....	....	....	....	....	....	....	.....	.....	....	.....
a–3	0	a–3	a–2	a–1	1	2	....	a–9	a–8	a–7	a–6	a–5	a–4
a–2	0	a–2	a–1	1	2	3	....	a–8	a–7	a–6	a–5	a–4	a–3
a–1	0	a–1	1	2	3	4	....	a–7	a–6	a–5	a–4	a–3	a–2

It is easy to make sure the operation **#-***product* on the set $A_a \backslash \{0\}$ makes up the finite cyclic group $A^{\#}$ of degree ***(a-1)***. Concerning the ***AS*** which is defined thus the following basic result takes place [5,54,79,84,88,90].

***Theorem 74*. There is an algebraic system <A_a; +; #>, in which «almost each» a–valued logic function defined in an alphabet A (a – composite integer) can be presented in the form of polynom $P_\#(n)$ (mod a) where:**

1) **(+) – traditional operation of addition modulo a (mod a);**
2) **(#) – operation of product, defined according to the above table 4;**

$$P_\# = \sum_{j=1}^{a^n-1} c_j\#X_1^{d_{j_1}}\#X_2^{d_{j_2}}\#\ldots\#X_n^{d_{j_n}} \quad (mod\ \ a) \ - \ a\ polynomial$$

3) *which is not containing dyadic expressions of the following kind :* **(17)**

$$p_d\#X_j^d + B_d\#X_j^{a-d-1} \quad (0 \le d_{i_j} \le a-1;\ \sum_{j=1}^{n} d_{i_j} \ge 1;\ X_j,\ c_j \in A_a;$$

$$p_d + B_d = a;\ p_d, B_d \ge 1;\ X_j^p = X_j\#X_j\#\cdots\#X_j;\ j=1..n;\ j=1..a^n-1;$$

$$\longleftarrow --- p --- \longrightarrow \quad d = 1..[(a-2)/2]$$

The given result has played important enough part in research of the ***DPDS*** for case of alphabet A_a ***(a –*** *composite number***)** and has allowed to receive a whole series of interesting enough results concerning the ***HS***–problematics, some from which are considered below. In addition, theorem ***74*** gives quite satisfactory analytical representation of a lot of ***a****–valued* logical functions in case of a composite ***a***–module. Even such simple enough logic function as:

$$R_1(x) = \begin{cases} 0, & if\ \ x = 0; \\ 2, & if\ \ x = 1; \\ 1, & otherwise \end{cases}$$

defined in alphabet A_6, can't be represented by means of a polynomial ***(mod 6)***, whereas in ***AS*** **<A_6; +; #>** its representation has the following simple kind, namely: $R_1(y) = P_\#(1) = y^2 + y^3$ ***(mod 6)***. A series of other rather interesting examples of similar character along with *comparative* analysis of the above ***AS*** and the classical algebraic system of the kind **<A_a;+;x>**, for which operations ***(+)*** and ***(x)*** are usual binary operations of addition and multiplication ***(mod a)*** accordingly the reader can find in our works [5,54–56,79,84,88,90,567,617,618,640–643].

On basis of the above *algebraic* system it is possible to define one more interesting enough type of the classical ***HS***-models possessing a rather high degree of reproducibility of finite configurations along with a lot of other interesting enough appendices. In view of the aforesaid for a certain classical model ***1–HS*** its local transition function is determined by the following parallel substitutions, namely:

$$x_1x_2x_3 \ldots x_n \to x_1^1 = 0, \quad if \quad (\forall k)(x_k = 0)$$

$$x_1x_2x_3 \ldots x_n \to x_1^1 = \prod_{k=1}^{n} \# \delta(x_k), \quad else; \quad x_1^1, x_k \in A \quad (k = 1..n) \qquad (18)$$

$$\delta(x_k) = \begin{cases} x, & if \quad x \neq 0 \\ 1, & else \end{cases}$$

where **#**–multiplication is being defined according to the table ***4***.

At the made assumptions we shall consider the set ***S(a,m)*** of all finite configurations $c = x_1x_2x_3 \ldots x_m$; $x_k \in A \backslash \{0\}$ ***(k=1..m)*** of size $\leq m$. Hence, cardinality of the set ***S(a,m)*** is $(a-1)\{(a-1)^m-1\}/(a-2)$, while cardinality of the set $\Sigma(a,m)$ of all finite configurations of size $\leq m$ is $(a-1)a^{m-1}$. So, the density of the set ***S(a,m)*** relative to the set $\Sigma(a,m)$ is defined as the expression $\Xi(a,m)=S(a,m)/\Sigma(a,m) \approx a(1-1/a)^m/(a-2)$ whose asymptotics is characterized by the following relations, namely:

$$\lim_{a \to \infty} \Xi(a,m) = 1 \qquad \lim_{m \to \infty} \Xi(a,m) = 0 \qquad (19)$$

$$\lim_{m \to \infty} \Xi(a,m) = \lim_{a \to \infty} \Xi(a,m) = e^{-p}, \quad if \quad \lim_{m,\, a \to \infty} m/a = p = const$$

Of the given relations it is possible to make sure, that in a series of the important enough cases the density of the set ***S(a,m)*** is quite sufficient to consider finite configurations composing it, as the self–reproducing configurations, i.e. to determine one more new class of the ***HS***–models substantially possessing the reproducibility property in Moore's sense of the finite configurations. On basis of the detailed analysis of *parallel* substitutions ***(18)*** it is possible to show, that global transition function appropriate to them of a structure ***1–HS*** possesses the ***NCF*** and ***NCF-3*** in the absence of the nonconstructability such as ***NCF–1;*** whereas each configuration $c^{**} \in S(a,m)$ is self–reproducing configuration in Moore's sense for such structure. This result does not conflict with the theorem ***68*** as the considered class of structures is characterized by existence of the property of essential or appreciable reproducibility in the Moore's sense of finite configurations but not universal reproducibility.

A classical nonlinear structure ***1–HS*** with the simplest neighbourhood index and state alphabet ***A=***{***0,1, ..., a–1***} presents one more interesting enough example of such kind; local transition function of the structure is defined by the following formula:

$$\sigma^{(2)}(x,y) = \begin{cases} x, & if \;\; y=0 \\ y, & if \;\; x=0 \\ x \cdot y \;\; (mod\; a), & otherwise \end{cases} \quad ; \quad x, y \in A$$

where $a = p^k$; $p \geq 3$ is a prime number and k is a positive integer.

It is simple to make sure, that the given structure does not possess the nonconstructability such as ***NCF–1***, possessing the nonconstructability such as ***NCF.*** Furthermore, in similar structure finite configurations of the kind $c = x_1x_2x_3 \ldots x_m$; $x_k \in A\backslash\{0\}$ $(j=1..m)$ are self–reproducing in the Moore's sense. More precisely [54]: ***For an arbitrary configuration*** $c^* \in C(A,1,\phi)$ ***exists such integer*** $w \geq 0$, ***that*** $c^*\tau^{(2)w}$ ***is a self–reproducing configuration in the Moore's sense.*** In addition, all the set $C(A,1,\phi)$ can be generated from a set $G \subseteq NCF$.

A series of special results in this direction can be found in our books [1,3,5,8,9]. The classical ***HS***–models with universal reproducibility are attractive in many respects. Results in this direction allow to discover a series of useful correlations between the nonconstructability and the universal reproducibility for the classical ***HS***–models, and to solve a series of mathematical problems [54–56,88,90,617,618,640–643].

Of the analysis of a whole series of classical ***HS***–models that in a great extent possess the property of reproducibility in Moore's sense of the finite configurations it is possible to assume that necessary condition – the local transition functions $\sigma^{(n)}$ of the classical ***HS***–models with such property will be defined by parallel substitutions of the kind, namely:

$$x_1x_2x_3 \ldots x_n \Rightarrow x^*_1 = \Phi(x_1, x_2, x_3, \ldots, x_n) \qquad x_k \in A \;\; (k = 1 \,..\, n)$$

where function Φ is based on such operations which form finite cyclic groups of the appropriate degree on a set A or its rather large subsets. The considered structures satisfy the above condition. Furthermore, in more wide context the classical structures known to date that possess the property of universal reproducibility in the Moore's sense, belong to a class of the structures possessing the nonconstructability such as ***NCF–1*** in the absence of the nonconstructability such as ***NCF.***

Along with continuation of studying of the classical ***HS***–models which possess the property of *universal reproducibility* it is quite expedient to define other classes of structures possessing a certain general property interesting enough from the theoretical and applied standpoints, and effectively to characterize the given classes in terms of new or earlier investigated concepts and categories. So in view of the given question, the research of the ***CSAG***-class of ***HS***–models with *symmetric* functions $\tau^{(n)}$ $\{\sigma^{(n)}\}$ seems rather interesting. The carried out analysis of a series of classical structures ***d–HS*** ***(d=1,2)*** of the ***CSAG***–class on basis of both theoretical approaches and by means of computer research [54-56] has allowed to formulate a rather interesting proposal, that in the ***CSAG–*** subclass there is infinitely a lot of structures possessing the property of universal or essential reproducibility in the Moore's sense of the finite configurations. Thus, the following rather interesting proposal seems quite convincing:

Among all classical structures d–HS (d ≥ 1) with symmetrical global transition functions not possessing the nonconstructability such as NCF (NCF–3) in the presence of the nonconstructability such as NCF-1 there is infinitely a lot of structures possessing attribute of universal or essential reproducibility in the Moore's sense.

It is well known, the quota of classical structures ***d–HS (d ≥ 1)*** that not possess the ***NCF (NCF–3)*** tends to ***one*** with growth of neighbourhood template size and/or size of an ***A***–alphabet of elementary automaton. Meanwhile, the class of ***HS***–models meeting conditions of the above proposal is quite representative. Thus, already for the classical binary ***1–HS*** the cardinality of this class not less $N(n) = 2^{2^{2n-3}}$. This estimation can be received, considering the classical ***1–HS*** whose ***LTF*** $\sigma^{(n)}$ is given by the following determinative relations, namely:

$$\begin{cases} (\forall < x_1, x_2, ..., x_{n-1} >)(\sigma^{(n)}(x_1, x_2, ..., x_{n-1}, x_n) \neq \sigma^{(n)}(x_1, x_2, ..., x_{n-1}, x_n^*)) \\ (\forall < x_1, x_2, ..., x_{n-1}, x_n >)(\sigma^{(n)}(x_1, x_2, ..., x_{n-1}, x_n) = \sigma^{(n)}(x_n, x_{n-1}, ..., x_2, x_1)) \\ x_k, x_k^* \in A = \{0, 1, 2, 3, ..., a-1\}; \quad k = 1..(n-1); \quad n \geq 2 \end{cases}$$

It is simple to be convinced that the classical structures defined thus possess completely by the symmetric ***LTF*** and don't possess ***NCF*** and ***NCF–3*** [90]. In case of classical ***1–HS(a,n),*** for each whole ***a > 1*** there is a ***n***–size of neighbourhood template since which number of structures with the *nonlinear* symmetric ***LTF*** which not possess the ***NCF (NCF–3),*** will grow quicker than the number of structures with *linear* symmetric ***LTF*** [90,640]. In particular, for binary structures ***1–HS*** $M(n) = 2^{n-2}$ and the specified relation is valid, starting already with $n = 5$. Prima facie, the class of classical ***HS***–models determined by completely symmetric ***LTF*** that not possess the ***NCF (NCF–3)*** in the presence for them ***NCF–1*** exhausts the various structures characterized by property of universal reproducibility of finite configurations in the *Moore* sense. However, as has been shown in[90], this statement most likely is incorrect and the class of structures with the specified self–reproduction property can be a few wider. The matter was investigated both by theoretical methods, and on the basis of computer simulation of dynamics of the respective classical structures of dimensionality ***1*** and ***2.***

So, for computer study of dynamics of classical structures ***1–HS*** with symmetric ***LTF*** the procedure, that allows to study reproducibility of arbitrary finite configuration depending on the ***LTF*** was been created in the system ***Mathematica 8*** [640]. The **HS[*Ltf, Cf, P*]** procedure has ***3*** formal arguments allowing to define the actual values for: ***Ltf*** – a local transition function which is given by the set of parallel substitutions; ***Cf*** – an initial finite configuration, and ***P*** – the demanded quantity of copies of ***Cf***. The successful conclusion of the analysis prints the initial ***Cf*** studied to reproducibility, with number of its copies, and quantity of steps, required for this purpose while through global variable ***CFfin*** is returned a final configuration on which ***P*** copies was reached.

```
In[2020]:= HS[LTF_List, CF_String, P_Integer] := Module[{n = 0, k,
                                     C1, Co, Cfo, CFf, H, t = 0, Z},
              n = StringLength[First[First[LTF]]]; C1 = ""; Co = "0";
                                   Do[C1 = C1 <> Co, {k, 1, n – 1}];
                                   CFo = C1 <> CF <> C1; CFf = "";
              Label[St]; T1 = TimeUsed[];
 Do[CFf = CFf <> StringReplace[StringTake[CFo, {k, k + n – 1}], LTF],
           {k, 1, StringLength[CFo] + 1 – n}]; t++; T2 = TimeUsed[];
                            If[T2 – T1 >= 15, Goto[Q], 7]; Goto[C];
     Label[Q]; Print["Steps: " <> ToString[t] <> "; CFfin: " <> CFf];
              Z = Input["Continue(y/n)?"]; If[Z == y, Goto[C], 7];
        Return["Job is canceled in step: " <> ToString[t]]; Label[C];
        If[H = Length[StringPosition[CFf, CF]]; H >= P, Goto[Fin],
              CFo = C1 <> CFf <> C1; CFf = ""]; Goto[St];
     Label[Fin]; Print["Initial configuration: " <> CF <> " is " <>
                         ToString[H] <> "– reproductable!"];
                  CFfin = CFf; Print["Steps: " <> ToString[t]]]

In[2021]:= Ltf8 := {"000" –> "0", "001" –> "2", "002" –> "1", "010" –> "2",
    "011" –> "1", "012" –> "0", "020" –> "1", "021" –> "0", "022" –> "2",
    "100" –> "2", "101" –> "1", "102" –> "0", "110" –> "1", "111" –> "0",
    "112" –> "2", "200" –> "1", "201" –> "0", "202" –> "2", "120" –> "0",
    "121" –> "2", "122" –> "1", "210" –> "0", "211" –> "2", "212" –> "1",
    "220" –> "2", "221" –> "1", "222" –> "0"}

In[2022]:= HS[Ltf8, "201021212020221", 80]
Initial configuration: 201021212020221 is 81–reproductable!
Steps: 1080
```

However, it is necessary to note, the given procedure along with other procedures intended for experimental research of dynamics of *classical* structures even in case of dimension ***1*** and ***2*** assumes use of productive enough classes of computers. By means of the procedure substantially our assumption was confirmed that ***the class of structures 1–HS that possesses the property of universal or essential reproducibility in the Moore sense is much wider than the class of linear structures.*** A lot of interesting enough results in this direction can be found in book [640]. So, in the set of ***CSAG*** the ***VS***–subset *(isolated relative to the composition operation)* of all symmetric global transition functions that not possess by ***NCF*** nonconstructability is naturally distinguished. In our opinion, exactly the subset ***VS*** is of special interest with standpoint of question of characterization of the ***HS***–models possessing property of universal or essential reproducibility in ***Moore*** sense of of finite configurations.

Hence, it is possible to assume that the ***symmetry*** of global transition functions along with absence for them of the nonconstructability such as ***NCF*** is one of prerequisites of universal or essential reproducibility of finite configurations in classical ***HS***–models. In this connexion the following rather interesting result can be formulated [54–56,640].

Among all classical structures d–HS with symmetrical GTF $\tau^{(n)}$ ***(d ≥ 1; n ≥ d + 1) which possess the nonconstructability such as NCF–1 in the absence of the nonconstructability such as NCF there is infinitely a lot of structures (not necessarily linear structures) which will possess the property of universal or essential reproducibility in the Moore's sense; at that, is supposed that the essential reproducibility will take place if reproducibility is intrinsic to more than half finite configurations.***

Thus, we receive well defined ***CSAG***–class of ***HS***–models possessing the specified general property; at that, in the ***1***–dimensional case such class is recursive and for it there is a constructive solving algorithm. It is necessary to note, that our research in this direction in a great extent confirms this proposal. The more detailed discussion of this question can be found in our works [54–56,79,88,90,617,618,640–643].

Meantime, the reproducibility is caused not only by symmetry of ***GTF.*** Linear global transition functions $\tau^{(n)}$ whose local transition functions are represented in the form $\sigma^{(n)}(x_1,x_2, ..., x_n)=\sum_j b_j x_j \pmod{p^k}$, where p is a prime, k – a positive integer; $b_1, b_n \in A \setminus \{0\}$, $x_j \in A$ $(j=2..n-1)$ possess the universal reproducibility property in the Moore's sense. Quantity of such global transition functions depending on values of parameters a and n is determined by the following formula, namely:

$$N(a,n) = (a-1)a^{(n-2)/2} * \begin{cases} 1, & \text{if } n \text{ - an even number} \\ \sqrt{a}, & \text{otherwise} \end{cases}$$

In addition among global functions of this class a quota of symmetric functions $\tau^{(n)}$ equals $1/[(a-1)a^{(n-k)/2}]$ where $k=2$ for an even integer n; otherwise $k=1$. So, with growth of value n and/or a their quota quickly decreases. Among symmetric and nonsymmetric global functions $\tau^{(n)}$ the functions which possess universal or essential reproducibility and which differ from linear ones exist [54–56,79,88,617,618,640–643].

It is simple to be convinced that by one of the simple examples of the nonlinear structures possessing property of universal reproducibility in the Moore's sense of finite configurations, the structure ***1–HS*** with the simplest neighbourhood index $X=\{0,1\}$, a state alphabet $A=\{0,1,2\}$ and whose local transition function $\sigma^{(2)}$ is defined by the formula:

$$\sigma^{(2)}(x,y)=\begin{cases} x+y \pmod 3, & \text{if } x=0 \\ x+y+1 \pmod 3, & \text{if } x=1 \\ x*y+1 \pmod 3, & \text{if } x=2 \end{cases}$$

can serves. Furthermore, it was shown that nonlinear structures from a class of structures ***1–HS*** with simplest neighbourhood index $X=\{0,1\}$, a state alphabet $A=\{0,1,2,3,\ldots,a-1\}$ (*a is a prime number*) and with local transition functions $\sigma^{(2)}$ defined by the formula:

$$\sigma^{(2)}(x,y)=\begin{cases} x+y \pmod a, & \text{if } x=0 \\ x*y+1 \pmod a, & \text{if } x=a-1 \\ x+y+1 \pmod a, & \text{otherwise} \end{cases}$$

will possess the property of *universal* or *essential* reproducibility in the Moore's sense of finite configurations.

The most common standpoint up till now was, *universal reproducibility* in the Moore's sense in linear classical structures has been associated with a state alphabet whose cardinality is defined by a number $a=p^k$, where p is a prime number and k is a positive integer. Meantime, the researches carried out by us which are based on theoretical results in combination with computer modeling have allowed to prove validity of the more general result [54–56], namely:

For each linear classical structure d–HS ($d \geq 1$) with a state alphabet $A=\{0,1,\ldots,a-1\}$ (where a can be presented in the form 2^k; k is a positive number) which possesses the property of universal reproducibility in the Moore's sense there is at least one nonlinear structure possessing the same property of universal reproducibility in the Moore's sense.

As a simple example it is possible to represent a subclass of classical structures ***1–HS*** with the simplest neighbourhood index $X=\{0,1\}$ and a state alphabet $A=\{0,1,\ldots,a-1\}$ where a number a can't be presented in the form p^k (*p is a prime number, and k is a positive integer*), whose local transition functions are defined by the following formula, namely:

$$\sigma^{(2)}(x,y)=\begin{cases}0, & if\ x=y\\ a-1, & if\ x+y=a-1\\ \sigma^{(2)}(y,x), & otherwise\end{cases};\quad x,y\in A=\{0,1,2,\dots,a-1\}$$

$$(\forall c)(\forall b,d)(b\neq d\rightarrow\sigma^{(2)}(c,b)\neq\sigma^{(2)}(c,d));\quad b,c,d\in A$$

It was shown that the structures of the given subclass will possess the property of universal or essential reproducibility in the Moore's sense of finite configurations. As an example the structure ***1–HS*** can serve with the simplest neighbourhood index, the state alphabet ***A=***{***0,1,2,3***} whose local transition function $\sigma^{(2)}$ is determined by the next parallel substitutions, namely:

$$00\rightarrow0\quad 10\rightarrow1\quad 20\rightarrow2\quad 30\rightarrow3\quad 01\rightarrow1\quad 11\rightarrow0\quad 21\rightarrow3\quad 31\rightarrow2$$
$$02\rightarrow2\quad 12\rightarrow3\quad 22\rightarrow0\quad 32\rightarrow1\quad 03\rightarrow3\quad 13\rightarrow2\quad 23\rightarrow1\quad 33\rightarrow0$$

The structure possesses the property of *universal* reproducibility in the Moore's sense and is nonlinear, differing from a linear structure with local function $\sigma^{(2)}(x,y)=bx+cy\ (mod\ 4);\ b,c,x,y\in A$, which possesses the same property. At that, frequently generating of the given number of copies of a finite configuration in the second case needs the essentially greater number of steps. So, generation ***10*** copies of the configuration «***0130232120010301312130321221З0***» by linear structure $\sigma^{(2)}(x,y) = x + y\ (mod\ 4);\ x,y\in A = \{0,1,2,3\}$ needs ***1985*** steps of the structure, while the above nonlinear structure possessing the same property of universal reproducibility in the Moore's sense of finite configurations demands only ***481*** step. At that, the similar ratio is fair for many configurations. Indeed, it is interesting to note, in a whole series of cases, the *nonlinear* structures essentially more effectively generate the specified quantity of copies of self–reproducing configurations than appropriate linear structures in the same state alphabet.

On the other hand, research of the above ***CSAG***–class of symmetrical transition functions has shown, that in it there are the structures that possess the property of universal reproducibility in the Moore's sense for case of composite numbers ***a*** as a cardinality of a state alphabet ***A***, while linear structures under the given condition do not possess such property. So, for case of cardinality ***6*** a linear structure with the above property do not exist, while the nonlinear structure with ***LTF*** defined by the following parallel substitution possesses such property [54]:

$00\rightarrow0$	$10\rightarrow1$	$20\rightarrow2$	$30\rightarrow3$	$40\rightarrow4$	$50\rightarrow5$
$01\rightarrow1$	$11\rightarrow0$	$21\rightarrow4$	$31\rightarrow2$	$41\rightarrow5$	$51\rightarrow3$
$02\rightarrow2$	$12\rightarrow4$	$22\rightarrow0$	$32\rightarrow5$	$42\rightarrow3$	$52\rightarrow1$
$03\rightarrow3$	$13\rightarrow2$	$23\rightarrow5$	$33\rightarrow0$	$43\rightarrow1$	$53\rightarrow4$

$04 \to 4$	$14 \to 5$	$24 \to 3$	$34 \to 1$	$44 \to 0$	$54 \to 2$
$06 \to 5$	$15 \to 3$	$25 \to 1$	$35 \to 4$	$45 \to 2$	$55 \to 0$

There is a whole series of other rather interesting examples of classical structures possessing the important property of universal or essential reproducibility in the Moore's sense of finite configurations. Some of them have been discovered on basis of computer modeling [54–56]. As a result of research of the problem of universal reproducibility in the Moore's sense we are inclined to come out with the suggestion:

The set of all classical structures possessing the property of universal reproducibility in the Moore's sense forms a subset of the structures, whose local transition functions meet the following condition:

$$\left\{(\forall c \in A)(<x_2,...,x_n> \neq <y_2,...,y_n> \to \sigma^{(n)}(c,x_2,...,x_n) \neq \sigma^{(n)}(c,y_2,...,y_n))\right\} \&$$
$$\left\{(\forall <x_2,...,x_n>)(b \neq d \to \sigma^{(n)}(b,x_2,...,x_n) \neq \sigma^{(n)}(d,x_2,...,x_n))\right\}$$
$$b,c,d \in A; \quad x_j, y_j \in A; \quad j = 2..n; \quad A = \{0,1,...,a-1\}$$

Number of such structures is at least $[(a-1)!]^2$. The performed research allows to speak about rather high degree of reliability of the *suggestion* for ***1***–dimensional case. Furthermore, in case of higher dimensionality the suggestion seems reliable too.

Meanwhile, use of non–standard approaches has allowed to discover a series of other interesting enough classes of ***HS***–models possessing the property of essential reproducibility of the finite configurations. In particular, an useful modification of the logic operation ***XOR*** over the positive integers is being determined as follows:

Operation 'x XOR1 y' with two positive integers x and y is defined as bit-by-bit operation XOR without a carrying over into the high–order digits with binary equivalents of the given integers x, y; in addition, a length l of binary representation is defined by length of representation of the maximal integer, i.e. $l = max\{|x|,|y|\}$, for example:

$$12 \;\; XOR1 \;\; 19 \equiv \begin{bmatrix} 01011_{12} \\ 10011_{19} \end{bmatrix} \equiv 11000 \equiv 24$$

It is obviously that in the binary case the introduced operation ***XOR1*** coincides with classical operation ***XOR***. While for set of integers ***A***={***0, 1,2,3***} the table of ***XOR1***–operation is defined by the following tables:

XOR1	***000***	***001***	***010***	***011***
000	***000***	***001***	***010***	***011***
001	***001***	***000***	***011***	***010***
010	***010***	***011***	***000***	***001***
011	***011***	***010***	***001***	***000***

XOR1	***0***	***1***	***2***	***3***
0	***0***	***1***	***2***	***3***
1	***1***	***0***	***3***	***2***
2	***2***	***3***	***0***	***1***
3	***3***	***2***	***1***	***0***

It is simple to make sure, that a set ***A***=\{***0,1, ..., h***\} ***(h*** *– an arbitrary prime)* forms finite additive ***Abel*** group with a neutral element «***0***» relative to the operation ***XOR1;*** in addition, each element of ***A*** possesses a single inverse element conterminous with itself. Simple procedure **&*XOR1,*** programmed in the environment of package ***Maple*** [97–118] provides execution of ***XOR1***–operation over an arbitrary finite set ***N*** of positive integers. In [617] the source code of procedure **&*XOR1*** along with its using is represented. So, for example, table of ***XOR1***–operation for set ***A***=\{***0,1, ..., 31***\} allows to establish its structural organization presenting self–dependent interest. The carried out experimental and theoretical analysis of the classical ***HS***–models has shown [88,90] that at the made assumptions the following suggestion can be formulated.

Theorem 75. An arbitrary classical structure d–HS (d ≥ 1) with a state alphabet A=\{0,1, ..., h\} (h – an arbitrary prime), whose local transition function $\sigma^{(n)}$ is defined by formula $\sigma^{(n)}(x_1, ..., x_n)$ = &XOR1(x_1, ..., x_n), will possess the property of universal reproducibility in the Moore's sense of finite configurations.

It has been shown that the given result is extrapolated also onto case of classical structures ***d–HS (d ≥ 1)*** with an arbitrary neighbourhood index. Thus, quite pertinently the following conclusion to formulate**:**

A class of structures d–HS (d ≥ 1) concerning the property of universal reproducibility in the Moore's sense wider than the class of structures defined by linear local transition functions and their superpositions. Quantity of such linear 1–HS(a, n) (a – prime) equals $(a-1)^2a^{(n-2)}$.

Theorem ***75*** in a great extent gives the answer to question of existence of the classical structures, distinct from linear structures, that possess the property of universal reproducibility in Moore's sense of the finite configurations. Moreover, an assumption which allows to treat more widely the class of ***HS***–models possessing the property of universal or essential reproducibility in Moore's sense of finite configurations will be formulated below.

In view of complex enough dynamics of a whole series of the classical structures ***1–HS*** even in case of the binary state alphabet of elementary automata the method of computer–based modelling with good reason can be referred to the basic components of the apparatus of researches of the classical structures. This method allows not only to empirically investigate dynamics of ***HS***–models and enough effectively to visually display it, but also gives good possibilities for formulation of different hypotheses, some of which have already received stringent theoretical substantiation whereas others stimulated a series of rather interesting researches. Along with the above procedures we programmed special tools of various complexity for computer researches of various aspects of the ***HS***–problematics, first of all, of dynamics of classical ***HS***–models of dimensions ***1*** and ***2*** [5,8,9,15,54-56,78,79,85,90,97-99,112,116-118]. So, this method has allowed to formulate and to appreciably approve one interesting enough hypothesis concerning the existence of a subset of the subset of ***1***–dimensional symmetrical global transition functions which not possess the nonconstructability such as ***NCF*** in the presence for them of the nonconstructability such as ***NCF–1***, and for which the property of *universal reproducibility* of finite configurations takes place. Researches in this direction seem to us rather interesting, allowing in case of positive solution to receive useful relations between algebraic properties of global transition functions and the nonconstructability.

In particular, computer simulation of ***1–HS*** with alphabet ***A***={***0,1,…,a***}, neighbourhood index ***X***={***0,1,…,n–1***} and local transition function

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = XOR1(x_1, x_2, ..., x_n); \quad x_j \in \{0,1,...,a\}; \;\; a - an\ integer;\ j = 1..n$$

allows to obtain a whole series of interesting enough results [641]. So, in such structures from arbitrary finite configurations ***W*** in alphabet ***A*** are generated the sequences of configurations containing *subsequences* of configurations of the form ***W0^jk^ … W^jk^S (jk≥0; k=1..∞), 0^jk^*** – a string ***0…0*** of length ***jk;*** at that, ***j1..jk*** is a palindrome. For simulation of such ***1–HS*** had been used two simple enough procedures programmed in ***Mathematica.*** The call **XOR1[{*a,…,c*}]** returns the result of application of the above operation ***XOR1*** to integers ***a,…,c*** whereas the procedure call **ReprodXOR1[*S,n,m*]** returns **2**-element list whose the *first* element defines the number of iterations of a structure with template size ***n*** that generates from a string ***S*** a *configuration* containing ***m*** copies of ***S*** while the *second* element defines real number of copies. The call with fourth optional argument ***g*** additionally through ***g*** *(indefinite variable)* returns the resultant configuration. The procedures codes are given below.

```
In[1959]:= XOR1[n_/; ListQ[n] && Length[n] >= 1 &&
                DeleteDuplicates[Map[IntegerQ[#] &, n]] == {True}] :=
                        Module[{a, b, c, d}, If[Length[n] == 1, {n}[[1]],
                                a = Map[IntegerDigits[#, 2] &, n];
   b = Sort[Map[Length[#] &, a]][[–1]];  b = Map[PadLeft[#, b] &, a]];
                  b = Map[Mod[#, 2] &, Total[b]];  FromDigits[b, 2]]
```

```
In[1960]:= XOR1[{1, 9, 4, 2}]
Out[1960]= 14
In[1962]:= ReprodXOR1[S_String, n_Integer, m_Integer, g___] :=
                                  Module[{a, b, c = "", h, k, d = 0, t},
               a = StringJoin[Map[ToString, NestList[Sin, 0, n – 2]]];
                                                        b = a <> S <> a;
       Label[AGN];     For[k = StringLength[b] – n + 1, k >= 1, k––,
       h = ToExpression[Characters[StringTake[b, {k, k + n – 1}]]];
       c = ToString[XOR1[h]] <> c]; If[t = StringCount[c, S]; t >= m,
       If[Length[{g}] != 0 && ! ValueQ[g], g = c]; {StringLength[c], t},
                    b = a <> c <> a;  c = "";  d = d + 1; Goto[AGN]]]
In[1963]:= ReprodXOR1["11331121131003233331123", 2, 2, g]
Out[1963]= {55, 2}
In[1964]:= g
Out[1964]= "1133112113100323333112300000000011331121131003233331123"
```

Procedure ***ReprodXOR1*** provides simulation of dynamics in ***1–HS*** of initial configurations containing symbols from alphabet ***A***={***0,1,2,…,a***} where ***a*** is a prime no more then ***7***. While for case of arbitrary alphabet ***A*** allowing to use the initial configuration with symbols from alphabet ***A***={***0,1,2,…,a***} ***(a*** – *an arbitrary integer)* as an initial configuration can be used a modification of the above procedure **ReprodXOR1** whose call **ReprodXOR11[*S,n,m*]** returns *two*–element list whose the first element defines the number of steps of a structure with *neighbourhood* template size ***n*** which generates from a list ***S*** defining the initial configuration a configuration containing ***m*** copies of configuration ***S*** while the second element defines *real* number of such copies. At that, the procedure call **ReprodXOR11[*S,n,m,g*]** with fourth optional argument ***g*** additionally through actual argument ***g*** *(indefinite variable)* returns the list defining the resultant configuration. The following fragment represents source procedure code along with typical examples its application.

```
In[1959]:= ReprodXOR11[S_List, n_Integer, m_Integer, g___] :=
                         Module[{b, c = {}, k, j = 1, d = 0, t, h = Length[S]},
                  b = PadLeft[PadRight[S, h + n – 1, 0], h + 2*(n – 1), 0];
                  Label[ArtKr];   For[k = Length[b] – n + 1, k >= 1, k––,
   c = Prepend[c, XOR1[b[[k ;; –j++]]]]]; If[t = ListCount[c, S]; t >= m,
               If[Length[{g}] != 0 && ! ValueQ[g], g = c]; {Length[c], t},
                              b = PadLeft[PadRight[c, Length[c] + n – 1, 0],
         Length[c] + 2*(n – 1), 0]; c = {}; d = d + 1; j = 1;  Goto[ArtKr]]]
```

```
In[1960]:= ReprodXOR11[{1, 4, 9, 8, 4, 0, 2, 10}, 4, 4, g]
Out[1960]= {128, 16}
In[1961]:= g
Out[1961]= {1, 4, 9, 8, 4, 0, 2, 10, 1, 4, 9, 8, 4, 0, 2, 10, 1, 4, 9, 8, 4, 0, 2, 10,
            1, 4, 9, 8, 4, 0, 2, 10}
In[1962]:= ReprodXOR11[{1, 4, 9, 8, 4, 0, 2, 10, 0, 1, 2, 3, 4, 5, 6}, 6, 23]
Out[1962]= {575, 24}
```

At that, function ***ListCount*** used by the procedure is contained in our freeware package ***AVZ_Package*** that contains more than ***600*** software tools [638]. The procedure **ReprodXOR11** along with its analog for ***2–HS*** and also other software allowed to obtain a number of interesting enough results concerning constructive possibilities of classical ***d–HS (d=1,2)*** including self–reproducibility property in the Moore's sense of finite configurations. Similar results on dynamical properties of ***d–HS (d=1,2)*** can be found, in particular, in our works [641,643].

Simulation approach with using of the above software has allowed to determine a series of types of classical structures ***1–HS*** which possess the property of essential reproducibility of finite configurations along with other rather interesting dynamic properties of structures of such class [88,536,567,641]. In particular, as a result of similar experimental research the structure ***1–HS*** has been discovered with neighbourhood index $X=\{0,1\}$, with alphabet $A = \{0,1,2\}$ and ***LTF*** $\sigma^{(2)}(x,y) = x\Theta y;\ x,y\in A$ where **Θ**–operation is defined by the following table, namely:

Θ	0	1	2
0	0	2	1
1	2	0	1
2	1	2	0

It is simple to make sure, that the given structure does not possess the nonconstructability such as ***NCF*** whereas any finite configuration such as $c^* = x_1x_2x_3 \ldots 21$ $\{x_1,x_2,x_3\in A\}$ is ***NCF–1*** for the structure [617]. The experimental research of the above structure ***1–HS*** has confirmed the existence of the property of essential reproducibility in Moore's sense of finite configurations for such classical structure.

Meanwhile, a whole series of researches in the given direction allows to formulate a series of interesting enough assumptions of which we shall mark the following. So, theoretical and experimental researches of classical structures ***1–HS*** have shown that the structures possessing the undermentioned general property as the structures possessing the universal reproducibility in Moore's sense of the finite configurations can be. As a classical structure ***1–HS*** the structure with neighbourhood index $X = \{0,1\}$ and alphabet $A = \{0,1, ..., a-1\}$ is chosen. On the set ***A*** the binary ⊗–operation determined by the following ⊗–table is given:

⊗	0	1	2	.....	a–2	a–1
0	0	$x_{0,1}$	$x_{0,2}$	.....	$x_{0,a-2}$	$x_{0,a-1}$
1	$x_{1,0}$	$x_{1,1}$	$x_{1,2}$	.....	$x_{1,a-2}$	$x_{1,a-1}$
2	$x_{2,0}$	$x_{2,1}$	$x_{2,2}$	.....	$x_{2,a-2}$	$x_{2,a-1}$
......	....	....	....	.....	....	....
a–2	$x_{a-2,0}$	$x_{a-2,1}$	$x_{a-2,2}$	.....	$x_{a-2,a-2}$	$x_{a-2,a-1}$
a–1	$x_{a-1,0}$	$x_{a-1,1}$	$x_{a-1,2}$	.....	$x_{a-1,a-2}$	$x_{a-1,a-1}$

Elements of the above table determining the ⊗–operation satisfy the following determinative conditions, namely:

$$(\forall h, j, k)(j \neq k \rightarrow x_{h,j} \neq x_{h,k}) \,\&\, (\forall h, j, k)(j \neq k \rightarrow x_{j,h} \neq x_{k,h})$$
$$x_{h,j},\ x_{h,k},\ x_{j,h},\ x_{k,h} \in A = \{0,1,...,a-1\};\quad h,k,j = 0..a-1$$

The essence of the given conditions consists in that that each column and each row of the ⊗–table contain strictly one entrance of elements of the alphabet ***A***. For example, we can be limited to the condition that ⊗-operation on the alphabet ***A*** forms the finite ***Abel*** group. Thus, from experimental and theoretical research of classical structures ***1–HS*** with simple enough neighbourhood indexes $X1=\{0,1\}$ and $X2=\{0,1,2\}$ along with alphabet $A=\{0,1,2, ..., a-1\}$ ***(a=2..5)*** exist rather forcible arguments to formulate the following rather interesting assumption:

A classical structure d–HS with neighbourhood index $X=\{0,1,2, ..., n-1\}$ and alphabet $A=\{0,1,2, ..., a-1\}$ whose LTF $\sigma^{(n)}$ is defined by formula of the kind $\sigma^{(n)}(x_o,x_1, ..., x_{n-1})=\otimes(x_o,x_1, ..., x_{n-1})$ possesses the property of universal reproducibility in Moore's sense of the finite configurations where x_j are coordinates of elementary automata of a neighbourhood template in the homogeneous space Z^d ($d \geq 1$; $j=0..n-1$).

For testing of the given assumption the computer modeling was used as a result of which a lot of rather interesting experimental results has been received [88,617]. Furthermore, it is possible to show [88,90], that in case of correctness of the above assumption in addition to the linear classical structures ***d–HS*** ($d \geq 1$) will exist not less $[(a-1)!]^2$ structures with alphabet $A=\{0,1, ..., a-1\}$, neighbourhood index $X=\{x_o,x_1, ..., x_{n-1}\}$, whose local transition functions $\sigma^{(n)}$ are defined by the above formula which possess the property of *universal reproducibility* in Moore's sense of finite configurations. The given result will allow to expand enough appreciably the class of ***HS***–models with the above rather interesting property of generating of the finite configurations; in turn, a series of ⊗–operations can represent a certain interest in a series of researches of the classical structures ***d–HS (d ≥ 1).***

The carried out analysis of similar examples together with the certain theoretical considerations allow to us to come to conclusion about the absence of some first cause of linearity of classical structures relatively to the property of universal or essential reproducibility in the Moore's sense of finite configurations. More precisely, property of universal or essential reproducibility in the Moore's sense of finite configurations has more deep hearts and their revealing presents undoubted interest.

In any case, on the assumption of the represented results, we receive interesting enough examples of the classical structures possessing the property of *universal* reproducibility of the finite configurations which are fundamentally different as from class of linear classical structures already becoming classical, and from wider class of structures formed by means of composition of their ***GTF*** also possessing the property of universal reproducibility in Moore's sense of the finite configurations. Meanwhile, research of questions of self-reproducibility in ***HS***–models in a lot of cases collides with problems of algorithmic solvability. For example, ***A. Leitsh*** has considered problems of self–reproduction and fertility relative to one subclass of *non–deterministic* ***HS***–models, using theory of recursive functions and has proved their unsolvability [306].

In accordance with the positive decision of the problem of universal reproducibility in the Moore's sense of finite configurations in *classical* structures possessing the given property, the nonconstructability such as ***NCF*** is absent in the presence of nonconstructability such as ***NCF–1.*** Consequently, an interesting enough question arises, namely: ***whether there are classical structures in which each finite configuration which is distinct from NCF would the self–reproducing in the Moore's sense or «almost all» finite constructible configurations, which are distinct from finite configurations such as NCF, will possess the property of reproducibility in the Moore's sense?***

This question is interesting in view of possibility of self–reproduction in case of narrowing of the set of all finite configurations up to the set ***C(A,d,∞)\NCF.*** Unfortunately, today the given question is open. The given question, meanwhile, in full measure concerns the problem of essential reproducibility in the Moore's sense, however our researches of it allows to expect for the question the negative answer.

Structures, different from *classical* ones, represent a certain interest for study of constructive possibilities, in particular, of self–reproducibility in the Moore sense of finite configurations. So, in this regard so–called structures with memory were investigated. Homogeneous structures with *memory* ***(HSM)*** are defined as follows. The state of an elementary automaton x_1^{t+1} at time t is defined by local transition function $\sigma^{(n)}$

$$\sigma^{(n)}(x_1^t, x_2^t, \ldots, x_n^t, x_1^{t-1}, x_1^{t-2}, \ldots, x_1^{t-p}) = x_1^{t+1} = \sum_{j=1}^{n} x_j^t + \sum_{k=1}^{p} x_1^{t-k} \ (mod\ a);$$

$$x_j^t, x_1^{t-k}, x_1^{t+1} \in A = \{0, 1, 2, \ldots, a-1\};\ j = 1..n;\ k = 1..p;\ t = 1..\infty$$

depending on its state and states of its neighbour automata according to a neighbourhood index at the moment t along with the states of this automaton which the automaton x_1 had received over the p previous steps. For study of self–reproducibility property in such structures the theoretic–experimental approach has been used. In particular, for case of ***1–HSM*** the experimental study is based on procedure ***HSM*** whose source code with example the following fragment represents.

```
In[2479]:= HSM[Cf_List, n_Integer, p_/; IntegerQ[p] && p > 0,
                                  m_Integer, g_Integer, v_Integer] :=
                  Module[{a = Flatten[Table[Table[0, {p + 1}], {n – 1}]],
                   b, c, d = {}, k, G, h, t, s}, G[x_List] := Mod[Total[x], m];
                     b = Join[{a}, Map[Flatten[{#, a[[1 ;; p]]}] &, Cf], {a}];
      Do[For[k = 1, k <= Length[b] – n + 1, k++, h = b[[k ;; k + n – 1]];
                   c = G[Flatten[{Map[Part[#, 1] &, h], b[[k]][[2 ;; –1]]}]];
                        d = AppendTo[d, Flatten[{c, b[[k]][[3 ;; –1]], c}]]];
                                                       t = Map[#[[1]] &, d];
                           If[s = ListCount[t, Cf]; s >= g, Return[{g, s}]];
                                         b = Join[{a}, d, {a}]; d = {}, {v}]]

In[2480]:= HSM[{1, 2, 1, 0, 2, 1}, 2, 4, 3, 3, 10000]
Out[2480]= {3, 4}
```

Procedure ***HSM*** has **6** formal arguments**:** ***Cf –*** a finite configuration in the list format, ***n –*** the size of neighbourhood index of a structure, ***p –*** memory depth, ***m -*** modulus of congruence, ***g -*** number of the required copies of configuration ***Cf*** in generated configurations, ***v –*** the limiting number of steps of the structure. The call **HSM[*Cf, n, p, m, g, v*]** returns **2**–element list whose first element is ***g*** and the second defines the real number of copies of the configuration ***Cf*** in generated configurations. At long non–performance of the condition ***(g)*** the procedure execution will end via ***v*** steps or can be interrupted manually.

The following rather interesting modification of the above structure is defined by the local transition function $\sigma^{(n)}$:

$$\sigma^{(n)}(x_1^t, x_1^{t-1}, \dots, x_1^{t-p}, x_n^t, x_n^{t-1}, \dots, x_n^{t-p}) = x_1^{t+1} = \sum_{k=0}^{p} \sum_{j=1}^{n} x_j^{t-k} \ (mod\ a);$$

$$x_j^t, x_1^{t-k}, x_1^{t+1} \in A = \{0, 1, 2, \dots, a-1\};\ j = 1..n;\ k = 1..p;\ t = 1..\infty$$

i.e. the state of an elementary automaton x_1^{t+1} at time ***t*** depend on its state and states of its neighbour automata according to a *neighbourhood* index at the moment ***t*** along with the states of these automata which they had received over the ***p*** previous steps. For study of constructive opportunities in similar ***HSM*** structures including self–reproducibility the theoretic–experimental approach has been used. In particular, for ***1***–dimensional case the experimental study is based on the procedure **ReprodHSM** whose source code with example of typical application the following fragment represents.

```
In[941]:= ReprodHSM[Cf_List, n_Integer, p_/; IntegerQ[p] && p > 0,
                    m_Integer, g_Integer, v_Integer] :=
      Module[{a = Flatten[Table[Table[0, {p + 1}], {n – 1}]],
                    b, c, d = {}, k, G, h, t, s, x = 0},
      b = Join[{a}, Map[Flatten[{#, a[[1 ;; p]]}] &, Cf], {a}];
                    G[x_List] := Mod[Total[x], m];
                    Do[For[k = 1, k <= Length[b] – n + 1, k++,
                    h = b[[k ;; k + n – 1]]; c = G[Flatten[h]];
             d = AppendTo[d, Flatten[{c, b[[k]][[3 ;; –1]], c}]]];
                    t = Map[#[[1]] &, d]; x++;
             If[s = ListCount[t, Cf]; s >= g, Return[{g, s, x}]];
                    b = Join[{a}, d, {a}]; d = {}, {v}]]

In[942]:= ReprodHSM[{1, 2, 1, 0, 2, 1, 1}, 2, 4, 3, 10, 10000]
Out[942]= {18, 20, 2469}
```

In contrast to the above procedure **HSM** the call **ReprodHSM[*Cf, n, p, m, g, v*]** returns ***3***-element list whose first element defines ***g***, the second defines the real number of copies of the configuration ***Cf***, and third – number of steps. The study of structures of ***HSM*** of type of dimension ***1*** and ***2*** allows to obtain a whole series of rather interesting results on self–reproducibility and other constructive opportunities in the ***HSM*** depending on neighbourhood index, memory depth, alphabet, type of ***HSM,*** initial finite configurations, and so on. [641–643].

At last, homogeneous structures with a variable neighbourhood index represent interesting enough class of ***d–HS*** $(d \geq 1)$, receiving a number of interesting interpretations [641,643]. In particular, it is possible to be limited to such structures whose neighbourhood index at the current time $t \geq 0$ is defined at the moment for each elementary automaton by its state at the the same time ***t.*** Let as a variable neighbourhood index X_j for an elementary automaton in state ***j*** will be $X_j=\{0,1,..,j+1\}$. While alphabet $A=\{0,1,\dots,a-1\}$ and local transition function for the structure with variable neighbourhood index X_j is defined as follows, namely:

$$\begin{cases} \sigma^{(2)}(x_1^t = 0, x_2^t) = x_1^{t+1} \\ \sigma^{(3)}(x_1^t = 1, x_2^t, x_3^t) = x_1^{t+1} \\ ================= \\ \sigma^{(a)}(x_1^t = a-1, x_2^t, \dots, x_a^t) = x_1^{t+1} \end{cases}$$
$$x_j^t, x_1^{t+1} \in A = \{0,1,2,\dots,a-1\};\ j = 1..a$$

For simplicity it is supposed that local transition functions **σ** for such structures are defined by identical parallel rules accurate to arity. We researched a number of questions in a class of structures with variable neighbourhood index ***(HSwVni)*** such as self-reproducibility and other constructive possibilities both theoretically and experimentally. So, for computer research of self–reproducibility in ***1***–dimensional structures of such type the procedure ***HSwVni*** can be useful enough. Source code of the procedure is represented below. The call **HSwVni[*A, Cf, m, g, v*]** returns ***3***–element list whose first element defines the desired number ***(g)*** of copies of a configuration ***Cf*** in an alphabet ***A,*** the second defines the real number of copies of ***Cf,*** and third element defines number of structure steps. Other arguments of the **HSwVni** are analogous to the arguments of same name in the above procedure **ReprodHSM.**

```
In[942]:= HSwVni[A_List, C_List, m_Integer, g_Integer, v_Integer] :=
       Module[{a = Table[0, {Length[A]}], b, c, d = {}, k, G, h, t, s, x = 0},
                         G[x_List] := Mod[Total[x], m];  b = Join[{0}, C, a];
    Do[For[k = 1, k < Length[b], k++, h = b[[k]]; h = b[[k ;; k + h + 1]];
                          c = G[Flatten[h]];  d = AppendTo[d, c]];  x++;
                          If[s = ListCount[d, C]; s >= g, Return[{x, g, s}]];
                          d = ListTrim[d, 0]; b = Join[{0}, d, a]; d = {}, {v}]]
In[943]:= HSwVni[{0, 1, 2}, {1, 2, 0, 2, 2}, 3, 18, 10000]
Out[943]= {531, 18, 18}
```

Modifications of procedures ***HSM, ReprodHSM*** and ***HSwVni*** allow to study of number of rather interesting questions on self-reproducibility and other constructive possibilities of the above structures type [641].

It is necessary to note, the researches of *extremal* dynamic properties of finite configurations play a rather important part in the mathematical theory of classical ***HS***–models. In this context it is desirable to consider other concepts of extremality, different from universal reproducibility in the Moore's sense or similar. In this direction a whole series of new concepts is considered by us. Thus, the first received results allow to say about their sufficient enough availability. In the sequent works we suppose to present the most interesting results in this direction.

Till now, we considered the classical ***HS***–models from the standpoint of their maximal generative opportunities concerning the set of finite configurations regardless of the order of their generating. However, a question about possibility of generating by a classical structure of the given history of finite configurations, i.e. a sequence of configurations $<c_o>[\tau^{(n)}]$ in its dynamics directly adjoins to the given problem. Thus, generally speaking, the given question can be formulated as follows:

Whether exists for the given history of configurations $\Omega = \{c_o \to c_1 \to c_2 \to c_3 \to c_4 \to \ldots \to c_k \to \ldots\}$ $c_o \in C(A,d,\phi)$***, which are given in a finite state alphabet A, a global transition function*** $\tau^{(n)}$ ***that is determined in the same state alphabet A and generates the above configurations history, i.e. whether the relation*** $<c_o>[\tau^{(n)}] = \Omega$ ***can take place?***

It is simple to make sure that the answer to this question is negative in general [1,3,5,640]. Furthermore, in algorithmic context the problem of definition of an opportunity of generating of an arbitrary Ω–history of the finite configurations by a classical ***HS***–model is unsolvable [3,55].

With important questions of more practical approach to the problem of implementation of self–reproducing industrial automata the reader can acquaint oneself in rather interesting popular scientific book [302], while in a series of works it is possible to acquaint oneself with other interesting discussions of self–reproducibility in ***HS***–models [536]. In light of rapidly developing of *nanotechnologies* the given problematics seems to us actual enough; the steady growth of intense interest to the problematics is undoubted evidence of it [536]. In addition from point of view of research of fundamental properties of ***HS***–models it would be utterly desirable to determine and investigate other certain fruitful concepts of the universality of ***HS***–models which are distinct from the reproducibility of the finite configurations and computability.

Investigation of properties of special types of configurations *(passive, periodical, vanishing, etc.)* in the classical ***HS***–models represent interest both highly specific and of this or that level of community for the ***HS***–problematics. So, the passive configurations play an important role in the case when the classic ***HS***–models are considered as algorithms of parallel words processing in finite alphabets and at embedding them in various types of processes and models. A whole series of questions relating to passive and vanishing configurations is discussed in [641]. From standpoint of investigation of stable trajectories of dynamics of classical ***HS***-models a sharply defined interest represent the periodical finite configurations. Of the theorem ***112*** follows, that the problem of determination, whether an arbitrary initial finite configuration as the periodic in a classical ***HS***–model will be algorithmically unsolvable. It is easy to show, that in the presence in a classic ***HS***–model of periodic configurations with minimal period ***p*** their set is infinite, and periodic configurations of infinite large size with the same period ***p*** exist. If in a ***HS***–model there are periodic configurations with minimal periods ***p*** and ***q (p≠q)***, then in it exist at least and periodical configurations with minimal period ***g = g(p, q) = LCM(p, q)*** where ***LCM*** – the least common multiple of ***p*** and ***q***. In this connexion the following **2** basic questions arise: ***(1)*** obtaining of *upper* bounds for the size of the minimal periods as a function of the basic parameters of a ***HS***–model, and ***(2)*** detection of the algorithmic solvability of the problem of *presence* in an arbitrary classical ***HS***–model of periodical configurations, except the trivial case of periodic zero configuration. In case of the classical ***HS***–models the low bound of size of minimal period was received which is expressed by the following result [3,5,88], namely:

There are classical d–HS with the Moore's neighbourhood index that have periodical finite configurations with minimal period p≥2|c|–2, where |c| – diameter of c–configurations. At that, there are classical 1–HS with the neighbourhood index X = {-3,-2,-1,0,1,2,3} that have the periodical finite configurations with minimal period p≥2L (|c|) where L–quantity is defined from relations (8).

The second part of the result indicates the existence in classical ***1-HS*** of periodic configurations of relatively small size with fantastically large size of minimal period. At that, the positive solution of the above *first* question entails algorithmic solvability of the *second* question whereas the algorithmic unsolvability of the *second*, in turn, entails the negative solution of the first question. Today both these questions remain open even for ***1***–dimensional case.

3.3. Universal and self–reproducing finite configurations for HSoS–models on the splitting

Homogeneous structures on the splitting ***(HSoS)*** defined in *section* ***1.2*** and in part considered above represent special interest for the problem of physical modelling, by allowing a rather simple programming of such fundamental property as ***reversibility*** of dynamics. In this connection concerning the class of ***HSoS***–models quite naturally arise questions of possibility of existence for them of *universal* and *self–reproducing* finite configurations which in a certain degree can characterize the extreme constructive possibilities of the given class of models. There and then once again it is necessary to note that the used term «***structures on the splitting***» is equivalent to the term «*neighbourhood index of* **Margolus**». The substantiation of our term can be found in the same place.

Above all, concerning the existence problem of *universal configurations* ***(UCF)*** in ***HSoS***–models the result analogous to the above case of the classical ***HS***–models takes place, namely [54–56,79,88,567,617,640]:

An arbitrary structure d–HSoS (d≥1) can't possess a finite set of UCF.

Meanwhile, quite other picture takes place concerning the question of existence in ***HSoS***–models of self–reproducing finite configurations in Moore's sense. Without loss of generality, we consider ***1***–dimensional ***HSoS***–models with state alphabet $A=\{0,1,2,3,\ldots,a-1\}$ on condition that $a=p^k$ where ***p*** – a prime number and ***k*** – a positive integer. In addition a local *block* transition function $\Psi^{(m)}$ that defines mapping of arbitrary configurations of some ***m***–block into new configurations of the same ***m***–block; i.e. the mapping $\Psi^{(m)}: A^m \Rightarrow A^m$, is determined by means of parallel block substitutions of the following kind, namely:

$$\left|x_1x_2x_3\ldots x_m\right| \;\rightarrow\; \left|\sum_{j=1}^{2}x_j\ (mod\ a)\ \sum_{j=2}^{3}x_j\ (mod\ a)\ \ldots \sum_{j=m-1}^{m}x_j\ (mod\ a)\ \ x_m\right|$$

$$\left|x_1x_2x_3\ldots x_m\right| \;\rightarrow\; \left|\sum_{j=1}^{m}x_j\ (mod\ a)\ \sum_{j=2}^{m}x_j\ (mod\ a)\ \ldots \sum_{j=m-1}^{m}x_j\ (mod\ a)\ \ x_m\right|$$

$$where\ a = p^k (p\text{ - }a\ \ prime, k\text{ - }a\ \ positive\ integer);\ \ x_j \in A = \{0,1,2,\ldots,a-1\}\ \ j = 1..m$$

It is easy to make sure, that the mapping $\Psi^{(m)}: A^m \Rightarrow A^m$, determined in this way, is one–to–one with ***a*** fixed points of kind ***X0 ... 0 → X0 ... 0*** ***(X∈A)***. Thus, the given mapping provides reversibility of dynamics of ***HSoS***–models of this type. It is shown that ***HSoS***–models determined by the above parallel block substitutions can possess the opportunity of ***universal*** or ***essential*** reproducibility in the Moore's sense of finite configurations [54,79,88]. A simple example will be presented below.

A *local block function* ***(LBF)*** $\Psi^{(2)}$ of such ***HSoS***–model is determined by parallel block substitutions as $\Psi^{(2)}: xy \Rightarrow \{x+y \ (mod\ a)\}\ y;\ x,y \in A$. It is easy to make sure that a mapping $\Psi^{(2)}: A^2 \rightarrow A^2$ determined by such ***LBF*** is one–to–one, whereas structure ***1–HSoS*** as a whole ***2***–simulates a classical structure ***1–HS*** determined in the same state alphabet ***A*** but with variable neighbourhood index ***X*** that depends on a coordinate of an elementary ***k***–automaton of space Z^1, i.e. $X_{2k} = \{0,1\}$, $X_{2k+1} = \{0,1,2\}$ ***(k=0, ±1; ±2;...)***, and linear ***LTF*** of the following kind accordingly:

$$\sigma^{(2)}_{2k}(x, y) = x + y \ (mod\ a), \quad \sigma^{(3)}_{2k+1}(x, y, z) = x + y + z \ (mod\ a);\ x, y, z \in A\ (k = 0, \pm 1, \pm 2, \ldots)$$

At the made assumptions can be shown, the model ***HSoS*** determined thus will possess the property of universal reproducibility in Moore's sense of finite configurations and its dynamics is reversible.

Meanwhile, today the given question is not worked out enough and it demands the further research similar to case of classical ***HS***-models. In particular, as an illustration we shall give a simple enough example of structure $HSoS \equiv \langle Z^1, A, 2, \Psi^{(2)}, \Xi \rangle$ with binary state alphabet ***A*** and ***LBF*** defined by parallel block substitutions in the form $\Psi^{(2)} = \{00 \Rightarrow 00,\ 01 \Rightarrow 11,\ 10 \Rightarrow 10,\ 11 \Rightarrow 01\}$. In the book [567] a simple fragment of generation of history of a certain finite configuration in classical ***1–HS*** equivalent to the given structure ***1–HSoS*** is presented in direct order, whereas in structure ***1–HSoS*** in reverse order. This example is simple enough and extremely illustrative. However, regularity of reproduction of copies of an arbitrary finite configuration in linear ***HSoS***-models of such type will be much more complex for a receiving of quantitative estimation, than in case of classical linear ***HS***–models which are characterized by the general property of universal reproducibility in Moore's sense of finite configurations. Therefore consideration of a generalized class of the linear classical ***HS***–models represented above also in certain cases meets with the certain impediments [5,54,79,88]. Hence for more deep studying of structures of the above classes the computer simulation is rather wide used along with theoretical approach. We have created a series of procedures programmed in ***Maple 8*** for empirical studying of similar problems, one of such procedures is ***HSoS*** [545,640–643].

The procedure call ***HSoS(co, LBF, n)*** returns a three–element sequence whose the first element determines initial configuration ***co*** in ***1–HSoS;***

the second **– *LBF*** given in the form of ***Maple*–**table, whereas the third element defines number of steps of a modelled structure ***HSoS*** for the purpose of generating ***n*** copies of ***co*.** In particular the given procedure serves both for testing of reproducibility and illustration of rapidity of generating of the interesting configurations in structures ***1–HSoS.***

Thus, in contrast to case of the classical ***HS*–**models, the phenomenon of universal reproducibility in Moore's sense of finite configurations that is directly linked with presence of the nonconstructability such as ***NCF–1,*** i.e. agrees with our understanding ***(****see definition* **12)** of absence of the reversibility of dynamics, whereas in case of ***HSoS*–**models the presence of the phenomenon of universal reproducibility in Moore's sense of finite configurations is directly connected to full reversibility of dynamics. At that, within this class of ***HSoS*–**models it is possible to observe a rather interesting phenomenon, namely**:**

Dynamics of self–reproducing finite configurations in such reversible HSoS–model is invariant relative to time – in this model an arbitrary finite configuration is self-reproducing configuration in Moore's sense both in direct and in reverse directions of the time scale.

Consequently, universal reproducibility in the class of ***HSoS*–**models is invariant concerning the time, illustrating existence of the principle opportunity «*to glance*» both at the future and at the past of history of an arbitrary self–reproducing finite configuration.

We find pertinent here to mention a rather interesting problem about possibility of existence of ***HSoS*–**models which can double an arbitrary finite configuration. Naturally, in case of existence of such models we would receive the important example of existence of the phenomenon of self-reproduction in ***HSoS*–**models along with decision of important enough formal problem. However, analogously to case of the classical ***HS*–**models here the negative result takes place [54–56,79,88]**,** namely**:**

There is not a structure 1–HSoS which can double an arbitrary finite configuration determined in the same state alphabet of the 1–HSoS.

However, the proof of this result turned out essentially more complex and tedious, than in analogous case for classical ***HS*–**models. Because of that for research of dynamics of ***HSoS*–**models the special software is more and more widely used. In particular, existence of a number of hardware emulators for example ***CAM*–**machines facilitates that [536]**.**

Furthermore, for support of block over–marking of a space Z^1 is used shift to the left and to the right onto one elementary automaton of the space of such ***HSoS.*** With the concept of ***HSoS***–models the reader can familiarize oneself in section ***1.2.*** Thus, at the made assumptions it is possible to show, the ***HSoS***–model determined in this way, possesses the property of universal reproducibility in the Moore's sense of finite configurations and its dynamics is completely reversible, i.e. for each current configuration of the model its predecessor is calculated quite uniquely on basis of one-to-one mapping $\Psi^{(m)}: A^m \Rightarrow A^m$ and a well–defined algorithm of block over–marking of the space Z^d. The result is generalized to case of ***d***–dimensionality ***(d≥1)*** as follows [54,79,88].

<u>*Theorem 76*</u>***. For an arbitrary integer d ≥ 1 there are structures d–HSoS that possess the property of universal or essential reproducibility in Moore's sense of finite configurations whose dynamics is reversible.***

The further researches in this directions seem to us rather interesting. Having considered the problem of *universal* reproducibility in Moore's sense of the finite configurations in classical ***HS***–models and ***HSoS,*** in brief we shall represent one more parallel processing system that it is quite possible to associate with two previous structures. Let's consider a class of grammars with parallel substitutions as the derivation rules. The requirement of parallel application of isotonic productions in so–called *isotonic structural grammars* ***(ISG)*** leads us to concept of parallel grammars ***(PG).*** Meanwhile, inasmuch as lengths of the right parts of *parallel productions* ***(PP)*** of the following kind, namely:

$$X_1^k X_2^k X_3^k X_4^k \ldots X_{m_k}^k \Rightarrow Y_1^k Y_2^k Y_3^k Y_4^k \ldots Y_{n_k}^k$$
$$(X_j^k, Y_q^k \in A;\ j = 1..m_k;\ q = 1..n_k;\ k = 1..p) \qquad (20)$$

can exceed one, for elimination of ambiguity in case of simultaneous application of the parallel productions to the words processed by the grammar, a special function ***W*** of one–valued choice of state is given

$$W(h_1, h_2, h_3, \ldots, h_v) \in A \qquad h_k \in A = \{0,1, \ldots, a-1\}; \quad (k=1\,..\,v;\ 1 \le v \le r)$$

that allows *unambiguously* to choose in points of uncertainty the states on basis of tuples $<h_1, h_2, \ldots, h_v>$ $h_k \in A = \{0,1,2, \ldots, a-1\}$; $(k=1..v,\ 1 \le v \le r)$ which will be unique states for each concrete uncertainty.

As one such class of the ***ISG*** we shall consider the *generalized* structure ***1–HS*** whose ***LTF*** $\sigma^{(n)}$ is defined by the above ***PP*** on the assumption of $m_k = n_k$; $m_k > 1$ and at existence the substitution of the kind ***0000...00 ⇒ 0000...00*** (*on the assumption of* $m_k > n_k = 1$ *a classical structure* ***1–HS*** *takes place)* among ***PP (20),*** whereas the function ***W*** of one–valued choice of state for the similar ***generalized*** structure ***1–HS*** assumes the following kind, namely:

$$W(h_1, h_2, ..., h_v) = \sum_k h_k \bmod a; \quad h_k \in A; \quad (k=1..m_k) \qquad (21)$$

The above *zero* substitution is an analogue to the quiescent state «***0***» in the classical structures. Inasmuch as global transition function $\sigma^{(n)}$ of a structure ***1–HS*** of the given ***generalized*** type replaces a configuration of neighbourhood template corresponding to a neighbourhood index $X=\{0,1,2,3, ..., m_k-1\}$, onto a block configuration of the same length m_k according to the specified ***PP*** then for an elementary automaton of the structure an ambiguity at choice of a state in following moment arises which is being resolved by means of a ***W***–function ***(21)*** of one–valued choice. At such definition of the generalized structure, its ***LTF*** $\sigma^{(n)}$ can define also a reversible mapping of configurations of blocks of size m_k under condition of its mutual uniqueness which is solvable. Results of the numerous computer experiments providing the analysis of similar generalized classical structures ***1–HS*** with the purpose of detection of existence for them of the property of *universal* reproducibility in sense of Moore of the finite configurations have allowed us to formulate the following assumption, namely:

The isotonic structural grammars (ISG) whose parallel substitutions in the form (20) which determine biunique mappings along with linear W–function (21) of one–valued choice quite can possess the properties of universal or essential reproducibility in the Moore's sense of finite configurations. An arbitrary ISG is simulated in strongly real time by an appropriate structure 1–HS with the same state alphabet A.

For case of an alphabet ***A*** provided that $a=p^k$ (***p*** – *a prime*, ***k*** – *a positive integer*) and $m_k=n_k=p$ this result has received theoretical confirmation, while for more general types of structures of this class, similar results carry, mainly, empirical character. At that, for experiments of similar type a series of procedures programmed for modeling of generalized classical structures ***1–HS*** can appear useful enough [88,545]. Thus, the property of universal and essential reproducibility is inherent to wide enough class of systems of parallel processing of finite words in finite alphabets. Therefore the further researches in this direction seem to us interesting enough, and in this direction we carry out a whole series of studies with results of which will be possible acquaint oneself later on. Meanwhile, the first received results in this and a number of adjacent directions seem interesting enough.

Chapter 4. Problem of complexity of finite configurations in classical homogeneous structures

Complexity in all its generality is one of the most intriguing and vague concepts of the modern natural sciences. In our opinion, the intuitive essence of the given concept is the main reason of it in a great extent. The most fundamental problem of development is the understanding how a system can self–complicate itself and as far as complexity of the initial system should be large for this purpose. One of complexities in the solution of this problem grandiose in many respects is absence of a satisfactory *measure* of complexity. At that, it is quite possible that to the general concept of complexity of the single approach simply does not exist, though in this direction a series of attempts has been done. Thus, researches in this direction are extremely desirable. Meanwhile, in view of use of classical ***HS***–models as formal basis of modelling in biology of development along with researches of the parallel discrete dynamic systems the questions connected to concept of complexity in ***HS***–models seem to us rather actual. The special urgency for the given problematics is given by that circumstance, that ***HS***–models find more and more wide use as conceptual models of the spatially–distributed dynamic systems from which various physical systems seem to us the most interesting [1,5,9,128,143-145,149,150,157,185,187,201,203,209,218, 225,536,545,567,640]. In the given chapter our basic results of research on *complexity* of finite configurations in the classical ***HS***–models along with the questions connected to it are represented.

For formal modelling of various discrete processes and phenomena in the environment of classical structures ***d–HS (d≥1),*** dynamics of initial finite configurations represents the greatest interest. Indeed, a certain modelled process is represented in *dynamics* of a classical structure by the appropriate history of the initial finite configurations. In the given context the question of complexity of finite configurations composing a history of process or object modelled in a classical structure ***d–HS*** all by itself arises. Today, three basic approaches to definition of concept «*quantity of information*» associated with concept of complexity of the finite objects are known: ***combinatory, probabilistic*** and ***algorithmic*** one basing on the theory of *recursive* functions and abstract automata. So, for the first time within algorithmical approach ***A.N. Kolmogorov*** has defined relative complexity by the minimal length of the program of deriving of some finite object ***A*** from a finite object ***B*** (*complexity of* *an object* ***A*** *relative to an object* ***B*)**. Furthermore as representatives of the comparable objects ***A.N. Kolmogorov*** has chosen their *binary* numbers in some formal numbering and as representatives of programs of their deriving – programs of work of appropriate Turing machines [257].

The approach suggested by us to definition of complexity of the finite configurations on basis of ***HS***–axiomatics by one's own essence also is algorithmic, however differs from ***A.N. Kolmogorov's*** approach [3,5,9, 56,68–73,82,83,88,90]. The essence of such approach to definition of the concept of *complexity* of finite configurations consists in an estimation of complexity of generating of an arbitrary finite configuration from a certain primitive configuration $c_p \in C(A,d,\phi)$ (*for example,* $c_p = 1$ *for* **1–*HS*)** by means of finite number of ***GTF*** $\tau^{(n_k)}$ from a certain fixed set of functions G_f, which we shall name a ***basic*** set. In the present chapter a definition of complexity concept of finite configurations is introduced on basis of ***HS***–axiomatics along with a number of interesting enough results connected to it. However, for strict definition of the complexity concept we need a series of fundamental results concerning dynamics of finite configurations in *classical* and *polygenic* ***HS***–models [5,90,536].

The *nonconstructability* problem takes place both for *monogenic* (*chapter* **2**), and for *polygenic* ***HS***–models. In the second case the given problem is known as the *completeness* problem and is defined by the following question: ***Whether an arbitrary finite configuration can be generated from the given primitive configuration by means of a finite sequence of global transition functions of a polygenic HS–model?*** This problem has attracted attention of a whole series of the researchers who have received many rather interesting results in this direction, whereas the following important result of ***A. Maruoka*** and ***M. Kimura*** has finished the solution of the completeness problem [244,245,536,618,640–643].

Theorem 77*. *An arbitrary d–dimensional non–zero configuration $c^{**} \in C(A,d,\phi)$ ***can be generated of a primitive configuration*** $c_p \in C(A,d,\phi)$ ***by means of an appropriate finite sequence of global transition functions*** $\tau^{(n_k)}$ ***of a polygenic structure d–HS*** ($d \geq 1$).

So, the *completeness* problem in the definite measure characterizes the constructive opportunities of the *polygenic* ***HS***–models and its positive decision proves wide enough opportunities of such class of structures concerning generating of the finite configurations. Indeed, basing on result of theorem 77, it is shown that of any ***d***–dimensional non–zero finite configuration $c \in C(A,d,\phi)$ by means of a finite sequence of global transition functions of some polygenic ***d–HS*** it is possible to generate any given finite configuration ***c**** [90]. Meanwhile, the following result directly follows of the results of ***M. Kimura*** and ***A. Maruoka,*** namely.

<u>Theorem 78</u>. An arbitrary d–dimensional configuration $c \in C(A,d,\phi)$ for a polygenic structure d–HS ($d \geq 1$) can be generated of a certain initial primitive configuration $c_p \in C(A,d,\phi)$ by means of application to it of a certain finite sequence of d–dimensional global transition functions $\tau^{(n_k)}$ of a certain fixed (base) set G_f of the global transition functions.

In addition, the given result along with theoretical interest represents also significant applied interest, for example, in systems of processing and storage of the graphic information of various type (*for example, in picture databases*), and also in various systems of coding and decoding of the information [5,9,83,88,90,92,132,135,146,190,360,371,536,567]. So, in systems of processing, storage and transfer of images of a different kind, computer graphics, cartography and a series of other important appendices the problem of compact representation of ***d***–dimensional configurations (*discrete images*) presents significant interest. At that, an approach to a solution of this problem along with an assumption that is associated with it is well coordinated with our presentations about the most general principles of functioning of developing systems: ***At the heart of developing systems a program of development lays more likely than the full description of the developed system.*** Perhaps, this problem is rather important and perspective from many standpoints, demanding the further more detailed research in this direction. With the problems of *complexity* of finite configurations and *completeness* in ***HS***–models also more applied problem of presentation and storage of the information in various picture databases in which the information is represented not by numbers and symbols but **2–** and ***3***–dimensional images of the different nature is naturally being linked [88,90]. So, this approach has been offered by us for solution of a series of problems of coding and data compression. In addition the approach has been used for the purpose of research of some biologically–motivated aspects.

Coming back to result of theorem ***77,*** we, on the other hand, should mention, that the following fundamental result describing dynamical properties of *classical* ***HS***–models and directly continuing results of the previous chapter on the general problem of existence of the universal configurations for classical ***HS***–models takes place [54–56]. Moreover, this result can even be considered as direct consequence of results of theorems ***81*** and ***82*** that can be enough easily received on their basis.

***Theorem 79**. There are no finite sets of d–dimensional configurations $c_k \in C(A,d,\phi)$ and global transition functions $\tau_k^{(n_k)}$ defined in the same finite state alphabet A which satisfy the following relation, namely:*

$$\bigcup_k <c_k>[\tau_k^{(n_k)}] = C(A,d,\phi) \qquad (k = 1..p)$$

We have represented several variants of the proof of theorem **79** with which it is possible to familiarize oneself, in particular, in [8,54–56,88]. Theorems *78* and *79* allow to give good enough grounds for the strict mathematical base of our concept of *complexity* of finite configurations on basis of ***HS***–axiomatics, and also for a whole series of other results in the given direction. For example, from the theorem **79** the following result enough easily follows, namely: *Even polygenic **HS**–models are not finitely axiomatized formal systems, i.e. it is impossible to determine a finite set of configurations (axioms) from which it would be possible to deduce the all set **C(A,d,φ)** of finite configurations by some finite set of global transition functions (derivation rules).* Let's pass on now straightway to definition of complexity concept of finite configurations, that presents doubtless theoretical and gnoseological interest.

Let's assume now, that G_f – a certain finite set of ***d***–dimensional global transition functions given in some finite state alphabet ***A*** by means of which during finite number of steps, an arbitrary finite configuration c^* can be generated from a certain *primitive* configuration $c_p \in C(A,d,\phi)$, i.e. there are the following derivation rules of the finite configurations from a certain primitive configuration, namely:

$$c^* = c_p \tau_1^{m_1} \tau_2^{m_2} \tau_3^{m_3} \ldots \tau_n^{m_n} \quad (\tau_k \in G_f;\ \tau_j \neq \tau_{j+1};\ k = 1..n;\ j = 1..n-1) \quad (22)$$

where m_k – multiplicity of usage of global transition functions $\tau_k \in G_f$ $(k = 1..n)$. We shall speak, that a configuration $c \in C(A,d,\phi)$ is generated from some *primitive* configuration $c_p \in C(A,d,\phi)$ at least during $r = \Sigma_k m_k$ steps of global transition functions $\tau_k \in G_f$ $(k=1..n)$. So, for the classical structures ***1–HS*** the configuration of the kind $c_p = 1$ can be chosen as a quite clear primitive configuration.

In addition, two arbitrary finite configurations $\tau_i, \tau_j \in G_f$ are supposed various $(\tau_i \neq \tau_j)$ only if the following relation $(\exists c \in C(A,d))(c\tau_i \neq c\tau_j)$ takes place. If in a derivation chain ***(22)*** there is ***(n–1)*** pairs of various global transition functions $<\tau_i, \tau_j>$ $(j=1..n-1)$, we shall speak, that in a chain of generating of configurations $c \in C(A,d,\phi)$ from a *primitive* configuration

$c_p \in C(A,d,\phi)$ there are $(n-1)$ levels L_k that are defined by the following signalling binary function, namely:

$$L_k = \begin{cases} 1, & \text{if } \tau_k \neq \tau_{k+1} \\ 0, & \text{otherwise} \end{cases} \qquad k = 1..n-1$$

The following diagram illustrates the described process of generating of an arbitrary finite configuration $c \in C(A,d,\phi)$ from a certain primitive finite configuration c_p *(fig. 13)*. In addition, the generating process of the finite configurations from a certain fixed *primitive* configuration c_p according to the above rules *(22)* underlies the following diagram.

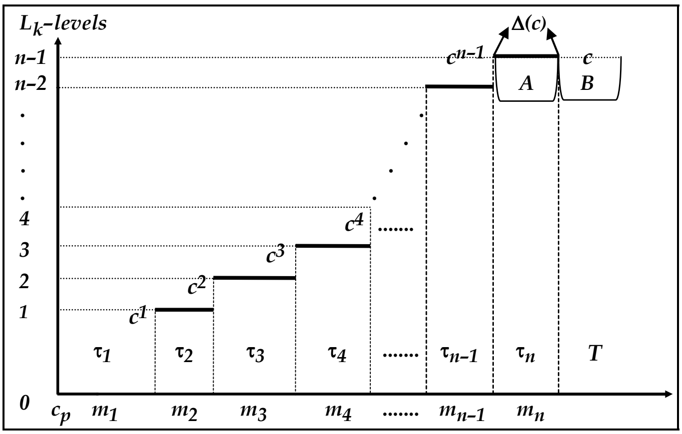


Fig. 13. The diagram explaining optimum strategy of deriving of any finite configuration $c \in C(A,d,\phi)$ [optimum graph HS(c) of derivation].

It is necessary to mark, the above diagram can serve as a good enough illustration for many researches connected to the introduced concept of *complexity* of finite configurations in classical ***HS***–models *(fig. 13)*. In view of told the complexity of an arbitrary configuration $c^{**} \in C(A,d,\phi)$ $(d \geq 1)$ can be determined as follows.

<u>*Definition 18*</u>. ***Complexity of an arbitrary configuration $c^{**} \in C(A,d,\phi)$ $(d \geq 1)$ on basis of HS–axiomatics is being evaluated according to the following general formula, namely:***

$$SL(c^{**}) = \min_{\tau_k \in G_k} \prod_{k=1}^{n-1} p_k^{m_k}$$

where p_k – k–th prime number and m_k is being defined on basis of the generating chains of finite configurations (22) of a certain polygenic structure d–HS (d ≥ 1).

The essence of the given concept of complexity is based on results of theorems ***78*** and ***79*** which assert ***(****on the one hand****)*** about the possibility of generating of an arbitrary finite configuration in a polygenic ***d–HS*** from a certain initial primitive configuration ***c_p∈C(A,d,ϕ),*** and also ***(****on the other hand****)*** about impossibility of determination of the finite sets of initial finite configurations and global transition functions of classical structures ***d–HS,*** which in aggregate generate all set ***C(A,d,ϕ) (d ≥ 1)*** of the finite configurations. On basis of this definition a lot of important enough properties of finite configurations in the *classical* and *polygenic* ***HS–***models characterizing them concerning introduced the *complexity* concept has been received [3–5,10,41,53–56,88,90]**.** A series of results in the given direction the following important theorem having a series of interesting enough appendices both in theoretical and applied aspects represents.

Theorem 80. For each integer d≥1 the set C(A,d,ϕ) of all d–dimensional finite configurations contains configurations of any given complexity concerning any finite base set G_f of d–dimensional global transition functions, defined in some finite alphabet A of a polygenic HS–model.

Of this theorem follows, that for any finite set ***G_f*** the configurations of any given complexity relative to it will still exist in the set ***C(A,d,ϕ).*** At that, other most characteristic properties of the introduced concept of *complexity* of finite configurations in ***HS–***models along with interesting enough consequences from them can be found in our works [5,9,10,41, 53–56,88,90]**.** On base of theorem ***80*** and some other our results in this direction it is possible to receive the result playing important enough role for research of dynamic properties of classical ***HS–***models and for the further development of the complexity concept, related with basic conception of classical ***HS–***models [5,53–56,79,88,90,536,618,640–643]**.**

Theorem 81. For an arbitrary dimension d ≥ 1 exist global transition functions τ∉G_f generating from the given configuration c∈C(A,d,ϕ) of the limited complexity the configurations of any given complexity in the meaning of definition 18.

This theorem asserts, if global transition functions composing the base set ***G_f*** generate finite configurations only of the *limited complexity*, then by means of global transition functions τ_j not belonging to the set G_f, finite configurations of any complexity can be generated. Result of the given theorem has generated a series of interesting enough questions, one of which is the question about number of finite configurations of the same complexity concerning the given base set G_f. The following important enough result allows in a great extent to clear up the given interesting question [53–56,79,88,90,640–643].

Theorem 82. There is infinite amount of base sets G_f of d–dimensional global transition functions, defined in an arbitrary finite alphabet A, concerning each of which the infinite sets F_j of finite configurations of the same complexity in the meaning of definition 18 exist.

Result of the theorem *82* allows to solve a whole series of interesting enough questions, formulated in our works [1,3]. Detailed research of the basic set G_f, used at definition of the complexity concept of finite configurations in classical ***HS***–models along with properties of global transition functions composing the set G_f allow essentially to clear up not only new properties of the introduced concept of complexity, but also will give effective enough apparatus of researches of dynamics of ***HS***–models such as *classical* ones, *polygenic* along with *nondeterministic* ones in a whole series of cases too.

So, in particular, it is rather important to investigate the minimal basic set G_f containing the *least* number of global transition functions $\tau_k^{(n_k)}$. Investigating the completeness problem in the polygenic ***HS***–models, ***A. Maruoka*** and ***M. Kimura*** have presented a constructive proof of the existence of base set G_f (*theorem 77*); however, at the same time, they did not use optimizing technics. In general case the detailed research of basic sets G_f of global transition functions hitherto is absent, while concerning the narrower class of binary one–dimensional ***HS***–models a whole series of interesting enough results in the given direction has been received [5,53–56,79,88,90,640–643].

Theorem 83. There is a minimal basic set G_f containing only 4 binary 1–dimensional global transition functions $\tau_k^{(n_k)}$; at least one of them possesses the nonconstructability such as NCF-1. Relative to minimal basic set G_f of 1–dimensional binary global transition functions there are infinite sets of finite configurations of the same complexity.

In a sense the given result carries result of the previous theorem *82* to case of minimal basic sets G_f while method of its proof appears useful enough at receiving of the following rather interesting theorem which has enough much important appendices at researches of dynamics of classical ***HS***–models [73,82,83,90,618,640–643].

<u>Theorem 84</u>. There are minimal basic sets G_f of 1–dimensional global transition functions $\tau_k^{(n_k)}$ in binary alphabet B in relation to each of which there are such infinite sets of functions $\tau_k^{(n_k)}$ of the same class together with configurations $c_k \in C(B,\phi)$ (B={0,1}), that configurations sequences $<c_k>[\tau_k^{(n_k)}]$ contain finite configurations of an arbitrary predetermined complexity. However, there is not a finite basic set G_f of 1–dimensional binary global transition functions $\tau_k^{(n_k)}$ concerning which any arbitrary sequence $<c>[\tau_k^{(n_k)}]$ ($c \in C(B,\phi)$; $\tau_k^{(n_k)} \notin G_f$) would contain binary finite configurations only of the limited complexity.

Theorem ***84*** allows to receive the answers to a series of questions and a little bit more deeply to reveal essence of the introduced complexity concept of finite configurations concerning ***HS***–axiomatics [76]. In this connection it is necessary to note, that the complexity concept of some algorithm depends on the concept of algorithm, and from its concrete realization. The conventional more precise definition today is absent. Thus the results relative to an estimation of complexity of algorithms can have essentially various character. So, for example, complexity of normal ***Markov*** algorithm is defined by the length of recording of all its formulas of substitutions, whereas under the complexity of Turing machine usually understand the product of quantity of internal states of the finite automaton and symbols of the alphabet of external tape.

Within the *complexity* concept an arbitrary function of algebra of logic from ***m*** variables can be realized by an appropriate normal algorithm with complexity of order ***2m*** and by Turing machine with complexity of order $2^m/m$. Meanwhile, defining complexity of a classical structure $<Z^d,A,\tau^{(m)},X>$ as a product ***dxmxa***, we can easily make sure, that each function of algebra of logic from ***m*** variables can be realized by means of an appropriate classical structure ***1–HS*** with complexity ***2m***. Hence, the above complexity concept of finite configurations quite essentially influences *comparative* characteristics of various classes of algorithms. Thus, should be given the more consideration to the conceptual basis of the compared formal algorithms [5,54–56,79,88,90,536,617,640].

The *complexity* concept of description of a certain algorithm is used, as a rule, for specification of a question about minimal complexity of the algorithm generating a certain finite object. Such minimal complexity very much is frequently named simply complexity of finite object *(at a concrete specification of complexity concept of the algorithm description)***.** As it was already marked, definition of complexity of a finite object ***G*** for the first time has been proposed by ***A. Kolmogorov.*** At the same time, between *complexity* ***K(G)*** of a finite object ***G*** according to ***Kolmogorov,*** *complexity* $M_q(G)$ of the same object ***G*** expressed by length of a *normal* ***Markov*** algorithm in alphabet with ***q*** symbols, and *complexity* $MT_q(G)$ expressed by a quantity of internal states of a Turing machine with an external alphabet of cardinality ***q*** asymptotically exact relations exist**:**

$$M_q(G) = K(G)/\log_2 q \qquad MT_q(G) = K(G)/(q-1)\log_2 K(G)$$

The approach suggested by us allows to estimate the complexity ***A(G)*** of such finite objects ***G*** as finite configurations in classical ***HS–***models. At the same time, here it is necessary to take into account a series of the essential moments connected to the above–mentioned conceptual circumstances [5,54–56,90,567,640]**.** So, the certain analogy takes place between complexity concepts of finite objects and complexity of finite configurations of the set ***C(A,d,***ϕ***).*** Both approaches to the complexity concept are algorithmic, however between them exist rather essential distinctions. For example, one of ***A. Kolmogorov's*** results in the given direction says that MT^s_q ***with a constant program of work will print on a final tape the binary words of only limited complexity.*** Whereas for case of our complexity concept a quite other picture whose reasons were considered in [9,567,618] takes place. These and a series of other distinctions once again accent a question on the essence of axiomatics of the complexity concept as a whole [90,536]**.** Meanwhile, the existing contradiction between the ***Kolmogorov's*** complexity concept of finite objects**,** on the one hand and our complexity concept of configurations in classical ***HS–***models can be removed, if to consider instead of finite configurations the *finite* ***block*** configurations, namely**:** there are binary ***1–***dimensional *classical* structures providing generating of the set of all such configurations from any finite initial configuration *(section* ***3.1).*** It is once more essential difference between both type of configurations.

At proof of theorems ***82–84*** the concept of minimal base set G_f and the certain dynamical properties of global transition functions entering in the set G_f were essentially used. At the same time, it seems that quite pertinently to present the more detailed properties of similar minimal base sets, considering their importance from standpoint of the further researches of deeper properties of dynamics of classical ***HS***–models. In this connexion we shall restrict oneself to a case of binary polygenic ***1–HS*** along with a set of finite binary configurations ***C(B,1,ϕ)*** and binary global transition functions $\tau_k^{(n_k)}$ determined by the appropriate local transition functions according to the following parallel substitutions:

		(a)	(b)	(c)
000	⇒	0	0	0
001	⇒	1	1	1
010	⇒	0	1	0
011	⇒	1	0	0
100	⇒	0	1	1
101	⇒	1	0	1
110	⇒	1	1	0
111	⇒	0	0	0

		(d)
00	⇒	0
01	⇒	1
10	⇒	1
11	⇒	1

(23)

In view of the done assumptions the following rather important result describing global transition functions of a minimal basic set G_f for the one–dimensional binary case of the polygenic ***HS***–models takes place, namely [5,9,45,54–56,79,90,617,618,640–643].

<u>Theorem 85</u>. Minimal base set G_f contains four 1–dimensional binary global transition functions $\tau_k^{(n_k)}$ whose appropriate local transition functions are defined by the above parallel substitutions (23.a–d); at that, the global transition functions composing the set G_f possess the types of the nonconstructability according to the following table 5.

Table 5

LTF\NCF	NCF	NCF–1	NCF–2	NCF–3
(23.a)	–	+	–	–
(23.b)	–	+	–	–
(23.c)	+	+	–	–
(23.d)	+	–	+	+

Minimal basic set G_f for a 1–dimensional non–binary case consists of global transition functions $\tau_k^{(n_k)}$ that possess the nonconstructability such as NCF and/or NCF–1, NCF–2 and, perhaps, NCF–3.

The choice of global transition functions $\tau_k^{(n_k)}$, determined by means of local transition functions with parallel substitutions ***(23.a–d)*** as the minimal base set G_f has been grounded in work [54]. Moreover, it is shown that not exists a set G_f of **1**–dimensional *binary* global transition functions, determined by local transition functions with the simplest neighbourhood index $X = \{0,1\}$ which could be chosen as the minimal basic set [5,88,90]. In the minimal base set G_f, whose global transition functions are determined by **4** local transition functions with parallel substitutions ***(23.a-d)***, the first two local functions ***(23.a-b)*** have defect ***one*** and **3** accordingly, belonging to the isolated subset concerning the composition operation of global transition functions, that possesses a series of interesting enough properties concerning dynamics of finite and infinite configurations [53]. This question is considered enough in detail in [3,90,567,640] along with discussion of the existence question of types of the nonconstructability for global transition functions, that compose the minimal set G_f.

So, result of theorem ***85*** has allowed to solve some problems from the monograph [5]; the result can be used for researches of the complexity problem of **1**–dimensional finite configurations in an arbitrary finite alphabet ***A*** [54–56,88]. In particular, on basis of results of this theorem it is possible to give the simplest validity of the introduced concept of complexity of finite configurations for **1**–dimensional binary case. The result of such substantiation takes the form of the following theorem representing also the important independent interest as a component of the apparatus of researches of classical ***HS***–models [54,86,90,640].

<u>Theorem 86</u>. Any 1–dimensional binary configuration $c^{**} \in C(B,1,\phi)$ is monotonously generated of primitive configuration $c_p = 1$ by means of global transition functions $\tau_{jk}^{(n_k)}$ from a certain fixed finite set VS. At the same time not exists such finite system of pairs $\{c_k, \tau_{jk}^{(n_k)}\}$ that the following determinative relation takes place, namely:

$$\bigcup_k \langle c_k \rangle \left[\tau_{j_k}^{(n_k)}\right] = C(B,1,\phi); \quad c_k \in C(B,1,\phi) \quad (n_k \in \{2,3\};\ j_k \in \{0,1,2,3\};\ k = 1..p)$$

Set VS of binary global transition functions can be chosen as base set G_f concerning which the complexity concept of 1–dimensional binary finite configurations of classical HS–models is being determined.

It is necessary to note that result of this theorem is generalized and to case of any finite alphabet ***A*** of internal states of elementary automata of an arbitrary model ***1–HS*** [90]. Moreover, on basis of the given result can be given more simple proofs, and in a series of cases constructive proofs of the previous theorems of this chapter can be received, along with other rather interesting results relative to the complexity of finite configurations for the binary structures ***1–HS*** [54–56]**.** In addition, the first part of the theorem ***86*** can be essentially improved, namely, the following basic result representing undoubted interest for the theory of both the classical, and the polygenic ***HS*–**models takes place [90]**.**

Theorem 87. ***A system of pairs in which any global transition function $\tau_{jk}^{(n_k)}$ is an arbitrary composition of finite number of functions of the set VS can be chosen as a certain finite system of pairs $\{c_k, \tau_{jk}^{(n_k)}\}$ of the theorem 86. There are subsets inside of the set $C(B,1,\phi)$ of all finite binary one–dimensional configurations {of cardinality on average 3/4 of cardinality of all set $C(B,1,\phi)$} which cannot be generated by means of finite number of pairs of the kind $\{c_k, \tau_k^{(n_k)}\}$ ($c_k \in C(B,1,\phi)$; $k = 1 .. p$).***

In a series of cases $\{c_k, \tau_k^{(n_k)}\}$ $c_k \in C(B,1,\phi)$ in formulations of theorems ***86*** and ***87*** it is possible to use a system of pairs in the equivalent form, supposing $(\forall k)(c_k \equiv c_p = 1\ )$, i.e. it is possible to use a single primitive configuration c_p ($k=1 .. m$). Indeed, we can easily make sure that for a finite configuration $c \in C(A,1,\phi)$ it is possible to designate such global transition function $\tau_c^{(h_c)}$ in the same alphabet ***A*** that $c_p\tau_c^{(h_c)}=c$. Hence for a finite system of pairs $\{c_k, \tau_k^{(n_k)}\}$ some equivalent system of pairs of the kind $<c_p, \{\tau_k^{(n_k)}\} \cup \{\tau_k^{(h_k)} \mid k=1 .. m\}>$ exists, for which the global transition function $\tau_k^{(h_k)}$ satisfies the relation $c_k = c_p\tau_k^{(h_k)}$ ($k = 1 .. m$). This approach allows to unify the set of initial configurations c_k of a system of pairs of generatrices, displacing the *basic* accent onto a finite set of base global transition functions. In view of told, formulations of theorems ***86, 87*** are modified in appropriate way and, first of all, the kind of a determinative relation of basic theorem ***79*** becomes simpler.

At the same time, these **2** equivalent forms of the above relation have own advantages in theoretical researches concerning the dynamics of classical ***HS*–**models. Most of all, the given moment become apparent at use of the introduced complexity concept of finite configurations in classical ***HS*–**models along with the results received on its basis as an apparatus of research of dynamic properties of classical and polygenic ***HS*–**models. It is necessary to note, that the complexity problem of the finite configurations in the classical ***HS*–**models, in spite of the results represented above and a number of other results, has a whole series of open questions and perspective directions for the further researches, needing solutions from the various standpoints [5,7–9,11,54–56,88,90]**.**

In conclusion of this section it is expedient to stop on two essentially various approaches to definition of the *complexity* concept of the finite configurations in classical ***HS***–models, namely**:** *configuration* approach and *block* approach**;** their essence was in brief marked in sections ***3.1 – 3.2.*** First of all, under the *configuration* complexity is being understood an opportunity of a ***HS***–model or a set of similar models of generating the set of all finite configurations from one or finite set of initial finite configurations. On basis of the definition ***18*** and theorem ***80*** follows, what for each integer $d \ge 1$ the set $C(A,d,\phi)$ of all ***d***–dimensional finite configurations contains configurations of an arbitrary predetermined complexity concerning any finite base set G_f of ***d***–dimensional global transition functions, defined in a certain finite alphabet ***A*** of a classical ***HS***–model**.** Of the given result follows, that at any definition of a finite base set G_f the finite configurations of any given complexity still will exist in the set $C(A,d,\phi)$ of all finite ***d***–dimensional configurations in ***A*.**

Completely other picture is quite allowable in case of definition of the block complexity when is being taken into account essentially wider possibility of generating not of finite configurations $c= hx_1x_2x_3...x_nh$ [– *configuration of infinite number of symbols* ***'0'***; $x_j \in A$, $j=1..n$; $h \in A \backslash \{0\}$] but of block configurations, i.e. configurations of blocks $<x_1x_2x_3 ... x_n>$ $\{x_j \in A,\ j=1..n\}$ of elementary automata. In case of such approach other situation is quite real. We shall illustrate essence of similar distinction by means of a binary structure ***1–HS*** whose local transition function is defined by the following formula, namely**:**

$$\sigma^{(3)}(x,y,z) = \begin{cases} x+y \quad (mod\,2), & if\ \ y=1 \\ x+y+z \quad (mod\,2), & otherwise \end{cases}$$

i.e. it is the above binary structure with discriminating number ***120.*** It is shown that the structure not possess the nonconstructability such as ***NCF (NCF–3),*** possessing the nonconstructible configurations such as ***NCF–1*** of the kind, for example, $c' = 10x_1x_2...x_n1$, $c= 1^40x_1x_2 ... x_n1$ $\{x_j \in B=\{0,1\},\ j=1..n\}$**;** i.e. quota of nonconstructible configurations such as ***NCF–1*** concerning all finite configurations is more than ***1/2.*** Along with that, the given structure does not possess the nonconstructability such as ***NCF–2;*** i.e. each finite configuration which is distinct from the ***NCF–1*** has a predecessor as from the set $C(B,1,\phi)$ and the set $C(B,1,\infty)$. Furthermore, this structure possesses the universal reproducibility in the Moore's sense of finite configurations.

In view of the aforesaid along with kind of global transition function of the considered structure it is possible to show, that for it there is an infinite set of configurations such as ***NCF–1*** $\left\{c_1^j\right\}$ $(j = 1..\infty)$, which in the aggregate generate all set ***C(B,1,ϕ)*** of finite configurations ***(24).*** Hence, the set of all configurations such as ***NCF–1*** of the above structure with number ***120*** generates the set ***C(B,1,ϕ)*** of all finite configurations**:**

$$\tau^{(3)}: \begin{cases} c_1^1 \to c_2^1 \to c_3^1 \to c_4^1 \to \ldots \to c_k^1 \to \ldots \\ ===================== \\ c_1^j \to c_2^j \to c_3^j \to c_4^j \to \ldots \to c_k^j \to \ldots \\ ===================== \\ c_1^k \to c_2^k \to c_3^k \to c_4^k \to \ldots \to c_k^k \to \ldots \\ ===================== \end{cases} \qquad (24)$$

$$\bigcup_{j=1}^{\infty} \left\langle c_1^j \right\rangle \left[\tau^{(3)}\right] = C(B,1,\varphi); \quad (\forall k,j)\left(k \neq j \to \left\langle c_1^k \right\rangle \left[\tau^{(3)}\right] \cap \left\langle c_1^j \right\rangle \left[\tau^{(3)}\right] = \varnothing\right)$$

In addition, as it was marked earlier, computer experiments with this structure in combination with a series of theoretical results basing on dynamic properties of classical structures ***1–HS*** caused by existence in them of the *nonconstructability* such as ***NCF–1*** in the absence of ***NCF–2*** and ***NCF*** as well as ***NCF–3*** have allowed us to formulate the following rather interesting assumption [88,640–643]**,** namely.

Proposal 6. The above classical binary 1–HS possesses the property of universal reproducibility in Moore's sense of finite configurations; in addition, an arbitrary finite configuration generates in the aggregate all set of finite binary block configurations.

In addition, in case of the *positive* answer we shall receive an example of simple enough binary classical ***HS***–model possessing the property of universal reproducibility in Moore's sense of finite configurations along with an example of structure for which any finite configuration generates all set of block configurations. The computer experiments carried out by us and other researchers persuade us at an idea about validity of this assumption, meanwhile, theoretical acknowledgement of it up till now is absent [5,88,90]**.** Validity of the given proposal once again will enough evidently illustrate essence of distinctions between universal finite configurations and the *block* configurations, generated by classical ***HS***–models along with a *distinction* between our approach to complexity definition of finite configurations in classical ***HS–models*** and approach of ***A.N. Kolmogorov*** to definition of complexity of finite objects. So, the possibility of generating by means of the above binary structure ***1–HS*** of all *block* configurations from *any* initial configuration could be a certain *analogue* of generating by means of Turing machine of sequences of binary words of the limited complexity [260,640]**.**

The *complexity* problem of finite configurations in classical ***HS***–models has great value not only in a context of their research as certain formal *deductive* systems, but also in case of embedding in them of *developing* systems of the cellular organization and their certain phenomena. In such case the given problem has the most direct attitude to a question of research of complexity of self-organizing biological cellular systems that is actual enough for modern biology of development. You know, till now at cybernetic research of biology of development we have no satisfactory enough approach to a question of estimation of *complexity* of developing biological systems. And our mathematical approach in the given direction can appear fruitful enough and perspective. Thus, the represented results along with other our results on the complexity problem of finite configurations in ***HS***–models not only form actually the problematics and solve a lot of its basic problems as a whole, but also formulate a whole series of ***open*** questions and rather perspective directions of the further research representing significant independent interest for theoretical and applied aspects of the ***HS***–problematics.

The results received by us concerning the complexity problem of the finite configurations in a context of ***HS***–axiomatics to a certain extent allow better to explain essence of the complexity concept depending on the used axiomatics. So, in axiomatics of the *classical* and *polygenic* ***HS***–models exist binary finite configurations of any given complexity, while in other axiomatics, for example, in ***A. Kolmogorov's*** axiomatics all binary words printed by means of an arbitrary Turing machine on final tape can have only the limited complexity. So, most likely, there is no any absolute complexity concept of finite objects along with the complexity concept as a whole; i.e., the complexity concept bears the pronounced relative axiomatic character in a great extent.

Similarly to case of classical ***HS***–models, in case of ***HSoS***–models it is quite naturally possible to define the complexity concept of the finite configurations that is based on an analogue of theorem ***79*** [5,6,8–10,54, 536]. The majority of the represented results concerning the *complexity* of the finite configurations is directly carried over to the ***HSoS***–models playing important enough role as an excellent environment of *physical* modelling, along with simulating and research of rather wide class of the spatially–distributed dynamic systems. The reader can familiarize oneself with the basic questions of the theory of this class of structures in works [8-10,54-56,88,90,150-152,268,273,536,617,618,640], a series of results concerning them is represented and in the present monograph.

Chapter 5. Parallel formal grammars and languages defined by homogeneous structures

The *theory of formal grammars* ***(TFG)*** fills central place in mathematical linguistics since it gives formal means for research of functioning of a *language*. At the same time, the ***TFG*** stands out against a background of other sections of mathematical linguistics by the essentially greater complexity of the used apparatus similar to apparatus of the theory of *algorithms* and apparatus of the general theory of *automata* with which has quite a few points of contacts and intersection, and by essentially greater complexity of mathematical problems arising in it. The formal grammars of most well researched types present the systems allowing to generate or recognize sets of chains, interpreted usually as the sets of grammatically correct sentences of some formal languages and also to associate description of their syntactic structure with chains which compose these sets in terms of the systems of components or trees of subjection [54–56,79,90,162,163,389,390,536,567,617,618,640–643].

Mathematical significance of the *generative grammars* is defined by the circumstance, that they represent one of means of effective definition of sets of words. A class of formal languages which are generated by means of any grammars will coincide with the class of all recursively–enumerable sets. From the given point of view the formal grammars of classical hierarchy of ***N. Chomsky*** represent here a special interest [389]. In this connection the essential significance receives studying of classes of abstract automata, which are equivalent to those or to other classes of formal grammars describing the same formal languages. In particular, for instance, automaton grammars are equivalent to finite automata, *context–**free*** grammars are equivalent to automata with *stack* memory while *context–**dependent*** grammars are equivalent to linearly limited Turing machines. Except the ***Chomsky*** grammars, today, there is a series of others interesting from the various standpoints of kinds of formal grammars, destined for description of the sets of words and other objects; among them at the latest years a particular attention the parallel formal grammars attract which give effective enough means for the linguistic description of some important parallel processes and objects [54–56,79,88,90,536,567,617,618,640–643].

Since the theory of formal grammars is a part of the automata theory then study of dynamics of ***HS***–models from the standpoint of the ***FGT*** undoubtedly deserves a separate attention hence a whole series of our works are devoted to these problems. At the same time, the theory of parallel formal grammars can be most advantageously used not only at creation of the theory of parallel programming and architecture of computing systems of high–parallel operation of new generations, but also at creation of a linguistic basis for the description of dynamics of various spatially–distributed systems of cellular nature [3-5,7-10,32,59, 60,71,75,80,86,162,163,233]. Therefore, for the purpose of studying the languages generated by classical ***HS***–models by us at ***1974*** have been introduced formal parallel grammars, named τ_n***–grammars*** [31,32]. At that, basically were studied the τ_n–grammars defined by classical and non–deterministic one–dimensional structures ***1–HS,*** however similar approach can be spread onto a case of structures ***d–HS*** ***(d≥2),*** and also other certain types of structures distinct from the above ones [5,54–56, 88,90,536,567,617]. At such approach the classical ***HS***–models can be considered as the formal parallel grammars ***(FPG)*** which not use non–terminal symbols and derivation of which is carried out by absolutely parallel manner. The grammars of this type are similar to well–known systems of ***A. Lindenmayer (L–systems),*** they can be quite successfully used for *formal* linguistic description of dynamics of various objects of cellular nature and many *parallel* discrete processes [5,136,163,233,251, 265,336,338]. Below, at a conceptual level the consideration of parallel τ_n–grammars is considered enough in detail according to traditions of the theory of formal grammars, as a consequence the reader receives a whole series of rather important characteristics of the formal parallel grammars of such class interesting from many standpoints.

Informally the parallel τ_n–grammars are being introduced as follows, namely. By analogy to basic concepts of the ***TFG*** the alphabet ***A*** of the internal states of an elementary automaton of a *classical* structure ***1–HS*** is named the alphabet of τ_n–grammar, its local transition function $\sigma^{(n)}$ defines a set of *parallel* productions or *derivation* rules of the grammar, an initial finite configuration of the structure defines an axiom, while finite configurations generated from the axiom are words of language defined by such parallel τ_n–grammar. Likewise with the usual formal grammar in a certain classical ***HS***–model from an initial configuration c_o ***(****axiom****)*** by means of a local transition function $\sigma^{(n)}$ ***(****derivation rules****)*** are being deduced new configurations ***(****words of language****).*** Meanwhile, between traditional formal grammars and parallel τ_n–grammars two rather essential differences take place, namely:

♦ ***derivation rules in a τ_n–grammar are being applied simultaneously and absolutely by parallel manner;***

♦ ***in a state alphabet A of the parallel τ_n–grammar are not being done any distinctions between terminal and non–terminal symbols.***

In the ***TFG*** a formal language is defined as a set of all terminal words generated from an axiom c_o by means of derivation rules of a certain grammar. $L(\tau_n)$–language is defined as a set of all finite configurations *(words)* generated from an initial configuration *(axiom)* by *simultaneous* application of parallel substitutions determined by means of the local transition function to all symbols of each current configuration *(word)*. Since a local transition function $\sigma^{(n)}$ uniquely defines global transition function, then global function of transition of a structure is frequently understood as the derivation rules in the formal parallel τ_n–grammar. The parallel formal τ_n–grammars, defined in a similar way, are similar to the above ***L***–systems; they can be used for the linguistic description of certain discrete developing systems and parallel processes. For the further presentation we need a whole series of definitions. In addition it is supposed that the reader is enough well familiar with basis of the ***TFG***. Thus, at presentation of the material of the chapter will be used standard terminology and designations of the recognized ***TFG*** [389].

We have investigated the τ_n–grammars in conformity with traditions of the ***TFG*** with respect to a series of important characteristics of such class of parallel grammars. The results in this direction are presented in our monograph, which contains the systematic exposition of the τ_n–grammars theory investigated by us [5].

5.1. The basic properties of the parallel languages determined by the classical homogeneous structures

In the present section the basic concepts and properties of the formal parallel languages determined by ***1***–dimensional classical ***HS***–models will be represented. So, in traditions of the ***TFG*** the τ_n–grammars and parallel formal languages determined by them have been investigated in detail enough what has found reflection in works [3,5,7–10,32,33,53, 59,60–63,71–73]. According to the traditions of the ***TFG***, we determine an arbitrary $L(\tau_n)$–language as a set of all words *(finite configurations)*, derived from an axiom $c_o \in C(A,1,\phi)$ *(initial configuration)* by sequential application to the axiom c_o of the derivation rules $\tau^{(n)}$ **(*GTF*** *of a certain classical* ***1–dimensional structure*)**. The more formally a parallel grammar τ_n and a $L(\tau_n)$–language are defined as follows, namely.

Definition 19. A parallel τ_n–grammar is the ordered tuple in the form τ_n = <n, A, $\tau^{(n)}$, c_o>, where its components have the following sense:

1) n – index of the grammar (size of neighbourhood template of some classical structure 1–HS with a state alphabet A);
2) A – a finite alphabet of the grammar (internal states of elementary automata in the classical structure 1–HS);
3) $\tau^{(n)}$ – derivation rules of the grammar (global transition function of the classical structure 1–HS with the state alphabet A);
4) c_o – an axiom of the grammar (an initial finite configuration in the classical structure 1–HS with the state alphabet A).

A formal parallel $L(\tau_n)$-language determined by τ_n-grammar is the set of all words (*finite configurations*), deduced from an axiom $c_o \in C(A,1,\phi)$ (*an initial configuration*) by means of sequential application to it of the derivation rules $\tau^{(n)}$ (*global transition function* $\tau^{(n)}$ *of a classical structure* ***1–HS***), i.e. the next relation $L(\tau_n) \equiv <c_o>[\tau^{(n)}]$ is here determinative. As it was already marked earlier, the especially *parallel principle* used by a τ_n–grammar for words processing is its essential feature appreciably distinguishing the τ_n–grammar from the traditional formal grammars. This *parallelism* reflects the basic applied motivations both on the part of computational and biological sciences, and on the part of a series of highly abstract models of the real physical world functioning at space and time. Moreover, τ_n–grammars as against the traditional grammars do not use the non–terminal symbols carrying a role of symbols which extend alphabet of a formal grammar.

Defining the parallel τ_n-grammar and $L(\tau_n)$-language thus, we receive a possibility to research dynamics of ***1***–dimensional *classical* structures within of the ***TFG*** what allows to look at it from an untraditional side. The received results of investigation of classical and non-deterministic structures ***1–HS*** from standpoint of the ***TFG*** are represented in a cycle of our works (*see reviews in* [5,7–10,32,53,59,61,86,90]), and in works of other researchers [162,163,184,233,251,254,264]. If the opposite not was said, then during of the present section the parallel τ_n–grammars and $L(\tau_n)$–languages defined by the classical structures $<Z^1,A,\tau^{(n)},X>$ with alphabet $A=\{0,1,2,3,...,a-1\}$ and neighbourhood index $X=\{0,1,2,...,n-1\}$ will be considered.

Research of the closure property of a certain class of formal languages concerning the operations, traditional in ***TFG,*** is classical approach to its mathematical characteristic. Two principal causes for consideration of these operations concerning the parallel $L(\tau_n)$–languages exist. First of all, a possibility to more deeply elucidate the distinctions between a family of $L(\tau_n)$–languages and traditional families of formal languages that on this way exist. Second, a set of operations natural to family of $L(\tau_n)$–languages is determined till now not up to the end. In addition, the following *basic* result determines behaviour of the $L(\tau_n)$–languages concerning the *traditional* operations researched in the classical theory of formal grammars [32,54–56,59,79,88,389,390,640–643].

Theorem 88. Class of parallel languages $L(\tau_n)$ ***is non–closed relative to such operations as a finite transformation, homomorphism, iteration, union, product, addition and crossing while the class of these parallel languages is closed concerning the operation of inversion.***

J. Dassow has investigated parallel τ_n–grammars and $L(\tau_n)$–languages associated with them, concerning four new set–theoretical operations and has shown that class of $L(\tau_n)$–languages is non–closed concerning these operations, interesting from the biological standpoint [233]. The above–mentioned results have been received, basically, by *constructive* methods which consist in construction of the appropriate examples of $L(\tau_n)$–languages. Furthermore, a rather important fact can be ascribed to most essential features of τ_n–grammars, namely, that the majority of approaches of standard techniques and apparatus of research in the ***TFG*** is *inapplicable* to the class of parallel τ_n–grammars, presuming use of new non–standard approaches. In particular, use of methods of the theory of recursive functions has allowed to solve certain questions of the theory of parallel τ_n–grammars [5,640–643].

The given approach is based on the introduced concept of ***G***–indexing relative to which *biunique* equivalence between some partial recursive word function $\tau^{(n)}: C(A,1,\phi) \to C(A,1,\phi)$ and a certain numerical partial recursive function $F_n(m): N \to N^* \subseteq N$ has been established. In this case the research of a parallel τ_n–grammar and $L(\tau_n)$–language associated with it, is reduced to studying of the appropriate numerical function

$F_n(m)$ and its range of values. Within the given approach a series of properties of the numerical function $F_n(m)$ has been investigated; for the function $F_n(m)$ the top *complexity* limit expressed by the following result takes place, namely: ***Any arbitrary numerical function*** $F_n(m)$ ***is majorized by a suitable primitive recursive function*** $J(n,m)$**.** The given result well conforms to the fact, that: ***A word function, determined by a parallel mapping*** $\tau^{(n)}: C(A,\phi) \rightarrow (A,\phi)$***, is primitive recursive function*** [10]. Meanwhile, still a series of questions connected to application of methods and results of the theory of recursive functions to problems of research of dynamic properties of the classical ***HS***–models remains; meantime, in the today's state the above approach provides essential assistance in this direction. So, for example, using the above approach an extremely simple proof of the following rather interesting result of the theory of finite automata has been received: ***Function defined by a finite automaton that correctly predicts an environment is primitive recursive*** [9]. In addition, some other rather interesting results can be received on basis of the above approach. Whereas for case of parallel $L(\tau_n)$–languages this approach allows to receive the following rather interesting results [3–5,32,54–56,79,88,90,536,640–643].

Theorem 89. Generally speaking, there is not a finite set of languages $L(\tau_n)$***, whose union forms addition of a certain language of the same class; moreover, addition of a finite set of languages*** $L(\tau_n)$ ***cannot be again formal parallel language of the same class.***

Of the represented results follows, that the family of languages $L(\tau_n)$ shows rather strong immunity to the *closing* concerning the operations traditional for the ***TFG***, and to a series of other operations presenting essential interest from the standpoint of the ***TFG*** itself and a series of interesting enough appendices. In this attitude it is interesting enough to compare among themselves ***L***–systems and τ_n–grammars. First of all, languages of ***L***–family in own majority as against languages $L(\tau_n)$ possess full immunity concerning the traditional operations of *closure*. At that, a rather interesting discussion of main reason of distinctions of τ_n–grammars and ***L***–systems both from the standpoint of biological appendices and as a whole can be found, for example, in [90,567,617].

At the same time, computing possibilities of parallel τ_n–grammars are equivalent to possibilities of the universal Turing machine, i.e. class of all parallel τ_n–grammars possesses the universal computability. Along with that, it is shown that each finite nonempty language is generated by a suitable τ_n–grammar whereas for each ***n***–index ***(n** is an integer* **> 1)** of grammar exist infinite regular languages and even finite languages which cannot be generated by means of τ_n–grammars, however quite can be generated by τ_{n+1}–grammars. Thus, by generative possibilities of parallel languages the τ_n–grammars form a hierarchy according to a ***n***–index of a concrete parallel τ_n–grammar.

Moreover, there are also non–recursive $L(\tau_n)$–languages and regular languages not being languages $L(\tau_n)$; $L(\tau_n)$–languages have nonempty crossings with regular languages, context–free languages and context–sensitive languages [389,390]. As consequence of the above–mentioned distinctions, the family of languages $L(\tau_n)$ is completely different from *traditional* families of formal languages in the *Chomsky's* hierarchy – the given family contains the certain non-context-free languages and even non–recursive languages in the absence in the family of great number of classes of regular languages [5,10,32,54–56,59,79,88,90,536,567,618]. In addition, the universally recognized method of comprehension of generative opportunities of some class of generative formal systems is reduced to comparison of it with already classical ***Chomsky*** hierarchy. Of the basic reasons in favour of that is the fact the *Chomsky's hierarchy* has been investigated in the ***TFG*** most in detail. Thus, a series of our results is devoted to ascertainment of interrelation of languages $L(\tau_n)$ with traditional languages in the ***Chomsky's*** hierarchy, in which the *regular* languages, *context–free* languages, *context–sensitive* languages, *recursive* and *recursively* enumerable languages are the basic classes in ascending order of their complexity [5,10,32,56,59]. It has been shown that the set of all parallel languages $L(\tau_n)$ forms *own* subset of set of all *Lindenmayer's* languages ***(L–**languages**)*** while languages $L(\tau_n)$ are own subclass of class of $L(T_n)$–languages determined by nondeterministic T_n–grammars, considered below.

So, our basic result establishes a relation between families of parallel formal languages $L(\tau_n)$ and $L(T_n)$, determined by parallel τ_n– and T_n–grammars of *classical* and *nondeterministic* structures ***1–HS*** accordingly within the *Chomsky's* hierarchy. Furthermore, with the purpose of the best understanding of place of languages $L(\tau_n)$ into the hierarchy the

<k,p>–languages of *Lindenmayer* have been additionally included [5,9, 10,32,56,59,88,90,536,567,640]**.** The following theorem defines place of languages $L(\tau_n)$ and $L(T_n)$ in classical Chomsky hierarchy concerning the basic traditional formal languages. Meantime, many of interesting enough properties of languages $L(\tau_n)$ concerning various operations with them are presented in works [5,8,32,59,60,233,536,640] and in the quoted references to original sources.

<u>Theorem 90</u>. The following diagram determines relations between the families of parallel formal languages $L(\tau_n)$ ***and*** $L(\tau_n)$ ***determined by classical and nondeterministic structures 1–HS accordingly within of the universally recognized Chomsky hierarchy.***

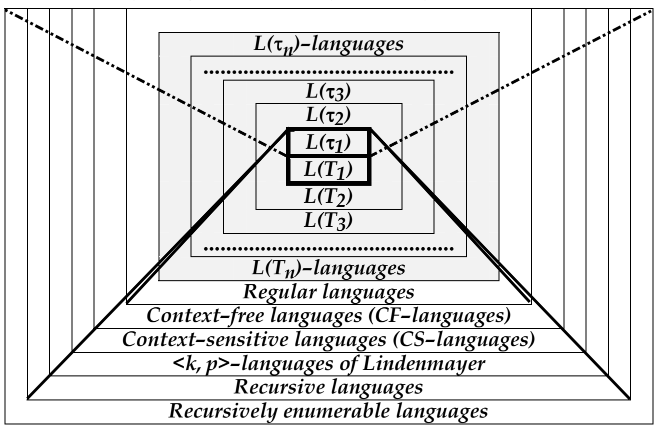


Fig. 14. Position of formal parallel languages $L(\tau_n)$ ***and*** $L(T_n)$ ***in the universally recognized Chomsky hierarchy.***

Finding a certain class of *recognizers* or *acceptors* admitting languages generated by the given grammars is traditional approach in the ***TFG.*** Obviously, the good *automaton* model of a family of formal languages gives for it strict enough characteristic. Furthermore, concerning such model it is necessary to do one important remark, generally speaking. All reasonable models of the given type ***(****at least in their classical sense****)*** have a finite automaton as a control device. Hence, a family of formal languages admitted by such model, should be closed concerning the operation of crossing with regular sets of words. Different classes of languages $L(\tau_n)$ have researched from such «***programmer***» standpoint. In this direction concerning languages $L(\tau_n)$ the following basic result takes place, namely [5,32,640–643].

Theorem 91. Class of parallel languages $L(\tau_n)$ ***is nonclosed concerning the operation of crossing with regular sets of finite words.***

So, from the result follows, that it is not possible to find an automaton model of an acceptor in the standard sense concerning class of parallel languages $L(\tau_n)$. Concerning the languages $L(\tau_n)$ there is a lot of other interesting questions with which it is possible to familiarize oneself in [5,32,57–60]. First of all, from standpoint of the *reversibility* problem of processes embedded into classical structures a question of existence of language $L^{-1}(\tau_h)$ which is reversible to a language $L(\tau_n)$ represents the undoubted interest. It is simple to make sure [32], that at existence of a biunique parallel global mapping $\tau^{(n)}: C(A) \rightarrow C(A)$ this question has the positive solution: ***For each language*** $L(\tau_n)$***, whose derivation rules correspond to a*** τ_n***–grammar possessing the given property, exists the reverse language*** $L^{-1}(\tau_h)$ ***of the same class.*** For example, the property of *reversibility* can be easily programmed for the above classical ***HSoS***–models. In addition, in case of existence for global transition functions $\tau^{(n)}$ of the nonconstructability such as ***NCF, NCF–3*** and/or ***NCF–1*** it is possible to determine the $L(\tau_n)$–languages having *reverse* languages of the same class too [32]. However, in general case the given question is solved negatively; about that the following basic result testifies.

Theorem 92. There are parallel formal languages $L(\tau_n)$***, for which the word sets*** $L^{-1}(\tau_h)$ ***are not languages of the same class.***

Researches of the property to stay by a $L(\tau_n)$–language concerning the operation of narrowing or broadening by some finite subset of words S from the set $C(A,1,\phi)$ also seem interesting enough. For many rather interesting cases the sets of words $L(\tau_n)$, $L(\tau_n) \cup S$ and $L(\tau_n) \backslash S$ are the formal languages of the same class, namely, the languages generated by parallel τ_n–grammars, whereas in general case the given assertion is incorrect; simple enough examples vindicate it. The more precisely, the following basic result takes place [5,32,57–60,640–643].

Theorem 93. There is a parallel formal language $L(\tau_n)$ ***and such finite subset of words*** $S \subset C(A,1,\phi)$***, that the sets*** $L(\tau_n)$, $L(\tau_n) \cup S$ ***and*** $L(\tau_n) \backslash S$

cannot be as formal languages of the same class.

Thus, results of theorems ***91–93*** represent a whole series of descriptive examples of nonclosure of the class of parallel languages $L(\tau_n)$ relative to the operations characterizing the important properties of dynamics of classical ***1***–dimensional ***HS***–models determining the τ_n–grammars corresponding to them. Together with other results about nonclosure of the *class* of parallel languages $L(\tau_n)$ concerning a series of important set–theoretic operations the theorems ***91–93*** confirm strong immunity of the given class of parallel formal languages in this direction [32,57–60,233]. The given phenomenon essentially distinguishes the class of parallel languages $L(\tau_n)$ from traditional families of formal languages considered in the classical ***TFG*** [5,32,54–56,79,88,90,389,390,536,640].

One of possible ways of research of structure of parallel τ_n–grammars is the approach consisting in imposing of *partial restrictions* directly on definitions of various their component with the subsequent studying of influence of these restrictions on the languages generated by these grammars. A series of the results in the given direction is represented in our monograph [5] and works [57-60]. The properties of parallel τ_n–grammars and languages $L(\tau_n)$ were considered earlier regardless of internal structure of the words composing such parallel languages. In the given connection the following rather interesting question arises.

An infinite sequence of words $S=\{c_k\}$ $(c_k \in C(A,1,\phi) \mid k=1,2, \ldots)$ is named formula sequence, if each its word $c_k \in S \subset C(A,1,\phi)$ can be structurally represented as one of finite number of formulas of the following kind:

$$C_k = C_{j_1}(k)C_{j_2}(k)C_{j_3}(k)C_{j_4}(k) \ldots C_{j_h}(k) \ldots C_{j_p}(k)$$
$$(\forall j_m)(\forall k)(C_k, C_{j_m}(k) \in C(A,1,\phi)); \quad j_m \in \{1,2,3,\ldots\};\ m = 1..p$$

Formula sequences of words are examples of $L(\tau_n)$–languages in which words composing them in certain respects contain a history of one's own development. A parallel $L(\tau_n)$–language is called *formula language* if the τ_n–grammar corresponding to it generates the formula sequence of words *(finite configurations)*. It is possible to show, that each parallel $L(\tau_n)$–language generated by appropriate τ_n–grammar, determined by a classical *linear* structure ***1–HS***, is *formula* language [73]. It is one more kind of the *general* characteristic of generative possibilities of this class of the ***HS***–models which is generalized to ***d***–dimensional case too. The parallel grammar τ_2=<2,{0,1},$\tau^{(2)}$,1> whose *derivation* rules are defined by the linear local transition function $\sigma^{(2)}(x,y)=x+y \ (mod\ 2)$ gives very simple example of formula language. At that, it is easy to make sure, that structure of words (*finite configurations*) c_k in $L(\tau_2)$–language that is generated by this τ_2–grammar is defined by the following recurrent formulas, namely:

$$c_o = 1 \ ; \quad c_1 = 11 \ ; \quad c_{k+j} = c_j 0^{k-j-1} c_j \ ; \quad \{j=0..(k-1),\ k=2^p,\ p=1,2,\ldots\}$$

where expression $c_j 0^n c_j$ designates concatenation of two words c_j and word 0^n out of n symbols «*0*». There is a series of much more complex examples of formula languages defined by *classical* ***HS***–models [88,90]. In particular, it is shown that ***1***-dimensional *binary* ***HS***-model different from linear model, and with neighbourhood index $X=\{0,1,2\}$ and local transition function $\sigma^{(3)}$ defined by the following formula, namely:

$$\sigma^{(3)}(x,y,z) = \begin{cases} x+y+z+1 \ (mod\ 2), & if\ xyz \in \{001, 011\} \\ x+y+z \ (mod\ 2), & otherwise \end{cases} \ ; \quad x,y,z \in \{0,1\}$$

produces a *formula* sequence of configurations from an arbitrary finite configuration c_o; at that, an arbitrary finite configuration c_o, excepting a case when c_o is simplest configuration ***1*** , is not a self–reproducing configuration in Moore's sense. It is easy to note that the above binary ***1–HS*** has the discriminating number ***57***. So, from the following three initial configurations c_o={***11011*** | ***10111*** | ***100111***} the above binary ***1–HS*** generates the next formula sequences of configurations accordingly:

$$\left\{ \begin{array}{c|c|c} c_{2k-1} = 100(10)^k 1 & c_{2k-1} = 101(10)^k 1 & c_{2k-1} = 1101(10)^k 1 \\ c_{2k} = 1(10)^k 11 & c_{2k} = 101(00)(10)^{k-1} 11 & c_{2k} = 10(010)^k 11 \end{array} \right\} (k = 1,2,3,\ldots)$$

While for initial configuration c_o = ***1111001011*** the structure generates the following formula sequence of configurations, namely:

$$c_1 = 11(10)^4 1,\ c_2 = 1100(10)^3 11,\ c_k = \begin{cases} (10)^{(k+9)/2} 1, & if\ k\ is\ odd\ number \\ (10)^{(k+8)/2} 11, & if\ k\ is\ even\ number \end{cases} ; k \geq 3$$

Thus, the concept of *formula* language well enough characterizes class of linear classical ***HS***–models along with a series of some other types of classical ***HS***–models. Furthermore, it is easy to show that any finite parallel language $L(\tau_n)$ is *formula* language, and any *formula* language $L(\tau_n)$ is recursive, however the converse assertions are false, generally speaking [15,32]. The more detailed consideration of formula parallel $L(\tau_n)$–languages with interesting examples can be found in [54,88,90].

The introduced concept of formula grammars and languages presents indubitable interest at researches of syntactic structure of the parallel languages generated by τ_n–grammars. Moreover, this concept is very closely connected to use of *classical* structures ***d–HS*** as an environment of modelling of various parallel processes, objects and phenomena. In this connection there is an actual enough problem of determination of formula representation of any parallel $L(\tau_n)$–language; the problem, in our opinion, is algorithmically *unsolvable*. In context of the considered concept the reverse problem arises, namely: ***It is necessary to define a parallel language $L(\tau_n)$ of the given formula structurization,*** which is solved negatively already for simplest types of formula presentation. In particular, the following set of finite formula words $L=\{c_o,c_1,...,c_m;$ $c_k=c_{k-2}c_{k-1} \mid k \geq m\}$ cannot be as a parallel language $L(\tau_n)$. In spite of a whole series of the received results, for today we have scanty enough information concerning the problem of formula representation of the parallel languages $L(\tau_n)$, hence the research in this direction represent the certain interest [5,15,32,57–60,82,83,90]. Now, we carry out a series of researches in this direction. Having presented the basic properties of parallel τ_n–grammars and languages determined by them, further we pass to consideration of their interrelations with other well–known grammars, including the parallel grammars of other types and classes.

5.2. Parallel grammars determined by the classical HS–models in comparison with formal grammars of other classes and types

Introducing the parallel τ_n–grammars, it is a quite natural to compare their *generative* possibilities with earlier investigated formal grammars of other types and classes. A series of results available in this direction allows not only to receive from many standpoints interesting enough comparative estimations of a new class of parallel grammars defined by the classical structures, but also, on the other hand, to estimate the parallel τ_n–grammars and the formal parallel languages generated by them. So, ***E.S. Scherbakov*** [261], occupying oneself with questions of development of the mathematical apparatus of modelling for biology of development at a cellular level, has defined a new class of parallel grammars named subsequently ***Sb(m)***–*grammars* which are defined as follows. In ***Sb(m)***–grammars the parallel productions of the following general kind are used as derivation rules, namely:

$$Sb:\begin{cases} x_1 x_2 x_3 \dots x_m \Rightarrow y_1 y_2 y_3 \dots y_p \\ 0\,0\,0\,0\,0 \dots 0 \Rightarrow 0\,0\,0\,0\,0 \dots 0 \end{cases} \qquad (25)$$

$$(x_k, y_j \in A = \{0,1,\dots,a-1\}; \quad k = 1..m;\ j = 1..p;\ 1 \le p \le m)$$

which are characterized by *simultaneous* application to any finite word in a certain finite alphabet ***A***. At that, inasmuch as lengths of the right parts of the parallel substitutions can exceed ***1***, then at determination of result of application of parallel productions of kind ***(25)*** to a certain word in the alphabet ***A*** occurrence of ambiguity in its some positions is quite possible. Therefore, to parallel substitutions ***(25)*** some simple ***function of choice*** of the posterior state is added, namely:

$$W(h_1, h_2, \dots, h_v) \in A \qquad h_k \in A; \quad (k=1 \,..\, v;\ 1 \le v \le r) \qquad (26)$$

which allows to unambiguously choose in points of ambiguity a state, single for the given concrete ambiguity, on basis of an arbitrary tuple $<h_1, h_2, \dots, h_v>$ of states. At the made assumptions the parallel ***Sb(m)***–grammar is introduced as follows.

<u>Definition 20</u>. A Sb(m)–grammar is the ordered tuple of the following kind Sb(m) = <A, c_o, Sb, W> whose components are defined as follows:

1) A – a terminal finite nonempty alphabet;

2) Sb – parallel productions of derivation in the form (25);

3) W – a function of choice of states in points of ambiguity (26);

4) c_o – an axiom of the parallel grammar.

A set of finite words generated by such Sb(m)–grammar is named the Sb(m)–language.

It is shown that by *generative* language possibilities the parallel ***Sb(m)***–grammars and τ_n-grammars are *equivalent*, namely the following basic result takes place [60,640–643].

<u>Theorem 94</u>. For each parallel Sb(m)–grammar exists τ_n–grammar that is strictly equivalent to the first {$n = max_k(m_k) + max_k(p_k) - 1$, if $m,p \ge 1$; and $n = 2m - 1$ otherwise}.

The result of theorem ***94*** is one more essential argument confirming a rather high degree of the generality of concept of classical structures ***1–HS*** as grammars with parallel substitutions defining *derivation* rules. Discussion of a series of questions of interrelation of τ_n–*grammars* with other types of parallel grammars (*isotonic structural grammars*, *parallel* *spatial grammars, parallel programmable spatial grammars, etc.)* along with some traditional grammars can be found in [90,567,640]. In particular, within the theory of parallel τ_n–grammars the formal model of ***Lipton*** of asynchronous linear structures which play quite defined part in the theory of programming has been considered. It is shown that the class of all grammars of such type composes own subclass of the class of all formal parallel τ_n–grammars.

Since the class of parallel languages $L(\tau_n)$ is own subclass of the class of languages of ***A. Lindenmayer*** that are generated by the ***L***–systems, a series of principal questions concerning more detailed determination of attitudes between both classes of these formal languages arises [10, 71–74,162,163]. So, ***J. Buttler*** has shown: ***Any classical structure 1–HS is modeled real time by a system PD(m,n) of A. Lindenmayer*** whereas ***any system PD(m,n) is modeled by an appropriate classical structure 1–HS not real time, generally speaking*** [163,251,254]. By basing on the results on computability and modeling in the classical structures ***d–HS*** ***(d ≥ 1)*** we proved even essentially more general result [5,72–74,536].

Theorem 95. ***An arbitrary L–system of Lindenmayer is modeled by an appropriate classical structure 1–HS not real time generally speaking, and vice versa.***

The interesting comparative analysis of both types of parallel systems of information processing in a context of their biological appendices is represented in our works [4,5,54–56,79,88,90,567,617,618,640–643].

To the problems of parallel τ_n–grammars the numerous works relative to application of ***HS***–models as acceptors and recognizers along with works on spatial grammars closely adjoin. First of all, that is related to questions of recognition by means of ***HS***–models of formal languages real time. In the given direction a lot of interesting enough results has been received. So, for example, ***A.R. Smith*** [265] has studied classes of ***1–*** and ***2–***dimensional formal languages, recognizable by ***HS***–models with restriction on the time. Many other interesting enough questions of recognition of various classes of formal languages both by *classical*, and *nondeterministic* ***HS***–models can be found in works of such famous researchers as ***A.R. Smith, H. Nishio, S. Seki, R. Vollmar, T. Jebelean, O. Ibarra, R. Sommerhalder, S. Westrhenen*** and others [15,135,138,166, 173–176,211,240,265,297,404,536,565]. The good review of results and methods in the given direction has been given by ***M. Mahajan*** [536]; in the same place it is possible to find and some additional examples of languages according to the above classification of ***HS***–models along with problems for the further research while the characteristic of most important of them is represented in the review [565]. In addition, a lot of rather interesting works is devoted to use of classical ***HS***–models as generators of languages of a special kind, for example, of fractal type [154]. Lately the spatial grammars attract significant enough attention; their languages represent the sets of spatial figures *(areas)* instead of ***1***-dimensional strings *(words)*. In particular, ***P.Wang*** and ***W. Grosky*** are one pair of the first researchers that considered interrelation between classical ***HS***–models, *parallel programmable spatial grammars* and *parallel spatial grammars* having large enough potentialities both in theoretical, and in applied aspects [263]. At that, it is shown that τ_n–grammars are equivalent to *parallel programmable spatial* grammars, and that there is a certain constructive unilateral bridge from parallel *spatial* grammars to τ_n–grammars. By a whole series of reasons the further researches in this direction seem to us rather perspective. Meantime, a whole series of questions in the given field remains open, however the questions of expansion of researches on parallel grammars, determined by various types of ***HS***–models are being represented the most important. In the following section certain results represented on parallel τ_n–grammars are generalized to case of nondeterministic ***1***–dimensional ***HS***–models.

5.3. Parallel grammars defined by nondeterministic homogeneous structures

A ***nondeterministic*** T_n***–grammar*** which in each discrete moment $t > 0$ admits one or more, but finite number of variants of choice of parallel derivation rules is an essential generalization of concept of parallel τ_n–grammar. A parallel nondeterministic grammar is defined as follows.

<u>Definition 21</u>. A parallel nondeterministic T_n***–grammar is the ordered tuple of the following kind*** $T_n = <n, A, W_n, c_o>$***, whose components are determined as follows, namely:***

1) n – index of the grammar (maximal size of neighbourhood template of the used 1–dimensional global transition functions);
2) A – a finite nonempty alphabet of the grammar;
3) W_n ***– an allowable finite set of derivation rules of the grammar;***
4) c_o ***– an axiom of the grammar (an initial finite configuration).***

A set of all words generated from an axiom $c_o \in C(A,1,\phi)$ by parallel derivation rules from an allowable set of global transition functions is named the parallel language $L(T_n)$.

From definitions of parallel τ_n–grammars and T_n–grammars it is easy to notice, the first ones are the special case of the second. Analogously to case of τ_n–grammars, we shall consider nondeterministic structures ***1–HS*** defined by parallel T_m–grammars corresponding to them. One of the most important results in the finite automata theory is the fact, that the class of formal languages determined by the nondeterministic finite automata, coincides with the class of all languages generated by completely determined finite automata. While in case of parallel T_m–grammars and τ_n–grammars a completely other picture characterized by the following basic result takes place [5,32,54–56,71,79,88,90,536].

<u>*Theorem 96*</u>. ***There are $L(T_m)$–languages that are not $L(\tau_n)$–languages while there are regular languages which are neither $L(T_m)$–languages, and nor $L(\tau_n)$–languages.***

Thus, as against the finite automata the nondeterminism of derivation rules of parallel grammars determined by *nondeterministic* ***HS***–models expands their generative language opportunities. Let's consider now the generative language possibilities of parallel nondeterministic T_m–grammars depending on their *derivation* rules W_n. At the earlier made definitions and assumptions in this direction the following base result takes place [3,5,32,54–56,71,79,88,90,536,567,617,618,640–643].

<u>*Theorem 97*</u>. ***Any finite set of words determined in a finite alphabet A is a language $L(T_m)$ for an appropriate parallel nondeterministic T_m–grammar. For any integer m≥2 there are infinite and finite regular sets of words which cannot be generated by any parallel nondeterministic T_m–grammar, however they can be generated both by an appropriate parallel T_{m+1}–grammar, and a parallel deterministic τ_{m+1}–grammar.***

Of result of the theorem **97** follows, that the non–determinism retains the generative language possibilities of parallel nondeterministic T_m–grammars relative to regular languages analogously to case of the τ_n–grammars. Similarly to the parallel τ_n–grammars for nondeterministic T_m–grammars a rather important result describing formal languages $L(T_m)$ from «***programmer***» standpoint holds true [54,71,79,88,90,536].

Theorem 98. The class of all nondeterministic languages $L(T_m)$ is non–closed concerning the crossing operation with regular sets of words.

Therefore, in case of nondeterministic parallel T_n–grammars also it is impossible to determine a satisfactory formal automaton model of the recognizers admissive of an arbitrary $L(T_n)$–language. Basically, it was necessary to expect the similar result, because such situation, in our opinion, is being defined, first of all, by absolutely parallel manner of application of derivation rules in this grammar whereas today's well–known traditional automaton models of the recognizers at own heart are based on pronounced serial principle of processing with the well defined centralized management. Whereas the ***HS***–models, being the parallel dynamic systems make use of decentralized control principle.

Let's consider now the question of closure of class of *nondeterministic* parallel languages $L(T_m)$ concerning a series of operations traditional for the classical ***TFG***. In the previous section it was marked, the class of all deterministic languages $L(\tau_n)$ is *nonclosed* concerning practically all basic operations considered as basic ones in the ***TFG*** [5,336,389,390, 536]. Meanwhile, the research of these questions concerning the class of nondeterministic languages $L(T_m)$ represents the essential enough interest. The following basic theorem represents some results received in the given direction [5,54–56,61,79,88,90,536,567,617,618,640–643].

Theorem 99. The class of all nondeterministic languages $L(T_m)$ is non–closed concerning such operations as union, addition, product, finite transformation, homomorphism and crossing, however this class will be closed concerning the operation of inversion.

Hence, and here for nondeterministic languages $L(T_m)$ the situation is completely similar to case of deterministic languages $L(\tau_n)$, however the given question needs the further research. Now let's introduce one more important enough operation named τ^*–operation concerning the languages $L(T_m)$. Let a subset of words $L \subset C(A,1,\phi)$ is some language $L(T_n)$, while τ^* – an arbitrary global transition function, defined in an alphabet ***A***. Let's define a set of the finite words as $\tau^*(L) = \{x \mid x=\tau^*(x');\ x' \in L\}$. An interesting enough question arises: ***Whether always the set $\tau^*(L)$ again will be language of the same class, where L – an arbitrary nondeterministic language $L(T_n)$?*** The answer to the given question is negative. Operations of the *left* and *right* division of a ***L***–language by a finite ω–word which are given in alphabet ***A***, are defined and denoted as $\omega\backslash L = \{x \mid \omega x \in L\}$ и $L\backslash\omega = \{x \mid x\omega \in L\}$ accordingly. It is shown that the class of all nondeterministic languages $L(T_m)$ is nonclosed concerning the above two operations [5,71]; the following theorem representing a certain interest from standpoint of grammatical properties of models T_m as generators of formal languages summarizes that.

<u>Theorem 100</u>. Class of nondeterministic languages $L(T_n)$ is non–closed concerning the τ*–operation and operation of the left (right) division by an arbitrary finite word determined in the same finite alphabet A.

Thus, the presented results confirm, that the class of nondeterministic parallel languages $L(T_m)$ possesses the same strong immunity relative to operation of *closure* concerning the major operations of the classical ***TFG*** analogously to the class of deterministic languages $L(\tau_n)$. At that, it is possible to familiarize oneself with other interesting properties of languages $L(T_m)$, for example, in [5,59,60,71,90,536,567,640], while the position of the given languages in the classical ***Chomsky*** hierarchy is represented on fig. ***14***. A whole series of received results on grammars τ_n and T_n had shown that traditional approach to implementation of programming languages for homogeneous computer systems will not allow to create actually effective parallel software that might use the maximal degree of paralleling permitted by similar structures of finite automata. In our opinion, grammars τ_n and T_n are powerful means of mathematical semantics both for microprogramming languages of the parallel microprogrammed computational structures and description of many kinds of cellular systems of the different nature. At present, along with the above–mentioned types of parallel formal languages we study some other languages of the same type that have interesting applications, including questions of their modeling by ***HS***–models.

On it we finish presentation of the basic received results in the theory of parallel grammars τ_n and T_m that are defined by the classical and nondeterministic one–dimensional ***HS***–models accordingly. Thus, we in the present monograph do not consider a lot of important enough and interesting questions connected to researches of the ***HS***–models as acceptors of the formal languages. The given questions were enough intensively investigated by such researchers as ***K. Culik, A.R. Smith, T. Jebelean, S. Kosaraju, M. Nordahl, A. Hemmerling,*** and by a whole series of others; works in the given direction can be found in [536].

5.4. Algorithmic problems of the theory of parallel grammars determined by homogeneous structures

Problems of *algorithmic solvability* play very important part in modern mathematics. Such problems are related to the so-called class of ***'mass'*** problems for which it is necessary to establish existence or absence of the common resolving algorithm. In the theoretical and mathematical cybernetics the algorithmically *unsolvable* problems frequently enough arise in problems of the analysis of dynamics of transducers of *discrete* information, for example, various infinite automata out of which the linguistic aspects of ***HS***–models are considered in the present chapter. Indeed, within finite systems the problems of algorithmical solvability do not carry such actual character at worst the many solutions can be received by methods of simple *exhaustive* search of the corresponding opportunities and variants. But, in the theory of generative grammars the algorithmic problems occupy the large enough place, for example, problems of existence of the algorithms recognizing relative to formal grammars of some class or type, whether a formal language generated by a certain grammar possess the given property.

Now, not onto all questions of algorithmic solvability in the theory of parallel grammars, determined by the *classical* ***HS***–models the answers exist. In the present section the results of solution of some algorithmic problems for parallel languages $L(\tau_n)$ and $L(T_m)$, which characterize, basically, constructive and dynamic properties of the homogeneous structures corresponding to them are represented. At that, a series of questions have remained beyond field of our sight. We shall give now mathematical formulations for the most known mass problems in the given direction.

1. *__The emptiness problem:__* *Whether exists a solving algorithm determining whether there will be a language generated by a formal grammar, empty?*

2. *__The completeness problem:__* *Whether exists a solving algorithm allowing to determine existence for any formal grammar of an opportunity to generate language which contains all nonempty finite words given in its alphabet?*

3. *__The finiteness problem:__* *Whether exists a solving algorithm allowing to determine, whether will be finite a language generated by a formal grammar?*

4. *__The membership problem:__* *Whether exists a solving algorithm allowing to determine the fact of belonging of a finite word to a language generated by the given grammar?*

5. <u>The identity problem</u>: *Whether exists any solving algorithm allowing to determine the identity of languages generated by two arbitrary grammars?*

6. <u>The simplicity problem</u>: *Whether exists a solving algorithm allowing to determine, whether will a language generated by the given formal grammar as a regular language, context–free language or context–sensitive language?*

7. <u>The formularity problem</u>: *Whether exists a solving algorithm allowing to determine, whether will a language generated by an arbitrary grammar as a formula language?*

8. <u>The limit problem</u>: *Let **G1** is some formal grammar. If for it there is such word **c*∈L(G1)**, that for each word **c∈L(G1)** of the language on the basis of derivation rules of the given grammar **G1** a sequence containing a word **c*** is generated, then it is quite natural to define the word **c*** as a **limit** of process of derivation in the given grammar. The limit for case of parallel grammars defined by the classical **HS**–models represents in a sense the **stability point** of process of derivation. In many cases the limit can be potentially achievable, for example, in case of unlimited growth of length of the deduced words. Now in view of these assumptions the problem is reduced to a question of existence of a solving algorithm for definition of existence for a language generated by an arbitrary grammar of a certain limit in the above sense.*

9. <u>Problem of emptiness of crossing</u>: *Whether exists a solving algorithm determining the emptiness of crossing of two languages generated by means of an arbitrary formal grammar?*

10. <u>The existence problem</u>: *Whether exists a solving algorithm determining whether will a set of finite words by a language of the given grammar?*

Undoubtedly, the above problems of algorithmic solvability are most important mathematical problems of the theory of parallel grammars, determined by the classical ***HS***–models, as, however, and other formal systems. On basis of a number of researches it is possible to formulate the following basic theorem [5,32,54–56,71–74,79,88,90,536,567,640].

<u>Theorem 101</u>. Problems of identity, finiteness, membership, emptiness of crossing, existence and existence of a limit are unsolvable for L(τ_n)–language generated by an arbitrary parallel τ_n–grammar, whereas the problems of emptiness and completeness are solvable.

The given assertions are true and for case of parallel languages ***L(T_m)***. At the same time in full or in part the above **2** problems of simplicity and formularity still remain open. At that, if the formularity problem still remains completely open, then concerning the simplicity problem an interesting enough result presented by the following basic theorem takes place [5,54–56,79,88,90,640–643].

<u>Theorem 102</u>. The simplicity problem relative to the regular languages and the context–free languages L(τ_n) is algorithmically unsolvable.

In view of result of the given theorem the unsolvability as a whole of the simplicity problem is quite real. Moreover, out of the theorem ***102*** and results of investigation on finite automata the unsolvability of the equivalence problem of an arbitrary parallel τ_n–grammar in a context of language opportunities generated by it to a {*nondeterministic*} finite automaton, and to an (*one–way nondeterministic*) automaton with stack memory follows. Thus, the comparison problem of the formal parallel languages ***L(τ_n)*** with known traditional recognizers of the ***Chomsky's*** hierarchy is algorithmically unsolvable, what represents strict enough result in the theory of formal parallel τ_n–grammars [5,54–56,79,88,90].

At that, analogously to case of parallel languages ***L(τ_n)*** the *algorithmic unsolvability* along with *solvability* of similar mass problems exists and for *nondeterministic* languages ***L(T_m).*** But since the class of all parallel languages ***L(τ_m)*** is own subclass of the class of all languages ***L(T_m),*** it allows to carry over the above results relative to algorithmic *solvability* and onto parallel nondeterministic languages ***L(T_m).*** In addition, it is necessary to have in view that unsolvability of a certain mass problem presumes absence of a solving algorithm in its modern understanding only. While for separate problems of this class the individual decision algorithms are quite possible. In particular, in case of the ***HS***–models similar situation is widespread enough phenomenon.

On it representation of the received results in the theory of grammars, defined by the *classical* and *nondeterministic* ***1***–dimensional ***HS***–models which allow not only to receive satisfactory *linguistic* characteristics of *dynamic* properties of models of such type but also to give new means of research of the ***HS***–models as a whole comes to the end. Meantime, in the given direction many open problems and perspective directions of research remain also. At that, a spreading of the received results of theory of parallel τ_n–grammars and T_m–grammars onto case of higher dimensions seems to us rather perspective [54–56,79,88,536,567,640].

The more partial problem of *identification* of the infinite ***HS***–models on basis of some results of their dynamics directly adjoins to the *existence* problem. Its base essence is reduced to definition of sought ***HS***–model in terms of its ***LTF*** on basis of known sequences $J_k=<c_k>[\tau^{(n)}]$ ***(k=1..v)*** of finite configurations generated by it; i.e. a constructive definition of the kind of ***LTF*** $\sigma^{(n)}$ of a model on basis of known J_k–sequences which are generated by the appropriate ***GTF*** $\tau^{(n)}$**.** This problem is similar to case of the theory of finite automata when in experiments with a finite automaton is being discovered its internal logic organization, i.e. the table determining *output* of the automaton on basis of its *internal* state and *input***.** Similar experiment will consist in submission to input of a finite automaton of the input sequences of symbols and the posterior analysis of the output sequences that are generated by the automaton**;** i.e. as a result of the given experiment the identification of automaton on basis of results of analysis of its reaction to the input sequences is made. In addition, the theory of finite automata deals with a series of experiments of various type and purpose for identification of the logic internal structure of finite automata or their separate components. The detailed enough discussion of the problematics can be found in works [226,266,281,282,536]**.** The identification problem in case of an infinite ***HS***–model is reduced to definition of its global transition function on basis of sequences J_k=<c_k>[$\tau^{(n)}$] of the finite configurations generated by the ***HS***–model, i.e. on basis of concrete history or finite number of histories of the model. Thus, the identification problem is reduced to determination of internal logic structure of a ***HS***–model on basis of its behaviour, namely**:** the input **(***the current configuration – global internal state***)** under the influence of ***GTF*** **(*LTF*)** is converted into the following internal state identified with output of this model**;** i.e. any ***HS***–model can be considered as the Moore's infinite automaton. At that the given problem, interesting by one's own essence, for the infinite ***HS***–models does not carry such comprehensive character, because in general case the problem is algorithmically unsolvable. However, in a whole series of rather interesting cases the problem is algorithmically solvable.

A. Adamatzky was the first who has started research of this problem for case of the ***HS***–models, having received a lot of interesting enough results concerning the identification problem for classical and other certain types of the finite ***HS***–models [161,183,189,267]**.** So, for case of the finite ***HS***–models the problem of both existence, and identification is algorithmically solvable because the ***HS***–models generate only finite languages $L(\tau_n)$**.** Generally speaking, both the existence problem, and the identification problem already for the classical structures ***1–HS*** are algorithmically unsolvable **(***the proof of the second assertion is immediate consequence out of the proof of the first one* [72]**)**. Meanwhile, algorithmic unsolvability of both mass problems gives a very good opportunity to use it as effective components of the general apparatus of research of mass problems of the dynamic theory of the classical ***HS***–models.

In spite of algorithmic unsolvability of the problems of existence and identification in general, the problem of experimental determination of a classical ***HS***–model generating the certain J_k–sequences of finite configurations presents the indubitable interest. An rather interesting experimental approach to solution of the *identification* problem can be found in [567,617]. The *identification* problem which in our case can be defined as follows, namely: ***In terms of dynamics of a HS–model with known alphabet A of internal states of its elementary automata it is necessary to determine its local transition function;*** the problem can be constructively solved in terms of an algorithm represented in [617], for example. This constructive algorithm has a very simple programm realization. Generalization of the algorithm to high dimensions allows to formulate the following assertion, namely: ***For a classical structure d–HS (d ≥ 1) the identification problem in our posing has constructive decision under the assumption of choice of a central automaton of the neighbourhood template of such structure.***

Thus, unsolvability of the problems of existence and identification for the classical structures as a whole presumes the further development of partial approaches to their decision for structures of a series of the certain types or classes. It would promote the occurrence of a series of the important enough theoretical and applied results as a whole. On the other hand, it is possible to show, that the identification problem for the finite structures ***d–HS (d ≥ 1)*** is constructively solvable and for this purpose there are effective enough computer methods [8,90]. The methods of identification of the finite structures ***d–HS (d ≥ 1)*** with help of artificial neural networks have been suggested; interesting enough questions of estimation and training of neural networks that are used for solution of the above identification problem for finite structures ***d–HS (d ≥ 1)*** have been considered [536]. As a whole, such problematics seems interesting enough from the certain standpoints, consequently in this direction a number of researches are carried out by us.

In the following chapter the questions presenting special interest from standpoint of use of classical structures as a perspective environment for modeling of parallel discrete processes and phenomena in various theoretical and applied fields of the *natural* sciences are considered. At that, the modeling concept is considered in various aspects presenting interest from different theoretical or applied standpoints.

Chapter 6. The modeling problem in the classical homogeneous structures along with related questions

The modeling problem in classical structures ***d–HS (d ≥ 1)*** represents the great theoretical and applied interest. The significant number of the works containing many rather interesting results is devoted to the given problem. One of basic directions of researches in the given field communicates with modeling of one structure ***d–HS (d ≥ 1)*** by another structure, namely**:** modeling real time**;** modelling with suppression of certain properties of the *modelled* structure ***d–HS (d ≥ 1)***, simplification of parameters of the *modeling* structure, etc. If in the earlier considered fields of the ***HS***–problematics the problems of optimization character, practically, were not put, then here at modeling the use of the certain optimizing technics is already supposed. A lot of the researchers was occupied with questions of modeling in the classical structures ***d–HS (d ≥ 1);*** of them it is necessary to note such researchers as ***J. Neumann, A.W. Burks, H. Yamada, H. Nishio, S. Amoroso, A.R. Smith, E. Banks, T. Toffoli, E. Codd, P. Sarkar, S. Cole, J. Buttler, R. Volmar, S. Ulam, A. Wuensche, A. Waksman, K. Culik, A. Adamatzky, A.S. Podkolzin, O.L. Bandman, V.Z. Aladjev,*** and a whole series of others [536]. More detailed information in this direction can be found in works [1,3–5,8,9, 35,37,54-56,68,73,75,79,80,83,86,90,114,128,130-135,138,146-151,156-161, 166,167,171,179,184–187,190, 199,230,240, 255,263,268, 285,288–292,295, 298,300,313,318,360,376,386,391,402,536,567,640] along with numerous references to other works available in them. And it in spite of the fact that these questions concern especially internal ***HS***–problematics only, without consideration of numerous enough works considering the ***HS*** as a modelling environment for numerous applied problems.

Of the first rather interesting results in this direction, not considering results of modelling in the ***HS*** of founders of the given problematics ***John von Neumann*** and of his direct followers, it is necessary to note the doctoral thesis of ***A.R. Smith*** [131], where he considers a series of principal questions of modeling of one classical structure ***d–HS (d ≥ 1)*** by other structure of the same dimension ***d,*** but with reduction of the size of neighbourhood template of the modelling structure. A plenty of other interesting enough results concerning modelling of the given and similar type for case of the classical structures ***d–HS (d ≥ 1)*** can be found in the works quoted above. With a deeper history of the given problematics it is possible to familiarize oneself, for example, in [640].

6.1. Concepts of modelling in classical homogeneous structures *(Cellular Automata)*

Above all, we shall make a general remark relative to **2** techniques of modelling in the class of homogeneous structures. Analogously to the founders of the ***HS***–problematics ***(J. von Neumann, S. Ulam, A. Burks, J. Holland, E. Codd, E. Banks, H. Yamada,*** *etc.****)***, a rather large number of researchers in this important direction uses for the purposes of both theoretical, and especially applied modeling immediately *homogeneous* structures, providing their by certain rules of functioning along with embedding in them of the modelled algorithms and processes. Such approach has appreciably *constructive* character when in a certain ***HS***–environment a single task of modeling can be reduced to a *composition* of subtasks composing it. Typical method of such type of modelling is a making in a ***HS***–environment of a series of blocks out of elementary automata that carry out determinate functions and cooperate among themselves by means of exchange of control impulses along specially organized information channels in the environment which are formed from elementary automata of the environment. The above approach is named *direct embedding* of a modelled problem into ***HS***–environment.

Whereas the second approach uses ***HS***–models at a level of the formal systems of parallel information processing, representing more general level of modelling of the researched algorithms and processes. In this respect both approaches to modelling on basis of the ***HS***–method, to a certain extent, it is possible to liken to well–known ways of modelling ***(****computability****)*** on basis of Turing machines and Markov's algorithms accordingly or other formal algebraic systems of words processing in finite alphabets. If the first approach is most suitable for the purposes of research of applied aspects of modelling on basis of the ***d–HS*** $(d \geq 1)$ while the second composes a basis of formal research of constructive, and computing possibilities of the ***HS***–models as abstract systems of parallel information processing, which at the axiomatic level provide properties of *homogeneity* and *localness* whereas at program level – the reversibility property of dynamics. At that, both these approaches can be mutually complementary with reasonable degree of admissibility. The second approach forms a basis for the further presentation of the various questions of modelling.

We shall begin the representation with the traditional approach to the concept of modelling going back to ***A.R. Smith*** [131]**.** Let c_t denotes a certain configuration of a classical structure ***d–HS*** ***(d ≥ 1)*** at a moment $t \ge 0$ and τ – a global transition function ***(GTF*** τ***)*** of the structure. Then, result of t–fold application of τ–function (*denotation* – τ^t) is determined by the following recurrent relation, namely:

$$\tau^o(c_o) \equiv c_o\tau^o \equiv c_o, \quad \tau^t(c_{t-1}) \equiv c_{t-1}\tau^t \equiv \tau(\tau^{t-1}(c_{t-1})) \equiv c_t \qquad (t \ge 1)$$

At the made assumptions we introduce the modelling concept of one structure ***d–HS*** by other structure of the same ***d***–dimensionality ***(d≥1).***

Definition 22. Let F_d ***is a certain set of all allowable global transition functions*** τ_j ***for classical structures d–HS (d ≥ 1). Now, let's consider two classical structures Z1 and Z2 out of the given set*** F_d ***with sets of configurations and global transition functions*** $c1,\tau_1$ & $c2,\tau_2$ ($\tau_1,\tau_2 \in F_d$) ***accordingly. Let's speak, that the structure Z2 simulates the structure Z1 real time k2/k1 if and only if there is such effectively computable injective mapping H: c1 → c2 and an effectively computable function*** $h: F_d \to F_d$***, that the following relation takes place, namely:***

$$\tau_2^{k2}(H(c_t)) \equiv H(\tau_1^{k1}(c_t)); \quad \tau_2 = h(\tau_1)$$

At fulfilling of the condition k2 = k1 we shall speak about modelling of classical structures d–HS (d ≥ 1) strictly real time.

In a series of cases at presentation of results concerning the modelling in ***d–HS (d ≥ 1)*** it will be convenient to use for an arbitrary structure ***d–HS*** ≡ $<Z^d,A,\tau^{(n)},X>$ designation ***(d,n,a)*** where the sense of parameters ***d, n*** and ***a*** fully corresponds to the definition of classical structures ***d–HS (d ≥ 1),*** not demanding any special explanations. By means of one simple enough procedure ***J. Buttler*** [313] on the basis of one sufficient condition of existence of algorithm of modelling at real time ***1/k*** of one classical structure ***d–HS (d ≥ 1)*** by means of other structure of the same dimension has proved the following rather interesting result which is enough frequently used in investigations of dynamics of the classical structures by means of modelling [536,618,640–643].

Theorem 103. For an arbitrary classical structure (2,n,a) there is such structure (2, p, a^{2kt}***) which simulates the first structure real time 1/k where t is length of side of minimal square containing neighbourhood template of the simulated structure.***

Without loss of generality, the modelling of a classical structure ***1–HS*** with alphabet ***A={0,1, ..., a–1}*** and neighbourhood index ***X={0,1, ..., n–1}*** by structures of the same class and dimensionality but with reduction of the size of neighbourhood template is represented in [90,640] with the purpose of illustration of one of possible approaches to modelling of classical structures ***d–HS*** $(d \geq 1)$. The represented scheme is pellucid enough, defining a principle of reduction of a source neighbourhood template of size n of the simulated structure and allows to receive the following result useful in a series of appendices.

Theorem 104. For an arbitrary classical structure (1, n, a) there is such structure (1, 2, ψ) which models it real time 1/(n–1), where alphabet of the modelling structure is $A^* = \cup_p A^p \cup A$ $\{p=1 .. (n-2)\}$ ***of a cardinality***

$$\psi = \# A^* = \sum_{p=1}^{n-1} a^p = (a^n - a)/(a-1)$$

A generalization of the given approach allows to receive the following useful enough result [527,617,618,640–643].

Theorem 105. For an arbitrary structure d–HS (d≥1) with alphabet $A = \{0,1,2, ..., a-1\}$ ***and neighbourhood template*** $n_1 x n_2 x \ldots x n_d$, ***there is such structure of the same dimension that simulates the first structure real time*** $1/(\sum_{j=1}^{d} n_j - d)$ ***with neighbourhood index*** $X = \{0,1\}$ ***and alphabet*** A' ***of the cardinality determined by the following formula, namely:***

$$\# A' = a + \sum_{j=1}^{d} \left(\frac{a^{\prod_{k=1}^{j} n_k + \varphi(j)} - a^{2\prod_{k=0}^{j-1} n_k}}{a^{\prod_{k=0}^{j-1} n_k} - 1} \right), \quad where \quad \varphi(j) = \begin{cases} 1, & if\ j < d \\ 0, & otherwise \end{cases}; \quad n_o = 1$$

The *optimization* problem of a modelling algorithm of similar or other type can be considered as an interesting enough problem. Using the above approach, it is possible to prove the following result useful for problems of modeling in the classical structures ***d–HS*** $(d \geq 1)$.

Theorem 106. Any classical structure d–HS (d≥1) with neighbourhood template in the form of hyperparallelepiped of size $n_1 x n_2 x \ldots x n_d$ ***and a state alphabet A={0,1, ..., a–1} is simulated real time 1/n by a structure of the same dimension with alphabet A* and neighbourhood template in the form of hypercube with edge of length 2 under the determinative conditions such as:***

$$n = \max_{k=1..d} \{n_k\} - 1 \qquad\qquad \# A^* = \sum_{k=1}^{n} a^{k^d}$$

In addition, a rather characteristic property of a modelling structure is inheritance by it of a series of basic dynamic properties of a modelled structure. The given circumstance is essential enough, allowing above all at the theoretical level to research dynamics of classical structures ***d–HS (d ≥ 1)*** with simple neighbourhood indexes with the subsequent spreading of the received results to more general types of the classical ***HS–***models. Naturally, the simplification of neighbourhood template of a researched ***HS–***model is reached, sometimes, owing to essential increase of alphabet cardinality of the modelling structure. In a series of cases, however, simplicity of topology of connections of elementary automata of the researched ***HS–***models speaks in support of a similar approach. The above results are characterized by the fact, the classical structures with rather large neighbourhood templates and with small state alphabets ***A*** can be simulated by the structures of the same type with the smaller neighbourhood templates and essentially large state alphabets ***A*,*** and vice versa.

Other strong enough result concerning modelling of arbitrary *classical* structures ***d–HS (d ≥ 1)*** by *binary* structures of the same dimension and type has been received by ***A.R. Smith*** [131]**,** namely.

<u>Theorem 107</u>. For an arbitrary classical structure (d, n, a) there is some structure (d, k, 2) which simulates the first structure strictly real time and for which the following relation takes place, namely:

$$|k| \leq (2^{a-1} - 1)(n + 2) + [log_2 a](n - 1) + 1$$

It is necessary to note, that at the proof of the above results of ***Buttler–Smith*** concerning of modelling in classical structures ***d–HS (d ≥ 1)*** any optimizing technics was not used and in some our works the attempts has been undertaken to receive definite optimization of parameters of the modelling structures. Questions of optimization of determinative parameters of the simulating structures have been considered also by a series of other researchers [199,262,281,536,573,617,618,640–643]**.**

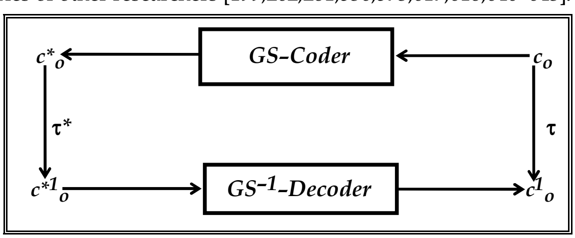


Fig. 15. The diagram illustrating the principle of 1–modelling in classical homogeneous structures.

Below, some of results in this direction will be represented. So, in this connexion the ***1–****modelling* concept has been introduced whose essence is illustrated by the diagram presented on fig. ***15*** and is reduced to the following [1,3–5,8]. This diagram allows to present quite evidently the essence of ***1–***modelling in environment of the classical structures ***d–HS (d ≥ 1)*** along with illustration of the used principle of modelling. In the modelled *classical* structure ***d–HS (d≥1)*** any configuration $c_o \in C(A,d,\phi)$, defined in a finite alphabet ***A*** as a result of action of global transition function τ is converted into the following configuration $c_o\tau = c^1{}_o$. Let ***GS(X)*** is some recursive *coding* method of symbols $X \in A$ while $GS^{-1}(Y)$ is some method of recursive decoding of a set of symbols of alphabet ***A**** of a modeling structure ***d–HS (d ≥ 1);*** let ***GS***[***X***] and $GS^{-1}(Y)$ define recursive methods of coding and decoding of configurations ***X*** and ***Y*** accordingly. Then for a modelling classical structure ***d–HS (d ≥ 1)*** any configuration $c^*{}_o = GS[c_o]$ as a result of action of the global transition function τ^* is converted into following configuration $c^{*1}{}_o$ for which a relation $GS^{-1}[c^{*1}{}_o] = c^1{}_o$ takes place. Let's speak, a classical structure ***d–HS*** is ***1–modeled*** by a structure of the same dimension if dynamics of the *simulated* structure and the *modelling* structure are subordinated to the conditions listed above. Let T_x is a length of edge of a minimal ***d–***dimensional hypercube containing neighbourhood template of the *modelled* classical structure ***d–HS (d ≥ 1).*** At the made assumptions the following result useful in many respects takes place [1,5,9,88,90,536].

__*Theorem 108*__***. For an arbitrary classical structure*** $<Z^d,A,\tau^{(n)},X>$ ***(d ≥ 1) with neighbourhood template of size*** T_X ***there is a classical structure*** $<Z^d,A^*,\tau^{(p)},X^*>$ ***with neighbourhood template of size*** T_{X^*} ***which will 1–simulate the first classical structure where*** T_{X^*} ***and a state alphabet A* satisfy the following determinative relations, namely:***

$$\begin{cases} T_{X^*} \le (T_X + 1)([\log_2 a] + 1) - 1, & \text{for } A^* = \{0,1,2\} \\ T_{X^*} \le (T_X + 1)(L + 5) - 1, & \text{for } A^* = \{0,1\} \\ \quad \text{where } L = [(\log_2 a - 1)/(\log_2 7 - 1)] + 2 & \end{cases}$$

As far as we know, the given result for today is the best among results of similar type. In monographs [1,3,5,90] a little the more special ways of simulation of one *classical* structure ***d–HS (d≥1)*** by another structure also have been considered. So, ***A.R. Smith*** has shown [131] that being based on definition **22,** generally speaking, it is impossible to model a classical structure ***d–HS (d≥2)*** by a structure of smaller dimension. On the other hand, we have disclosed, that at the certain assumptions it is possible to model an arbitrary classical structure ***d–HS (d≥2)*** even by means of appropriate classical structure ***1–HS*** [1]**.** One opportunity of similar modelling of classical structures ***d–HS (d≥2)*** by means of the classical structures ***1–HS*** will be constructively illustrated below. This approach uses a special ***1***–dimensional presentation of ***2***–dimensional finite configuration of classical structures ***2–HS.***

Certain works relating to the special properties of classical structures ***d–HS (d≥1)*** directly relate to the modelling problem too. In particular, some researchers dealt with questions of standardization of structure of neighbourhood templates of classical ***HS***–models. The problematics is represented enough important and from the theoretical standpoint, and from standpoint of numerous appendices, especially in view of use of the ***d–HS*** as a rather perspective environment of modelling. So, questions of the given direction regularly were investigated in works of ***H. Yamada*** and ***S. Amorozo*** on basis of the introduced concepts of ***structural*** and ***behavioural*** isomorphism. It has been shown that the certain equivalent relations retaining one or both forms of *isomorphism* lead to *standardizations* of structure of neighbourhood templates in the ***HS***–models. At receiving of these results the rather important concepts of ***blocking*** and the ***blocked structure*** of elementary automata of a ***HS***–model play the central role. In addition, definition of the weak form of the ***behavioural isomorphism*** has led to the further simplifications of standard structure of neighbourhood templates in classical structures. These and other related questions can be found in works [130,536,640]**.**

In many problems of modelling in classical structures ***d–HS (d≥1)*** and, first of all, from the standpoint of questions of research of algorithmic properties of the structures the ***T–modelling*** concept appears a rather effective. It is well–known that a classical ***HS***–model presents a certain *parallel* algorithm of processing of words in finite alphabets. Inasmuch as research of the classical ***HS***–models from this standpoint represents indubitable interest, then it is enough expedient to define essentially important concept, namely ***– «one algorithm (weakly) T–models other algorithm»;*** its definition consists in the following.

Let M_1 is some algorithm of processing of words in a finite alphabet A whereas M_2 is an algorithm of processing of words in a finite alphabet A^* $(A \subseteq A^*)$. Let's assume that $M^k_1 s = s^k$ $(M^o_1 s = s)$ is a result of k–fold processing of a word ***s,*** given in the alphabet A by means of algorithm

M_1. For an arbitrary finite word ***s*** in the alphabet ***A*** the algorithm M_1 generates a certain sequence of finite words of the following kind:

$$M^o{}_1s,\ M^1{}_1s,\ M^2{}_1s,\ M^3{}_1s,\ \dots,\ M^k{}_1s,\ \dots \qquad (27)$$

Let now ***s**** is an arbitrary finite word in the alphabet ***A**** which in the alphabet ***A*** coincides with word ***s***, and algorithm M_2 generates of the word ***s**** a certain sequence of words of the following kind, namely:

$$M^o{}_2s^*,\ M^1{}_2s^*,\ M^2{}_2s^*,\ M^3{}_2s^*,\ \dots,\ M^k{}_2s^*,\ \dots \qquad (28)$$

<u>Definition 23</u>. Let's speak, that an algorithm M_2 T–models a certain algorithm M_1 if and only if there is a recursive procedure, permitting for each finite word s, defined in a certain alphabet A, to choose out of sequences of words of the kind (28) such subsequences of words of the following kind, namely:

$$M^o{}_0s^*,\ M^{j1}{}_2s^*,\ M^{j2}{}_2s^*,\ M^{j3}{}_2s^*,\ \dots,\ M^{jk}{}_2s^*,\ \dots$$

that in the alphabet A the relation of the kind $(\forall k\in N)(M^{j}k_2s^* \equiv M^k{}_1s)$ & $(j_k=T+k)$ takes place. At that, if value j_k in the modelling algorithm M_2 depends on length of the processed words s $\{j_k = f(|S|)\}$, we shall speak – the algorithm M_2 weakly T–models the source algorithm M_1, in particular, functional algorithm of homogeneous structures.

The introduced concept of ***T***–modelling is used by us widely enough, possessing a whole series of positive features. In particular, in view of the introduced concepts it is enough easy to make sure, the concept of ***1***–modelling is a special case of ***T***–modelling, representing, however, independent interest, above all, because of following **2** circumstances, namely: ***(1)*** the method of ***1***–modelling defines the mode of *strictly real* time, and ***(2)*** simple method GS^{-1} of decoding of the words generated by the modelling algorithm.

Meanwhile, along with the concepts introduced above for theoretical research of a lot of important enough questions, linked with modeling in classical structures ***d–HS***, we have defined essentially more abstract ***WM–modeling*** concept [53] which subsequently has received essential generalization in the form of the concept of ***W–modelling***, allowing to solve subsequently a lot of interesting enough problems of the general problem of modelling for classical structures ***d–HS (d≥1)***.

Let ***M*** is an arbitrary *classical* structure ***d–HS (d≥1)*** in which any finite configuration $c^*\in C(A,d,\phi)$ has a sharply defined dynamics $<c^*>[\tau^{(n)}]=$

$\{c^*\tau^{(n)k} \mid k = 0,1,2, ...\}$. We choose some finite alphabet E and determine a recursive coding function F that converts an arbitrary configuration $c \in C(A,d,\phi)$ into some set $F(c) = \{c^e{}_p \in C_E\}$ of such configurations, that $F^{-1}(c^e{}_p) = c$ $(p=1,2,3,4, ..., r;\ 1 \le r \le \infty)$. Set $S = \bigcup_{c \in C(A,d,\phi)} F(c)$ we shall name the *signaling* set and shall suppose that it is recursive. Then, if there is such coding algorithm F and a classical structure d–HS $(d \ge 1)$ W with alphabet E, that any dynamics of configurations $<c^e>[\tau^{(v)}]$ ($c^e \in S$; $\tau^{(n)}$ – *a global transition function in the classical structure* W) contains a certain *maximal* subsequence of configurations $\{c^e{}_k \in S\}$ for which *determinative* relation of the kind $\{F^{-1}(c^e{}_k)\} = <c>[\tau^{(n)}]$ takes place we shall speak, the classical structure M is ***W–modeled*** by means of the structure W of the same type [5,53–56,79,88,90,536,617,618,640–643].

At that, it is necessary to note that in view of the made presumptions the choosing out of an arbitrary histories $<c^e>[\tau^{(v)}]$ of configurations of the maximal subsequences $\{c^e{}_k\}$ is possible in view of recursiveness of the signaling set S. Meanwhile, the introduced concept of modeling enough essentially expands the ***WM***–modelling concept, covering the extremely wide class of methods of modeling in classical structures d–HS $(d \ge 1)$. A series of the used methods for simulation in environment of the classical structures d–HS $(d \ge 1)$ corresponds to the W–modeling concept. In addition, with a whole series of the related questions it is possible to acquaint oneself in works [5,8,53–56,79,88,90,640–643].

The W–modelling concept is rather wide, covering not only essentially classical structures d–HS $(d \ge 1)$. As an example we shall consider the following algorithm of modelling of a classical structure 1–HS with an alphabet $A=\{0,1,2, ..., a-1\}$ and neighbourhood index $X=\{0,1\}$ by means of the structure 1–HS^* with the same alphabet A and neighbourhood index $X^*=\{-1,0,1\}$, whose local transition function ***(LTF)*** $\sigma^{(3)}$ is defined not only by configuration of the neighbourhood template, but also by coordinates of its central automaton. More precisely, a local transition function $\sigma^{(3)}$ of the 1–HS^* is being defined by the following formula:

$$\sigma^{(3)}(x_{-1}, x_o, x_1) = \begin{cases} \sigma^{(2)}(x_{-1}, x_1), & if \quad [x_o] \ is\ even\ number \\ x_{-1} \otimes x_o \otimes x_1, & if \quad [x_o] \ is\ odd\ number \end{cases}$$

where: $\sigma^{(2)}(x,y)$ – a local transition function of the modelled structure 1–HS, $\otimes$ - operation of addition modulo a, and $[x_j]$ - a coordinate of an elementary j–automaton of the modelling structure 1–HS^*.

Depending on the definition of local function of the structure ***1–HS****, it is simple to make sure that structure ***1–HS* 1***–models a structure ***1–HS*** on elementary automata with even coordinates. It is possible to show [8,88,90], that concerning this class of the ***HS***–models the first criterion of the nonconstructability that is based on the general concept of pairs of the ***MEC*** is valid ***(****theorem* **16).**

The carried out analysis of existence of pairs of the ***MEC*** for structures such as ***1–HS**** that are intended for modeling of an arbitrary structure ***1–HS*** has allowed to formulate even a little stronger result [79,88,90].

Theorem 109. Class of the structures 1–HS* with neighbourhood index X={0,1,2, ..., n–1} and alphabet A={0,1,2, ..., a–1} whose local transition functions are defined both by configurations of their neighbourhood templates, and coordinates of their central automata can't model an arbitrary classical structure 1–HS by means of a structure 1–HS* that does not possess the nonconstructability such as NCF.

Further in the course of the present chapter some other approaches to modelling in the classical structures ***d–HS*** $(d \geq 1)$ will be used whose essence will be clear or out of the principle of modelling itself, or will be explained in case of need.

6.2. Modelling of the well–known formal processing algorithms of words in finite alphabets by means of classical homogeneous structures

As classical structures ***d–HS*** $(d \geq 1)$ are parallel processing algorithms of ***d***–dimensional words in finite alphabets, it is rather interesting to compare them with known formal sequential algorithms. One of ways of the comparative approach of such type is a modeling of one type of algorithms by others, and vice versa. Above all, now we shall present results of ***T***–modelling of well–known formal processing algorithms of finite one–dimensional words in finite alphabets by means of classical structures ***1-HS,*** and vice versa. Moreover, an optimizing technics that was used at modelling has allowed to receive rather good proportions between basic parameters of the modelled and modelling algorithms, what allows to carry out the comparative estimations of algorithms of such type. Further, as a parallel algorithm a classical structure ***1–HS*** $\equiv$ $<Z,A,\tau^{(n)},X>$ is chosen, while as the first sequential algorithm – a well–known Turing machine MT^s_q with alphabet S $(\#S=s)$ of symbols on a final tape and state alphabet ***Q*** ***(#Q=q)*** of internal states of some finite automaton (*a control device*); MT^s_q represents the most popular formal model of sequential calculations. At that, we shall consider a machine MT^s_q with an final tape, infinite into both sides; the given machine is a modification of the conventional Turing machine and is completely equivalent to it. The reader can familiarize oneself with other certain equivalent modifications of the Turing machine, for example, in [535]. In the given direction the following result rather useful to the further research of dynamics of classical ***HS***–models takes place [54,79,88,90].

<u>*Theorem 110*</u>***. For any machine*** MT^s_q ***exists a classical structure 1–HS with neighbourhood index X = {–1,0,1} and alphabet A of cardinality (s+q+9) which 8–simulates the first. For any machine*** MT^s_q ***there is a classical structure 1–HS with neighbourhood index X = {–1,0,1} and an alphabet A of cardinality (s+2q) which 2–simulates the first. For any machine*** MT^s_q ***there is a classical structure 1–HS with neighbourhood index X = {–2,–1,0,1} and an alphabet A of cardinality (s+q) which its 1–simulates. For an arbitrary machine*** MT^s_q ***with k (k ≥ 1) final tapes there is an appropriate classical structure 1–HS with neighbourhood index X={–1,0,1} and a state alphabet A of cardinality*** $s^k(q+1)^k$ ***which 1–simulates an arbitrary machine*** MT^s_q***.***

For comparison the modeling problem of an arbitrary MT^s_q by means of a structure $HSoS \equiv <Z^1, A, 2, \Psi^{(2)}, \Xi>$ was considered. This class of the structures is widely used for creation of physical models of a various sort and the interrelation of this type of structures with homogeneous structures was considered earlier [15,150–152,165,187,273,378,386,536]. The result can be represented by the following theorem [88,567,640].

<u>*Theorem 111*</u>***. An arbitrary machine*** MT^s_q ***is 2–simulated by means of a structure*** $HSoS \equiv <Z^1, A, 2, \Psi^{(2)}, \Xi>$ ***with #A=(s+2q) and*** $A=S \cup Q \cup Q^*$***, and by a structure*** $HSoS \equiv <Z^1, A, 3, \Psi^{(3)}, \Xi>$ ***with alphabet*** $A=S \cup Q$ ***and #A=(s+q), where #G is cardinality of an arbitrary finite set G.***

Determining the *complexity* of a machine MT^s_q and a *classical* structure ***1–HS*** as $S_{MT}=sxq$ and $S_{HS}=axn$ accordingly, we receive corresponding values for three types determined by the theorem ***110,*** of the classical structures simulating a machine MT^s_q as $S_{HS}=3(s+q+9)$, $S_{HS'}=3(s+2q)$ and $S_{HS''}=4(s+q)$ accordingly. Then on basis of minimal estimation of complexity for MT^s_q that for today equals S_{MT}**=24** ***(s=4, q=6; s=6, q=4)*** [624,628–630] it is easy to make sure, that S_{HS}**=57,** $S_{HS'}$**=42** & $S_{HS''}$**=40.**

In the meantime, along with the algorithmical complexity concept, the complexity concept of modeling which includes temporal and spatial costs of a modelling algorithm is interesting enough**;** i.e. the ***modelling*** complexity in case of classical structures ***d–HS (d≥1)*** can be defined by the formula $SM_{HS}=d \times T \times a \times n$ where ***d*** **–** dimensionality of homogeneous space of a structure**;** ***a*** & ***n*** **–** cardinality of a state alphabet ***A*** and size of neighbourhood template accordingly**;** ***T*** **–** time of modelling of one step of an arbitrary modelled algorithm.

At the made assumptions the difference for three represented cases of modelling by means of ***1–HS*** becomes more striking, namely**:** $SM_{HS} = 1 \times 8 \times 3 \times (s+q+9) = 456$, $SM_{HS'} = 1 \times 2 \times 3 \times (s+2q) = 84$, $SM_{HS''} = 1 \times 1 \times 4 \times (s+q) = 40$. So, if concerning the algorithm complexity the third way of modeling appears the most simple concerning the modelling complexity the full superiority of the third one takes place**;** moreover, for it values of the algorithm complexity and the modelling complexity coincide $S_{HS''} \equiv SM_{HS''} = 40$. For estimation of the modelling complexity by means of ***HSoS*** we can use such parameter as $SM_{HSoS} = \#A \times m$ which for case of modelling of the universal MT^s_q gives value $SM_{HSoS} = 36$. However, an estimation of influence on this parameter of **Ξ–**procedure of block over–marking of space Z^1 of a modelling structure is difficult enough. As a consequence from the theorem ***110*** a series of interesting enough results concerning algorithmic unsolvability of certain mass problems linked with dynamics of finite configurations in classical ***HS***–models follows [1,3,5]**.** Some of these problems are discussed a little bit below, but for this purpose we need to introduce a series of new concepts.

Definition 24. Each configuration $c_o \in C(A, d, \phi)$ ***for a global transition function*** $\tau^{(n)}$ ***of a classical structure d–HS (d≥1) is named accordingly: limited, if the relation*** $(\exists p)(\forall k)(c_k \in \langle c_o \rangle[\tau^{(n)}] \rightarrow d(c_k) \leq p)$ ***takes place, where d is minimal diameter of a block containing configuration*** c_k***; (k–m)-periodic, if the relation*** $(\exists m)(\exists k)(c_o\tau^{(n)m} = c_o\tau^{(n)k})$ $(m>0;\ k-m>1)$ ***takes place; periodic configuration, if the relation*** $(\exists k > 1)(c_o\tau^{(n)k} = c_o)$ ***takes place, and passive configuration for k=1; at that, two last make a class of all fully periodical configurations; vanishing configuration, if the relation*** $(\exists m)(c_o\tau^{(n)m} = «\ »)$ ***takes place, where c* = « ».***

In view of the given definition the following important enough result which determines the solution of a series of mass dynamics problems of the classical ***HS***–models can be formulated**;** the result is of interest also for the further research of the given problematics [1,19,20,640]**.**

Theorem 112. The following problems are algorithmically unsolvable for an arbitrary classical structure d–HS ($d \geq 1$), namely:

– problem of recursiveness of a set $<c_o>[\tau^{(n)}]$ ***of finite configurations;***

– problem of limitation of an arbitrary configuration $c_o \in C(A,d,\phi)$***;***

– problem of (k-m)-periodicity or periodicity of a finite configuration;

– problem of existence of passive and/or vanishing configuration in an arbitrary sequence $<c_o>[\tau^{(n)}]$ ***for a classical structure d–HS ($d \geq 1$);***

– problem of existence for a classical structure d–HS ($d \geq 2$) with GTF $\tau^{(n)}$ ***of such configurations c* that*** $c^*\tau^{(n)} = c^{\infty}_r$***, where*** $c^* \in C(A,d,\infty)$ ***and*** c^{∞}_r ***– an arbitrary infinite configuration consisting only of states*** $r \in A$***.***

At proof of this theorem the ***T***–modelling opportunity of the universal Turing machine by classical structures ***1–HS*** which directly follows of result of the theorem ***110*** was essentially used. The given result along with independent interest can be used as the auxiliary apparatus that bases on ***T***–modeling at the solution of some mass dynamics problems of the classical ***HS***–models representing as the theoretical, and applied interest. In particular, from this result follows, that the determination problem of type of a graph of global states of an arbitrary *classical* ***HS***–model relative to its initial configurations is algorithmically *unsolvable***.** Whereas the unsolvability of the last problem of the theorem is based on unsolvability of the general «***domino***» problem considered in [617]**.**

On basis of the ***T***–modelling concept certain questions of modelling by means of classical ***1***–dimensional structures of such known processing algorithms as ***TAG***–systems and ***LAG***–systems, *Büchi* regular systems, ***SS***–machines, *Marcov* normal algorithms, *Post* production systems and some others have been considered rather minutely [1,5,8]**.** At receiving of the results along with use of the ***T–****modeling* principle the *optimizing* technics consisting in use of special optimizing procedures of parallel programming in classical structures ***1–HS*** to some extent was applied. It has allowed to receive appreciably optimum relations between base parameters of the modelling and the modelled algorithms. Definitions of the processing algorithms of words in finite alphabets, used below, are known enough and the reader can familiarize oneself with them in

[1,3,5,9,180,181], therefore they carry a rather schematic character with the purpose of explanation of certain base parameters of the modelled sequential algorithms. Meanwhile, it is necessary to have in view, the modelling is made at especially formal level, considering the modeled algorithm and the modelling algorithm as formal processing systems of finite words in finite alphabets without any special internal coding.

Let, a modelled formal algorithm has alphabet $C = \{c_1, ..., c_m\}$. Then an arbitrary ***TAG***–system has ω–number of truncation and ***m*** elementary transformations of the kind $c_k \Rightarrow b_k$ $(k=1..m)$, where b_k – words in the alphabet ***C;*** the Post productions system has the same alphabet ***C,*** and ***p*** basic productions of the kind $a_jW \Rightarrow Wb_j$ $(j=1..p)$, where a_j, b_j, W are finite words in the alphabet ***C.*** The detailed description of ***SS***–machine can be found in monographs [1,5,180] whereas with concept of *Markov* normal algorithms it is possible to familiarize oneself in the excellent monograph containing enough detailed presentation of base elements of the theory of algorithms and recursive functions [3,180]. The Büchi regular system has the alphabet ***C*** and ***p*** basic transformations of the kind $a_jW \Rightarrow b_jW$ $(j=1..p)$, where a_j, b_j, W – finite words in the alphabet ***C.*** Other natural expansion of the class of ***TAG***–systems are the ***LAG***–systems determined in the alphabet ***C*** by means of transformations of the following general kind, namely:

$$r_j = c_{j1}c_{j2}c_{j3} \dots c_{jq}; \quad r_jW \Rightarrow Wb_j \qquad (j = 1..p;\ p \le m^q)$$

where b_j, W – finite words in the alphabet ***C;*** in addition, if the first ***q*** symbols of a certain word ***s**** processed by the system coincide with a subword r_j then its first symbol c_{j1} is deleted and to right end of the word ***s**** is added a subword b_j. Obviously, for $q = 1$ the systems ***TAG*** and ***LAG*** coincide. Thus, at the made assumptions the following basic result presenting the certain interest takes place, bearing in mind that the received relations were result of the certain optimizing procedure.

<u>Theorem 113</u>. An arbitrary TAG–system is weakly T–simulated by an appropriate classical structure 1–HS; at that, the following relations take place, namely:

$$a = \omega + m + \sum_{k=1}^{m} |b_k| + 3; \quad T = |s^*| + |b_k|; \quad s^* = c_k s; \quad s^* \in C \ (k = 1..m)$$

An arbitrary Post productions system is weakly T–simulated by an appropriate classical structure 1–HS; at that, the following relations take place, namely:

$$a = 3\sum_{j=1}^{p}[|a_j|+|b_j|]+3p+m+10; \quad T = 4|a_j|+2|s^*|+2|b_j|; s^* \in C \quad (j=1..p)$$

An arbitrary SS–machine is weakly T–simulated by an appropriate classical structure 1–HS; at that, the following relations take place: a= $2n_1+n_2+4$ and T = 2|s*|+2, where n_1, n_2 – quantity of instructions {P_o, P_1} and SD(k) of the SS–machine accordingly. An arbitrary Büchi regular system is weakly T–simulated by means of an corresponding classical structure 1–HS; at that, the following relations take place:

$$a = 3\sum_{j=1}^{p}[|a_j|+|b_j|]-6p+m+10; \; T = |a_j|+max\{|a_j|,|b_j|\}; \; s^* \in C \; (j=1..p)$$

In all previous cases the modelling classical structure 1–HS possesses neighbourhood index X={–1,0,1}. An arbitrary LAG–system is weakly T–simulated by an appropriate structure 1–HS with a neighbourhood index X={0,1,2, ..., q+1}; at that, the following relations take place:

$$a = m+\sum_{j=1}^{m}|b_j|+3; \quad T = |s^*|+|b_j|; \quad s^* = r_j; \quad s^* \in C \; (j=1..m)$$

An arbitrary Markov normal algorithm is weakly T–simulated by a classical structure 1–HS with neighbourhood index X={0,1} ≡ {–1,0}. At that, it is necessary to keep in mind that for all presented cases s* is a processed word of a modelled algorithm & |b| – length of a b–word.

In particular, an interesting enough consequence follows of modelling of an arbitrary ***SS***–machine by an appropriate ***1–HS***, namely [1].

Proposal 7. There are 1–dimensional classical structures whose sets of finite configurations generated into null configuration are creative.

Hence, there are *classical* structures whose sets of finite configurations converted directly into the null configuration are nonrecursive. In this connexion there is very interesting question about existence of *classical* structures, whose analogous sets of finite configurations are simple or maximal, and what are the values of base parameters for structures ***1–HS*** of such type. Thus, from the theorem ***113*** follows, in spite of use in its proof of an optimizing technics of modeling for known sequential processing algorithms, it is not possible to get rid of condition of weak modelling at use of structures ***1–HS*** as the modelling algorithms. An interesting enough discussion of such situation can be found in [640].

On basis of the Turing machines with ***k*** heads ***(TM***[k]***)*** a whole series of formal models of parallel information processing is considered. So, one of such models allows to more simply analyze a whole series of situations arising in systems of parallel information processing [12]. In addition the model receives rather interesting interpretations in terms of multiprocessing computing systems [15]. In the deterministic cases the given model of calculations on basis of ***TM***[***k***] is easily reduced to traditional single–head ***TM,*** however there is the question of studying of acceleration of processing due to parallel work of ***k*** scanning heads of the machine. With principle of functioning of ***TM***[***k***] the reader can familiarize oneself, for example, in [53,640]; ibidem a discussion of the related questions can be found. As a modelling algorithm the classical structure ***1–HS*** with neighbourhood index ***X***={***–1,0,1***} has been chosen. At the made assumptions the following base result takes place [5,53].

***<u>Theorem 114</u>. An arbitrary Turing machine TM*[*k*] *with work program of length out of s commands is simulated by means of an appropriate classical structure 1–HS with neighbourhood index X=*{*–1,0,1*} *within no more than t* = 2{[*s/2*](*p* + *1*) *– p*} (*p ≥ k*) *steps of the structure, where p – an initial distance between extreme scanning heads of the TM*[*k*].**

A discussion of the results of modelling by means of classical ***1–HS*** of multihead Turing machines can be found, for example, in [5,617,640].

Modelling on basis of the classical structures ***d–HS (d≥1)*** is linked with the problem of *generality degree* of either concept of a structure of such type whose discussion has been started in section ***1.2.*** In monographs [1,3,5,90] and other our works the question of generality degree of the concept of classical ***HS***–models was investigated and the basic method of the given research was and there is the modelling. Further we shall continue discussion of this question concerning one rather interesting generalization of the *parallel transformations* that have been introduced and investigated by ***V.M. Glushkov*** and his colleagues to a case of the homogeneous synchronous parallel processes. In this direction as the given generalization ***G. Tseitlin*** has offered the so–called *heterogeneous periodically defined transformations* **(*HPDT)*** whose essence is reduced to the following [227,324,406,617,618,640–643].

Let ***R*** – a bilateral infinite abstract register divided into segments each of which contains ***r*** individual elements. Then the formal object ***HPDT***

$$\Theta_s^R = \begin{bmatrix} f_1^{t_1}, & f_2^{t_2}, & \cdots & , f_r^{t_r} \\ k_1, & k_2, & \cdots & , k_r \end{bmatrix}$$

is defined by a shift function ***s(q) = r*q + k (****mod* ***r)*** along with a system of generating functions $\{f^t_j \mid j=1..r\}$ where t_j – arity of f^t_j–functions that corresponds to the integral factors k_j ***(j=1..r).*** In addition, the result of application of the ***HPDT*** to an arbitrary state W^* $(W_q \mid -\infty < q < +\infty)$ of the ***R***–register is such new state $b^* = \Theta^R{}_s(W^*)$, that $b^* = (b_q \mid -\infty < q < +\infty)$, where states of individual elements of ***q***–th segment of the ***R***–register are calculated according to the following general formulas, namely:

$$b_{s(q)+p-1} = f_p^{t_p}(a_{s(q)+k_p}, a_{s(q)+k_p+1}, \ldots, a_{s(q)+k_p+t_p-1}) \quad (p = 1 .. r)$$

Informally speaking, the shift function ***s(q)*** defines a periodicity of the distribution of segments on the ***R***–register, while the ordered system of generating functions with factors relating to them defines changes of states of elements of ***q***–th segment of the ***R***–register. In this context the concept of the ***HPDT*** is easy generalized and to case of the higher dimensions. This formal model seems a rather useful at consideration of a lot of such classical problems of parallel programming as pipeline translation, «*writers – readers*», the rifleman problem, etc. In particular, ***G. Tseitlin*** has shown, that on such model it is possible to sort a string of ***n*** symbols during no more than ***n*** steps; earlier the similar result for case of classical structures ***1–HS*** has been received by us in work [8,37] on basis of other approach. This result carries not only the theoretical character but it has appendices at practical use of parallel processors [57]. It turned out that such ***d***–dimensional abstract ***R***–register with the ***HPDT*** which are defined on it is modeled by means of an appropriate classical structure ***d–HS*** ***(d ≥ 1)*** at quite moderate assumptions [5,53].

<u>*Theorem 115*</u>. ***Any heterogeneous periodically defined transformation defined in a state alphabet A on a d–dimensional abstract R–register is 1–simulated by means of an appropriate classical structure d–HS (d ≥ 1) with a state alphabet A∪{b} (b ∉ A).***

The given result once again confirms a significant enough generality degree of the concept of classical ***HS***–models and importance of its use in particular for problems of parallel programming, that already finds a reflection in the developed concept of *parallel* microprogramming [5, 9,71,162,536,586]. Indeed, establishing equivalence of concepts of ***HS***–models and ***HPDT*** on abstract ***R***–registers, it is possible to spread the results, methods and approaches of the rather advanced theory of the classical structures to research of theoretical questions of the parallel programming investigated by the abstract heterogeneous periodically defined transformations. With a number of other interesting examples of modelling of formal algorithms by means of the classical structures ***d–HS*** ***(d ≥ 1)*** it is possible to familiarize oneself, for example, in works [1,3,5,8,9,53-56,88,90,131-135,184-187,190,240,255,308,325,360,536,567].

It is easy to be convinced, an arbitrary classical structure ***1–HS*** with a neighbourhood index $X=\{0,1, ..., n-1\}$ and alphabet $A=\{0,1, ..., a-1\}$ is ***T–***modeled by a structure of the same dimension with the characteristics (***A**** – *an alphabet and* ***X**** – *neighbourhood index of a modelling structure*)**:**

$$\#A^* = \sum_{j=0}^{T-1} a^{k^j},\ X^* = \{0,1,2,...,k-1\};\ T = \left[\frac{n-k}{k-1}\right] + Sg\left(\frac{n-k}{k-1} - \left[\frac{n-k}{k-1}\right]\right) + 1;\ n > k$$

where ***#A*** – the cardinality of a set ***A*** and ***Sg(x)*** – the sign function. The given result allows to simply generalize the theorems ***113, 114*** to case of *classical* structures ***1–HS*** as a modelling algorithm with the simplest neighbourhood index $X = \{0,1\} \equiv \{-1,0\}$.

In view of the represented problems it is rather interesting to consider the *reverse* problem of modeling of the classical structures ***1–HS*** by the above processing sequential algorithms of words. Above all, now we shall consider the biologically motivated algorithms to which recently a specific attention in theoretical biological sciences has been devoted. In view of studying of dynamic mathematical theories, isomorphic to biological developing systems the principle of biological ***epimorphism*** is actual enough, which reduces to the question about opportunity of mapping of one algorithm determined by a certain development onto another. In this case we can speak about *epimorphism* or *isomorphism* of two algorithms. For example, because of a lot of the biological reasons linked with research of applicability of *dynamic* mathematical theories for modeling of the development biology we defined and investigated the so–called ***A–algorithms*** formulated as follows [31,640–643]**.**

Let ***G*** is a certain finite nonempty alphabet and P_j, Q_j $(j=1 .. k)$ – finite words that can be empty in the alphabet ***G*** whereas symbols **(⇒)** and **(∗)** do not belong to the alphabet ***G*.** Then an ***A***–algorithm is a certain finite set of productions of the following kind, namely**:**

$$P_j \Rightarrow Q_j;\quad P_k \Rightarrow *Q_k \qquad (j = 1 .. k-1) \qquad \textbf{(29)}$$

which is named ***schema*** of the algorithm, where $P \Rightarrow (*)\ Q$ can be the *simple* production $P \Rightarrow Q$ or the *final* production $P \Rightarrow *Q$. Functioning of an ***A***–algorithm with the given schema will consist in the following. Let S_o is an arbitrary finite word defined in a finite alphabet ***G*.** Then on the first step of algorithm $S_1 = A(S_o)$ the first entrance of a subword P_1 into S_o is replaced by a subword Q_1**.** Then the process repeats with word S_1 and so on, finishing on some word S_v which does not contain entrances of subword P_1**.** Next, to the word S_v the above procedure is applied but already concerning entrances of a subword P_2, and so on. Processing of the word S_o by the *A*–algorithm finishs on a certain step *f* if and only if the word S_f will not contain entrances of subwords P_j *(j = 1..k-1)* or word S_{f-1} contains entrance of a subword P_k. In this case a word $S_f = A(S_o)$ is named the result of processing *(calculation)* of the word S_o by means of the *A*–algorithm with the above schema ***(29)***. If not, it is considered that the *A*–algorithm is inapplicable to the word S_o. Concerning the *A*–algorithms it is shown that they are epimorphic to the classical structures ***1–HS*** and have rather interesting biological interpretations [5,31,46]. Now, using the above-mentioned definitions and designations along with definition of the semi–Thue system [180], we can formulate the basic result concerning modeling of the classical structures ***1–HS*** by the mentioned sequential processing algorithms of words [1,3,5,8,9,88]; at that, a ***1–HS*** has state alphabet ***A={0,1, ..., a–1}***.

<u>Theorem 116</u>. Any classical structure 1–HS is weakly T-modeled by an appropriate LAG–system; at that, the following relations take place:

$$C = A\bigcup\{\nabla\};\quad q = n;\quad p = 2*\sum_{j=1}^{n} a^j - a^n;\quad T = |s^*| + n - 1$$

Any classical structure 1–HS with the simplest neighbourhood index X={0,1} is weakly T–simulated by a Post productions system; at that, the following relations take place: #C=3a+5 and T=[(3|s*|+10)/2]. A classical structure 1–HS with the simplest neighbourhood index X={0, 1} is weakly T–simulated by the Markov normal algorithms; at that, the following relations take place #C=2a+5 & T=3|s*|+2. A classical structure 1–HS with neighbourhood index X={0,1, ..., n-1} is weakly T–modeled by an arbitrary MT^s_q ***under the assumption T = |s*| +n–1. An arbitrary classical structure 1–HS with the simplest neighbourhood index X = {0,1} is weakly T–simulated by an appropriate A–algorithm defined in alphabet G = A∪{b, ∇} (∇, b∉A). A classical structure 1–HS with the simplest neighbourhood index X = {0,1} is weakly T–modeled by means of an appropriate*** MT^s_q***; in addition, the following relations take place, namely: s = (a+1), q = 2(a+1), sxq =*** $2(a+1)^2$ ***and T = |s*| +1.***

At that, designations of theorem ***116*** fully correspond to designations of theorem ***113***, meantime a classical structure ***1–HS*** with alphabet ***A= {0,1,2,3, ..., a–1}*** is understood as a modelled structure. The optimizing technics of ***T***–modelling used at the solution of the given problems has allowed to receive substantially optimum relations between a series of main parameters of the ***modelled*** and ***modelling*** algorithms; that gives strong reasons for a ascertaining of the fact that excepting the Turing machines, it is not possible to narrow condition of weak ***T***–modelling up to condition of ***T***–modelling, what is conditioned by the principal complexity of embedding of especially sequential algorithms into the highly-parallel computing environment such as the ***HS***–models. Along with it, the presented results of mutual modelling to a certain extent can characterize the relative complexity of corresponding algorithms within of the used concepts of ***T***-modeling and classical ***HS***–models as a whole. More detailed discussion of the given question can be found, for example, in works [5,54–56,79,88,90,567,617,618,640–643].

Moreover, on basis of the represented results of the modelling we can receive a whole series of rather interesting consequences, first of all, of theoretical character. In particular, results of ***T. Yaku*** [224] along with our result of ***T***–modelling of ***SS***–machines by classical structures ***1–HS*** ***(****theorem* ***113)*** have allowed to receive the following important enough result deciding one of mass dynamics problems of classical structures ***d–HS (d ≥1)*** that is linked with so-called *vanishing* finite configurations ***(****definition* ***8)*** [1,3,5,8,9,54–56,79,88,90,640–643].

<u>***Theorem 117***</u>***. For an arbitrary classical structure d–HS (d ≥ 1) and an arbitrary configuration h∈C(A,d,ϕ) the problem of definition whether will the given finite configuration h by the vanishing configuration is algorithmically unsolvable.***

Moreover ***T. Yaku*** on basis of a special technics of embedding into the classical ***HS***–models has proved *unsolvability* of the existence problem of *vanishing* configurations for an arbitrary structure ***d–HS (d≥2)*** [224]. In particular, our proof of the given result is based on the algorithmic unsolvability of well–known «***domino***» problem considered in [640].

However, for case of the classical structures ***1–HS*** the given problem is algorithmically solvable what easily follows from theorem ***22*** which is valid as well for the passive configurations. Along with a whole series of other consequences the given result confirms nonequivalence of the classical structures ***1–HS*** and ***d–HS (d≥2)*** also concerning the *solvability* problems, emphasizing existence of sharply expressed differentiation of the set of all classical ***HS***–models concerning their dimension [5,90].

Meanwhile, along with the above–mentioned questions of modelling, the essential attention is given also to questions of modelling of one classical structure by other structure of with satisfaction of the certain conditions. Similar questions are considered in the following section.

6.3. Modelling of the classical homogeneous structures by means of structures of the same class

By present time within the considered problems the greatest number of works is devoted to modelling of one classical structure ***d–HS (d≥1)*** by another structure of the same type under the given conditions. The given problem represents significant theoretical and applied interest since results of researches in this direction allow to determine various types of standardization of all or separate classes of the ***HS***–models, to successfully solve various optimization problems, to simulate classical structures ***d–HS (d≥1)*** with suppression of those or other properties of the initial structures, and so on. So, from the applied point of view the modelling problem of the classical structures ***d–HS (d ≥ 1)*** by means of binary structures of the same dimension represents a special interest, dictated on the part of computing sciences and a whole series of other appendices. Generally, this problem can be formulated as follows:

For an arbitrary classical structure d–HS (d ≥ 1) with alphabet A and neighbourhood template, contained in a minimal hyperparallelepiped of size $n_1 x n_2 x \dots x n_d$ it is necessary to build a binary classical structure of the same dimensionality and with possibly smaller neighbourhood template which will simulate the initial structure.

As a rule, the problems of *optimization* character in all areas are related to a category of complex enough problems. The above problem is not exception therefore for its solution other method of research has been used [5,8,9,53–56,88,90]. Using certain experimental results along with a series of intuitive arguments, a certain special optimizing technics of modelling of the classical structures ***d–HS (d ≥ 2)*** by binary structures with taking into account of influence of dimensionality of the *modelled* structure onto the given process has been determined. The suggested approach has allowed to receive the following basic result [5,9,640].

<u>Theorem 118</u>. An arbitrary classical structure d–HS is 1–simulated by means of an appropriate binary structure of the same dimensionality (d ≥ 2) and with neighbourhood template of the following size L:

$$L = (L_1)^{d-1}(L_d+1)\prod_{k=1}^{d}(p_k+1);\ L_1 = \left[V = \sqrt[d]{\log_2(a-1)+2}\right];\ L_d = L_1 + [2(V - L_1)]$$

where $p_1 x p_2 x \dots x p_d$ – size of minimal hyperparallelepiped containing neighbourhood template of the modelled structure on the assumption of ensuring of the following relation, namely: $\log_2 \log_2 4(a–1) \ge d$.

Therefore, the edge of ***d*–**dimensional neighbourhood template of the modelling structure ***d–HS* (*d* ≥ 2)** at ensuring of the relation of theorem ***118*** is asymptotically decreased in $\sqrt[d]{log_2(a-1)+2}$ times with growth of ***d*–**dimensionality. We have considered the earlier formulated problem separately for **2** cases of the classical structures ***1–HS*** and ***d–HS* (*d* ≥ 2)** because of influence of dimensionality of structures upon technics of optimizing modelling. Earlier the nonequivalence of structures ***1–HS*** and ***d–HS* (*d* ≥ 2)** concerning certain phenomena was already marked**;** it concerns also the modelling problem in the classical structures ***d–HS*** **(*d* ≥ 1).** A whole series of aspects of discussion of this question can be found in [88,90,567,640]**.** We have paid a special attention to the given moment therefore the modelling technique began to proceed from the influence of *dimensionality* of a classical structure ***d–HS* (*d* ≥ 1)** upon an optimizing factor. For more optimal modeling some other approaches are needed.

So, for ***1*–**dimensional case the optimum technics which is as much as possible taking into account specificity of functioning of the classical structures ***1–HS*** has been offered [3,5,9,54]**.** The given technics is based on a principle of ***maximal*** approach of characteristics of the modelling structures to the appropriate characteristics of the potentially optimal modeling structures. At that, under potentially optimal structures are understood the modelling structures whose values of base parameters can be unattainable, however capable be as a good reference point for prospect of researches in this direction and for estimation of values of parameters of the earlier defined modelling structures. The structure with neighbourhood template of size $L_{opt} = (n+1)[log_2 a] + 2$, which, moreover, is unattainable is supposed as a potentially optimal binary modelling structure for the classical structures ***1–HS*** with an alphabet ***A*** and neighbourhood template of size ***n*.** From the standpoint of this estimation the problem of receiving of an optimal classical structure ***1–HS*,** the closest to the potentially optimal modelling structure of the same dimensionality exists. Researches in the given direction allowed to define a modelling binary structure ***1–HS*** with the neighbourhood template of size $L = (n+1)[log_2 a+1+\omega]+2$, where $0<\omega<1$. For receiving of estimation of the affinity degree of this modelling binary structure to the potentially optimal structure it is possible to take advantage of the following obvious relation, namely**:**

$$\frac{L}{L_{opt}} = \frac{(n+1)[log_2 a]+(n+1)[1+\varpi]+2}{(n+1)[log_2 a]} = 1+\frac{1+\varpi}{log_2 a}+\frac{2}{(n+1)[log_2 a]}; \quad (0<\varpi<1)$$

Of the given relation it is easy to draw a conclusion about satisfactory affinity of the received modelling structure to some kind of standard structure even at moderate enough cardinality of an alphabet ***A*** and a size of neighbourhood template of a *modelled* classical structure ***1–HS.*** Furthermore, the value ω depends on a series of conditions and since the computer estimations reveal that in case of $a \leq 2^{19}$ the value ω does not exceed ***1,*** in calculations it is entirely possible to accept value $\omega = 1$. For practical purposes it is entirely comprehensible approach because already such number of states of elementary automaton of a modelled classical structure ***1–HS*** is practically immense. In general case a value ω does not exceed two. So, in view of the aforesaid the following basic result for arbitrary classical structures ***d–HS*** $(d \geq 1)$ can be formulated [5,53–56,79,88,90,617,618,640–643].

Theorem 119. A classical structure d–HS with alphabet $A=\{0,1, ..., a-1\}$ ***and neighbourhood template contained in the minimal d–dimensional hyperparallelepiped*** $p_1 x p_2 x p_3 x \ldots x p_d$ ***is 1–simulated by an appropriate binary classical structure d–HS*** $(d \geq 1)$ ***with neighbourhood template of size*** $L = (p_1+1)[\log_2 a + p_1+\lambda)] x p_2 x p_3 x p_4 x \ldots x p_d$ ***where*** $\lambda = 4$ ***for*** $a \leq 2^{19}$ ***and*** $\lambda = 5$***, otherwise.***

The method of proof of this theorem allows to model quite effectively classical structures ***1–HS*** which have big enough alphabet ***A*** and small neighbourhood templates by means of *binary* classical structures ***1–HS*** with quite reasonable size of neighbourhood templates. Now we shall illustrate it by one interesting example having important independent value. We shall speak, that a classical structure ***d–HS*** $(d \geq 1)$ is *universal* or possesses *universal computability* if it ***T–***models the universal Turing machine. Such structures possess property of universal computability and play important enough role at researches of the *classical* structures ***d–HS*** $(d \geq 1)$ as a formal model of parallel calculations. In principle, the *universal* computability in ***HS–***models can be defined and by means of other equivalent ways.

In connexion with definition of the concept of universal computability on basis of the ***T–***modelling concept arises a rather important question about the *minimal* complexity of a classical structure that ***T–***models the universal Turing machine, or in more general posing about the most simple classical structure ***1–HS*** that possesses the property of *universal* computability. As a measure of complexity of an universal structure ***d–HS*** $(d \geq 1)$ it is quite natural to use value of product $d x a x n$ in which ***3*** factors define values of base parameters of such structure: dimension, cardinality of alphabet ***A,*** size of neighbourhood template. So, in our opinion, determination of the universal structure ***d–HS*** ***(d ≥ 1)*** with the *minimal* value of ***dxaxn*** is not lesser problem than determination of the universal MT^s_q with minimal value of ***sxq,*** making up today the value ***sxq = 26.*** Though the given problem not seems to us such fundamental scientific problem as a whole [54-56,79,88,90,536,567,617,624,628-630].

For classical structures ***1–HS*** the best result in the given direction has been received by ***A.R. Smith,*** who proved availability of the universal structures with the following values of ***axn,*** namely: ***2x40,3x18,6x7,8x5, 9x4,12x3*** and ***14x2*** [131]. At that, it is necessary to note existence of the universal classical structure ***1–HS*** with simplest neighbourhood index. So, we have the universal classical structure ***1–HS*** with value ***axn = 28*** that almost coincides with best today's minimal value ***sxq = 26*** for the universal machine MT^s_q. The best result of similar type for case of the universal classical structures ***2–HS*** has been received by ***E. Banks*** who proved existence of the universal structures with value ***dxaxn = 2x2x5= 20*** at use of an infinite initial configuration of the modelling structure, and ***dxaxn=2x3x5=30,*** otherwise [132]. Meantime, results of ***A.R. Smith*** and ***E.R. Banks*** have allowed to make, among other things, a series of interesting enough conclusions about influence of dimensionality and type of the used initial configurations of the structures ***d–HS*** upon the results of modelling in them. A number of similar results in the given direction has been received by ***A.S. Podkolzin*** and by others a little bit later [158,199,200,281,536,589]. So, for example, ***A.S. Podkolzin*** proved existence of the universal structures ***2–HS*** with values ***dxaxn=2x2x9=36*** and ***dxaxn = 2x3x5 = 30*** [590]; thus, this result repeats the ***Banks's*** result. The main principle of the proof is based on embedding into structures of the basic logic circuits that realize functions of disjunction, trigger, etc. Furthermore, ***A.S. Podkolzin*** has proved the following result.

<u>Theorem 120</u>. The recognition problem of an universal structure d–HS in the class of structures d–HS (d ≥ 2) is algorithmically unsolvable.

On basis of realization in the classical structures ***2–HS*** of the base logic functions the universality of binary structures ***2–HS*** with very simple neighbourhood templates has been proved [409]; it is shown that well-known game ***Life*** is equivalent to the universal Turing machine [429], what is conditioned by an opportunity of determination for the game of processes, equivalent to the universal calculations. Depending on definition of this game we naturally come to existence of the universal classical binary structures ***2–HS*** with the value ***dxaxn = 2x2x9 = 36.***

The used optimizing method of modeling of classical structures ***1–HS*** by binary structures of the same class gives rise to rather good results and gives good comparative characteristics concerning the potentially optimal modelling structures. However, its efficiency depends in the certain limits on values of the base parameters of the modelled classic structures, namely**:** size of neighbourhood template and cardinality of an alphabet ***A*.** Therefore, in individual cases it is possible to use some special modifications of this method which allow to receive the results of modelling the most close to optimum. The given modelling aspect has been considered concerning a problem of discovery the of *minimal* universal classical structures ***1–HS*** with alphabets ***A***={***0,1,2,3, ..., a–1***} of cardinality $a \in \{2,3,4,8,14\}$, in particular**.** Classical ***HS***–models with the given characteristics present the certain interest from a whole series of theoretical and applied standpoints. The basic result can be presented here as follows [3,5,8,9,53–56,79,88,90,527,617,618,640–643]**.**

<u>Theorem 121</u>. An arbitrary classical structure 1–HS with an alphabet of cardinality a≥6 and neighbourhood template of size n is 1–modeled by a binary classical structure 1–HS with neighbourhood template of size $n[2\log_2(a+2)+1]-2$***. An arbitrary classical structure 1–HS with an alphabet of cardinality*** $4 \le a \le 21$ ***and neighbourhood template of size n is 1–modeled by a suitable classical structure 1–HS with alphabet of cardinality 3 and neighbourhood template of size 6n – 1. An arbitrary classical structure 1–HS with an alphabet of cardinality*** $5 \le a \le 14$ ***and neighbourhood template of size n is 1–modeled by means of a suitable classical structure 1–HS with a state alphabet of cardinality four and neighbourhood template of size 5n – 2.***

It is necessary to note that modeling in rigorously real time is essential enough characteristic of the given result. Using method of the proof of theorem ***121,*** it is easy to show, that the universal classical structure ***1–HS*** with value ***axn = 14x2*** is ***1***–modeled by means of a binary structure ***1–HS*** with neighbourhood template of size $L=2[2\log_2 16+1]-2=16$, i.e. for the modeling structure ***1–HS*** the value ***axn=2x16*** is allowable. That gives rise to the following result [53]**.**

<u>Theorem 122</u>. There are the universal classical structures 1–HS with a characteristic value axn = {2x16 | 3x11 | 4x8}; these classical structures were obtained as a result of simulation in rigorously real time.

This result proves existence of universal classical structures ***1–HS*** with relatively small neighbourhood templates along with cardinality of a state alphabet ***A;*** in addition, during a long enough time it was as one of the best results in the given direction [88,567]. Meantime, ***M. Cook*** at ***2000*** has shown that classical binary structure ***1–HS*** with distinctive number ***118*** ***(****in English notation it appears under number* ***110)*** possesses the universal computability [536]. In view of the introduced ***4*** types of the nonconstructability for the classical structures ***(NCF, NCF-1, NCF-2*** *and* ***NCF–3)*** can be shown that the given structure determined by local transition function $\sigma^{(3)}$ of the following kind, namely:

$$\sigma^{(3)}(x,y,z) = \begin{cases} x+y+z+1 \ (mod\ 2), \ if\ x=1\ or\ <yz> = <11> \\ x+y+z \ (mod\ 2), \ otherwise \end{cases}; \quad x,y,z \in \{0,1\}$$

possesses the nonconstructability such as ***NCF, NCF–1*** and ***NCF–2.*** As simple examples of nonconstructible configurations of such types the following finite configurations can be presented, namely ***c1= <11001>, c2 = <1>, c3 = <01010>,*** accordingly ***NCF–2, NCF–1*** and ***NCF.*** At that, the given structure possesses the ***γ–CF*** with minimal size ***1*** and pairs of ***MEC*** {***<01|01|10>, <01|10|10>***} with ***IB*** of minimal size two along with ***NCF, NCF–1*** and ***NCF–2*** of minimal size ***5, 1*** and ***5*** accordingly.

Among all *binary* structures ***1–HS*** with neighbourhood index ***X=***{***0,1,2***} up till now only one structure with property of universal *computability* has been found, although other certain structures of such simple kind also could possess by the given property. So, there are classical binary structures ***1–HS*** with neighbourhood index ***X=***{***0,1,2***} that possess the property of universal computability. In this sense their complexity is determined by the value ***axn = 2x3 = 6.*** However, the ***Cook's*** result has provoked the certain doubts; unfortunately, we have not any detailed enough information in this direction. Thus for today, the above binary structure ***1–HS*** can be considered as an example of the *elementary* ***HS–*** model possessing the property of universal computability, allowing to formulate the following proposal.

Proposal 8. There are universal classical binary structures 1–HS with neighbourhood index X = {0,1,2} of complexity axn = 2x3 = 6.

Meanwhile, revealing of *minimal* universal structures ***d–HS,*** especially of binary structures ***1–HS*** presents in our opinion more gnoseological, than theoretical and applied interest. Especially, if artificial technique of the proof uses occasionally as a whole disputable agreements.

Meanwhile, it is interesting to investigate properties of such universal structures ***1–HS*** from point of view of the *nonconstructability* problem. The carried out analysis in this direction has allowed to formulate the following rather interesting result [54–56,79,88,617,618,640–643].

<u>Theorem 123</u>. There are one–dimensional universal classical structures which possess the nonconstructability of all four types, namely: NCF, NCF–1, NCF–2 and NCF–3.

Discussion of results of theorems ***118–123*** together with the associated rather interesting questions can be found, for example, in [567,618]. In particular, the above results allow to essentially simplify a modelling technology of the complex processes demanding appreciable enough efforts for their embedding into classical structures ***1–HS.*** The essence of such technology is represented both at the formal level, and on two interesting enough examples, namely: problems of the limited growth in classical ***HS***–models, and of periodic finite configurations with the maximal periods. In this direction a whole series of other instructive enough examples exists [5,54–56,79,88,90,536,567,617,618,640–643].

The *first* example is enough closely linked with the problem of limited growth that has been considered for case of structures ***1–HS*** in section ***1.2.*** Parallel algorithms of growing of chains of active automata of the fantastic length, received here with use of structures such as ***1–HS**** are embedded into *classical* structures ***1–HS*** with neighbourhood template of size ***3*** and with an alphabet of cardinality $4m + 29$, where ***m*** – speed of transfer of control information in ***HS****–models. Then on basis of the theorem ***121*** it is rather simple to draw the conclusion about existence of the equivalent binary classical structures ***1–HS*** with neighbourhood templates of size $3[\log_2(4m + 31)] - 2$.

The discovering problem of the greatest possible minimal sizes of the periods for periodic finite configurations in classical binary structures ***1–HS*** can successfully serve as the *second* example. In the monography [5] it is shown that there is a structure such as ***1–HS**** whose functional algorithm admits for any integer $m \geq 3$ the existence of periodic finite configurations of length ***m*** with minimal period $p=2^m-2$ at cardinality of alphabet ***A*** and number of control impulses ***I,*** which equal ***3.*** In the same place it is shown that this structure ***1–HS**** is equivalent to some classical structure ***1–HS*** with neighbourhood index $X=\{-1,0,1\}$ together with alphabet ***A**** ***(#A*=#A+#I=6).*** The subsequent application to these results of the proof method of theorem ***121*** gives rise to discovering of existence of binary classical structures ***1–HS*** with the neighbourhood template of size $n=16$, that possess the periodic finite configurations of length $(5m+3)(m \geq 3)$ with the minimal period $p = 2^m-2$. So, three basic stages of the above type of modeling in classical structures ***d–HS*** ***(d≥1)*** are, generally speaking, looked through, namely:

d–HS* $\Rightarrow$ ***classical structures d–HS*** $\Rightarrow$ ***binary classical structures d–HS***

Estimation for neighbourhood template of a modeling *binary* structure ***1–HS*** in the theorem ***121*** can be generalized that in combination with technics of modeling of the classical structures ***1–HS*** offered in proof of the theorem ***121*** allows to obtain an interesting estimation of size of neighbourhood template of the modeling binary structures for case of higher dimensionalities too.

Theorem 124. Any classical structure d–HS (d ≥ 1) with alphabet A={0, 1,2, ..., a–1} and neighbourhood template that is contained in minimal d–dimensional parallelepiped $p_1 x p_2 x \ldots x p_d$ ***is 1–modeled by means of an appropriate binary classical structure of the same dimension with neighbourhood template of size*** $\{p_1[2\log_2(a+2)+1]-2\} x p_2 x p_3 x \ldots x p_d$.

A whole series of aspects of discussion of the modelling technics used in the proof of theorems ***119–124*** along with certain related questions can be found in the works [3,5,8,9,53–56,79,88,90,536,567,617,618,640].

Question of modelling of the classical structures ***d–HS (d≥1)*** by means of structures of the same class, but with decreasing of dimensionality of the modeling structures represents rather essential both theoretical, and applied interest. In the work [532] one interesting approach to the problem of realization of of modelling of the classical structures ***3–HS*** by means of structures ***2–HS*** that has used some results of the works [533,534] has been represented and analysed. A generalization of this approach allows to model structures ***d–HS (d ≥ 3)*** by structures ***2–HS.*** Meanwhile the given approach does not work for ***1***–dimensional case, not allowing to simulate an arbitrary structure ***2–HS*** by means of the corresponding structure ***1–HS.*** The following theorem presents other approach which provides modeling of the classical structures ***2–HS*** by means of structures ***1–HS*** of the same type.

Theorem 125. Dynamics of finite configurations of classical structures 2–HS can be simulated by the corresponding classical structures 1–HS with neighbourhood index X = {–1,0,1}.

Essence of proof of the theorem is reduced to the following. Above all, it is known that an arbitrary classical structure ***d–HS (d ≥1)*** is modeled by an appropriate classical structure ***d–HS*** of the same dimension with simplest neighbourhood index ***X*** of the kind [3,5]:

$$X=\underbrace{\{\underbrace{(0,0,0,\ldots,0,0)}_{d},\ \underbrace{(1,0,0,\ldots,0,0)}_{d},\ \ldots,\ \underbrace{(0,0,0,\ldots,0,1)}_{d}\}}_{d+1}$$

i.e. its neighbourhood template contains ***d+1*** elementary automata out of which one automaton is central whereas each of ***d*** automata adjoin to it along axes of the coordinates E^d. Therefore, it is quite enough to be limited oneself to a case of a classical structure ***2–HS*** with alphabet $A = \{0,1,2, ..., a-1\}$ and neighbourhood index $X = \{(0,0),(1,0),(0,1)\}$. Then, writing down an arbitrary finite **2**–dimensional configuration on final tape of a MT^sq by a special way with using of **2**–level structurization of states of cells of final tape of similar Turing machine, we determine an appropriate program of its work that will simulate dynamics of an arbitrary finite configuration in the modeled classical structure ***2–HS.***

The program of the modelling MT^sq is represented in [567,640]. Thus, it is shown that any arbitrary classical structure of ***2–HS*** with simplest neighbourhood index $X=\{(0,0),(1,0),(0,1)\}$ and alphabet of cardinality ***a*** is modeled by means of machine MT^sq with parameters $\#S=2a^2+3a+3$ and $\#Q=a^2+2a+15$ where $\#V$ is cardinality of a set ***V.*** At that, one step $c_o\tau$ of a modeled structure ***2–HS*** an appropriate MT^sq models during $(n+1)m^2+(2n^2+13n+15)m+2n(n+9)+28$ steps, where $(n{\times}m)$ – size of the minimal rectangle containing an initial configuration $c_o \in C(A,2,\phi)$. Of that follows, that a cost of the given modelling is rather essential time costs determined by quantity of commands required for the modeling MT^sq for realization of one step of the modeled structure of ***2–HS.*** On the other hand, according to the theorem ***110*** follows, that for a MT^sq there is an appropriate classical structure ***1–HS*** with neighbourhood index $X=\{-1,0,1\}$ and alphabet A^* of the cardinality $\#A^*=s+q+9$ which ***8***–simulates the MT^sq. Thus, an arbitrary classical structure ***2–HS*** with neighbourhood index $X=\{(0,0),(1,0),(0,1)\}$ and alphabet of cardinality ***a*** is simulated by an appropriate classical structure ***1–HS*** with alphabet of cardinality $3a^2+5a+27$ and neighbourhood index $X=\{-1,0,1\}$.

At that, approach used in the theorem ***125*** is intended for modeling of dynamics of configurations $<c_o>=\{c_o\tau^{(3)k} \mid k=0,1,2,3, ...\}$ $[c_o \in C(A,2,\phi)]$, however with a rather small modification works and in the case when configurations co are periodic in structural, instead of dynamic sense. In this case we shall deal not with a finite configuration c_o but with its finite period. So, the above reduction of the represented results allows us to formulate the following rather interesting assertion [567,617,640].

<u>Theorem 126</u>. Dynamics of finite configurations of classical structures d–HS (d ≥ 1) with an arbitrary finite state alphabet A is simulated by

means of an appropriate nonbinary classical structure 1–HS with the neighbourhood index X = {–1,0,1} or binary classical structure 1–HS.

At the same time it is necessary to emphasize once again, the method used in the above theorem ***125*** ensures modelling only of dynamics of finite and/or structural–periodic configurations in classical structures ***d–HS (d ≥ 2)*** and not spreads to more general case. Discussion of some features of such modelling can be found in [527,617]**.** In particular, the way of *structurization* of states of the finite automaton of the modeling ***MT^s q*** and symbols on its tape is a rather productive, allowing in a lot of cases it is essential to simplify programming of the ***MT^s q,*** and also to embed various processes, phenomena and objects into ***HS***–models. Meanwhile, it has and own faults, first of all, of *optimization* character**:** on the one hand, simplifying process of programming in ***HS***–models and in certain cases optimizing temporal characteristics**;** at the same time, it complicates ***HS***–models, first of all, by increase, in a series of cases enough essentially, of cardinality of a state set of the elementary automaton of ***HS***–models. For this reason similar way can be enough successfully used, above all, for the conceptual decision of problems of ***HS***–models, instead of the optimization problems connected to the complexity of ***HS***–models. In particular, use of this approach allows to receive simply enough the following result [567,617,640]**,** namely**:**

A classical structure 1–HS with alphabet of cardinality s(s+1)(q+1)+1 and neighbourhood index X={0,1} 2–simulates any machine MT^s q. Any machine MT^s q with 2–dimensional tape is 1–simulated by means of an appropriate classical structure 2–HS with neighbourhood index of the Neumann and a state alphabet A of cardinality q(s+1)+1.

The more detailed discussion of a whole series of principal questions of mutual modelling of such formal computers as machines ***MT^s q*** of sequential action along with the ***HS***–models of highly–parallel action can be found in works [54–56,79,88,90,567,617,618,640–643]**.**

6.4. The formal parallel algorithms determined by classical one–dimensional homogeneous structures

Parallel algorithms of processing of the words, that are determined by *classical* structures ***d–HS (d≥1)*** are being studied especially intensively the recent years what is caused not only by their independent interest within of the general theory of algorithms, but also by means of use of classical structures ***d–HS*** as formal models in such areas of the modern natural sciences as mathematical modelling, physics, systems theory, development biology, discrete synergetics, computing sciences, etc.

The parallel algorithms determined by classical structures ***d–HS (d≥1)*** play rather essential role in the formal description of a whole series of biological development processes and various programmable systems basing on computational homogeneous structures. Meanwhile, owing to indubitable interest for the solution of the important problems of a designing of languages of *multiprocessing*, investigations of the formal language models functioning in especially parallel manner present a special importance. With this purpose in chapter **5** have been defined the formal parallel τ_n–grammars and formal parallel $L(\tau_n)$–languages appropriate to them.

While in the algorithmic attitude the *parallel* algorithms defined by the *classical structures* ***d–HS (d-PAHS)*** represent the further research of the classical structures ***d–HS (d≥1)*** as parallel processing systems of words in the finite alphabets. This problematics has the most direct attitude to the question of modelling in classical structures of ***d–HS (d ≥ 1)*** both by the considered problems and by a whole series of base methods of research. In the given section the class of parallel algorithms ***1–PAHS*** is defined, and questions of their complexity concerning a number of well–known formal processing algorithms of words are discussed.

By definition the parallel algorithms ***1–PAHS(a,n)*** operate with words {*configurations of the set* ***C(A,1,ϕ)***} which are defined in an alphabet ***A=*** {***0,1,2, ..., a–1***}. The manner of functioning of a ***1–PAHS(a,n)*** is given by a neighbourhood index ***X=***{***0,1,2, ..., n–1***} and global transition function $\tau^{(n)}$ of a certain classical structure ***1–HS*** defined by the corresponding local transition function $\sigma^{(n)}$ with parallel rules of substitutions of the following general kind, namely:

$$\underbrace{00\ldots0}_{n} \Rightarrow 0 \quad x^j_1x^j_2\ldots x^j_n \Rightarrow x^{*j}_1; \quad x^{*j}_1,\, x^j_k \in A\ (k=1\,..\,n;\ j=1\,..\,a^n-1)$$

which are simultaneously applied to each word ***S*** of the set ***C(A,1,ϕ)*** of all finite **1**–dimensional configurations defined in an alphabet ***A,*** i.e. a word ***S*** is processed by a certain parallel algorithm ***1–PAHS(a,n).*** Thus a parallel algorithm ***1–PAHS(a,n)*** is fully defined by the above parallel *substitutions* corresponding to local transition function $\sigma^{(n)}$ of a certain classical structure ***1–HS.***

For each word ***c∈C(A,1,ϕ)*** a parallel algorithm ***1–PAHS(a,n)*** defines a sequence of words **<*c*>**[$\tau^{(n)}$] in which a word c_k is named the ***final,*** if for it the following relation $c_{k+1}=c_k\tau^{(n)}=c_k$ takes place. Let $C(A,1,\phi)$ is a set of words processed by a parallel algorithm ***1–PAHS(a,n),*** and ***F*** **–** a partial word function in an alphabet ***A*** if for some word $c_j \in C(A,1,\phi)$ the relation $F(c_j) = c^*_j \in C(A,1,\phi)$ takes place. For such word ***F***–function the domain of existence and the range of values is accordingly E_F and V_F. In view of the made assumptions we shall speak, that the word ***F***–function determined in alphabet ***A****, is ***PAHS–computable*** if there is such algorithm ***1–PAHS*** in the alphabet $A=\{b\}\cup A^*$ $(b \notin A^*)$ that for any word $c^*_o \in C(A^*,1,\phi)$, if c_o – representation of the word c^*_o of the kind $c^*_o = \ b^{p+2}0^1b^20^{p+1}c^*_ob^{p+2}$, then the following two determinative conditions are being satisfied, namely:

1) if a word ***c**** belongs to domain of existence E_F of the ***F***–function, the configurations sequence $<c_o>[\tau^{(n)}]$ will be contain the ***final*** word c_f of the following general kind, namely:

$$c_f = \ b^{p+2}0^1b^10^{p+2}F(c^*_o)b^{p+2} \ , \qquad F(c^*_o) \in V_F; \tag{30}$$

2) if the word c^*_o does not belong to domain of existence E_F of the ***F***–function, the sequence $<c_o>[\tau^{(n)}]$ does not contain the final word c_f of the above kind ***(30).***

In view of the given definition it is shown that a word ***F***–function will be ***PAHS***–computable if and only if it is computable on an appropriate Turing machine [3]. Equivalence of strict formalization of the intuitive concept of ***PAHS***–computable functions with class of Turing machines represents one more strong enough argument in favour of the known algorithmical thesis of ***A. Church.*** Inasmuch as in the theory of formal algorithms the large attention is given to the questions of computing complexity, therefore with respect to the class of parallel algorithms ***1–PAHS*** in this direction the following interesting enough result has been received [3,5,9,35,53–56,79,88,617,618,640–643].

<u>Theorem 127</u>. An arbitrary partial recursive word function W defined in an arbitrary finite alphabet A is PAHS–computable in the enlarged alphabet A* = {b}∪A (b∉A).

Of the result of theorem ***127*** follows, the parallel algorithms ***1–PAHS*** in sense of complexity of Markov–Nagorny are equivalent to normal Markov algorithms. We have analysed enough in detail the questions of parallelism of the class of algorithms ***1–PAHS*** and as a consequence of this we have defined the most interesting directions for the further researches: parallelism classes, a detailing and specification of essence of parallelism, choice of algorithms most suitable for effective enough realization in the computational ***HS***–models, etc. [53]. So, in particular, a rather interesting class of so–called *locally realizable algorithms* ***(LRA)*** whose essence consists in a possibility to present a general algorithm of processing as local identical algorithms over separate subwords of each processed word has been determined. The problem of symbolic sorting can be indicated as a simple example of such algorithm. The carried out analysis has allowed to formulate the assumption that the algorithms of the above class ***LRA*** can be executed in computational ***HS***–models by the most effective manner. In this connection there is a rather interesting question concerning existence of other classes of the algorithms most effectively executed in the computational ***HS***–models and how they can be characterized.

Coming back to the problem of symbolic sorting we shall estimate the efficiency of its realization by algorithms such as ***1–PAHS.*** At that, the general problem of symbolic sorting is defined as follows. Let ***G1*** is an arbitrary finite word in an alphabet ***A*** whose symbols is allotted by a certain hierarchy *(a priority principle)*. It is necessary to define a certain symbol-by-symbol algorithm that will quickly sort any finite word ***G1*** according to the given priority principle of symbols of the alphabet ***A***. It is known that sequential algorithms for the solution of this problem require no more, than $M=\alpha|G1|^2$ viewings of a word ***G1***, where α is a constant and $|G1|$ is length of the word ***G1*** whereas an appropriate ***1–PAHS*** can solve the given problem for strictly linear time, namely:

There is an algorithm 1–PAHS which sorts an arbitrary finite word G defined in an arbitrary finite alphabet A during no more than L steps, where L is length of the word G.

The known problem of *French flag* is a formalization of the problem of regulation and differentiation of real biological cellular structures; it directly adjoins to the similar problem of sorting, and in details it has been discussed in our works [3,5,26,27,33,567]. In the same aspect the problem of mirror inversion of an arbitrary finite symbolic string by a MT^s_q and a classical structure ***1–HS*** is of interest too. So, it is simple to make sure that for mirror inversion of a string ***X*** of length ***m*** by means of MT^s_q is required about $2m^2$ steps, whereas an appropriate classical structure ***1–HS*** can solve the same problem for linear time. At that, use of a classical structure ***1–HS*** with an alphabet of the structured states of elementary automata allows to effectively solve this problem and a series of other problems [567,640]. The used way of structurization of state alphabet of the modelling structures [53] allows to solve enough effectively in the temporal attitude a whole series of rather interesting problems of modelling, by being non–optimal from standpoint of the complexity of classical ***HS***–models; i.e. the given approach is effective enough in temporal attitude, leading generally speaking to the certain *complication* of a modeling classical structure. As a result the following assertion can be formulated [53–56,617,618,640–643], namely:

An appropriate classical structure 1–HS with simplest neighbourhood index X = {0,1} decides the problem of mirror inversion of an arbitrary finite configuration of length m for time t=2m–1. A classical structure 1–HS decides the symmetry recognition problem of an arbitrary finite configuration of length m for time about t=[m/2]. A classical structure 1–HS with simplest neighbourhood index X={0,1} doubles an arbitrary finite configuration determined in a certain narrowing of an alphabet A during t = β|m| steps, where β – a constant and |m| – length of the processed configuration.

It is quite natural to assume that paralleling gives rather essential time advantage when temporal complexity of some algorithm nonlinearly depends on the input data of a problem as, in particular, in case of the sorting problem. However even in the linear case the paralleling can give rather essential temporal advantage. We shall consider the well–known problem of finding of a sample on strings as rather interesting illustrating example. Let ***S*** and ***G*** are two strings containing ***n*** and ***m*** symbols of an arbitrary finite alphabet ***A*** accordingly; it is necessary to check up membership of the substring ***S*** to the string ***G (S⊂G?).*** In this regard ***D. Knuth, P. Pratt*** and ***A. Moris*** have offered a solution of the given problem for time no more, than ***0(n+m).*** Meantime, it is possible to simply prove, that the following assertion takes place [5,37,640]:

An appropriate classical structure 1–HS with neighbourhood index X= {-1,0,1} can solve the problem of finding of a sample on a string during no more, than 0(|n–m|) steps, where n and m are lengths of the string and the sample accordingly.

A lot of other interesting enough examples of similar character takes place [5,53–56,88,90,536,640]. The class ***1–PAHS*** of parallel algorithms determined by the classical structures ***1–HS*** represents own subclass of the class of all local algorithms ***(LA),*** i.e. the algorithms establishing properties of elements of some set, at that, by using the information in each step only about a certain neighborhood of a word, processed at present. In terms of the local algorithms the problems about existence or lack of effective algorithms for various discrete extremal problems are naturally formulated and are solved [53–56]. Hence, it is desirable to apply and to investigate the results and technique of the ***LA*** theory concerning the class ***d–PAHS*** ***(d ≥ 1)*** of highly–parallel algorithms.

The increasing number of the works, devoted to questions of parallel algorithms as a important enough component of the general theory of algorithms is being observed at the late years. In this attitude we have researched the question of temporal complexity of parallel algorithms ***1–PAHS*** concerning a whole series of well–known computing formal sequential models [5,35,37,47,54–56,79,88,90,536,617,618,640–643].

<u>Theorem 128</u>. If a partial recursive word function F can be computed by means of a parallel algorithm 1–PAHS(a,n) during t steps, then an appropriate Turing machine can calculate the same function F during no more, than $\{(n+1)t^2+(n-1)t\}/2$ ***steps, where n – length of template of neighborhood of a structure corresponding to the 1–PAHS(a,n).***

The carried out analysis of the given result shows a sufficient degree of closeness of the received estimation to the optimal. Hence, if some partial recursive word function is ***PAHS***–computable for time ***t***, then a suitable Turing machine can calculate it for time no more than αt^2 ***(α –*** *a constant)*. So, for formal computers on basis of the *classical* structures ***1–HS*** and Turing machines the temporal difference of calculations has the quadratic order. Computing opportunities *(in view of their temporal complexity)* of the parallel algorithms ***1–PAHS*** more evidently can be illustrated by the following example. It is known that a two–way stack automaton ***(TWSA)*** is equivalent to an one–head Turing machine with time of work no more than $t=|x|^{\alpha|x|}$ steps where x – an input word, α – a certain constant. The detailed description of the two–way stack automaton and principle of its functioning can be found, for example, in [9,37,640]. Characteristic of the two–way stack automata in terms of the parallel algorithms ***1–PAHS*** gives essentially best result relative to the characteristic of their temporal complexity [5,37,617,618,640].

<u>Theorem 129</u>. If a two–way stack automaton admits some set of finite words during t steps, then an appropriate parallel algorithm 1–PAHS can admit the same set of words during no more, than $2t^2$ ***steps.***

Thus, by means of the parallel algorithms ***1–PAHS*** for a whole series of computing algorithms it is possible to receive essentially more best temporal results than on basis of the Turing machines. Moreover, it is necessary to note, the high degree of parallelism inherent in parallel algorithms ***1–PAHS*** was used only at a level of the ***T***–modelling of one algorithm by another**;** in addition, in most cases a parallelism inherent in the modelled algorithms was essentially not used. Thus, the further research in this direction seems to us a rather perspective and actual.

Let's speak, a Turing machine admits a set ***G*** of the finite words given in an alphabet ***A,*** if the machine containing on final tape an arbitrary finite **ω–**word determined in the alphabet ***A*** passes into a final state ***q'*** after the analysis of this word if and only if **ω∈*G.*** Let ***S(c)*** will be a set of all finite ***predecessors*** of a certain configuration ***c∈C(A,1,ϕ)*** for some classical structure ***1–HS*** with an alphabet ***A;*** at that, configuration ***c**** is a predecessor of the configuration ***c∈C(A,1,ϕ),*** if $c^*\tau^{(n)}=c$**.** The problem of finding of predecessors for finite configurations in the classical ***HS–*** models plays important enough role, above all from the standpoint of research of the dynamics reversibility property, fundamental at use of classical structures as conceptual models of the spatially–distributed discrete dynamic systems out of which physical systems represent the greatest interest. In light of the made assumptions a rather interesting result having a series of important appendices takes place [55,640]**.**

<u>Theorem 130</u>. For any configuration c∈C(A,1,ϕ) an appropriate Turing machine MT^s_q ***admits a set S(c) during no more than*** $2(|c^*|+n)^2$ ***steps where c* – a predecessor of c–configuration having maximal length n.***

Researching the *reversibility* questions of dynamics of the classical ***HS–*** models, ***T. Toffoli*** has shown, that in case of classical structures ***d–HS (d=1, 2)*** for any configuration ***c∈C(A,d,ϕ)*** the set ***S(c)*** can be generated by a certain *nondeterministic finite automaton* ***(NFA),*** what allows us to do the defined conclusions about temporal complexity of generating and recognition by means of the ***NFA*** and MT^s_q of predecessors for an arbitrary finite configuration in classical ***HS–***models [273,318,640]**.**

Now, a series of works concerning the research of various concepts of stack automata is known [5,390,536]**.** So, so-called ***bc–****automata* present essential significance for structural programming**;** they have received wide enough popularity owing to works linked with representability of formal languages in them. On basis of the ***bc–****automaton* description it is easy to make sure, that for it exists *three–head* Turing machine that models it without temporal delay, and exists *one–head* Turing machine that models it for time $2(t+1)^2$ where ***t*** **–** time of processing of an input word by the ***bc*–**automaton**.** The following result allows to estimate the temporal costs of a parallel algorithm ***1–PAHS*** for performance of the same work, as a certain ***bc*–**automaton [37,640–643]**.**

Theorem 131. If a certain bc–automaton demands t steps of processing of an input finite word then an appropriate parallel PAHS–algorithm can perform the same processing during no more than $2(t+1)^2$ ***steps.***

The modelling results received in this direction have a various degree of closeness to optimum, however they allow to carry out to a certain extent comparative estimations of temporal complexities of classical ***HS*–**models and other known sequential formal computing models. In combination with results of a whole series of other researchers in this direction the given results allow to receive full enough picture in field of computing complexity of the classical ***HS*–**models. So, for example, ***A. Hemmerling*** has presented a rather interesting review of researches concerning the comparative analysis of computing complexity of the classical ***HS*–**models and ***d*–**dimensional Turing machines $(d \geq 1)$ [171]**.** However, in comparison with the theory of sequential algorithms, the theory of parallel computing ***HS*–**models is not so advanced.

In conclusion it is necessary to mark once again the following a rather important circumstance concerning the results presented above. We used modeling at a level of classical structures as the abstract *algebraic* parallel systems of processing of words **(***configurations***)** without use of *maximum parallel* modeling in the cellular environment of such objects **(***for example***,** *by means of immersing in them of appropriate algorithms such as logic networks***,** *etc.***).** Naturally, our approach has established results, that are a little remote from possible ones, however us has interested such approach. In the following chapter the complexity questions of ***HS*–**models will be considered in a few other aspect characterising the properties of the models not on a level of a set of finite configurations, but on a level of global transition functions defined by them, allowing to differentiate the class of ***HS*–**models from the other standpoint.

6.5. Special questions of modelling in the classical homogeneous structures concerning their dynamics

It is known that modelling in the environment of classical structures is the multifold problem including such important enough questions as modeling real time, *optimal* modelling according to the chosen criteria of optimization**;** methods and principles of simplification of process of modelling, receiving of estimations of complexity of mutual modeling of structures, modelling of individual algorithms, objects, phenomena and processes, modelling in the definite classes of structures and also at various conditions, etc. In the previous sections of this chapter the questions of modelling in classical structures without any additional conditions for the modelling structures have been considered. Below, we shall present a series of results of such modelling when the certain restrictions having one or another sense will be imposed on modelling classical structures together with interpretations of such restrictions.

Studying of dynamic properties of classical ***HS*–**models depending on type of their *local transition functions* ***(LTF)*** represents the indubitable interest. So, in the monograph [3] two large classes of ***HS*–**models with ***symmetric (SF)*** local transition functions and ***asymmetric (ASF)*** local transition functions are differentiated. It is possible to show that the ***SF*–**class of ***HS*–**models composes a subset concerning the composition operation. At that, composition of a symmetric and asymmetric local transition function always gives an *asymmetric* function whereas exist asymmetric local transition functions, whose compositions give some *symmetric* function. In particular, two simple local transition function, defined by formulas of the following kind**:**

$$\sigma_1^{(2)}(x,y) = xy^2 \pmod a; \qquad \sigma_2^{(2)}(x,y) = x^2y \pmod a \qquad (a \geq 3)$$

as a result of the composition give symmetric local transition function $\sigma^{(3)}(x,y,z) = x^2y^5z^2 \pmod a$. Concerning the classes ***SF*** and ***ASF*** of ***HS*–** models it is shown that in sense of computing opportunities they are *equivalent***,** i.e. both specified classes possess the universal classical ***HS*–** models. Meanwhile, by a series of other characteristics the classes ***ASF*** and ***SF*** can essentially differ. So, for example, the essential distinctions take place concerning the constructive structures possibilities and sets of *nonconstructible* configurations in classical structures with *symmetric* and *asymmetric* local transition functions what undoubtedly should be taken into account in many model appendices [3,5,8,9,53–56,88,536]**.**

Indeed, enough many processes and algorithms have the pronounced asymmetric character, although in their basis at the lowest levels the elements of a various degree of *symmetry* can take place, and they can be much simply embedded into classical ***HS*–**models with asymmetric local transition functions. Naturally, in view of the told such processes and phenomena by classical ***HS*–**models both with *symmetric* and with *asymmetric* local transition functions are simulated, however functions of the first type will demand as a rule of such outlay as increase of the *base* characteristics of a modeling structure relative to similar structure of the second type, namely: cardinality of alphabet, time of modelling, size of neighbourhood template.

As an example we shall represent an interesting result of modelling of any Turing machine by a classical structure ***1–HS*** with neighbourhood index $X = \{-1,0,1\}$ and symmetric local transition function [53]. At that, a ***1***–dimensional local transition function $\sigma^{(n)}$, determined by parallel substitutions of the following kind is considered the symmetric:

$$x_1x_2 \dots x_n \to x^*_1 \quad \textit{and} \quad (x_1x_2 \dots x_n)^R \to x^*_1 \quad (x_k, x^*_1 \in A;\ k=1 \,..\, n)$$

where X^R – tuple symmetric to a tuple ***X.*** Naturally, generalization of the *symmetry* concept of local transition function to case of the higher dimensionality of any difficulties does not entail.

<u>*Theorem 132*</u>***. If a Turing machine*** MT^s_q ***realizes a certain algorithm S during time t then exists an appropriate classical structure 1–HS with neighbourhood index*** $X = \{-1,0,1\}$***, symmetric local transition function*** $\sigma^{(3)}$ ***and a state alphabet A of cardinality*** $2s+4q+2$ ***which models the same algorithm during time not more than 4t.***

Method of proof of the theorem allows to spread the received result to case of classical ***HS***–models of the higher dimensionality also, but then the necessity of expansion of alphabet of a modelling structure arises. The received result not only once again corroborates the equivalence of classical ***HS***–models with *symmetric* and *asymmetric* local transition functions relative to their *computing* possibilities but in a certain extent illustrates complexity of modeling as a whole of *asymmetric* algorithms by means of symmetric structures ***d–HS*** $(d \geq 1)$. In the given direction interesting enough results have been received also by ***H. Schwerinsky*** [156] and ***Y. Kobuchi*** [9,395]; they proved an opportunity of modeling real time of an arbitrary classical structure ***1–HS*** by a structure of the same dimension with neighbourhood index $X=\{-1,0,1\}$ and symmetric local transition function. Moreover, the modelling in their works has been considered relative to the set $C(A,1,\phi)$ of all finite configurations only without any serious optimization what had an influence on some parameters of the modeling structures. At that, we have used a certain optimization at modelling of asymmetric algorithms by symmetric.

In this connexion, we would like once again to focus the attention, the both classes ***SF*** and ***ASF*** of classical ***HS***–models possess a lot of *specific* features but if to start with more practical reasons then between them there are two basic differences, namely**:** the class of ***HS–***models with symmetric local transition functions appears much easier for practical realization and is of interest from standpoint of a number of biological interpretations **(***so***,** *the symmetry can be associated with absence of a certain gradient at modelling by such structures of either biological phenomenon***;** *in environment of such type of* ***HS–****models are more naturally represented and neuro-like systems***,** *etc***.)** while the classical ***HS–***models with asymmetric local transition functions are generally essentially better adapted to a modelling of various processes and algorithms, i.e. possess the greater degree of *constructive* possibilities by a series of major factors. Indeed, as shows experience, for strongly pronounced asymmetric processes, generally speaking it is impossible to well solve in environment of the *symmetric* structures the problems of optimization character [536,567]**.**

At modelling in classical structures ***d–HS (d ≥ 1)*** the problem of *optimal* modelling of different objects, algorithms or phenomena is important enough. Optimization is considered, as a rule, concerning such basic parameters of a modeling structure as a neighbourhood template size, alphabet cardinality, structure dimensionality and modelling time. In this attitude the optimization of a classical structure ***d–HS (d ≥ 1)*** with symmetric local transition function which simulates a structure of the same dimension with an arbitrary local transition function represents indubitable interest. In **1–**dimensional case in the given direction the following basic result takes place [5,53,54–56,79,88,90,618,640–643]**.**

Theorem 133*. *A classical structure 1–HS with neighbourhood index X= {-1,0,1} is modeled by an appropriate classical structure 1–HS with the same neighbourhood index, alphabet of cardinality* $2(4a^2+5a+12)$ *and symmetric local transition function for time no more, than 4L; where L – length of a processed finite configuration in the modelled classical structure and a is cardinality of alphabet A of the modelled structure.

Investigations on the optimization problem of basic parameters of the modelling classical structures ***d–HS*** represent essential interest and it is necessary to pay corresponding attention to them although similar problems are presented rather complex as, however, and the majority of the optimization problems as a whole. Along with a series of rather interesting results on mutual modelling of classical structures in book [158] the modelling of an arbitrary classical structure ***2–HS*** by means of an appropriate classical structure ***2–HS*** with neighbourhood index of Moore, alphabet of cardinality ***7776*** and symmetric local transition function has been represented. It would be rather interesting to lower essentially this value which, in our opinion, is greatly exaggerated. In addition, in the sequel we need an useful enough property of classical structures ***1–HS*** with the simplest neighbourhood index $X = \{0,1\}$ and symmetric local transition functions $\sigma^{(2)}$.

__*Lemma 4*__***. A classical structure 1–HS with symmetric local transition function*** $\sigma^{(2)}$ ***and the simplest neighbourhood index*** $X = \{0,1\}$ ***possesses the nonconstructability such as NCF and/or NCF–1.***

Proof of the lemma is extremely simple and consists in the following. Let's suppose that a classical structure with alphabet $A=\{0,1, ..., a-1\}$ of the above type does not possess the nonconstructability such as ***NCF.*** But then according to the criterion of nonconstructability such as ***NCF*** on basis ***γ–CF*** the number of parallel substitutions $xy \rightarrow 0$ $(x,y \in A)$ that define local transition function $\sigma^{(2)}$ of such structure should be equal ***a***. Hence, here three cases are possible, namely: ***(1)*** $\sigma^{(2)}(x,x)=0,\ x \neq 0$, ***(2)*** $\sigma^{(2)}(x,y) = \sigma^{(2)}(y,x)=0$ $\{x \neq y\ \&\ x+y \neq 0;\ x,y \in A\}$ or ***(3)*** simultaneously both cases. It is easy to make sure that at the done *premise* the structure will possess the *nonconstructability* such as ***NCF–1*** if the structure does not possess the nonconstructability such as ***NCF,*** because the set $C(A,1,\infty)$ for it will be nonclosed relative to parallel mapping $\tau^{(2)}$ defined by the local transition function of the structure. Hence, an arbitrary classical structure ***1–HS*** of the above type possess the nonconstructability such as ***NCF*** and/or ***NCF–1.*** On the other hand, for classical structures ***1–HS*** with simplest neighbourhood index ***X*** and asymmetric local transition functions this result generally speaking is incorrect.

On basis of this result can be shown, the result of theorem ***133*** already for rather general methods of modeling is unimprovable from point of view of reducing to simplest neighbourhood template of a modelling classical structure ***1–HS*** with *symmetric* local transition function. More precisely, in contrast to theorem ***133*** any classical structure ***1–HS*** can not be modeled by means of an appropriate classical structure ***1–HS*** with symmetric local transition function and simplest neighbourhood index $X=\{0,1\}$ at use of rather wide class of the modelling concepts. In the heart of the given affirmation the fact is put, that in conditions of a lot of the modelling concepts such properties of the modelled classical structures as presence/absence of the nonconstructability such as ***NCF (NCF–3), NCF–1*** along with ***MEC*** and ***γ–CF*** are kept. Furthermore, the given result is used at studying a series of aspects of the *decomposition* problem of global transition functions of ***HS***–models. Thus, a classical ***1–HS*** is simulated by an appropriate structure ***1–HS*** with the simplest neighbourhood index and symmetrical local transition function but only at having the nonconstructability such as ***NCF–1*** and/or ***NCF.*** In addition, this result with all evidence once again confirms the earlier assumption [19,20] about more preferable constructive possibilities of modelling in the environment of classical structures ***d–HS (d ≥ 1)*** with asymmetric local transition functions [5,54–56,79,88,90,536,567,640].

At studying of questions of the universal computability and modeling of physical processes in classical structures the dynamics reversibility problem in such structures arises. A detailed enough discussion of the given problematics is represented in works [3,5,90,273,378,536]. In this connexion the question of interrelation of the properties of *reversibility* and *universal computability* in classical structures is a rather interesting. With this purpose we use the concept of «*reversibility*» of dynamics of classical structures in conformity with definition ***11*** (*chapter* **2**). In this connection the question about interrelation of the properties of *mutual erasability* and *universal* computability in classical structures is a rather interesting. Whereas a calculation generally speaking is a irreversible process then is seemed quite natural that existence in the structures of pairs of ***MEC–1*** and hence ***NCF*** and/or ***NCF–1*** should be closely linked with property of the universal computability in classical structures.

Meanwhile, already simple enough classical structures possessing the pairs of ***MEC–1*** do not admit the universal computability [5,88]. Thus, presence for a classical structure of ***MEC–1 (NCF*** *and/or* ***NCF–1)*** is not sufficient condition for possessing the universal computability. On the other hand, on basis of the above approach to definition of universal computability it has been shown that existence of ***MEC–1 (NCF*** *and/or* ***NCF–1)*** is a *necessary* condition in order an arbitrary classical structure ***1–HS*** possessed the given property on the assumption of use of finite configurations only [1,3,5,11]. This result in one's time has provoked a wide enough discussions and stimulated the further researches in the given direction [1,3–5,7,29,30,43,53-56,63,66,68,70,73,77,79,80,88,90,160, 184–187,258,259,263,268,273,314,318,536,617,618,640–643].

Meanwhile, on basis of rather other approaches ***K. Morita*** has proved existence of a reversible structure ***1–HS*** which simulates any structure ***1–HS,*** including irreversible structures, whereas ***J. Dubacq*** has proved an opportunity of simulation of the Turing machines by the reversible structures ***1–HS*** [536]. ***T. Toffoli*** has proved the fact of an opportunity of modelling of an arbitrary structure ***d–HS*** by means of a reversible structure ***(d+1)–HS (d≥1),*** having proved thus computing universality of the reversible structures ***d–HS*** $(d \geq 2)$ [268]. Whereas ***K. Morita*** and others have proved computing universality of the *reversible* structures ***1–HS*** [321,322]. Therefore, in connection with the told the question of modelling of irreversible classical structures ***d–HS*** by reversible ones along with the question of existence of the *reversible* universal classical structures ***d–HS*** $(d \geq 1)$ represents a special interest. At that, the above results have associated the reversibility mainly with absence of ***NCF.***

So, ***T. Toffoli,*** by working by results of ***Aladjev–Smith*** that are linked with the *computability* problem in the classical ***HS***–models, has shown, in spite of the results received by them, the *reversible universal* classical ***HS***–models exist. More precisely, ***T. Toffoli*** has proved, that a classical structure ***d–HS*** can be constructively embedded into some reversible classical structure ***(d+1)–HS,*** for which problem of generation of finite configurations of unlimited size is algorithmically solvable. At that, ***T. Toffoli***'s given result [430] does not contradict our results which have been received earlier and the specified situation is caused by a series of rather essential factors determining these contradictions; first of all, it is connected to use of different concepts of reversibility of dynamics of classical ***HS***–models whose essence is discussed enough in detail in the present monograph as well.

Completely other approach allows to model classical structures ***d–HS*** $(d \geq 1)$, including structures possessing the nonconstructability such as ***NCF,*** by means of classical structures ***(d+1)–HS*** that do not possess the nonconstructability such as ***NCF,*** allowing to formulate the following interesting enough result for a whole series of appendices.

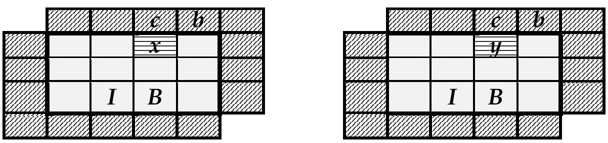
<u>*Theorem 134*</u>***. A classical structure d–HS (d ≥ 1) with an alphabet A is 1–modeled by means of a classical structure (d+1)–HS with the same alphabet; at that, the modelling structure (d+1)–HS does not possess the nonconstructability such as NCF and keeps also dynamics history of an arbitrary finite configuration of the modeled structure d–HS.***

We shall consider such approach, without loss of generality, on basis of classical structures ***1–HS.*** Let a certain ***1–HS*** is a classical structure with alphabet $A=\{0,1, ..., a-1\}$, simplest neighbourhood index $X=\{0,1\}$ and a local transition function $\sigma^{(2)}(x,y)=x^*;\ x,y,x^* \in A$. It is well–known that an arbitrary classical structure ***1–HS*** is simulated with time delay and expansion of alphabet ***A*** by an appropriate structure of the same class, the same dimension and with neighbourhood index $X=\{0,1\}$. In addition, let's define a classical structure ***2–HS*** which will simulate the structure ***1–HS*** with the same alphabet ***A,*** the simplest neighbourhood index $X^* = \{(0,0), (0,1), (1,1)\}$ and a local transition function $\sigma^{(3)}$ whose rules ***(GS)*** are determined by the parallel substitutions defined below.

$$S(x,y)_{t+1} = \sigma^{(2)}(S(x,y+1)_t, S(x+1,y+1)_t) \otimes S(x,y)_t$$

where: $\sigma^{(2)}(c,d)$ *– LTF of the modelled classical 1-HS;*
$c \otimes d$ *– computes* $c + d \pmod a$; $c,d \in A = \{0,1,2,...,a-1\}$ ***(GS)***
$S(x,y)_t$ *– a state of an elementary automaton with coordinates (x,y) at a moment* $t \geq 0$ $x,y \in \{0,\pm1,\pm2,\pm3,...\}$; $t = 0,1,2,3,...$.

First of all, we shall show, that the modelling classical structure ***2–HS*** determined by such manner does not possess the nonconstructability such as ***NCF.*** Indeed, otherwise according to the criterion of existence of the *nonconstructability* such as ***NCF*** (*theorem* ***16***) the above structure ***2–HS*** should possess pairs of ***MEC*** of the following general kind:



In the assumption of presence for the above classical structure ***2–HS*** of the pairs of ***MEC*** of the specified kind, we choose a pair of ***MEC*** of the rectangular form with minimal ***IB*** in which each corresponding sides (*without loss of generality we choose the upper side*) contain at least ***1*** pair of elementary automata that are being in various states $x \neq y$ $(x,y \in A)$.

Having chosen now in the given pair of ***MEC*** the most *right upper* pair of corresponding automata of ***IB*** in different states $x \neq y$ $(x,y \in A)$, it is simple to make sure that on basis of local transition function $\sigma^{(3)}$ ***(GS)*** of the modelling structure at the following moment of time t we again receive for the chosen pair of elementary automata different states; i.e. more precisely, the following relation takes place: $(\forall c, b \in A)(x \neq y \rightarrow \sigma^{(2)}(c,b) \otimes x \neq \sigma^{(2)}(c,b) \otimes y)$. Thus, the modelling classical structure ***2–HS*** with local transition function $\sigma^{(3)}$, determined by relations ***(GS)***, does not possess the pairs of ***MEC,*** what according to the above criterion of nonconstructability says that such structure also does not possess the nonconstructability such as ***NCF.***

It is simple to make sure that the above structure ***2–HS*** simulates each structure ***1–HS*** with state alphabet $A=\{0,1, ..., a-1\}$, the neighbourhood index $X = \{0,1\}$ and local transition function $\sigma^{(2)}(x,y) = x^*$; $x,y,x^* \in A$. By placing, for example, in string with coordinates $\{(0, j) \mid j = 0, \pm1, \pm2, ...\}$, any finite or infinite (*without loss of generality a finite*) a **1–**dimensional configuration $c=x_1x_2x_3 \ldots x_n$ $(x_1, x_n \neq 0;\ x_j \in A;\ j=1..n)$, it is easy to trace dynamics of its development under the influence of ***GTF*** $\tau^{(3)}$, defined by appropriate ***LTF*** $\sigma^{(3)}$ ***(GS)*** of a modelling structure. The following visual scheme illustrates the fact of modeling of an arbitrary ***1–HS.***

	...	0	0	0	0	0	0	...	0	0	0	...
t=0	c_o=	...	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
t=1	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	$x^1{}_o$	$x^1{}_1$	$x^1{}_2$	$x^1{}_3$	...	$x^1{}_{n-1}$	$x^1{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
t=2	...	0	0	$b^1{}_o$	$b^1{}_1$	$b^1{}_2$	$b^1{}_3$	...	$b^1{}_{n-1}$	$b^1{}_n$	0	...
	...	0	$x^2{}_{-1}$	$x^2{}_o$	$x^2{}_1$	$x^2{}_2$	$x^2{}_3$	...	$x^2{}_{n-1}$	$x^2{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
t=3	...	0	0	$b^1{}_o$	$b^1{}_1$	$b^1{}_2$	$b^1{}_3$	...	$b^1{}_{n-1}$	$b^1{}_n$	0	...
	...	0	$b^2{}_{-1}$	$b^2{}_o$	$b^2{}_1$	$b^2{}_2$	$b^2{}_3$	...	$b^2{}_{n-1}$	$b^2{}_n$	0	...
	...	$x^3{}_{-2}$	$x^3{}_{-1}$	$x^3{}_o$	$x^3{}_1$	$x^3{}_2$	$x^3{}_3$	...	$x^3{}_{n-1}$	$x^3{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...

==

$$x_j^1 = \sigma^{(2)}(x_j, x_{j+1});\quad x_j^{k+1} = \sigma^{(2)}(x_j^k, x_{j+1}^k);$$
$$x_j, x_j^k, x_j^{k+1}, b_j^k \in A;\ k = 1, 2, \ldots;\ j = 0, \pm1, \pm2, \ldots$$

Detailed enough description of the modeling algorithm that underlies proof of the given result can be found, for example, in [88,90,640]**.** It is necessary to mark, the result of theorem ***134*** along with the result of ***T. Toffoli*** determines a high enough price of such modelling **–** increasing of dimensionality of a modelling classical structure on ***one*** concerning the dimensionality of the modeled classical structure. Of the results of theorem ***126*** directly follows, that each classical structure ***d–HS*** $(d \geq 1)$ within of dynamics of finite and**/**or structural**–**periodic configurations is simulated by means of an appropriate classical structure ***1–HS*** with neighbourhood index ***X***={***–1,0,1***}**.** At the same time, the approach used at the proof, provides the modelling of dynamics only in the classical structures ***d–HS (d ≥ 1)*** and onto more general case of modelling does not spread. Thus, in view of the told along with the result of theorem ***134*** it is possible to formulate the following rather interesting offer.

<u>*Theorem 135*</u>***. Any classical structure d–HS (d ≥ 1) within of dynamics of the finite and/or structural–periodic configurations is simulated by means of a certain classical structure 2–HS that does not possess the nonconstructability such as NCF and has the simplest neighbourhood index X = {(0,0), (0,1), (1,1)}.***

At the same time, under the reversibility ***T. Toffoli*** along with a series of other researchers understand absence for a modelling structure of the mutual erasability ***(****the nonconstructability such as* ***NCF*** *and* ***NCF–3)*** while we understand also absence for the reversible structures of the nonconstructability such as ***NCF–1,*** the substantiation of such premise is submitted earlier. Whereas the approach used by ***T. Toffoli*** not only demands an increase in dimensionality of a modelling structure, but also does not free it from the nonconstructability such as ***NCF–1,*** not allowing to count dynamics of such modelling structure reversible in the full measure. Along with it, ***T. Toffoli*** for creation of the reversible ***HS–***models used also a structural approach, presenting an elementary automaton of a model by a simple logic circuit out of ***3*** elements [318]**.** Meantime, the analysis of the given approach shows that reversibility is reached owing to implicit increase in cardinality of a state alphabet and concerns to its some subset. Whereas the real reversibility relative to the expanded alphabet is not reached. On the other hand, we under ***real*** reversibility understand the *reversibility* of dynamics of a classical ***HS–***model concerning the set $C(A,d,\phi)$**.** Here again enough pertinently to discuss two levels of the ***reversibility – real*** and ***formal.***

So, under the ***formal*** *level* is understood the *reversibility* of an arbitrary finite configuration ***c,*** namely existence for the $c \in C(A,d,\phi)$ of such sole configuration $c' \in C(A,d,\phi)$ irrespective of the set $C(A,d,\infty)$ that relation $c'\tau^{(n)} = c$ takes place**.** Meantime, under the ***real*** level is understood the reversibility relative to the finite configurations only**;** i.e. existence for an arbitrary finite configuration ***c*** of such sole configuration c^* of the set $C(A,d,\phi)$ only, that the relation $c^*\tau^{(n)} = c$ takes place. Consequently, depending on the *criterion* of existence of the nonconstructability such as ***NCF*** basing on the concept of ***MEC*** or ***γ–CF,*** it is easy to make sure that the presence of *formal* reversibility can entail the *real irreversibility* whereas the converse proposition is generally speaking incorrect. The given circumstance is based on existence of two *nonequivalent* types of the *nonconstructability* in classical ***HS***–models – ***NCF*** and ***NCF–1*** which have been considered earlier along with the real reversibility concept according to the definition ***13.*** So, if for a formally reversible classical structure really reversible configurations exist, then similar structure can possess the nonconstructability such as ***NCF–1;*** i.e. the structure is *really* irreversible; on the other hand, at lack of the nonconstructability such as ***NCF*** for a certain classical structure, the structure quite can be really irreversible. Thus, each classical structure can't simultaneously possess the properties of formal reversibility and real reversibility, i.e. these properties as a whole are mutually exclusive ones. Evidently the real reversibility in classical structures entails the formal reversibility while converse sentence is false generally speaking.

One of motivations for introduction of concept of the real reversibility of dynamics of finite configuration in classical structures is provoked as well by a quite natural requirement that each single predecessor in a prehistory $\{c\tau^{(n)k} \mid k = -1, -2, \ldots\}$ of a configuration $c \in C(A,d,\phi)$ should be calculated during a finite quantity of steps. In particular, under the assumption of belonging of fully zero configuration c_o = « » to the set $C(A,d,\phi)$ of finite configurations the possessing by a classical structure by the nonconstructability such as ***NCF–1*** does its really–irreversible.

From the point of view of definitions ***6*** and ***10*** of two types of erasable configurations the following rather useful result can be presented that gives a criterion of **2** types of reversibility in classical structures [79].

<u>Proposal 9</u>. A classical structure d–HS is formally (really) reversible if and only if the structure not possesses the pairs of MEC (MEC–1); i. e. not possesses the nonconstructability such as NCF (NCF & NCF–1).

In connection with the told an interesting question arises: ***Whether can an arbitrary classical structure d–HS (d ≥ 1) be simulated by means of a reversible structure d–HS?*** In turn, this question brings up a series of accompanying questions, that in some extent describe the reversibility problem in classical ***HS***–models. Generally speaking, similar questions make up the modeling problem of arbitrary classical structures ***d–HS (d ≥ 1)*** by classical structures of the same dimensionality that suppress the given properties of the simulated structures. Relative to the formal reversibility characterized by presence of the nonconstructability such as ***NCF–1,*** the next basic result has been received, that plays essential enough part at researches of dynamic properties of ***d–HS*** [5,8,640].

<u>Theorem 136</u>. An arbitrary classical structure d–HS (d≥1) is 1-modeled by an appropriate structure of the same type with minimal expansion of a state alphabet A; in addition, the modelling structure d–HS does not possess the nonconstructability such as NCF–1, but possesses the nonconstructability such as NCF–2.

Detailed enough description of the modeling algorithm that underlies proof of the given result can be found, for example, in [5,8,88,90,640]. Much more difficultly affair gets on for case of the nonconstructability such as ***NCF*** that makes up together with type ***NCF–1*** the basis of the reversibility concept of classical ***HS***–models. Within of research of the given question, the concept of ***WM***–modelling covering a rather wide class of techniques of modelling of one classical structure by another structure of the same class and dimensionality has been defined. On its basis a result describing its opportunities for problems of modeling and useful in a series of theoretical researches has been received [53].

<u>Theorem 137</u>. A classical structure d–HS (d≥1) cannot be WM-modeled by means of an appropriate reversible structure (in sense of absence of nonconstructability such as NCF) of the same class and dimension.

Of the concept of ***WM***–modelling and the given result the conclusion directly follows, what for an opportunity of modelling of an arbitrary classical structure by means of a reversible structure of the same class and dimensionality it is necessary to use the coding methods of finite configurations for the modelled structure that admit infinite quantity of equivalent representatives for the modelling structure. That makes up a certain test of the first level for an admissibility of one or another way of modeling by structures with the above property of *reversibility*. Moreover, it follows that the known traditional methods of modelling in the classical structures covered with the concept of ***WM***–modelling, cannot lead to the necessary purpose, therefore new nonconventional approaches here are required. By us in process of the further research the concept of ***W***–*modelling* expanding essentially the concept of ***WM***–*modelling* and covering extremely wide class of known and potentially allowable modes of modelling in classical structures has been defined. At that, the modelling of classical structures by structures of the same dimension is considered. Meantime, and that has not allowed to solve positively in one's own networks the modeling problem by means of an appropriate classical *reversible* structure of the same dimensionality what the following basic result testifies [5,53–57,79,88,90,567,617,640], that represents quite independent interest.

<u>Theorem 138</u>. An arbitrary classical structure d–HS (d ≥ 1) can't be W–modeled by means of a reversible structure (in sense of absence of the nonconstructability such as NCF) of the same class and dimension.

Thus, even within of such general enough concept as ***W***–modelling it is impossible to model an arbitrary classical structure ***d–HS (d ≥ 1)*** by a *reversible* structure of the same dimension. Questions of distinctions of nonconstructability of types ***NCF, NCF-1, NCF-2*** and ***NCF-3*** in *classical* structures have been considered enough in detail above. Once more, if the modeling problem of the structures possessing nonconstructability such as ***NCF–1*** by structures without this type of nonconstructability is solved rather simply ***(****theorem* ***136),*** then in case of nonconstructability such as ***NCF (NCF–3)*** the question is a rather difficult, namely**:** at least within of two important enough concepts of ***WM***–modelling as well as ***W***–modelling which cover a broad enough spectrum of the modelling algorithms interesting both from applied, and theoretical standpoints, the given problem has the negative decision. In addition, the concepts of ***W***– and ***WM***–modelling in the best way conform to the algorithms of modeling of classical homogeneous structures by means of classical structures of the same dimensionality with suppression of existence of pairs of mutually–erasable configurations and **γ**–configurations in the simulating structures [54-56,79,88,90]. In addition, the following result having a series of interesting enough appendices takes place [8,79,88].

<u>Proposal 10</u>. Within finite configurations any classical structure d–HS (d≥1) can't be modeled by means of an appropriate classical structure not possessing the nonconstructability such as NCF and NCF–1, i.e. by an appropriate really–reversible classical structure that not possesses the property of universal computability and universal reproducibility in the Moore's sense of finite configurations.

So, concerning the modelling possibilities the classical structures that do not possess the nonconstructability such as ***NCF*** and ***NCF–1*** are not of any especial interest, composing in addition a narrow enough class. Meanwhile, possessing substantial breadth of coverage the modelling algorithms of the above *two* classes are far from being exhaustive ones therefore the further search of algorithms of modelling of irreversible classical structures by reversible structures is productive enough. For example, overstepping the limits of finiteness of a state alphabet, there is an opportunity to model an arbitrary classical structure by means of structures of the same dimension at absence of the nonconstructability such as ***NCF.*** The following result can be formulated in this direction.

<u>Proposal 11</u>. A classical structure d–HS is modeled strictly real time by a classical structure d–HS (d≥1) with an infinite state alphabet A^{∞} in the absence of the nonconstructability such as NCF.

Entirely other approach allows to model any classical structures ***d–HS (d ≥ 1)*** by means of classical structures of the same dimension but with solvable existence problem of the nonconstructability such as ***NCF–1*** and ***NCF.*** The detailed enough description of the modelling algorithm that underlies proof of the given result can be found in [79,88,90,640].

<u>Theorem 139</u>. An arbitrary classical structure d–HS (d≥1) is 1-modeled by means of appropriate classical structure d–HS* which has a simple neighbourhood index X and possesses the nonconstructability such as NCF and NCF–1; besides, the existence problem of nonconstructability of above types for the modelling structure is algorithmically solvable.

The result turns out rather interesting for a whole series of appendices including theoretical ones. Moreover this theorem absolutely does not contradict the result that generally speaking the existence problem of the nonconstructability such as ***NCF (NCF–3)*** for an arbitrary classical structure ***d–HS (d ≥ 2)*** is algorithmically unsolvable [5,88,90]. By using now the above results of ***K. Morita, J. Dubacq*** and of a series of others [8,9,321,322,536] together with the result of theorem ***126*** can be proved the following interesting enough result [90,567,617,618,640–643].

<u>Theorem 140</u>. An arbitrary classical structure d–HS (d≥1) is simulated by means of an appropriate formally–reversible structure 1–HS.

On basis of modelling of the universal Turing machine in the classical structures ***1–HS*** it has been shown that, generally speaking, a series of mass problems for them is algorithmically unsolvable ***(****see, for example, theorem* ***112).*** Moreover, ***A.R. Smith,*** being based on one concept of the *universal* computability in classical ***HS***–models has proved that for ***HS***-models possessing the property of *universal computability* the problems of ***limitation*** and ***passivity*** of sequences ***<c'>***$[\tau^{(n)}]$ are algorithmically unsolvable. Meanwhile, as a consequence of the proof of theorem ***139*** the result interesting enough for the subsequent research on dynamics of classical ***HS***–models easily follows.

<u>Theorem 141</u>. There are universal classical structures 1–HS that have neighbourhood index X = {–1,0,1} along with algorithmically solvable problem of limitation and passivity of an arbitrary sequence <c>$[\tau^{(3)}]$ ***of the finite configurations, where c∈C(A,1,ϕ).***

This contradiction in results is seeming, it is caused by distinctions of approaches to definition of the concepts of both the modeling, and the universal computability in classical structures. So, more precisely, the computability in ***HS***–models we can define or on basis of the theory of word recursive functions directly, or on basis of modelling of a certain known formal algorithms **(***Turing machines***,** *Post machines***,** ***SS***–*machines***,** ***TAG***–*systems***,** ***LAG***–*systems***,** *etc.***).**

Within of the general problem of modeling in *classical* structures ***d–HS*** ***(d ≥ 1)*** the question of modeling of *real* structures by means of classical structures represents a rather considerable applied interest. Under the ***real*** structure such structure is understood which differs from *classical* structure if its elementary automaton at transition in the next moment into a new state determined by local transition function, can pass and into some other state out of the same state alphabet, i.e. a new state of the automaton will generally differ from a state expected according to local transition function of the classical structure. Similar behaviour of the elementary automaton in a real structure can be explained by a lot of factors, namely**:** chance failure, automaton malfunction, emergency etc. Therefore, structures of the given type can be named the ***real*** with full grounds. They represent significant enough interest at research of some questions of practical realization of parallel computers on basis of ***HS***–models along with a lot of other important enough motivations.

A whole series of works has been devoted to investigation of the real ***HS***–models, more exactly, to the reliability problem of functioning of structures of the given type**.** The ***Nishio–Kobuchi*** approach is the most known technics of errors correction of functioning of real ***HS***–models, its base idea consists in simulation of work of an arbitrary elementary automaton of some real ***HS***–model by means of ***3*** corrective neighbour automata [319]**.** In this case a classical structure which simulates a real structure on basis of the information of ***3*** neighbours of an elementary automaton in the moment ***t > 0*** correctly determines a new state of the automaton in the next moment ***t+1.*** However, a choice function of the states in this case is also supposed quite correct.

The block coding whose essence consists in embedding of states of an elementary automaton into some coding block organized by a special manner is presented as the most natural approach for this purpose. It corresponds to replacement of a real ***HS***–model with a neighbourhood index by means of an appropriate modelling *classical* ***HS***–model whose neighbourhood template includes neighbourhood template of the real model whereas its organization and way of operation of a modelling

HS*–**model allow to restore infringements occuring in the *real* model**.** A series of considerations relative to principles of realization of classical structures correcting mistakes of functioning of *real* ***HS*–**models can be found, for example, in [3,90,567,640]**.** The approach suggested by us to the organization of reliable functioning of real ***HS*–**models constitutes one of problems of the general problem of modelling of one classical structure ***d–HS (d ≥ 1) by other structure of the same dimension with suppression of a certain property of a modelled structure in context of absence of the given property for a modelling structure.

The *reliability* problem of functioning of structures of such type which consist of *real* elementary automata in some ways concerns the *general* problem of modeling in classical structures. Till now it was supposed that the structures of ***d–HS (d ≥ 1)*** represent especially abstract model, while in the *real* conditions an operation of structures can be subject to infringements of different sort what entails the extremely undesirable consequences. In the given connexion there is the important enough problem of such organization of an arbitrary ***HS*–**model which would allow to correct for many important cases possible failures arising in process of functioning of the real computing ***HS*–**models.

We shall name a structure the ***self-correcting*** structure, if the structure during own functioning has possibility to eliminate consequences of failures in operating of the elementary automata and the information channels connecting them. It is natural that for the objects such as ***HS*–**models consisting of infinite number of elementary automata, systems of switching of any elementary automaton with its direct neighbours, determined by the neighbourhood index and complex local transition functions a big enough variety of malfunctions arising at functioning of the real ***HS*–**models is quite real. Here, we shall consider only two most important classes of malfunctions arising in the real ***HS*–**models**:**

♦ *malfunction* at definition of the next state of an arbitrary elementary automaton of a certain real ***HS*–**model**;** i.e. malfunction of functioning of its local transition function**;**

♦ *malfunction* at gathering the information about states of elementary automata of a certain real ***HS*–**model that compose its neighbourhood template**;** i.e. malfunction in means of switching of the ***HS*–**model.

Considering properties of the ***HS*–**models on behavioural ***(dynamical)*** level, instead of structural level, we quite can limit oneself only to two given types of malfunctions that to some degree are abstraction of real conditions. Moreover, in view of the principle of functioning of a ***HS*–**

model when for reception of the information about a configuration of neighbourhood template a time is not spent we quite can limit oneself to consideration only of the first type of malfunctions. In general case in the real ***HS***–models during their functioning a whole series of other malfunctions can arise however we put into the term «***real HS-model***» only the above sense. Meanwhile, analysis of malfunctions of a series of other types is a rather complex and important problem demanding the detailed development at practical elaboration of ***HS***–models [536]. We shall introduce now the ***reliability*** concept of a real ***HS***–model.

<u>Definition 25</u>. An arbitrary real structure d–HS (d ≥ 1) will possess the reliability (1–1/h^d) if in each d–dimension hypercube with edge of size h no more than one elementary automaton may be subject to different malfunctions at the same moment.

It is obvious that at boundary values ***h=∞*** or ***h=1*** we deal with reliable ***(classical)*** structure and unreliable ***(real)*** structure accordingly. While all other intermediate values ***h*** give *real* structures of a various level of reliability. Certain methods of the organization of functioning of real ***HS***–models doing their by the self-correcting computational structures have been offered. So, one of similar methods allows to represent the following interesting enough result [53–56,567,617,618,640–643].

<u>Theorem 142</u>. For a real structure d–HS of reliability (1-1/h^d) (d≥1,h≥3) with an alphabet A there is an appropriate self-correcting structure of the same dimensionality with alphabet A∪{F} and a global transition function $\tau^{(q)}\tau^{(p)}$, which 2–simulates the first structure, where $\tau^{(p)}$ is a reliable corrective function with neighbourhood index of Moore and F is a certain marker state.

The correction problem of real structures enough essentially becomes simpler if to assume that occurrence of *failures* in individual automata of a structure is identified by them themselves by transition into some signaling state ***g∉A.*** At such assumption it is possible to considerably simplify coding of states of some modelling structure together with a corrective function $\tau^{(p)}$. Interesting enough possibilities for support of correction can be received at the assumption that individual automata in a ***F***–state operate without failures. In this case the states of automata of a real ***HS***–model are coded by means of linear configuration in the form «***xxF***» ***(x∈A, F∉A).*** Thus, other things being equal the elementary automaton of the structure should not be provided by the property of identification of failures. This approach allows to simplify algorithm of functioning of each elementary automaton of a modelling structure what allows to formulate the following useful enough result [53,640].

***Theorem 143*. For a real structure d–HS of reliability (1-1/h^d) (d≥1,h≥3) with an alphabet A there is an appropriate self-correcting structure of the same dimensionality with alphabet A∪{F,g} and global transition function $\tau^{(q)}\tau^{(p)}$, which 2–simulates the first structure, where $\tau^{(p)}$ is a reliable corrective function with 1–dimensional neighbourhood index of Moore and F, g are a marker state and signaling state accordingly. Under the condition of reliability of functioning of automata in state F for an arbitrary real structure d–HS of reliability (1–1/h^d) (d≥1,h≥3) with an alphabet A there is an arbitrary self–correcting structure of the same dimensionality with alphabet A∪{F} and a global transition function $\tau^{(q)}\tau^{(p)}$ which 2–simulates the first structure where a reliable function $\tau^{(p)}$ has 1–dimensional neighbourhood index X={–2,–1,0,1,2}.**

The further researches of the *self–restoration* problem of real structures of different type represent significant enough applied and cognitive interest, and to the given direction the corresponding attention should be paid. Researches on stability of the *real* ***HS***–models to failures of the various kind can be attributed to this direction too. In this respect the structures out of so–called class of ***threshold*** structures, for which the local transition functions are carried out by the principle of a *threshold* element represent quite definite interest. The local transition functions of such type already in principle of own functioning contain a rather significant element of reliability, i.e. stability to possible failures. May be, it – one of the reasons of *high reliability* of functioning of real *neuro*–like structures of various nature [536]. Discussion of a whole series of practical conclusions out of the presented results and most interesting themes of research on *reliability* of real structures can be found in [567, 640]. Above all, the determination of the most effective self–correcting structures relative to various concepts of failures is of interest. At that, satisfactory enough solution of the reliability question of computing elements, devices and systems on basis of the ***HS***–models has the great applied significance too.

Meanwhile, the above approaches not only carry the general formal character, but also allow to consider the reliability problem of cellular systems of a various sort and nature from the various formal points of view. So, the suggested receptions of correction of the real ***HS***–models present a certain, most likely, theoretical interest, carrying features of the general base approach but for the purposes of *practical* application they are insufficiently effective by the required resources. Hence, for practical problems it is necessary to develop more effective methods of correction of failures, i.e. increasing of reliability of functioning of the real ***HS***–models and concrete devices realizable on their basis. So, in practical realizations of computing ***HS***–models with purpose of the reliability control, the *structural* approach whose base essence consists in providing of the elementary automaton and, perhaps, of switching system of neighbourhood template of the real ***HS***–models by special correcting logic circuits is represented as the most natural. These logic circuits on basis of the input information and the current state of each elementary automaton should have a possibility enough effectively to carry out the local analysis ***(****testing****)*** of reliability of the automaton and if necessary to carry out the corresponding diagnostic or a correcting procedure. For these purposes the available results on self–correcting codes can be quite successfully used. Here we have rather wide field of activity**;** and some useful enough ideas can be used also out of the above theoretical considerations.

Results on *self–correcting* ***HS***–models can appear fruitful not only from the standpoint of computing sciences, but also in a context of research of mechanisms of restoration that take place at appearance of various damages in real biological cellular structures. Other rather interesting interpretations in the given direction also are possible. At present, on account of the above reasons to the problem of elaboration of the self–recovering ***HS***–models is paid considerable enough attention and first of all in connexion with engineering on their base of different objects and devices with use nanotechnology [536,618,640–643]**.**

6.6. Software simulation of homogeneous structures

In spite of such extremely simple concept of the *classical* homogeneous structures, they have, generally speaking, complex enough dynamics. In very many cases theoretical research of their *dynamics* collides with essential enough difficulties. Therefore, computer simulation of these structures that in the empirical way allows to research their dynamics is extremely powerful tool. For this reason the given question is quite natural for a considering within the present section. The discussion of the question represented below will carry schematic enough character while its detailed enough discussion can be found in [79,90,567,640]**.**

At present, the problem of computer modeling of structures is solved at two basic levels: ***(1)*** *software modelling dynamics on computing systems of traditional architecture*, and ***(2)*** *hardware architecture which as much as possible corresponds to the **HS**–concept; so-called **HS**-oriented architecture of computing systems*. Computer simulation of ***HS***–models plays a rather essential role at theoretical researches of their dynamics, however it is even more important at practical realizations of ***HS***–models of various processes. By present time a whole series of rather interesting systems of software and hardware for the help to researchers of different types of ***HS***–models has been developed [5,54–56,79,88,90,536,617,618,640].

J. von Neumann and ***S. Ulam*** perhaps were the first who have awared of large possibilities of computer modeling. In particular, ***S. Ulam*** has considered heuristic use of computers for a series of rather interesting applications. Obviously, ***S. Ulam, J. Holladay*** along with ***R. Schrandt*** are pioneers of effective computer simulation of ***HS***–like models [431–434]. These authors used rather powerful computers for generation of huge number of configurations and have considered a whole series of properties of their morphology in space and time. Majority of results in this direction are empirical however as yet there are not too many general opportunities which can be received theoretically. The similar picture takes place for ***HS***–models as a whole, thus software modeling becomes one of the basic research methods in ***HS***–problematics and of especially applied directions of this type of dynamic objects [536,640].

The subsequent stage in development of the computing approach to research of ***HS***–like models goes back to researchers who have created the first hardware–software system for heuristic investigations of the ***HS***–models [128]. ***R. Brender*** was, maybe, the first, who has developed the programming system for simulation of ***HS***–models [174]. The next development of the computer approach is characterized by creation of a plenty of program systems of different character for simulation and empirical research of different dynamic aspects of ***HS***–models. So, in our works many programs in environment of different programming languages for different computer platforms has been represented [8,9, 15,19,20,54–56,90,94,98,103,118]. In particular, means of mathematical system ***Mathematica*** support *algebraic* substitutions rules which easily enough model the local transition functions of the classical structures ***1–HS*** [93]. In this context many interesting programs for simulation of ***HS***–models in the environment of system ***Mathematica*** can be found, for example, in the book [167]. On the basis of computer simulation a whole series of interesting enough theoretical results on the theory of classical ***HS***–models and their applications in such fields as computer sciences, mathematics and developmental biology has been received.

In addition, it is necessary to note four basic prerequisites of computer modeling of classical homogeneous structures, namely: ***(1)*** monitoring and illustration of dynamics of the initial finite configurations during generating from them of sequences of configurations in environment of a ***HS***–model, ***(2)*** receipt of kind and other interesting estimations of concrete objects in a ***HS***–model ***(****minimal size* ***of MEC, NCF, NCF–1*** *and their form, etc.****)***, ***(3)*** investigation of dynamics of modeled structure for formulation of hypotheses in this or that direction, and ***(4)*** research of complex models basing on concept of homogeneous structures, which represent direct applied interest from many standpoints. In a number of cases the computer modeling had allowed to formulate interesting enough problems in the ***HS***–problematics, and also to draw up a plan of solving of some of them [5,11–13,19,20,48-50,52,54-56,79,88,90,640].

So, we can mark such theoretical results as types of mutually–erasable configurations in classical structures ***1–HS***, certain dynamic aspects of ***HS*** with refractority, *Steinhaus* combinatory problem, etc. [3,4,19,20,54] while out of especially applied ones it is possible to mark such results as differentiation, regulation and regeneration of cells, the *French flag's* problem, regulation of axial structures, etc. [1,4,17,26,27,31]. At that, in these works the questions of development of effective software means of research of ***HS***–models are discussed; in works [212,435,536] also it is possible to find the detailed enough discussion of similar questions. At present the given computer approach to researches of dynamics of the ***HS***–models is characterized by three basic directions, namely:

(1) creation of the specialized programming languages for an effective embedding of structures ***d–HS (d ≥ 1)*** of various types along with their practical realizations into an appropriate computing environment;

(2) creation of programs providing computer simulation of structures ***d–HS (d ≥ 1)*** of the special types describing those or other processes;

(3) creation of program systems and complexes providing computer simulation of a wide enough class of structures ***d–HS (d ≥ 1)***.

In this context on one essential enough moment it makes sense to stop separately. It is supposed the base of researchers of various aspects of homogeneous structures consists of experts in natural fields and first of all mathematicians and physicists which in the right degree not all possess by the programming. Meanwhile, the given situation becomes greatly simpler in view of existence enough developed systems of the computer mathematics whose undoubted leaders – ***Mathematica*** and

Maple [631-636]. The vast majority of the modern mathematicians and physicists to some extent use these or similar systems for professional activity; furthermore, the mastering of programming in these systems shouldn't represent especial difficulties. Therefore similar systems can serve as software environment for creation of the software oriented on the experimental study of various classes and types of homogeneous structures. Complexity of such software can fluctuate in wide enough ranges depending on the studied objects and their dynamics.

In particular, for experimental study of dynamics and other aspects of both classical, and some other types of the homogeneous structures we substantially used the software environment of the above *Mathematica* and *Maple* systems. On that ground, a rather large set of the software tools solving different problems of experimental research of classical homogeneous structures of dimension ***1*** and ***2*** was created. The given tools bear both highly specialized, and the more all-purpose character; they are arranged or as separate programs, or grouped into the special libraries; some of them are included in universal libraries for ***Maple*** or packages for ***Mathematica*** [637,638,641]. Many of them is presented in our books, scientific reports and preprints on the mathematical theory of homogeneous structures and their appendices [1,3,5,6,9,11,12,54–56, 88,640–643]. At that, several examples of such means are given and in the present book, illustrating some possibilities of experimental study of dynamics of ***1***–dimensional classical homogeneous structures.

Our early software for experimental study of dynamics of the classical homogeneous structures have been developed in the environment of such languages as ***Assembler, Basic, Pascal, PL/1*** and ***Reduce,*** then in the environment of such systems as ***Maple*** and ***Mathematica.*** The late ***10*** years we use only these two systems for the above purposes. At the same time both systems are of the same class, but in most cases for the problems of experimental study of classical structures, in our opinion, ***Mathematica*** system is more preferable. First of all, more developed software along with significantly most prolongation of programming language is the main reason of such conclusion.

Of means of the ***first*** group we can mark the following software. First of all, once more the ***R. Brender's*** programming system can be marked which was intended for simulation of the classical ***HS***–models. Much interpreters has been created for simulation of ***HS,*** in particular, of ***R. Vollmar*** [1,5,253], *SIBICA* of ***J. Pecht*** [250], etc. However, the software support of practical realizations of computing systems on basis of ***HS***–models in their purest kind, above all, from the practical point of view is represented more interesting. So, for software of ***ML***–processors the Hungarian researchers have created the language *InterCELLAS* which is well adapted to simulating of cellular automata [243]. In the works [169,175] it is possible to familiarize oneself with architecture of such ***ML***–processors and their software.

In turn, software of machines *CAM* is supported by a special subset of language *Forth* (*CAM Forth*), providing above all determination of the local transition functions. Hither it is directly possible to attribute the program *CAMEX* that covers rather extensive collection of ***HS***–models of dimensions ***1–3***. While *CAM*–simulator created at university *ELTE* (*Hungary*) represents the program simulating the cellular automata on base of *CAM–6*. At the general level with *CAM* and their software it is possible to familiarize oneself in the excellent book [150], whereas for details is recommended to address to works [151,152,165,376,394,430].

Language of parallel substitutions **(*LPS*)** can be a good enough example of a rather convenient algebraic means for description and analysis of the parallel microprograms. Meantime, the ***LPS*** is a certain linguistic formalism of a narrow enough class of the computing ***HS***–models that is based on the concept of *systems of parallel substitutions* defining local transition functions for the classical ***HS***–models. We have shown, that concerning the systems of parallel substitutions the following general result takes place, namely: ***The consistency problem of algorithms of parallel substitutions which are determined by means of an arbitrary system of parallel substitutions is constructively solvable*** [54–56].

The given result has allowed to receive a series of interesting enough consequences, including the applied ones for development of certain control microprogrammable systems [54–56,79,88,90,536,567,617,640].

The Italian researchers have elaborated the high–level programming language *CARPET* with a whole series of additional constructions for the description of local transition functions of a cellular environment. Language *CARPET* [428] provides a rather effective support of *parallel* information processing in the computing ***HS***–models. Next language *Cellang*, created by ***D. Eckart***, can be considered as a rather interesting software for simulation of ***HS***–models [436]. *Cellang* – the specialized language for a programming of a wide enough class of ***HS***–models. In our opinion, the *Cellang* – a rather useful programming system that is oriented, first of all, the computational structures ***d–HS (d=1..3)***.

Of other interesting enough programming languages of ***HS***–models it is possible to mark such languages as *CEPROL* [438], *CELIP* [439], *CAL*

[440] and some others. So, *CDL*–language for cellular processing along with language *ALPACA* for descriptions of the ***HS***–models represent a certain interest [536]. The specified software is intended for empirical research of a lot of the important dynamic aspects of the classical ***HS***–models. With its help many interesting enough results of both applied and theoretical character have been received [536,567]. Our experience received in process of both creation, and use of software of the above type have allowed to formulate the general concept of an interactive program system *SVEGAL*, which is intended for computer research of ***HS***–models of a broad enough set of types [78,85]. Unfortunately, the given system both for subjective and objective reasons remains up till now only at a level of the conceptual design [8,54–56,79,88,441–442].

The ***second*** group of means for today is most numerous and allows to experimentally investigate and to conveniently visualize the required dynamic properties of the ***HS***–models for concrete applications. At the same time the majority of means of this group serves as for simulation of known game «***Life***» or its various modifications and extensions. At present, the game «***Life***» is probably the most popular example of the ***HS***–models along with good enough object for computer modelling [5, 80,168,172,239,444,536]. Of software for simulation of the game «***Life***» such programs as *Life32* of ***J. Bontes***, *WinLife* of ***J. Harper***, and *LifeLab* of ***A. Trevorov*** can be mentioned. Themes of the game «***Life***» is rather extensive, the interested reader can receive rather full information on it in [536], and in the Internet by a key phrase «*Conway's Game of Life*». Many researchers for an experimental research of the ***HS***–models have used interesting enough programs out of which it is possible to mark means represented in works [5,280,329,446,447]. As simple simulators of ***HS***–models it is possible to mark such programs as *CAPOW*, *LCAU*, *CALAB*, etc. [366]. Into the operating system *Mandriva Linux 2008* a lot of software for work with ***HS***–models is included. We likewise to a lot of other authors used the special programs developed in environment of such known programming languages as *C+*, *Basic*, *PL/1*, *Pascal* and *Reduce*, and also in environments of known computer algebra systems *Mathematica* and *Maple* with the purpose of empirical research of a lot of concrete dynamic and applied aspects of classical structures ***d–HS (d=1 .. 3)*** [5,8,9,11,15,54–56,96,102,112,118,536]. In particular, a series of procedures for investigation of certain special aspects of dynamics of structures ***d–HS (d=1,2)*** is included in library [545], which in accord to site '*www.**123**–free–download.com*' is more popular than ***78%*** of another software from category '*Components, Libraries*' and more popular than

70% of another software from high level category *'Software Developer'*.

At last, means of the ***third*** group provide computer investigation of a rather wide class of ***HS***–models. So, here we can mark very interesting simulator *MCell* of ***M. Wojtowicz*** that is oriented a wide enough class of types of ***HS***–models of dimension ***1*** and ***2*** [448]. Of other simulators of the similar type it is possible to mark such simulators as *SARCASim* [450], *CelLab* [449], *CASE* [451], etc. So, program *Dr.Cell* is intended for research of dynamics of structures ***1-HS*** and ***2-HS*** with local transition functions and neighbourhood templates defined by the user. Program *SRCA* allowing to research dynamics of ***HS***–models for a lot of rather complex local transition functions characterized by a large cardinality of a state alphabet of a model ***($a \geq 2^{32}$)*** represents the definite interest. Quite certain interest the program *CABuilder* of creation of structures ***3–HS*** simultaneously with program *CAReader* that allows to carry out a satisfactory analysis of the structures created by the first program is represented. A full enough information on the specified means can be found in the extensive bibliography [536,640–643].

Simulation system *Trend* **&** *jTrend* is intended for modeling in classical structures ***2–HS*** of *self–reproduction* process, i.e. the basic component of any alive system. The system has an advanced enough opportunity of tracking of *reverse* dynamics of a ***HS***–model; perhaps, the system – one of the most opportune simulators of structures ***2–HS***. With description of these and a lot of other rather interesting ***HS***–simulators the reader can familiarise oneself in works [452-454,536]. The appreciable interest represents also package *CAT* intended for research of some features of the programming paradigm [536] on parallel computers basing on the concept of computing ***HS***–models. *CAT* can serve also as a convenient enough tool for creation of different ***HS***–models. At last, for empirical research of ***HS***–models a certain interest can present package ***DDLab*** oriented onto researches of dynamics of finite binary networks – from binary ***HS***–models up to probabilistic boolean networks. A number of both typical, and special supplements of ***DDLab*** for research of binary ***HS***–models and networks along with the related questions the reader can find in works [269,279,455,456,484]. For computer research of the ***HS***–models by a series of other researchers the program simulators of those or other purposes and a level of complexity have been produced [54–56,79,88,90,167,174,185–187,243,250,253,255,329,399–401,536,640].

A new approach to computer researches of ***HS***–like models consists in creation for their description of effective enough linguistic, i.e. highly–

parallel programming languages. However, in the given direction the rather complex problems of organization and description of parallel information processing in similar ***HS***–models arise**;** they are discussed, for example, in [90,567,640]**.** The carried out by us analysis shows that proceeding to highly–parallel calculations supposes a serious enough revision of many our traditional approaches to calculations. Here, the complexity of a paralleling procedure is defined, naturally, directly by internal specific features of some parallelizable algorithm. Multitude of very interesting examples of paralleling is represented in numerous works, for example, in [11,12,15,36,40,48,49,54,202,357,459–462,536]**.** A lot of rather interesting problems concerning the *paralleling complexity* of algorithms, including algorithms in computing ***HS***–like models, is discussed in works [465,466]**,** and also in a whole series of articles and transactions of conferences [463–467,536]**.** Similar questions have been discussed in our works [79,90,567,617,618,640–643] too.

Consequently, the theory of parallel algorithms should have structure substantially similar to structure of the theory of *sequential* algorithms. Theoretical aspects of the parallel algorithms are researched a little bit better, whereas work on applied aspects is in a great extent in a stage of becoming [8,202,356,365,372,470–474]**.** Now, the given field rapidly enough develops, and many new appendices discover crossings with a lot of the important scientific disciplines [11,202,353,356-358,361-363, 367,373,460-463,475,476,536]**.** Thus, a formal parallel computing model demands the specialized linguistic means which with the maximally possible efficiency should describe a paralleling level admitted by an arbitrary model of calculations or information processing as a whole. The linguistic means for the modern commercial parallel computing systems based on various formal computing models **(***excepting* ***HS–like*** *models***),** and also prospect of their development are discussed enough in detail in a lot of works, in particular, in [5,11,12,96,369,477–482,536]**.** Now we have very great number of hardware and software systems, intended for parallel processing. The given systems use different both parallel hardware architecture, and parallel software [5,11,12,13,48–50, 52,353,356-359,361-363,365,367,369,372,373,460-462,470,471,473,474]**.** In our works [9,42,52] the *Parallel Control System* and the *Parallel System of Information Processing* for homogeneous computing systems have been represented**;** it is shown that on basis of these systems the information paralleling allows to create data processing systems of high efficiency right up to the systems with direct economic effect.

In work [51] the description of the above *Parallel Control System* **(*PCS***) in terms of *systems of algorithmic algebras* ***(SAA)*** has been represented. This representation is based on an automaton model of the ***PCS*** and allows to use the ***SAA*** for optimization of the ***PCS*** software. At that, the above works at the most general level have used the ***HS***–concept as a formal parallel model of calculations. Meanwhile, a series of the parallel software uses parallel algorithms inherent in the computing ***HS***–models. So, for example, the above ***PCS*** uses a whole series of the parallel algorithms inherent in a biological ***HS***–model [26], and which are based on the principle of globally–local action [23]. An interesting enough approach to use of *highly–parallel* ***HS***–algorithms can be found in [459]. At present, there is a series of parallel software oriented onto manifold parallel architectures along with a series of highly–parallel programming languages, oriented, first of all, onto the computing ***HS***-models. These languages form both linguistic means, and a toolkit for research of a wide enough class of parallel computational ***HS***–models [54–56,79,88,241,243,250,428,438–440,453,483,536,567,617,618,640].

The practical realization of the computing ***HS***–models occupies here a special place. So, group of the *Hungarian* researchers under leadership of ***T. Legendi*** in process of work on cellular processors has essentially simplified and modified the cellular model of ***J. Neumann*** and of a lot of his followers [124,128,169,170,175,288–291]. The further researches in this direction have led them to creation of the practical realizations of the computing ***HS***–model such as «*Legendi's cellular processors*», next of cellular ***ML***–coprocessors for ***IBM***–compatible personal computers [171,175]. The long-term cooperation of research groups of ***R. Vollmar*** *(Germany)* and ***T. Legendi*** *(Hungary)* became the reason of creation of both functioning commercial models of cellular processors, and their satisfactory theoretical researching. Within of team–work on the given specified problematics a whole series of widely known international conferences *PARCELLA* ***(PAR****allel* ***CELL****ular* ***A****utomata)* was conducted, as well as collections of articles and scientific reports were issued [156, 165,179,308,536]. In the *PARCELLA* proceedings a series of interesting enough results relative to the specific computing architectures as well as separate devices based on ***HS***–models is submitted [464-467]. Of the Soviet practical works on realization of the computing ***HS***–models it is possible to mark interesting enough results of groups of researchers from Novosibirsk, Saint–Petersburg, Taganrog, Moscow and Kishinev [8,15,176–178]. Perspective enough works executed in Taganrog under leadership of ***A.V. Kaljaev*** and in Novosibirsk under leadership of ***O. L. Bandman*** represent the especial interest [555].

Another interesting and perspective approach to practical realization of the computing ***HS*–**model ***T. Toffoli*** and ***N. Margolus*** have offered, which have created a series of so-called «*Machines of Cellular automata*» ***(C****ellular* ***A****utomata* ***M****achines* **– *CAM)*** [150-152,165,318,376]. The popular description of the ***CAM*** along with experiments with them the reader can find in the fine book [150]. Practical application of machines ***CAM*** appeared a rather effective at modelling of complex enough problems of hydrodynamics, ecology and research of mathematical properties of ***HS*–**models, of models of images processing, physical modeling, of generating of special effects and a lot of others [5,166,366,384,392,394]. Thus, a series of machines ***CAM*** has carried over the ***HS*–**problematics onto qualitatively new unique level, having added formal ***HS*–**models by their direct computing analogues.

By present time a great number of other interesting enough practical realizations of computing ***HS*–**models exists, from them we shall mark only several [410,436,485–495]. All abundance of materials on applied problematics of computing ***HS*–**models does not allow to present their any satisfactory analysis. Meantime, to the interested reader it would be possible to recommend the proceedings of such conferences as [51, 360–363,473,476,479,480,493] and the similar conferences as well as the numerous articles in various periodicals of the corresponding themes [371,536]. Furthermore, owing to a great successes of microelectronics and prospects of use of nanotechnology a lot of authors already now offer practical approaches to realization of very large-scale integration on basis, above all, of the concept of ***HS*–**models which can be created in the near future. In view of development of technology of integrated circuits and new approaches to creation of perspective architecture of the computer facilities ***(****quantum cellular processors****,*** *nanocomputers****,*** *etc****.)*** with wide use of nanotechnology the interest to the ***HS*–**problematics steadily grows. The themes connected to the *cellular logic* which deals with mathematical models and technics for the analysis and synthesis of digital networks on basis of computing ***HS*–**models adjoin directly to the considered questions too. However, in view of extensiveness of the given theme its consideration is beyond the present monography. For more full acquaintance with such problematics are recommended sources [496,536] where the most full bibliography concerning the ***HS*–** models, parallel calculations and information processing is submitted, along with the related themes, including books, articles, journals, etc. Considering rather essential complexity of theoretical research of ***HS*–** models, in their research the computer modeling plays serious part.

Chapter 7. The decomposition problem of global transition functions in the classical HS–models

The problem of *decomposition* of *global transition functions* ***(GTF)*** for the ***HS***–models presents the considerable enough theoretical and applied interest. The main goal of decomposition of global transition functions in ***HS***–models consists in finding of effective procedures, that allow on basis of the given global transition function to define a composition of more simple functions that is equivalent to the initial global transition function. The given problem is similar to the problem of partition of a complex system onto more simple subsystems, presenting rather large interest for many directions of the ***HS***–problematics. In particular, this problem directly adjoins to the above problem of *complexity* ***A(X).*** The problem has the most direct attitude to the questions of constructive complexity playing a rather important part also in concrete realization of ***HS***–models of different sort and destination.

The first posing and results concerning the decomposition problem go back to ***S. Amorozo*** and ***J. Epstein*** which have shown that in the set of all *binary* ***1***–dimensional global transition functions there are functions not representable in the form of composition of finite number of more simple functions of the same type and class [316]. Using rather simple numerical procedure, ***J. Buttler*** has shown that in the set of all global transition functions of ***d***–dimensionality ***(d≥1)*** also there are functions not representable in the form of so–called ***minimal*** *composition* of finite number of more simple global transition functions out of the same set of functions [141,142]. In some our works the decomposition problem has received the further development [3–5,8,9,11,53–56,90]; the results received in this direction have allowed to consider the problem from new rather interesting standpoints considered a little bit below.

Above all, the decomposition problem in a certain extent concerns the complexity problem of global transition functions, namely:

Whether can an arbitrary global transition function be represented by means of some composition of finite number of more simple functions out of the same class and in the same state alphabet?

At that, below we shall speak, that a global transition function $\tau^{(n)}$ is *simpler* than a global function $\tau^{(m)}$ ***(****both functions are defined in the same finite alphabet and have identical dimensionality****)*** if $n<m$; $n<m$ determines relation between quantity of the elementary automata making up the

neighbourhood templates of both structures. Meantime, it turned out that such problems as ***complexity*** of finite configurations, *completeness* for polygenic structures along with ***decomposition*** of global transition functions are enough closely connected, presenting a rather extensive and perspective field for the further researches [5,8,9,54–56,88,90,536].

Our first results concerning the *decomposition* problem were based on earlier results on the *nonconstructability* in *classical* structures and have allowed to solve this problem for classical structures ***1–HS*** [41,44]. The basic result in this direction was the proof of existence of **1**–dimension global transition function with an arbitrary neighbourhood index and state alphabet for which the decomposition problem has the negative decision. Meanwhile, for the further discussion of the decomposition problem of global transition functions, we need certain new concepts and definitions introduced as required.

Definition 26. In toto, the decomposition problem of global transition functions (d–PDF) $\tau^{(n)}$ is reduced to the question of an opportunity of representation of an arbitrary global transition function in the form of composition of finite number of more simple functions of the same class and in the same state alphabet $A = \{0,1,2, ..., a-1\}$, namely:

$$\tau^{(n)} = \tau^{(n_1)}\, \tau^{(n_2)}\, \tau^{(n_3)} \ldots \tau^{(n_m)} \qquad (n > d+1;\ n_j < n;\ j = 1..m) \qquad (31)$$

where the global transition functions $\tau^{(n)}$, $\tau^{(n_j)}$ (j=1..m) have identical dimensionality and are defined in the same state alphabet A; at that, for global transition functions $\tau^{(n_j)}$ multiple occurences are admitted in representation (31). In case of 1–dimensional functions $\tau^{(n)}$ for the decomposition (31) the relation $n = \sum_j n_j - m + 1$ (j = 1..m) takes place.

At that, it is necessary to pay attention to spurious misunderstanding which can arise from the fact of negative decision of the ***d–PDF***, on the one hand, and *universality* of the classical ***HS***–models with the *simplest* neighbourhood indexes, on the other hand. Really, any structure ***d–HS*** is simulated by means of a structure of the same dimension and with the simplest elementary neighbourhood index, whose neighbourhood template contains only ***(d+1)*** elementary automata. However, in case of modelling we deal, as a whole, with various sets of configurations, on which a modeled structure and a modeling structure operate while a global transition function, and functions making up its *decomposition* ***(31)*** operate with the same set of configurations and in the same state alphabet; thus, the given distinction has very principal character.

A rather large help in study of the problem of decomposition of global transition functions is computer modeling [641]. In particular, the next procedure provides of composition of ***1***–dimensional binary functions.

```
In[2433]:= ComposeGTF[g__] := Module[{j = 1, cgtf, v = {g}, r, u, x,
                                                    s = Length[{g}]},
   If[s < 4 | | DeleteDuplicates[Flatten[Map[{IntegerQ[v[[2*# – 1]]],
      IntegerQ[v[[2*#]]] && 0 <= v[[2*#]] <= 2^2^v[[2*# – 1]] – 1} &,
                                      Range[1, Floor[s/2]]]]] != {True},
                                      Return[Defer[ComposeGTF[g]]],
                       cgtf[n_Integer, m_/; IntegerQ[m], n1_Integer,
                  m1_/; IntegerQ[m1]] := Module[{a = NfToLtf[n, m],
               b = NfToLtf[n1, m1], c = n + n1 – 1, d = {}, h, k = 1, p, t},
                       h = Map[StringJoin, Map[Map[ToString, #] &,
                    Flatten[Map[Tuples[{0, 1}, #] &, Range[c, c]], 1]]];
                             For[k, k <= Length[h], k++, p = h[[k]];
          t = StringTake[p, Map[{# + 1, n + #} &, Range[0, n1 – 1]]];
                        t = StringJoin[Map[StringReplace[#, a] &, t]];
                  d = AppendTo[d, Rule[p, StringReplace[t, b]]]];
            {c, FromDigits[ToExpression[Map[Part[#, 2] &, d]], 2]}];
                                    r = cgtf[v[[1]], v[[2]], v[[3]], v[[4]]];
                                         For[j = 3, j <= Floor[s/2], j++,
                           r = cgtf[r[[1]], r[[2]], v[[2*j – 1]], v[[2*j]]]];
      If[EvenQ[s], r, u = Map[StringJoin, Map[Map[ToString, #] &,
          Flatten[Map[Tuples[{0, 1}, #] &, Range[r[[1]], r[[1]]]], 1]]];
               x = Map[ToString, IntegerDigits[r[[2]], 2, 2^r[[1]]]];
                           Table[Rule[u[[j]], x[[j]]], {j, 1, 2^r[[1]]}]]]]
In[2434]:= ComposeGTF[2, 6, 2, 6]
Out[2434]= {3, 90}
In[2435]:= ComposeGTF[2, 6, 2, 6, 72]
Out[2435]= {"000" –> "0", "001" –> "1", "010" –> "0", "011" –> "1",
            "100" –> "1", "101" –> "0", "110" –> "1", "111" –> "0"}
In[2436]:= ComposeGTF[3, 105, 2, 6, 3, 125]
Out[2436]= {6, 9060916908255837630}
In[2437]:= ComposeGTF[3, 105, 2, 6, 3, 125, 72]
Out[2437]= {"000000" –> "0", "000001" –> "1", "000010" –> "1",
     "000011" –> "1", "000100" –> "1", "000101" –> "1", "000110" –> "0",
     "000111" –> "1", "001000" –> "1", "001001" –> "0", "001010" –> "1",
     "001011" –> "1", "001100" –> "1", "001101" –> "1", "001110" –> "1",
```

```
"001111" -> "0", "010000" -> "1", "010001" -> "1", "010010" -> "0",
"010011" -> "1", "010100" -> "0", "010101" -> "1", "010110" -> "1",
==================================================
"111011" -> "1", "111100" -> "1", "111101" -> "1", "111110" -> "1",
"111111" -> "0"}
```

The procedure call **ComposeGTF[*g*]** returns the **2–**element list, whose first element defines the neighbourhood template size and the second defines the discriminating number of ***GTF*** that is result of *composition* of two or more global transition functions, whose the local transition functions are determined by neighbourhood template sizes {***n1, …, nk***} and discriminating numbers {***m1, …, mk***} respectively. In addition, in the call **ComposeGTF[*g*]** the pairs {***nj,mj***} ***(j=1..k)*** are used as argument ***g,*** i.e. **ComposeGTF[*n1, m1,…,nk, mk*];** number of such pairs should be at least two**.** Meanwhile, if the procedure call contains odd number of actual arguments ***(****as the last argument can be used an arbitrary expression****)*** the list of parallel substitutions defining the ***LTF*** of the resulting ***GTF*** is returned. The procedure ***ComposeGTF*** processes the basic especial and erroneous situations.

This procedure is quite simply generalized to the alphabet of internal states of the elementary automaton of a homogeneous structure which is other than the binary alphabet. Procedure is useful enough at study of a whole series of questions of the problem of decomposition of one-dimensional global transition functions. In particular, it can be useful at solution of the question of definition of possibility of ***decomposition*** of an arbitrary global transition function onto functions from the given set of global transition functions. Now, we develop similar procedure for study of *decomposition* of **2-**dimensional global transition functions, being based on a modification of algorithm of the above procedure.

In addition, if **ComposeGTF** solves the composition problem of global transition functions, the decomposition problem of an arbitrary global transition function is algorithmically solvable, its *constructive* decision is reduced to programm analysis of all possible variants, whose set is large enough but finite [641–643]**.**

It is simple to be sure that a set of all linear global transition functions which have been given in an arbitrary alphabet $A = \{0,1,2,\dots,a-1\}$ ***(a –*** *a prime number****)*** and whose local transition functions are determined by the above function, is the *closed* concerning the composition operation. The given assertion can be obtained on the basis of definition of global transition function $\tau^{(n+m-1)}$ **–** the result of composition of two global transition functions $\tau_1^{(n)}, \tau_2^{(m)}$ determined by local transition functions $\sigma_1^{(n)}$, $\sigma_2^{(m)}$ accordingly, namely:

$$\begin{cases} \sigma_1^{(n)}(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} x_j \ (mod\ a); \qquad \sigma_2^{(m)}(y_1, y_2, \dots, y_m) = \sum_{j=1}^{m} y_j \ (mod\ a); \\ \sigma^{(n+m-1)}(z_1, z_2, \dots, z_{n+m-1}) = \sum_{k=1}^{m} \left(\sum_{j=k}^{k+n-1} z_j \ (mod\ a) \right) (mod\ a) \\ x_n, y_m, z_2 \in A = \{0, 1, \dots, a-1\};\ a - a\ prime;\ j = 1..n;\ k = 1..m;\ p = 1..n+m-1 \end{cases}$$

So, the resulting global transition function $\tau^{(n+m-1)}$ of composition of **2** global transition functions $\tau_1^{(n)}$, $\tau_2^{(m)}$ determines by the above local transition function $\sigma^{(n+m-1)}$. The question of determination of closed subsets of the above set of linear global transition functions represents a positive interest. In particular, the subset of binary global transition functions with the *connected* neighbourhood index isn't closed relative to the *composition* operation. Along with theoretical approach for study of the question the computer modelling had been used.

```
In[2387]:= GtfMod2[n_Integer] := FromDigits[Map[Mod[#, 2] &,
 Map[Total[#] &, Flatten[Map[Tuples[{0, 1}, #] &, Range[n, n]], 1]]], 2]
In[2388]:= GtfMod2[4]
Out[2388]= 27030
In[2389]:= GtfMod2Q[n_Integer, m_Integer] := Module[{a},
                    a = ComposeGTF[n, GtfMod2[n], m, GtfMod2[m]];
                         If[a[[2]] == GtfMod2[n + m – 1], True, False]]
In[2390]:= GtfMod2Q[3, 7]
Out[2390]= False
```

In particular, the call **GtfMod2[*n*]** returns the *discriminating* number of a structure ***1–HS*** with ***LTF*** $\sigma^{(n)}=\Sigma_j x_j \ (mod\ 2);\ x_j \in \{0,1\};\ j=1..n$, while the procedure call **GtfMod2Q[*n, m*]** returns ***True*** if composition $\tau_1^{(n)}\tau_2^{(m)}$ of ***2 1***–dimensional ***GTF*** with ***LTF*** of the above form is a ***GTF*** $\tau^{(n+m-1)}$ with ***LTF*** of the same form, and ***False*** otherwise. This test persuades in nonclosure of a set of ***GTF*** of such type relative to composition.

Along with the known ***d–PDF*** it is rather interesting to investigate so–called generalized problem of decomposition of ***d***–dimensional global transition functions ***(d–GPDF)***, that consists in the question about an possibility of representation of an arbitrary global transition function $\tau^{(n)}$ in the form of the composition ***(31)*** provided that global transition functions $\tau^{(n_j)}$ of the representation are not connected with obligatory restriction ***($n_j < n$) ($j = 1$ .. k),*** permitting the sign of equality, excepting trivial cases of the representation. Thus, in case of the ***d–GPDF*** in the representation ***(31)*** the use of arbitrary global transition functions $\tau^{(n_j)}$ ***($n_j \le n$),*** that have both dimensionality and the state alphabet, identical with the initial function $\tau^{(n)}$ is permitted. It is obvious that the *positive* decision of the ***d–PDF*** for any global transition function $\tau^{(n)}$ entails the positive decision the ***d–GPDF*** for the function also, while the converse proposition, generally speaking, is incorrect. Hence, the ***d–GPDF*** and the ***d–PDF,*** generally speaking, are nonequivalent problems. At that, along with the ***d–GPDF*** the question of *special* representation of global transition functions in the form of a composition of a finite number of more simple functions represents significant interest. Further under a ***special*** we shall understand any representation of an arbitrary global transition function $\tau^{(n)}$ in the form of ***(31)*** under the condition that the *initial* function $\tau^{(n)}$ and functions $\tau^{(n_j)}$ that make up its decomposition are chosen of the given class of functions, but some special restrictions having a certain interpretation are imposed on them. In particular, the question of interrelation of properties of the nonconstructability of an arbitrary global transition function $\tau^{(n)}$ and global transition functions $\tau^{(n_j)}$ ***($n_j < n$)*** entering in its representation ***(31)*** will absolutely naturally interest us. In this direction the following result takes place [79,88,90].

<u>Theorem 144</u>. An arbitrary global transition function $\tau^{(n)}$ determined in an arbitrary state alphabet possesses the nonconstructability such as NCF if and only if at least 1 global transition function $\tau^{(n_j)}$ ($n_j < n$) possesses NCF; if at least one global transition functions $\tau^{(n_j)}$ ($n_j < n$) possesses NCF–1, their composition $\tau^{(n)}=\tau^{(n_1)}\tau^{(n_2)}...\tau^{(n_k)}$ will possess the nonconstructability such as NCF and/or NCF–1. The set of all GTF without NCF is closed relative to the composition operation whereas the set of all GTF possessing the NCF not. If a GTF $\tau^{(n)}$ in composition $\tau^{(n+m-1)}=\tau^{(n)}\tau^{(m)}$ possesses the pairs of MEC of a minimal size w, the GTF $\tau^{(n+m-1)}$ will possess the pairs of MEC of the same minimal size.

At that, it is necessary to accent attention on transferability of that and a series of results represented below on the ***PDF*** onto the ***HS–***models distinct from classical ones. Furthermore, a whole series of the results presented below is generalized to case of higher dimensionalities.

Along with the above mentioned concept of decomposition of global transition functions in some cases a certain interest other approaches to definition of this concept represent too. In our works [90,641–644] a whole series of similar approaches is considered. So, in particular, an approach, mentioned below, to definition of this concept is interesting from the standpoint of its influence on the questions connected with the *nonconstructability* problem as a whole. The approach to definition of composition of global transition functions is defined as follows. The composition of two global transition functions $\tau 1^{(n)}$, $\tau 2^{(n)}$ is designated as $\tau^{(n)} = \tau 1^{(n)} \oplus \tau 2^{(n)}$ and defined by the following relation for their local transition functions, namely:

$$\left(\forall \langle x_1x_2...x_n\rangle\right)\left(\sigma^{(n)}(x_1,x_2,...,x_n) = \sigma_1^{(n)}(x_1,x_2,...,x_n) + \sigma_2^{(n)}(x_1,x_2,...,x_n) \quad (mod\ a)\right)$$
$$x_j \in A = \{0,1,2,...,a-1\}; \quad j = 1..n$$

The following simple example illustrates this concept of composition:

$$\begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \\ x_1x_2x_3 \end{bmatrix} \to \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \sigma_1^{(3)} \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ \sigma_2^{(3)} \end{bmatrix} = \begin{bmatrix} 0+0\ (mod\ 2) = 0 \\ 1+0\ (mod\ 2) = 1 \\ 1+1\ (mod\ 2) = 0 \\ 0+1\ (mod\ 2) = 1 \\ 1+0\ (mod\ 2) = 1 \\ 0+1\ (mod\ 2) = 1 \\ 0+0\ (mod\ 2) = 0 \\ 1+1\ (mod\ 2) = 0 \\ \sigma^{(3)} \end{bmatrix}; \quad x_1, x_2, x_3 \in A = \{0,1\}$$

representing the scheme of composition of **2** local transition functions $\sigma_1^{(3)}$ and $\sigma_2^{(3)}$ with binary alphabet $A=\{0,1\}$ and neighbourhood index $X=\{0,1,2\}$, and with discriminating numbers ***105*** and ***53*** accordingly as a result we obtain the local transition function $\sigma^{(3)}$ with discriminating number ***92***. At that, the global transition functions $\tau 1^{(n)}$, $\tau 2^{(n)}$ and $\tau^{(n)}$, corresponding to them possess the ***NCF*** & ***NCF-1***, ***NCF*** without ***NCF-1***, and ***NCF*** & ***NCF–1*** accordingly. It is easy to see, that a arbitrary global transition function $\tau^{(n)}$ can be represented as a ⊕–composition of finite global transition functions of the same type ***(****identical alphabet of internal states and neighbourhood index****);*** the set of such ***GTF*** is closed relative to ⊕–operation. Moreover, ⊕–composition can be used together with the standard composition for creation of the more complex constructions:

$$\tau^{(n)} = \tau_1^{(n_1)} ... \tau_p^{(n_p)} \oplus ... \oplus \tau_1^{(m_1)} ... \tau_t^{(m_t)}$$
$$n = \Sigma_{j=1}^{j=p} n_j - (p-1) = ... = \Sigma_{j=1}^{j=t} m_j - (t-1)$$

In this direction a number of interesting enough results concerning the nonconstructability problem relative to the above type of composition of global transition functions has been received [90,641–644].

7.1. Decomposition of special global transition functions in the classical HS–models

Before consideration of questions of *decomposition* of global transition functions of special classes we again shall return to case of classical ***1–***dimensional structures for which ***1–PDF,*** generally speaking, has the negative decision, represented in our works [3,5,44,53–56,63,65,67,77].

Theorem 143. For arbitrary integers a > 2 and n ≥ 3 exist 1–dimensional global transition functions $\tau^{(n)}$ ***given in an alphabet A={0,1,...,a–1} for which 1–PDF has the negative decision.***

How it was marked earlier, the given result for the first time has been received by us on basis of results on the nonconstructability in *classical* structures ***1–HS*** and then was overproved by much more constructive methods on basis of results of ***H. Yamada*** and ***S. Amoroso*** concerning the completeness problem for *polygenic* structures in combination with our results on the *nonconstructability (ch.* ***3*** [3,54–56]***).*** Having received the negative answer to solution of the ***1–PDF*** in general case, we leave aside a lot of the important questions connected to structure of the set of all global transition functions not having of the above presentation ***(31),*** with influence on a possibility of the solution of the *decomposition* problem of *base* parameters and properties of *classical* structures ***1–HS.*** For research of the marked and other questions on the decomposition problem we need a series of new approaches and methods, the part of which will be considered a little bit below.

First of all, we shall consider the class of all binary structures ***1–HS*** for which ***1–PDF*** in accord with theorem ***145*** in general has the negative decision. Within of this class we have researched *influence* on decision of this problem of the restriction of the problem on case of all binary global transition functions that do not possess the nonconstructability such as ***NCF,*** i.e. injective global functions on the set ***C(A,1,***ϕ***)*** of finite ***1–***dimensional configurations. Of the results on nonconstructability in classical structures ***1–HS*** it is known that a sphere of such structures which do not possess ***NCF*** is insignificant and rather quickly *decreases* with growth of size of neighbourhood template of a ***1–HS.*** At that, the class of these structures is closed concerning the *composition* operation what is rather essential for the solution of ***1–PDF.*** The following result gives the answer to the earlier put question [5,54–56,79,88,90,640].

Theorem 146. In the class of all binary injective 1–dimensional global transition functions the 1–PDF in general case has negative decision.

So, narrowing of the class of all ***1***–dimensional binary global transition functions up to own subclass of injective functions keeps negativity of decision of the ***1–PDF.*** Along with the ***d–PDF*** of the general kind the significant interest represents the question of special representation of global transition functions in the form of composition of finite number of more simple functions. Some of *special* representations of such type are considered a little bit below.

In connection with the negative decision of the ***d–PDF*** in general case a lot of interesting accompanying problems arises among which quite naturally it is possible to single out the following ones. Let ***M(A,d,SH)*** will be a certain subset of the set of all ***d***–dimensional global transition functions, determined in a finite alphabet ***A,*** and which possess some general property ***SH.*** Therefore the partial problem of decomposition is reduced to the question about an opportunity of presentation of any global transition function out of the set ***M(A,d,SH)*** in the form of some *composition* ***(31)*** of functions out of the same set ***M(A,d,SH).*** The given partial problem ***d–PDF*** presents considerable theoretical and applied interest depending on an appropriate choice of the determinative set ***M(A,d,SH)*** ***(d≥1)*** of global transition functions of classical ***HS***–models.

First of all, we shall consider the ***1–PDF*** concerning the known class of all ***1***–dimension linear global transition functions which are defined in section ***3.2.*** Functions of such class possess the general property ***SH*** of the universal reproducibility in Moore's sense of finite configurations.

The set ***SL*** of linear ***1***–dimensional global transition functions, whose appropriate local transition functions are defined as follows

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = b_o x_1 + b_n x_n + \sum_{j=2}^{n-1} b_j x_j \quad (mod\ a); \quad x_j \in A = \{0,1,2,...,a-1\}$$

$$b_o, b_n \in A \setminus \{0\}; \quad a = p^k, \ where\ p - a\ \ prime,\ \ k - a\ positive\ \ integer; \quad j = 1..n$$

composes a *commutative* subset concerning the composition operation and whose elements possess the property of universal reproducibility in Moore's sense of finite configurations. Obviously, for an arbitrary ***n*** and a state alphabet ***A***=***{0,1,2, ..., a–1}*** the number of such structures is $a^{n-2}(a-1)^2$. In addition, for arbitrary positive integers ***a*** and ***n*** there are global transition functions $\tau^{(n)} \in SL$ for which the *decomposition problem* has the negative decision, i.e. these functions cannot be represented in the form of composition of more simple global transition functions of the set ***SL*** [79,88]. As a simple example we shall consider linear binary global transition functions $\tau^{(4)} \in SL$ whose *local* transition functions are defined by the following formulas, namely:

$\sigma^{(4)}(x_1,...,x_4)=x_1+x_2+x_3+x_4 \ (mod\ 2)$ $\sigma^{(4)}(x_1,...,x_4)=x_1+x_4 \ (mod\ 2)$

$\sigma^{(4)}(x_1,...,x_4)=x_1+x_2+x_4 \ (mod\ 2)$ $\sigma^{(4)}(x_1,...,x_4)=x_1+x_3+x_4 \ (mod\ 2)$

Obviously, for representation of all global transition functions $\tau^{(4)} \in SL$, taking into account the commutativity of subset ***SL***, it is expedient to consider the following two compositions of more simple functions only, namely: $\tau^{(2)3}$ and $\tau^{(2)}\tau^{(3)}$, where $\tau^{(2)}, \tau^{(3)} \in SL$ and for ***GTF*** $\tau^{(3)}$ **2** neighbourhood indices $X=\{-1,0,1\}$, $Y=\{-1,1\}$ take place. The following fragment enough visually illustrates the result of such compositions:

$$\tau^{(2)3}: \left|\begin{matrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{matrix}\right| \rightarrow \left|\begin{matrix} x_1+x_2+x_2+x_3 \ (mod\ 2) \\ x_2+x_3+x_3+x_4 \ (mod\ 2) \end{matrix}\right| \rightarrow x_1+x_2+x_3+x_4 \ (mod\ 2)$$

$$\tau^{(2)}\tau^{(3)}: \left|\begin{matrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{matrix}\right| \rightarrow x_1+x_4 \ (mod\ 2); \qquad x_j \in A=\{0,1\}; \ j=1..4$$

$$\tau^{(2)}\tau^{(3)}: \left|\begin{matrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{matrix}\right| \rightarrow x_1+x_2+x_3+x_4 \ (mod\ 2); \quad Y=\{-1,1\}$$

Thus, for **2** of ***4*** global transition functions $\tau^{(4)} \in SL$ the decomposition problem concerning the set ***SL*** has the negative decision. In addition, for these two functions with discriminating numbers ***23205*** and ***26265*** the decomposition problem has the negative decision also as a whole. Similar picture takes place for any set of the global transition functions $\tau^{(n)} \in SL$ concerning an arbitrary integers $a \ge 2$ and $n \ge 3$ [79,88,640].

Class ***W*** of ***1***–dimensional structures ***HS*** with alphabet $A=\{0,1,\dots,a-1\}$, neighbourhood index $X=\{0,1,2,\dots,n-1\}$ and global transition functions $\tau^{(n)}$ defined by the local transition functions as follows

$$\forall (x_1,x_2,...,x_{n-1})(h \ne t \rightarrow \sigma^{(n)}(x_1,x_2,...,x_{n-1},h) \ne \sigma^{(n)}(x_1,x_2,...,x_{n-1},t))$$
$$h,t,x_j \in A=\{0,1,2,...,a-1\}; \quad j=1..n-1$$

represents essential enough interest with standpoint of the property of self-reproducibility in *Moore* sense of finite configurations. Shown that structures, excluding trivial ones, of this class possess the property of essential or full self–reproducibility of finite configurations [641–643].

The given class is a rather interesting with standpoint of composition of global transition functions. Without loss of generality, a scheme that illustrates composition of *two* global transition functions $\tau 1^{(n)}$ and $\tau 2^{(p)}$ defined by local transition functions $\sigma 1^{(n)}$ and $\sigma 2^{(p)}$ accordingly will be presented. The scheme shows that composition of **2** global transition functions from the above class ***W*** gives a function of the same class.

$$
\begin{array}{lll}
\sigma 1^{(n)}: & x_1x_2x_3 \ldots x_n x_{n+1} x_{n+2} \ldots x_{n+p-2}(x_{n+p-1} \equiv 0) & \to \; y \\
\sigma 2^{(p)}: & x_1^1 x_2^1 x_3^1 x_n^1 \ldots x_p^{11} & \to \; x_1^2 \\
\sigma 1^{(n)}: & x_1x_2x_3 \ldots x_n x_{n+1} x_{n+2} \ldots x_{n+p-2}(x_{n+p-1} \equiv 1) & \to \; z \\
\sigma 2^{(p)}: & x_1^1 x_2^1 x_3^1 x_n^1 \ldots x_p^{12} & \to \; x_1^3
\end{array}
$$

$$y, z, x_p^{11}, x_p^{12}, x_k^1, x_1^2, x_1^3, x_j \in \{0,1\};\; j = 1..n+p-1;\; k = 1..p; \quad y \neq z \;\&\; x_1^2 \neq x_1^3$$

The given scheme represents a result of application of local transition function $\sigma^{(n+p-1)}$ that corresponds to global transition function defined by composition of global transition functions $\tau 1^{(n)}\tau 2^{(p)}$ to two tuples of the form $x_1x_2 \ldots x_n \ldots x_{n+p-2}0$ & $x_1x_2 \ldots x_n \ldots x_{n+p-2}1;\; x_j \in \{0,1\},\; j=1..n+p-2.$ Without loss of generality, the scheme is based on the binary alphabet $A=\{0,1\}$. Whereas for the local transition functions $\sigma 1^{(n)}$ and $\sigma 2^{(p)}$ takes place the following relation

$$\forall(x_1, x_2, \ldots, x_{n-1})(\sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, 0) \neq \sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, 1))$$
$$\forall(y_1, y_2, \ldots, y_{p-1})(\sigma^{(p)}(y_1, y_2, \ldots, y_{p-1}, 0) \neq \sigma^{(p)}(y_1, y_2, \ldots, y_{p-1}, 1))$$
$$x_j, y_k \in A = \{0,1\}; \quad j = 1..n-1; \quad k = 1..p-1$$

it is a rather easily be sure that the local transition function $\sigma^{(n+p-1)}$ is defined by the parallel substitutions as follows, namely:

$$\forall(x_1, x_2, \ldots, x_{n-1})(\sigma^{(n+p-1)}(x_1, x_2, \ldots, x_{n+p-1}, 0) \neq \sigma^{(n+p-1)}(x_1, x_2, \ldots, x_{n+p-1}, 1))$$
$$x_j \in A = \{0,1\}; \quad j = 1..n-1$$

Thus, composition of two global transition functions $\tau 1^{(n)}$ and $\tau 2^{(p)}$ as a result gives a global transition function $\tau^{(n+p-1)}$ of the same class that is interesting enough from many standpoints. And what is more, takes place the following proposal, namely:

the aforesaid set W of global transition functions is noncommutative and closed concerning the composition operation. This proposition is valid for 1-dimensional structures with alphabet different from binary and neighbourhood index of an arbitrary form. Generally, composition operation on a set of global transition functions is noncommutative.

Indeed, as a rather simple example the *composition* of two ***1***-dimension structures with binary alphabet ***A*** and neighbourhood index ***X=***{***0,1,2***} whose local transition functions have discriminating numbers ***86*** and ***90*** can be represented, namely:

```
In[2433]:= G = ComposeGTF[3, 90, 3, 86, 72]
Out[2433]= {"00000" -> "0", "00001" -> "1", "00010" -> "0", "00011" -> "1",
"00100" -> "1", "00101" -> "0", "00110" -> "0", "00111" -> "1", "01000" -> "0",
"01001" -> "1", "01010" -> "0", "01011" -> "1", "01100" -> "0", "01101" -> "1",
"01110" -> "1", "01111" -> "0", "10000" -> "0", "10001" -> "1", "10010" -> "1",
"10011" -> "0", "10100" -> "1", "10101" -> "0", "10110" -> "1", "10111" -> "0",
```

```
"11000" -> "1", "11001" -> "0", "11010" -> "0", "11011" -> "1", "11100" -> "1",
"11101" -> "0", "11110" -> "1", "11111" -> "0"}
In[2434]:= S = ComposeGTF[3, 86, 3, 90, 72]
Out[2434]= {"00000" -> "0", "00001" -> "1", "00010" -> "0", "00011" -> "1",
"00100" -> "1", "00101" -> "0", "00110" -> "0", "00111" -> "1", "01000" -> "0",
"01001" -> "1", "01010" -> "0", "01011" -> "1", "01100" -> "1", "01101" -> "0",
"01110" -> "0", "01111" -> "1", "10000" -> "0", "10001" -> "1", "10010" -> "0",
"10011" -> "1", "10100" -> "1", "10101" -> "0", "10110" -> "0", "10111" -> "1",
"11000" -> "1", "11001" -> "0", "11010" -> "1", "11011" -> "0", "11100" -> "0",
"11101" -> "1", "11110" -> "1", "11111" -> "0"}
In[2435]:= Select[Gather[Join[G, S], Part[#1, 1] == Part[#2, 1] &&
                        Part[#1, 2] != Part[#2, 2] &], Length[#] > 1 &]
Out[2435]= {{"01100" -> "0", "01100" -> "1"}, {"01101" -> "1", "01101" -> "0"},
{"01110" -> "1", "01110" -> "0"}, {"01111" -> "0", "01111" -> "1"}, {"10010" -> "1",
"10010" -> "0"}, {"10011" -> "0", "10011" -> "1"}, {"10110" -> "1", "10110" -> "0"},
{"10111" -> "0", "10111" -> "1"}, {"11010" -> "0", "11010" -> "1"}, {"11011" -> "1",
"11011" -> "0"}, {"11100" -> "1", "11100" -> "0"}, {"11101" -> "0", "11101" -> "1"}}
```

In the previous fragment **2** compositions of the above global transition functions are evaluated by means of the procedure **ComposeGTF** that is considered below. Then, tuples $<x1 \ldots x5>$ ($xj \in \{0,1\}$; $j=1..5$) on which the resultant functions have *different* values are given. From the result follows, that the composition operation concerning the above **2** global functions is noncommutative. So, certain sets of global functions that are interesting enough with many points of view are *closed* concerning the composition operation however are *noncommutative.*

The basic results concerning the decomposition problem for classes of global transition functions, including functions of a set ***M(A,d,SH)*** will be represented below. Here, we shall consider some deeper questions of the decomposition of global transition functions for one interesting subset of the set ***M(A,d,SH)*** – the *linear* **1**–dimensional global transition functions $\tau^{(n)}$, whose local functions $\sigma^{(n)}$ are defined as follows:

$$\sigma^{(n)}(x_1, x_2, \ldots, x_n) = (x_1 + x_n + \sum_{2}^{n-1} b_k x_k) \pmod a;$$

$$a = p^h;\ p \text{ - a prime, } h \text{ - an integer;} \quad b_k \in \{0,1\};\ x_1, x_n, x_k \in A \quad (k = 2..n-1)$$

The given subset of the set ***M(A,d,SH)*** we shall denote as ***M*(A,1,SH).*** In this connection we shall consider the closure of the set ***M*(A,1,SH)*** relative to the composition operation; at that, it is expedient to present such set in the form of union of two noncrossing subsets $M^*_1(A,1,SH)$ and $M^*_2(A,1,SH)$ (*henceforward for brevity* M^*_1 *and* M^*_2) accordingly of the subsets of global transition functions $\tau^{(n)}$ ($n \geq 2$) with *connected* and *disconnected* neighbourhood templates of size n; i.e. $M^*_1 \cap M^*_2 = \varnothing$ and $M^*(A,1,SH) = M^*_1 \cup M^*_2$, where $\varnothing$ – the empty set. We shall designate the global transition functions out of these subsets accordingly as $\tau_1^{(n)}$ and $\tau_2^{(n)}$. At the made assumptions the following base result having a whole series of interesting enough appendices takes place [53–56,640].

***Theorem 147*. The subset $M^*_1 \in M^*(A,1,SH)$ is nonclosed concerning the composition operation of global transition functions $\tau^{(m)}$ which form them. A global transition function $\tau_1^{(n)} \in M^*_1$ can't be presented in the form of a composition of two more simple functions of the set M^*_1.**

In spite of result of this theorem, it is possible to show that expansion of a set of global transition functions that are admitted as elements of representation **(31)** for an arbitrary function out of the set M^*_1, onto the set M^*_2 allows to solve the ***1–PDF*** positively for global transition functions $\tau^{(n_j)}$ out of the set M^*_1 [54], namely:

***Theorem 148*. An arbitrary global transition function $\tau_1^{(2k)} \in M^*_1$ can be represented in the following form $\tau_1^{(2k)} = \tau_1^{(k)} \tau_2^{(k+1)} = \tau_2^{(k+1)} \tau_1^{(k)}$ where $\tau_1^{(k)} \in M^*_1$ and $\tau_2^{(k+1)} \in M^*_2$ with neighbourhood index $X_2 = \{0,k\}$ ($k=2, \ldots$). An arbitrary global transition function $\tau_1^{[p(2k+1)]} \in M^*_1$ can be presented in the following form $\tau_1^{[p(2k+1)]} = \tau_2^{[p(2k+1)-2]} \tau_1^{(p)} = \tau_1^{(p)} \tau_2^{[p(2k+1)-2]}$ ($p=3,5,7,9, \ldots; k=1,2,3,4, \ldots$). An arbitrary global transition function $\tau_1^{(n)} \in M^*_1$ on the assumption that $n = pxq$ can be presented in the following form $\tau_1^{(n)} = \tau_2^{(n-p+1)} \tau_1^{(p)} = \tau_2^{(n-q+1)} \tau_1^{(q)}$ where functions of the set M^*_2 that participate in representation of a global transition function $\tau_1^{(n)}$ have symmetric neighbourhood indices.**

Of this result follows, an arbitrary global transition function $\tau_1^{(n)} \in M^*_1$ on the assumption of a composite number n can be represented in the form of a composition of two more simple global functions out of the set $M^*(A,1,SH) = M^*_1 \cup M^*_2$. For the final decision of the question with the set M^*_1, it is quite sufficient to consider a case of global transition function $\tau_1^{(n)}$ on the assumption of a prime integer n. So, by not going into numerous and quite laborious details of the analysis of the global transition functions $\tau_1^{(n)}$ of such type, we shall present an interesting enough result expressed by the following theorem [53,618,640–643].

***Theorem 149**. A global transition function $\tau_1^{(n)} \in M^*_1$ can be presented in the form of the composition of finite number of more simple global transition functions of the set $M^*(A,1,SH)$ if n is a composite number; for example, $\tau_1^{(n)}=\tau_2^{(p)}\tau_3^{(n-p+1)}=\tau_4^{(q)}\tau_5^{(n-q+1)}$ where $\tau_2^{(p)},\tau_4^{(q)} \in M^*_1$; $\tau_3^{(n-p+1)},\tau_5^{(n-q+1)} \in M^*_2$ $(n \geq 4)$ under the condition $n=p{\times}q$ and connected and disconnected neighbourhood template of size p,q and $n-p+1,n-q+1$ along with neighbourhood indices $X_2=\{0,1,\dots,p-1\}, X_3=\{kp \mid k=0,1,\dots,q-1\}$ and $X_4=\{0,1,\dots,q-1\}$, $X_5=\{kq \mid k=0,1,\dots,p-1\}$ accordingly.*

So, the *decomposition* problem of an arbitrary global transition function $\tau_1^{(n)} \in M^*_1$ concerning the set $M^*(A,1,SH)$ is solvable, and in case of its positive decision the satisfactory constructive solving algorithms exist. Entirely other picture takes place in case of global transition functions out of the set M^*_2. A great many global transition functions $\tau_2^{(n)} \in M^*_2$ can be presented in the form of a *composition* of finite number of more simple functions out of the set $M^*(A,1,SH)$ *irrespective* of the nature of the number defining the size of neighborhood template of a *researched* global transition function. Though it is necessary to bear in mind that with neighborhood template of size n $2^{n-2}-1$ various global transition functions $\tau_2^{(n)} \in M^*_2$ exists, and by far not all of them have the above representation. In particular, out of *7 binary* global transition functions $\tau_2^{(5)} \in M^*_2$ the above presentation from finite number of more simple functions out of the set $M^*(B,1,SH)$ *5* functions have while of *15 binary* functions $\tau_2^{(6)} \in M^*_2$ only ***six*** functions have such representation, etc. Along with that, for an integer $n \geq 5$ and an arbitrary alphabet A, exist global transition functions $\tau_2^{(n)} \in M^*_2$ that have the representation ***(31)*** with appropriate restrictions [5,53–56,77,79,88,90,536,618,640–643].

***Theorem 150**. For an arbitrary integer $n \geq 5$ and an arbitrary alphabet A there are at least $n-4$ global transition functions $\tau_2^{(n)} \in M^*_2$ which can be presented in the form of a composition $\tau_2^{(n)}=\tau_2^{(n-1)}\tau_1^{(2)}$ of two global transition functions. An arbitrary 1–dimensional binary global transition function $\tau^{(n)}$, whose local transition function is defined as $\sigma^{(n)}(x_1, \dots, x_n) = \sum_k x_k \pmod 2$ with connected neighbourhood template can be represented in the form of composition $\tau^{(n)} = \tau^{(2)(n-1)}$ where $\tau^{(2)}$*

is the simplest binary linear function and $n = 2^k$ ***(k = 2,3,4, ...).***

However, in spite of this result, the carried out analysis of big enough quantity of global functions of the set M^*_2 along with some theoretical results have allowed to confirm the following proposal [79,88,640]**:**

For an integer $n \geq 4$ ***and a state alphabet A there are global transition functions of the set*** M^*_2 ***which can't be represented as a composition of finite number of more simple functions from the set M*(A,1,SH).***

So, the *decomposition* problem of global transition functions relative to the set ***M*(A,1,SH)*** in general has the *negative* decision. Thus, research of the *decomposition* problem of global transition functions, it is rather interesting to generalize to cases of the higher dimensionalities and to observe change of the results presented above depending on increase of dimensionality of global transition functions out of the considered class ***M*(A,d,SH).*** As distinct from the ***M*(A,1,SH)*** the set ***M*(B,1,SH)*** of ***1–***dimension binary *linear* global transition functions makes up own subclass of the class of all ***1–***dimensional linear global functions that is closed concerning the composition, forming the ***subset*** concerning the given operation. Therefore, the above results can be entirely naturally spread onto the set ***M*(B,1,SH),*** representing in a whole series of cases applied, theoretical and independent interest.

On the example of research of the ***1-PDF*** relative to the set ***M*(A,1,SH)*** of linear global transition functions a series of rather deep properties of global transition functions concerning the given problem has been investigated, namely**:** influence of the type of neighborhood template **(***connected or disconnected***),** number defining its size **(***composite or prime***),** and also type of functions making up representation ***(31).*** Within the further development of the given problematics it is possible to extend similar researches onto the general case of the set ***M(A,d,SH)*** of linear functions, and also to determine a number of sets ***MW(A,d,W)*** of the global transition functions with a certain general ***W–***property that are interesting from the theoretical and applied standpoints, in particular, *symmetry***.** One more interesting *subclass* of the class of all classical ***HS–***models relative to the *decomposition* problem of their global transition functions is represented by structures with refractority.

The above ***HS–***models have a series of biomedical interpretations, and recently they begin to be used also for the problems of recognition of images, researches of properties and topology of digital figures, etc. In our works a number of interesting theoretical and empirical results on dynamics of structures ***d–HSR(r,P)*** {*these structures have been defined in ch.* ***1***} can be found [1,4,5,8,15,45,78,85,90,640]**.** Below, these structures are considered concerning the decomposition problem of their global transition functions. Each structure ***d–HSR(r,P)*** is determined by such important parameters as *excitation threshold* ***(P)*** and *depth of refractority* ***(r).*** On basis of these parameters and differentiation of a state alphabet ***A*** a rather good possibility of research of peculiar dynamic properties of such structures consisting in excitations spread in environment of the elementary automata of the structures arises. The following result represents a solution of the decomposition problem for case of global transition functions concerning the class of all structures ***d–HSR(r, P)*** [5,53–56,63,78,79,85,88,90,617,618,640–643]**.**

<u>Theorem 151</u>. The decomposition problem has the negative decision in the class of all structures with refractority d–HSR(r, P) (d ≥ 1).

This result speaks about the *negative* decision of the ***d–PDF*** in the class of all global transition functions with refractority without restrictions on excitation threshold and depth of refractority of the structures**;** i.e. the class of such global transition functions is considered, whose local functions are characterized by a *switching* of an elementary automaton being in an excited state into a refractority state. Now we shall assume that some global transition function with refractority has an arbitrary excitation threshold ***1 ≤ P ≤ n–1*** and refractority depth ***r≥1.*** On account of the made assumption, the following important enough result takes place [54–56,79,88,90,536,617,618,640–643].

<u>Theorem 152</u>. Each global transition function out of a class B(r) of all 1–dimensional global transition functions with refractority depth r≥1 cannot be represented in the form of a composition of finite number of functions out of the same set B(r); B(r) is the set of global transition functions with refractority that are isolated concerning the operation of composition.

Hence, the result of theorem ***152*** gives the exhaustive decision of the decomposition problem in the class ***B(r)*** of all one–dimensional global transition functions with *refractority* of ***r–***depth. Moreover, each global function out of the class ***B(r)*** is located at own complexity level which is defined by its excitation threshold ***P*** and neighbourhood index. So, research of many important dynamic properties of distribution of the excited states in environment even of structures ***1–HSR(r,P)*** has much more individual character depending on the base characteristics of the structures of such type. The result of theorems ***151–152*** is generalized to structures ***d–HSR(r,P) (d>1)*** with neighbourhood index of a rather general kind [5,54–56,88,90,536]. Many other rather interesting special presentations of global transition functions in the form of composition of finite number of more simple functions is considered in our works and the subsequent sections of the monograph [5,8,9,53–56,88,90,114].

7.2. Some approaches to solution of the general decomposition problem of global functions

In the previous section some questions of the decomposition problem concerning a series of *special* classes of global transition functions have been considered, and also results of solution of the ***1–PDF*** have been presented on basis of approaches utilizing the results of ***Amoroso*** and ***Yamada*** on the ***completeness*** problem together with our results on the *nonconstructability* problem in classical ***HS***–models. In this section we shall present results of research of the general decomposition problem on basis of the new *perspective* approaches basing on use of results and methods of the theory of functions of algebra of logic, ***a***–valued logics, and also on use of the formal apparatus of the group theory, algebras and semigroups along with an essential generalization of a method of solution of the ***d–PDF*** on basis of researches on the *nonconstructability* problem in classical ***HS***–models. Along with it we will show the close enough connection of the decomposition problem of global transition functions with the complexity problem of finite configurations in the classical ***HS***–models and with the completeness problem for polygenic ***HS***–models. Approaches and methods of solution of the ***d–PDF*** which are here discussed represent the certain interest at researches of some other questions of the ***HS***-problematics and a lot of its applied aspects.

Above all, for solution of the *decomposition* problem the approach that is based on use of the ***Shannon***'s function is offered; the approach was entered for an estimation of the complexity of realization of functions of algebra of logic. It is known that each function of algebra of logic is realized by the corresponding logic circuit consisting of certain basic logic elements. As the given basic elements, the elements realizing the following well–known logical operations are chosen, namely: *negation*, *conjunction* and *disjunction*.

For description of the *complexity* of logic circuits we act as follows. Let, to each logic circuit ***M*** which realizes a certain function of algebra of logic, a non–negative number ***L(M)*** is attributed – a complexity of the circuit, while ***L(σ) –*** is a minimum of complexities of all circuits ***M*** that realize the given logic **σ–**function. At that, let $L(n) = max_{\sigma} L(\sigma)$ and the maximum is taken in all local transition functions $\sigma^{(n)}$ that depend on ***n*** logic variables. If a maximum is achieved, then ***L(n)*** is such minimal number, that by means of circuits of complexity no more than ***L(n)*** we can realize an arbitrary *function* $\sigma^{(n)}$ of algebra of logic, in other words define the corresponding local transition function of a structure ***d–HS.*** Function ***L(n)*** for the first time has been introduced by ***C. Shannon*** for contact networks and he has received the first significant results on it. Subsequently, such function ***L(n)*** has received the name of the author and began to be rather widely used for an estimation of complexity of function circuits. The research of behaviour of functions ***L(σ)*** and ***L(n)*** is the important enough problem**;** in addition, both aspects of research appear closely interconnected among themselves [5,8,54–56,88,90,536]**.**

At present, the upper bounds that are asymptotically equal to the low bounds are received only for rather few types of functional circuits. In addition, under type of a circuit is understood, above all, a certain set of base functional elements of which is designed the circuit realizing a demanded function $\sigma^{(n)}$ of algebra of logic. Inasmuch as our problem is reduced to a comparative analysis of local transition functions $\sigma^{(n)}$, then it is quite possible to restrict oneself to some concrete set of base functional elements, i.e. concrete type of functional circuits on basis of which is being carried out the corresponding comparative analysis. In this connection the circuits out of functional elements can be chosen in basis ***G=***{***And, Or, Not***}**;** for such circuits the asymptotical expressions for the ***Shannon's*** function under the condition of asymptotic equality of the upper and lower bounds of their complexity have been found.

Let's name a quantity of elements of a functional circuit ***M*** realizing a logic function $\sigma^{(n)}$ its complexity and to denote as $S = L(\sigma^{(n)})$**.** Let now $L(\sigma^{(n)})$ is the *least* complexity of the circuits realizing an arbitrary local transition function $\sigma^{(n)}$ and $L(n) = max_{\sigma} L(\sigma^{(n)})$ where the maximum is taken in all functions $\sigma^{(n)}$**.** In other words, a value ***L(n) –*** the minimal number of elements in a functional circuit, sufficient for realization of an arbitrary boolean function from ***n*** variables. At that, at the proof of our result the ***O. Lupanov's*** result fundamental in the theory of *boolean* functions is enough substantially used, namely [6]**:**

The asymptotic equality $L(n) = 2^n/n$ ***takes place; in addition, for any value*** $\delta > 0$ ***the quota of logical functions*** $\sigma^{(n)}$ ***for which the following***

relation $Q=L(\sigma^{(n)}) \le (1-\delta)x2^n/n$ is valid will tend to zero with growth of value n.

In spite of possible generalizations of this result, it is quite possible to restrict oneself to it for the further considerations. It is obvious that an arbitrary local transition function $\sigma^{(n)}$ given in binary alphabet can be considered as a certain function of algebra of logic. As a composition of global transition functions $\tau^{(n_j)}$ is uniquely determined by a special superposition of the corresponding local transition functions $\sigma^{(n_j)}$, in further this question is reduced to *superposition* of local functions $\sigma^{(n)}$ in binary alphabet. For that for each integer ***n*** a certain local function $\sigma^{(n)}$ with greatest possible complexity ***L(n)*** is chosen. Then, on basis of a sought composition of finite number of binary global functions $\tau^{(n_j)}$ $(n_j<n; j=1..k)$ for an arbitrary function $\tau^{(n_j)}$ an appropriate Σ–circuit of superposition of local transition functions is defined and its *complexity* $L(\Sigma)$ is estimated, which is compared next with complexity ***L(n)*** of the initial local transition function $\sigma^{(n)}$. The comparative analysis carried out after that allows to formulate the following basic result [53–56,88].

<u>Theorem 153</u>. For any given finite base set G_f of 1–dimensional binary global transition functions $\tau^{(n)}$ exists such integer $n_o > 0$ that for each integer $n \ge n_o$ there is at least one binary global transition function for which 1–PDF has the negative decision.

It is necessary to note that the similar result for binary case is received by us on basis of results on universal reproducibility in Moore's sense of finite configurations in classical structures ***1–HS*** [41]. Thus, result of theorem ***153*** has been received on the assumption that a local function $\sigma^{(n)}$ has an arbitrary kind, i.e. the obligatory determinative condition $\sigma^{(n)}(x, ..., x)=x$ is not required, allowing to spread the result of solution of the ***d–PDF*** onto case of the ***nonstable*** structures ***d–HS***. Moreover, at proof of the theorem the concept of *dimensionality* did not participate, but only neighbourhood template, represented by its neighbourhood index. At that, it is necessary to have in view that in case of dimension $d \ge 1$ an arbitrary neighbourhood index $X=\{x_o, x_1, ..., x_{n-1}\}$ is ***n***–tuple of ***d***–dimensional points defining coordinates of elementary automata of neighbourhood template of a structure ***d–HS*** $(d \ge 1)$. This remark play a rather essential part at generalization of results to case of more high dimensionality when there is a possibility of special numbering of the elementary automata making up neighbourhood template of a ***d–HS*** ***($d \geq 1$).*** Consequently, it is rather simple to make sure that the result of theorem ***153*** is valid also for global transition function in an arbitrary binary structure ***d–HS ($d \geq 1$).*** Once again it is necessary to note, that to a composition of global transition functions $\tau^{(n)}$ the corresponding superposition of local transition functions corresponds which is rather essential ***narrowing*** of the general superposition concept of functions. The given fact causes the impossibility of direct carry–over of results concerning ***a***–valued logics ***($a > 2$)*** to case of local transition functions determined in a state alphabet ***A*** of general kind though methods of ***a***–valued logics are represented rather perspective for research of a lot of properties of dynamics of classical ***HS***–models. Detailed discussion of the given aspect can be found, for example, in [88,90,536,567,640].

Inasmuch as in the *general* case of state alphabet of a classical structure it is not possible to directly generalize the above results regarding the *binary* global transition functions, we are compelled to address, first of all, to ***a***–valued logic ***($a>2$)*** relative to which it is necessary to note that at solution of a whole series of its problems we collide with the rather large complexities and especial circumstances [5,8,53,88,536]. So, in an ***a***–valued logic the research of an arbitrary system of functions relative to its *completeness* is connected to the large technical difficulties. While the proofs of completeness of concrete systems in a set of functions of an ***a***–valued logic are carried out, as a rule, by means of the method of their reduction to deliberately complete systems of functions what in many cases meets rather essential difficulties [5,79,88,90,536,567,640].

Let's consider now a local transition function $\sigma^{(n)}$, defined in a certain alphabet ***A***={***0,1,3,4, ..., a-1***} ***($a > 2$)*** as a function of ***a***–valued logic. Let's assume, that there is such finite base set ***G*** of local transition functions that *each* local function $\sigma^{(n)} \notin G$ can be represented in the form of finite number of superpositions of local transition functions out of the set ***G.*** Using terminology and designations of work [53] and designating the set of all functions of ***a***–valued logic through ***L(a),*** we simply conclude that closure of the set ***G*** coincides with ***L(a),*** i.e. [***G***]≡***L(a).*** It is obvious that the set ***G*** is complete system in ***L(a).*** But inasmuch as the set ***G*** is finite under the assumption, then we can always choose such minimal subset ***G***⊆***G,*** that the set ***G*** is complete system in ***L(a),*** while any its subset will not be by a complete system in ***L(a).*** Thus, the assumption about existence for the set ***L(a)*** of a finite basic subset ***G*** is equivalent to existence of a finite *basis* in ***L(a).*** Hence, the decomposition problem of global transition functions, determined in a finite state alphabet ***A***, can be reduced to the problem of functional completeness in ***a***–valued logic whose research is essentially connected to so–called ***precomplete classes*** [10,53]. Questions of researches on ***a***–valued logics are enough entirely represented, in particular, in work [323].

Definition 27. A class S of functions belonging to a closed class R of the set L(a), we shall name precomplete in R if S is incomplete system in R, however a supplement to it of any function $f \in R \setminus S$ transforms S in a complete system of functions in R.

Of the known results on ***a***–*valued* logics it is easy to make sure that the number of *precomplete* classes in ***L(a)*** is finite and does not exceed 2^{a^a}. But it is possible to show, it contradicts the condition what in case of a fixed integer ***a*** in ***a***–valued logic always will exist a basis consisting of any but finite number of functions, including the suggested *basis* ***G**** in class of functions ***L(a)*** [54]. Hence, in the above class of functions ***L(a)*** can't exist a finite basis ***G*.*** We need only to estimate influence of this fact upon the question of existence of a finite basis in a narrower class ***L(a,0)*** of functions of ***a***–valued logic that satisfy the stability condition $\sigma^{(n)}(0,0, ..., 0)=0$, i.e. determinative condition of classical structures. As a result of the fulfilled estimation, absence of a finite basis in the class ***L(a,0)*** of functions has been proved that allows to formulate the result being analogue of the theorem ***153*** for case of functions of an ***a***–valued logic and representing the certain theoretical interest [54].

Theorem 154. The class L(a,0) of local transition functions $\sigma^{(n)}$ under the condition $n \geq 2$ and $a > 2$ does not possess any finite basis.

On basis of this result similarly the binary case it is easy to receive the negative decision of the decomposition problem for a global transition function $\tau^{(n)}$ as a whole [5,54–56,79,88,90,617,618,640–643].

Theorem 155. Among all structures d–HS (d≥1) defined in an arbitrary finite state alphabet A and with any neighbourhood indices exists an infinite set of global transition functions for which the d–PDF has the negative decision.

The similar result can be received also on basis of the above results on the complexity problem of finite configurations in classical structures ***d–HS (d≥1)*** [54-56]. The impossibility of positive decision of the ***d-PDF*** for an arbitrary *global* transition function allows to naturally introduce the complexity concept for global transition functions similarly to case of finite configurations in classical structures ***d–HS (d ≥ 1).*** Thus, of the analysis of the complexity concepts of finite configurations and global transition functions in classical structures ensues – the impossibility of existence of certain finite basic sets for finite configurations and global transition functions underlies accordingly.

Of the told with all clarity it is possible to draw a conclusion about an opportunity of fruitful use in researches in this direction of apparatus and methods developed in algebra of logic, and ***a***–valued logics along with results on the complexity problem of finite configurations in the classical structures ***d–HS (d≥1).*** Besides, for research of the problem of decomposition of global transition functions some algebraic methods also can be used [54–56,79,88,90,567,617,618,640–643].

It is known that the global transition functions that realize a mapping of configurations out of the set ***C(A,d)*** onto themselves form a certain *semigroup* concerning the composition operation; let ***L(a,d)*** denotes the semigroup of all such ***d***–dimensional global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$. It is possible to show that ***L(a,d)*** is *noncommutative* semigroup with the *group identity (for definiteness defined by* $\sigma^{(2)}(x,y)=x$*)* that leaves any ***LTF*** $\sigma^{(n)}$ by invariable within leading variables. Hence, researches of properties of *composition* of global transition functions $\tau^{(n)}$ in many cases can be reduced to investigation of the corresponding properties of an appropriate semigroup ***L(a,d).*** Therefore, for the further we shall define a series of necessary algebraical concepts.

We shall call a subgroup ***G*** of some semigroup ***S*** with identity by the ***maximal*** subgroup of the semigroup ***S*** if it is strictly not contained in any other subgroup of the given semigroup. Existence of the maximal subgroups for the first time has been proved by ***J. Schwarz*** for *periodic semigroup* along with ***Wallace*** and ***Kimura*** for an arbitrary semigroup. Let's give one more example of existence of the maximal subgroups in semigroups, defined by global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ in the classical structures ***d–HS (d≥1).*** We shall show, that semigroup ***L(a,d)*** contains one maximal group ***G*** where under the maximal group such group ***G*** contained in semigroup ***L(a,d)*** is understood, which cannot be expanded by means of addition to ***G*** of new elements out of the set ***L(a,d)\G.*** With one–to–one global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ the problem of research of properties of classical ***HS***–models by algebraic methods is enough closely connected.

It is known that the set of all mappings $\tau^{(n)}: C(A,d) \to C(A,d)$, generally speaking, does not meet the group axioms, rather, the axiom about an inverse element, even on the assumption of exception of global *parallel* mappings $\tau^{(n)}$ possessing the nonconstructability of types ***NCF, NCF–1*** and ***NCF–3*** [3]. However, considering a set ***G(d)*** of all ***d***–dimensional global transition functions whose the corresponding global mappings $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$ are one-to-one, it is possible to show, that the set ***G(d)*** forms a group concerning the composition operation [3,54]. This result allows to apply the group methods for research of dynamics of classical structures ***d–HS (d≥1),*** i.e. to reduce research of a whole series of properties of such classical structures to research of the appropriate properties of the *group* ***G(d),*** and also of *subgroups* making up it. So, in particular, from the definition of the set ***G(d) (d≥1)*** it is easy to receive and *maximality* of the group ***G(d)*** belonging to the semigroup ***L(a,d)*** of all global parallel mappings $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$.

Under the ***decomposition*** of a semigroup ***S*** is understood a possibility of its presentation in the form of *union* of noncrossing sub-semigroups S_k ***(k=1,2, ...).*** In order that such decomposition represents the certain significance for research of its structure and determinative properties, it is necessary in order the *sub–semigroups* S_k were by the more special semigroups than ***S,*** in particular, by simple semigroups or groups. In this respect the semigroup ***L(a,d)*** of mappings decomposes into union of noncrossing *semigroup* ***L*(a,d)*** and *maximal* group ***G(d),*** namely: ***the following determinative relation takes place, namely:***

$$L(a,d) = L^*(a,d) \cup G(d) \quad \text{and} \quad L^*(a,d) \cap G(d) = E$$

where E – an unit group consisting only of one unit element – identity of the semigroup. The solution of very many questions concerning the opportunities of the semigroup ***L(a,d)*** can be reduced to the solution of the corresponding questions for the *semigroup* ***L*(a,d)*** or *group* ***G(d),*** what and has been made.

Let each global transition function $\tau^{(n)} \in L(a,d)$ can be presented in the form of a composition of finite number of more simple functions out of some finite set $G_f \subset L(a,d)$. In this case ***L(a,d)*** will be by a semigroup with the finite number of *generators*. Indeed, the set $G_f \subset L(a,d)$ will be a system of generators for the semigroup ***L(a,d)*** if only any element of the set can be presented by not less than by one manner in the form of a *composition* of finite number of degrees of elements out of the set G_f. But inasmuch as according to the above assumption the set G_f is finite then out of it our previous conclusion easily follows.

A system of generators for the set G_f is named as *irreducible* one if any true subsystem for it is not a system of generators for the semigroup ***L(a,d)***. Further under the set $G_f \subset L(a,d)$ we shall understand a certain *irreducible* system of generators and name its as a *basis* of the *semigroup* ***L(a,d)***. The above consideration, in full measure can be ascribed to the group $G(d) \subset L(a,d)$. We shall call a subgroup $G^*(d)$ of the group $G(d)$ closed, if composition g_1g_2 of any two its elements $g_1, g_2 \in G(d) \backslash G^*(d)$ does not belong to the subgroup $G^*(d)$. Further we shall need a series of rather simple results presented in our works [5,54–56,88,618,640].

First of all, if the semigroup $L(a,d)$ has a finite basis G_f, then maximal group $G(d) \subset L(a,d)$ also has such finite basis G^* that the next relations take place $G_f = G^* \cup G^\#$ and $G^* \cap G^\# = \varnothing$ where $G^\#$ – a finite basis of the semigroup $L(a,d) \backslash G(d)$. In addition, if the group $G(d)$ have not a finite basis, then the semigroup $L(a,d)$ will not have a finite basis too. If the group $G(d)$ contains n of the closed subgroups, then the number N of elements of basis G_f of the group not less, than n $(N \geq n)$. Of the given result it is easy to receive the important consequence for the further: ***If the group G(d) contains infinite number of the closed subgroups, then it cannot have a finite basis.*** The given results are carried over to case of arbitrary algebraic semigroups too. Hereinafter, without loss of the generality, we shall identify the global transition functions $\tau^{(n)}$ of the classical structures ***d–HS*** $(d \geq 1)$ with the corresponding parallel global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$.

Let's consider now a little bit more in details one representation of the semigroup $L(a,d)$ according to our results on the *nonconstructability* in classical structures ***d–HS*** $(d \geq 1)$ (*ch.* **2**). It is well–known that the set of all parallel global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ can be presented in the form of union of seven noncrossing subsets, that possess the next basic determinative characteristics concerning the component parallel global mappings $\tau^{(n)}$ {*global transition functions* $\tau^{(n)}$ – ***GTF***}, namely:

- G_1: ***GTF*** $\tau^{(n)}$ ***possess all four types of the nonconstructability such as NCF–1, NCF, NCF–2 and NCF–3;***
- G_2: ***GTF*** $\tau^{(n)}$ ***possess the nonconstructability such as NCF (NCF–3) and NCF–1 without the nonconstructability such as NCF–2;***
- G_3: ***GTF*** $\tau^{(n)}$ ***possess the nonconstructability such as NCF (NCF–3) and NCF–2 without the nonconstructability such as NCF–1;***

G_4 : GTF $\tau^{(n)}$ possess the nonconstructability such as NCF–2 only; at that, global mappings defined by these GTF are not one–to–one;

G_5 : GTF $\tau^{(n)}$ possess the nonconstructability such as NCF–1 only;

G_6 : GTF $\tau^{(n)}$ possess nonconstructability such as NCF (NCF–3) only;

$G_7 \subset G_6$: GTF $\tau^{(n)}$ define one–to–one parallel global mappings of the following kind, namely: $\tau^{(n)}$: C(A,d) → C(A,d); (d ≥ 1).

It is possible to make sure, concerning the composition operation the sets G_k ***(k=1..6)*** make up the *noncommutative* semigroups while the set G_7 makes up the group [53]. Thus, the semigroup ***L(a,d)*** of all parallel global mappings $\tau^{(n)}$***: C(A,d) → C(A,d)*** in the classical structures ***d–HS (d ≥ 1)*** can be represented in the form of union of finite number of the noncrossing semigroups and the group, i.e. $L(a,d)=\cup_k G_k$ ***(k=1..7).*** The carried out analysis of structures of semigroups G_j ***(j = 1..6)*** and group G_7 has allowed to formulate the following interesting enough primal result concerning the decomposition operation of semigroup ***L(a,d)*** of mappings $\tau^{(n)}$***: C(A,d) → C(A,d)*** for classical structures ***d–HS (d ≥ 1).***

<u>*Theorem 156*</u>***. A semigroup L(a,d) of all parallel global mappings $\tau^{(n)}$: C(A,d) → C(A,d), determined by classical structures d–HS (d ≥ 1), can be presented in the form of union of 6 noncrossing sub–semigroups G_k (k=1..6), that have not finite systems of generators, and one maximal group G(d). Sets G_h (h=4..6) relative to the semigroup L(a,d)\G(d) are isolated sub–semigroups.***

Concerning the group ***G(d)*** one rather essential remark it is necessary to make. Analysis of a lot of ***1***–dimensional binary parallel mappings $\tau^{(n)}$***: C(B,1) → C(B,1)*** and, above all, of the mappings forming the sub–semigroup G_4, has allowed to express the assumption that ***G(d)*** is the ***unit*** group, i.e. will consist only of identical global mappings. At that, the further researches have shown that the question of structure of the group ***G(d)*** remains to some degree open. Undertaken more detailed research of the binary classical structures ***1–HS*** for detection of one-to-one mappings $\tau^{(n)}$***: C(B,1) → C(B,1)*** that differ from the identical ones, has appeared quite successful. The following theorem represents the best result received in this direction, presenting the certain theoretical interest [54–56,79,88,90,618,640]. Theorem ***157*** presents a quite certain interest for formal researches of ***1***–dimendional classical ***HS***–models.

Theorem 157. A semigroup L(a,1) of all 1–dimensional parallel global mappings $\tau^{(n)}$: C(A,1) → C(A,1) (a≥3) determined by means of classical structures 1–HS, can be represented in the form of union of 6 pairwise non–equivalent noncrossing sub–semigroups G_k (k=1..6) that have not finite systems of generators and 1 maximal group G(a) which is union of subgroup T* of all identical mappings $\tau^{(n)}{}_o$ (n ≥ 2) with some finite system P(a,2) of generators together with the next relation $\tau^{(n)(a-1)!} = \tau^{(2)}{}_o$ and, perhaps, of some subgroup of biunique mappings which are distinct from the above–mentioned ones.

Results of theorems ***156*** and ***157*** speak about necessity of continuation of research in this direction, taking into account variety of possibilities already for one–dimensional binary case. At present, our researches in this direction quite definitely say in favour of this supposition. So, the next result illustrates variety of possibilities already for case of binary classical structures ***1–HS*** [19,53–56,79,88,90,536,617,618,640–643].

Theorem 158. For an arbitrary integer n≥3 exists not less 2^{n-1}-n binary 1-dimensional global transition functions $\tau^{(n)}$ each of which possesses the following properties simultaneously, namely:

♦ ***for such global transition functions the nonconstructability such as NCF, NCF–3 and NCF–1 is absent in the presence of NCF–2;***

♦ ***an arbitrary configuration c∈C(B,1,ϕ) for similar global transition functions is periodic; there are periodic configurations with minimal p–period, whose size is defined by relation (|c|+n–2)/(n–1) < p = 2^k;***

♦ ***global parallel mappings $\tau^{(n)}$: C(B,1,∞) → C(B,1,∞) defined by such global transition functions (GTF) are not one–to–one;***

♦ ***for these GTF $\tau^{(n)}$ the 1–PDF has the negative decision.***

The given result is essential generalization of some lemmas out of the work [3], however it does not give complete solution of the question even concerning structure of the group ***G(1)*** participating in the above presentation of the semigroup ***L(a,1)*** of ***1***-dimension global mappings. Along with it, this result once again illustrates all variety and riches of forms of behaviour of finite configurations already for case of classical binary structures ***1-HS.*** Like that, from the results of theorems ***156-158*** follows, that the group ***G(d)*** in representation of the semigroup ***L(a,d)*** should contain nontrivial identical one–to–one global mappings while for each of six sub-semigroups G_j ***(j=1..6),*** determined by presentation ***L(a,d),*** the ***d–PDF*** generally speaking will have the negative decision.

In closing of the section on basis of the concept of the *infinite mutually-erasable configurations* ***(∞–MEC)*** we shall define one more approach to the solution of the *decomposition* problem of global transition functions in classical ***HS***–models. In this connection the sub–semigroup G_4 of ***1***–dimensional global parallel mappings $\tau^{(n)} : C(A,1) \rightarrow C(A,1)$ has been investigated enough in detail; for the G_4 a series of interesting enough dynamic properties has been received. For the further we need certain new concepts and definitions [5,54–56,79,88,90,617,618,640–643].

<u>Definition 28</u>. Two configurations $c_1, c_2 \in C(A,d,\infty)$ ***are called a pair of infinite mutually–erasable configurations (∞–MEC) if and only if for them the following relation*** $c_1\tau^{(n)} = c_2\tau^{(n)} = c_3 \in C(A,d,\infty) \neq$ ***takes place where*** $c_1 \neq c_2$ ***and « » is completely zero configuration of the space*** Z^d***, which in accordance with the postulate belongs to the set*** $C(A,d,\phi)$***.***

We shall speak, that a *binary* global transition function $\tau^{(n)}$ realizes so–called «*flip–flop*» if on some pair of tuples $<x_1x_2 \ldots x_{n-1}0>, <x_1x_2 \ldots x_{n-1}1>$ the corresponding local transition function $\sigma^{(n)}$ will accept the values $\sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, x_n) = x_n+1 \ (mod\ 2)$. The number of «*flip–flops*» for an arbitrary global transition function $\tau^{(n)}$ is called its defect that to some extent characterizes a degree of its deviation from the identical global transition function $\tau^{(n)}{}_o$. We shall define one useful class $E^\#$ of global transition functions $\tau^{(n)}$ of the *sub–semigroup* G_4 whose local transition functions $E^{(n)}$ are defined as follows, namely:

$$E^{(n)}(x_1, x_2, \ldots, x_n) = \begin{cases} 0, & \text{if } x_j = 0, \quad x_{n-1} = x_n = 1 \quad (j = 1 \,..\, n-2) \\ 1, & \text{if } x_j = 0, \quad x_{n-1} = 1 \qquad (j = 1 \,..\, n-2,\ n) \\ x_n, & \text{otherwise} \end{cases}$$

The given class of global transition functions because of a whole series of *specific* dynamic properties represents a quite certain interest for the further research even regardless of the decomposition problem. Now, considering a subclass of global transition functions of the set $E^\# \subset G_4$ that have defect ***1*** it is proved that composition of two such functions is again function of the set G_4, but with the defect distinct from ***1*** [53]. On basis of the analysis of sets of pairs of ***∞–MEC*** for global transition functions of the set $E^\#$ it is possible to establish an exact kind of such pairs for each of function of class $E^\#$, allowing to prove the following interesting enough result, namely:

For an arbitrary integer $n \geq 3$ the global transition functions $E^{(n)}$ from the class $E^{\#}$ have, on the whole, the negative decision of the 1–PDF.

The given result allows to receive a constructive negative decision of the ***1–PDF***, having other interesting enough appendices [536]. A little bit below some of them are considered in a little bit other context.

So, for decision of the existence question for the sub–semigroup G_4 of a certain finite basis one class $V \subset G_4$ of global transition functions $\tau^{(n)}$, concerning which the detailed enough research of structure of pairs of ∞–*MEC* has been carried out, is introduced. The suggested approach is constructive and allows to do persuasive enough assumptions relative to real possibility of carrying over of the results received in the given direction onto general case of classical structures ***1–HS***. While the base result for the further research the next theorem represents [5,53-56,88].

Theorem 159. ***For an arbitrary integer $n \geq 3$ exist 1–dimensional binary global transition functions $\tau^{(n)} \subset G_4$ which will possess a set of pairs of ∞–MEC of the following kind only, namely:***

$$C_1^{\infty} = \ldots\ldots g_{j+p+1}^1\, g_{j+p}^1 \cdots g_{j+1}^1\, g_j^1\, g_{j-1}^1\, g_{j-2}^1 \cdots g_1^1 X$$
$$C_2^{\infty} = \ldots\ldots g_{j+p+1}^2\, g_{j+p}^2 \cdots g_{j+1}^2\, g_j^2\, g_{j-1}^2\, g_{j-2}^2 \cdots g_1^2 X$$
$$\text{where } (g_j^1, g_j^2) - \text{ an arbitrary pair of the 2-element set}$$
$$\left\{(x_1 x_2 \ldots x_{n-1}, x_1^1, x_2^1, \ldots, x_{n-1}^1), (x_1^1, x_2^1, \ldots, x_{n-1}^1, x_1 x_2 \ldots x_{n-1})\right\}$$
$$(x_k \in \{0,1\},\ k = 1..n-1;\ j = 1,2,3,\ldots)$$

where X^{∞} is an arbitrary infinity binary 1–dimensional configuration and tuples $<x_1x_2 \ldots x_{n-1}>$, $<x^1{}_1x^1{}_2 \ldots x^1{}_{n-1}>$ are various and are defined by the kind of binary tuples of length n on which the global transition functions $\tau^{(n)} \subset G_4$ will realize «flip–flops».

On basis of the certain class of global transition functions $\tau^{(n)} \subset G_4$ that is defined by a special way and whose functions satisfy the conditions of theorem ***159***, using the detailed analysis of structures of pairs of ∞–*MEC* existing for them we can formulate the following result useful in a whole series of appendices of theoretical character [53]:

Sub-semigroup G_4 of 1-dimensional binary global transition functions does not possess any finite basis.

Of the detailed analysis of global functions $\tau^{(n)} \subset G_4$, which satisfy the conditions of theorem ***159***, it is possible to deduce the conclusion that among functions of the sub–semigroup G_4 there are the functions that possess the pairs of ∞–***MEC***, that consist of subconfigurations having quite concrete set of components of the minimal length ***(n–1)***. At that, for such functions there are pairs of ∞–***MEC*** that consist from periodic configurations with minimal period ***(n–1)***. Furthermore, a ***constituent (K)*** of configurations of this pair ∞–***MEC*** to some extent is similar to the *internal block* ***(IB)*** of classical ***MEC*** that are defined and considered in section **2.3** of the book. Moreover, it is interesting to mark, that the received lower bounds for the minimal sizes of ***K*** of pairs of ∞–***MEC*** for global transition functions $\tau^{(n)} \subset G_4$, ***IB*** of the classical pairs of ***MEC*** and ***NCF–1*** for case of classical structures ***1–HS*** coincide and are equal ***(n–1)***, i.e. on ***1*** less size of neighbourhood template of the appropriate classical structure ***1–HS***. At last, on basis of detailed enough analysis of the set of global transition functions $\tau^{(n)} \subset G_4$ that satisfy conditions of theorem ***159*** and possess the pairs of ∞–***MEC*** with minimal period ***(n–1)***, a new solution of the ***1–PDF*** can be received [54].

<u>Theorem 160</u>. The sub–semigroup L(a,1) of all 1–dimensional parallel global mappings $\tau^{(n)}: C(A,1) \to C(A,1)$ does not possess a finite basis. For an arbitrary integer $n \geq 3$ there is 1–dimensional global transition function $\tau^{(n)}$, defined in an arbitrary finite state alphabet A for which the 1–PDF has the negative decision.

The analogue of our theorem ***160*** on basis of an *algebraic* approach has been received by ***C.A. Bodnarchuk*** and ***G.E. Tseitlin***. Working on the *transformations* that are equivalent to classical structures ***1–HS***, they on basis of research of one class of semigroups, determined by the given transformations, have proved, that such semigroups hence also and ***1–***dimensional global transition functions can't have *finite basis* [324,325]. Such algebraic approach allows to solve the *decomposition* problem of global transition functions for ***HS***–models only at the principal level – opportunity or impossibility of the positive decision of the ***d–PDF***. On the other hand, the research method of the ***1–PDF*** on basis of theorem ***159*** allows to receive stronger result including elements of *constructive* determination of the kind of global transition functions $\tau^{(n)}$, for which the ***1–PDF*** has the negative decision. Besides, a series of methods used by us for decision of the decomposition problem also bear to a certain extent a constructive character. In this respect it would be interesting enough to spread the given approach also to general case of classical

HS*–**models. At that, it is necessary to mark, the suggested method of solution of the ***1–PDF that bases on the concept of ***∞–MEC*** is essential generalization of the decision method of the *decomposition* problem of global transition functions on basis of results on the *nonconstructability* problem in classical ***HS*–**models [54–56,90,536,567,617,618,640–643]**.**

In the next sections on basis of a *polynomial* representation of the local transition functions $\sigma^{(n)}$ in classical ***HS*–**models the further discussion of the *general* problem and the *generalized* problem of *decomposition* of global transition functions, and also the questions connected to them concerning the complexity of global transition functions in ***HS*–**models along with algorithmic solvability of the decomposition problem will be considered.

7.3. Questions of solvability of the decomposition problem for global transition functions

In this section on basis of one algebraic approach representing broad enough interest for the mathematical theory of ***HS*–**models along with its numerous appendices certain questions of researches of the *general problem* ***(d–PDF)*** and the *generalized problem* ***(d–GPDF)*** of *decomposition* of global transition functions are considered. However, first of all, we once again shall note principal difference between the *general* problem and the *generalized* problem. As it was marked earlier, the generalized *decomposition* problem differs from the *general decomposition problem* by the condition, what in the above representation ***(31)*** is admitted use of global transition functions with neighbourhood templates of the same size, what and an assumed global transition function $\tau^{(n)}$, i.e. instead of condition $n_j < n;\ j=1..m$ the condition $n_j \le n;\ j = 1..m$ is used. At that, both decomposition problems are non-equivalent – for a certain global transition function the ***d–GPDF*** can has a decision whereas the ***d–PDF*** no**.** We can adduce some simple enough examples of such decision as the compositions of binary ***1*–**dimensional global transition functions, namely: $\tau^{(3)}_{120} = \tau^{(2)}_{10}\tau^{(3)}_{30}$, $\tau^{(3)}_{84} = \tau^{(2)}_{10}\tau^{(3)}_{42}$ and $\tau^{(3)}_{20} = \tau^{(2)}_{9}\tau^{(3)}_{136}$; i.e. global transition functions with *discriminating* numbers ***120, 84*** and ***20*** can be represented in the form of composition of **2** global transition functions – with numbers ***30,42*** and ***136*** accordingly along with global transition functions $\tau^{(2)}_{10}$ and $\tau^{(2)}_{9}$ whose local transition functions are determined as $\sigma^{(2)}_{10}(x,y) = y+1 \pmod 2$ and $\sigma^{(2)}_{9}(x,y) = x+y+1 \pmod 2$.

Can be adduced a great deal of similar examples. Therefore, there are global transition functions for that the ***d–PDF*** has the *negative* decision at presence of a *positive* decision of the ***d–GPDF.***

In particular, with the purpose of analysis in this attitude of binary ***1–***dimensional both classical and non–classical structures with index of neighbourhood ***X***={***0,...,3***} the following rather simple procedure ***PDF*** programmed in ***Maple*** had been used. Procedure call ***PDF(m)*** returns a set in the form {[a,b],[c,d,g],...,[e,f],[l,s]} whose sublists with **2** and/or ***3*** elements determine the discriminating numbers of global transition functions making up all allowable compositions from the more simple global functions for an assumed global transition function with given number ***m,*** if similar compositions exist; otherwise, the procedure call returns the empty set {}. Incoming text of this procedure is presented below while for its comprehension entirely sufficiently of familiarity with ***Maple*** within our books [112–115] along with our library [545].

```
PDF := proc(m::nonnegative)
local  a, b, c, k, h, j, p, n, v, x, q, f, Res, Res1, Res2, Res3;
  if 32767 < m  then error  "GTF number should be in range  0..32767
    but had received %1", m  end if;
  assign(c = cat("", convert(m, 'binary')), Res = {});
  c := convert(cat(seq("0", k = 1 .. 16 – length(c)), c), 'list');
  Res1 := {seq(seq([k, j], k = 1 .. 8), j = 1 .. 128)};
  Res2 := {seq(seq([k, j], k = 1 .. 128), j = 1 .. 8)};
  Res3 := {seq(seq(seq([k, j, q], k = 1 .. 8), j = 1 .. 8), q = 1 .. 8)};
  a := table([seq(op([assign('h' = cat("", convert(k, 'binary'))),
      cat(seq("0", j = 1 .. 4 – length(h)), h)]) = c[k + 1], k = 0 .. 15)]);
  c := [seq(op([assign('h' = cat("", convert(k, 'binary'))),
      cat(seq("0", j = 1 .. 4 – length(h)), h)]), k = 0 .. 15)];
  b := [seq(op([assign('x' = cat("", convert(k, 'binary'))),
       assign('x' = convert(cat(seq("0", k=1 .. 8 – length(x)), x), 'list')),
       subs({1 = x[1], 2 = x[2], 3 = x[3], 4 = x[4], 5 = x[5], 6 = x[6],
       7 = x[7], 8 = x[8]}, table(["000" =1, "001" =2, "010" =3, "011" =4,
       "100" = 5, "101" = 6, "110" = 7, "111" = 8]))]), k = 0 .. 127)];
  f := [seq(op([assign('x' = cat("", convert(k, 'binary'))),
      assign('x' = convert(cat(seq("0", k = 1 .. 4 – length(x)), x), 'list')),
      subs({1 = x[1], 2 = x[2], 3 = x[3], 4 = x[4]}, table(["00" =1, "01" =2,
      "10" = 3, "11" = 4]))]), k = 0 .. 7)];
  for n to 16 do for k to 8 do for j to 8 do for q to 8 do
          if f[q][cat(seq(f[j][cat(seq(f[k][c[n][p .. p + 1]], p = 1 .. 3))
```

```
            [v..v+1]],v=1..2))] <> a[c[n]] then Res3 := Res3 minus {[k,j,q]}
            end if
          end do
        end do
      end do
    end do;
    Res := {op(Res), op(Res3)};
    for n to 16 do for k to 128 do for j to 8 do
            if f[j][cat(seq(b[k][c[n][p .. p + 2]], p = 1 .. 2))] <> a[c[n]]
            then Res2 := Res2 minus {[k, j]}
            end if
         end do
      end do
    end do;
    Res := {op(Res), op(Res2)};
    for n to 16 do for k to 8 do for j to 128 do
            if b[j][cat(seq(f[k][c[n][p .. p + 1]], p = 1 .. 3))] <> a[c[n]]
            then Res1 := Res1 minus {[k, j]}
            end if
         end do
      end do
    end do;
    Res := {op(Res), op(Res1)}
end proc:   # Maple 8, for other versions an adaptation can be needed
> PDF(27030);   → {[7,7,7], [91,7], [7,91]}
> PDF(32767);   → {[128,8], [8,96], [8,128], [80,8], [94,8], [96,8], [112,8],
                   [124,8], [126,8], [8,8,8]}
> PDF(32767);   → {[128,8], [8,96], [8,128], [80,8], [94,8], [96,8], [112,8],
                   [124,8], [126,8], [8,8,8]}
> PDF(20104);   → { }
> m :=[]: for k from 2601 to 4000 do if PDF(k) <> {} then m := [op(m),k]
  end if end do: m;
[2640, 2758, 2766, 2816, 2827, 3012, 3023, 3024, 3072, 3074, 3075, 3084,
 3087, 3104, 3105, 3116, 3119, 3120, 3123, 3132, 3135, 3264, 3267, 3276,
 3279, 3292, 3312, 3315, 3324, 3327, 3340, 3341, 3377, 3504, 3582, 3583,
 3586, 3598, 3644, 3696, 3826, 3838, 3840, 3843, 3852, 3855, 3888, 3891,
 3900, 3903, 3967]
> evalf(nops(m)/(4000 – 2600));    →  0.036
=====================================================
 [4, 5, 6, 7, 9, 10, 11, 13, 14, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33,
```

35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 50, 52, 53, 54, 56, 57, 58, 59, 61, 62, 65, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 188, 190, 193, 194, 196, 197, 198, 199, 201, 202, 203, 205, 206, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 241, 242, 244, 245, 246, 248, 249, 250, 251]

> evalf(nops(%)/256, 2); → 0.76

It is necessary to note, that all ***Maple*** procedures presented in the book have been implemented in software of ***Maple 8***, therefore for more new ***Maple*** versions a suitable adaptation of the procedures can be needed.

This procedure relatively simply is adapted as to more general kinds of ***1***–dimensional structures and for more simple structures**;** a series of its modifications the reader can find in the library [545]**.** Naturally, the more universal program of such purpose even for ***1***–dimensional case is appreciably more volumetric, however the essence of implemented algorithm of exhaustive search is simply perceived in the represented text of its concrete realization. Computer experiments carried out with help of the given procedure have allowed to receive diverse statistics relative to representability of *binary* global functions of dimensionality ***1*** and ***2*** in the form of *composition* ***(31)*** of more simple global transition functions given in the same state alphabet [88,90]**.** Only certain results of the gathered statistics are mentioned below.

In particular, in the above example of use of a procedure modification the list of numbers of all ***1***–dimensional binary global functions with neighbourhood index ***X***={***0,1,2***} which not possess compositions from more simple global functions is given. Very simple count shows that quota of structures with similar global transition functions is ***0.76,*** i.e. it exceeds ***3/4.*** In particular, among all classical ***1***–dimensional binary structures with neighbourhood index ***X***={***0,1,2***} that possess attribute of universal reproducibility in Moore's sense of finite configurations only three with numbers ***60, 90, 102*** can be represented in the form of compositions of two more simple functions with numbers {[7, 4], [4, 7], [13,7], [10,13]}, {[7, 7],[10,7]} and {[10,11], [7,6], [6,7], [11,7]} accordingly. At that, linear global function with number ***105*** that along with others possesses such attribute can't be presented in the form of composition of two more simple global transition functions.

On the other hand, use of the procedure has shown that linear binary global function $\tau^{(4)}_{27030}$ has **3** presentation in the form of composition of more simple functions, namely: $\tau^{(4)}_{27030}=\tau^{(2)3}_{6}=\tau^{(2)}_{6}\tau^{(3)}_{90}=\tau^{(3)}_{90}\tau^{(2)}_{6}$; at that, existing difference in *one* in indexing of both composing global functions is conditioned by algorithm of the procedure ***PDF.*** Thus, if the first representation follows of a result represented below whereas the second and third have been received by experimental way.

In addition, using a rather simple modification of the above procedure ***PDF,*** it is possible to make sure that the given estimation satisfactorily enough conforms to analogous estimations for classical ***1–***dimensional binary structures with an arbitrary neighbourhood index. Besides, the fulfilled numerous computing experiments with by far more complex global transition functions of dimensionality ***1*** and ***2*** have shown, that with growth of values of cardinality of a state alphabet and/or size of neighbourhood index the quota of classical structures for which the ***d–PDF*** has the positive decision very quickly decreases. So, for example, already minimal complication of the above binary structures up to ***1–***dimensional binary structures with neighbourhood template of size ***4*** reduces the quota of structures for which the ***d–PDF*** has the positive decision up to ***0.025,*** instead of ***0.242*** for the previous case. The call of *Mathematica*–procedure ***QdecompGTF*[*ltf*** {***, w***}**]** returns ***True*** if ***PDF*** for ***1-HS*** with ***LTF*** given by the list ***ltf*** of parallel substitutions of the form ***x1x2x3…xn → x1′ (xj, x1′∈A)*** has positive solution**,** and ***False*** otherwise**,** whereas through optional argument ***w*** the list of all such decisions is returned. The above parallel substitutions are standardized for more convenient programming of the ***QdecompGTF*** procedure [641,644]**.**

Form of representability of global transition functions also represents considerable enough interest. Specifically, of ***62 1–***dimensional binary global transition functions with neighbourhood index ***X=***{***0,1,2***} which have the positive decision of the ***1–PDF*** only two transition functions have symmetrical representations, namely: $\tau^{(3)}_{240}=\tau^{(2)}_{4}\tau^{(2)}_{13}=\tau^{(2)}_{13}\tau^{(2)}_{4}$ and $\tau^{(3)}_{170}=\tau^{(2)}_{6}\tau^{(2)}_{11}=\tau^{(2)}_{11}\tau^{(2)}_{6}$; in addition, global functions allowing single representations in the form ***(31)*** are absent. While of ***823*** binary transition functions with neighbourhood index ***X=***{***0,1,2,3***} which also have the positive decision of the ***1–PDF*** already ***185*** global transition functions have symmetrical representations, and ***382*** global functions have single representations as compositions ***(31).*** At last, quantities of admissible compositions ***(31)*** for both types of functions $\tau^{(3)}$ and $\tau^{(4)}$ are {***2,4,6,34***} and {***1..13,15,20,31,482***} accordingly**;** in addition**,** *maximal* values relate to the functions whose local functions have number ***0.***

A rather significant potential for the decomposition problem of global functions provides use of global functions in the same alphabet along with the unconnected neighborhood indices. And above all, it applies to the case when the decomposition problem is considered relative to the assigned subclasses of global transition functions. Now, consider the given proposition on an example of interesting class of linear ***1–HS*** whose local transition functions are defined by the following formula**:**

$$\sigma^{(n)}(x_1,\dots,x_n)=\sum_{1}^{n} b_k x_k \ (mod\ a);\ x_k, b_k \in A=\{0,1,\dots,a-1\};\ (k=1..n),\ a - integer \quad (sv)$$

This interesting class of ***d–HS*** is considered in the book with different standpoints. In particular, the simple linear ***1–HS*** with local transition function $\sigma^{(4)}(x_1, x_2, x_3, x_4) = x_1+x_2+x_3+x_4\ (mod\ a);\ x_j \in \{0,1,\dots,a-1\};\ j=1..4;$ ***a –*** prime can't be represented in the form of composition of the more simple structures of the same class with the connected neighbourhood indices in a whole. Whereas this problem is easily solved by means of usage of global transition functions with unconnected neighbourhood indices, suppose that $\sigma^{(4)}(x_1, x_2, x_3, x_4) = b_1x_1+b_2x_2+b_3x_3+b_4x_4\ (mod\ a);$ $b_jx_j \in \{0,1, \dots, a-1\};\ j=1..4;$ ***a –*** prime defines a global transition function $\tau^{(4)}$ and connected neighbourhood index $X4 = \{0,1,2,3\}$**.** Easy to see that two global transition functions $\tau^{(3)}$ and $\tau^{(2)}$ defined by local transition functions $\sigma^{(3)}$ and $\sigma^{(2)}$ with neighbourhood indices $X3 = \{0,3\}$, and $X2 = \{0,1\}$ accordingly, namely**:** $\sigma^{(3)}(x_1, x_2, x_3)=x_1+x_3\ (mod\ a)$ and $\sigma^{(2)}(x_1, x_2)= x_1+x_2\ (mod\ a);$ ***a –*** prime solves the ***1–PDF*** for ***GTF*** $\tau^{(4)}$ as $\tau^{(4)} = \tau^{(3)}\tau^{(2)}$**.**

The reader can make sure in justice of the following proposal, namely**:** ***For an arbitrary even integer n the solution of decomposition problem for GTF*** $\tau^{(n)}$ ***defined by LTF*** $\sigma^{(n)}$ ***in the form (sv) is defined as follows:***

$$\tau^{(n)} = \tau_1^{(n/2+1)}\tau_2^{(n/2)} = \tau_3^{(2)}\tau_4^{(n-1)}$$

where GTF $\tau_1^{(n/2+1)}, \tau_2^{(n/2)}, \tau_3^{(2)}, \tau_4^{(n-1)}$ ***are defined by local transition functions*** $\sigma_1^{(n/2+1)}, \sigma_2^{(n/2)}, \sigma_3^{(2)}, \sigma_4^{(n-1)}$ ***with neighbourhood indices*** $X_1= \{0, n/2\}, X_2 = \{0, 1, \dots, n/2-1\}, X_3 = \{0, 1\}, X_4 = \{2k\ |\ k=0..n/2\}$ ***accordingly.***

Therefore, the problem of detection of rather interesting classes of ***HS–*** models whose global functions admit the positive decision of ***d–PDF*** becomes enough actual owing to a significant enough oneness of this phenomenon. More precisely, as distinct from the initial posing of the ***d–PDF*** whose decisions were reduced to proofs of existence of global transition functions which not admit representation in the form of ***(31)*** of more simple functions, now with good reason it is possible to speak about search of rather interesting classes of global transition functions allowing such representations.

At last, it is necessary to mark, that the *general problem* of *decomposition* ***(d–PDF)*** of global transition functions represents the most appreciable interest from all standpoints, while the *generalized* problem ***(d–GPDF)*** has, rather, the more perceptual character and has been introduced by us for more exhaustive investigation of the decomposition problem of global transition functions. At the same time the ***d–GPDF*** perhaps has rather interesting appendices too except especially theoretical interest.

Questions of algorithmic solvability play in the modern mathematics extremely important part and make up wide enough class of so–called ***mass*** problems. A series of problems of this class that are connected to homogeneous structures has been considered above. Below, questions of algorithmic solvability concern both problems of decomposition of global transition functions for case of classical ***HS***–models. The *general* problem of algorithmic solvability of decomposition is reduced to the following rather interesting question, namely**:**

Whether exists a constructive algorithm determining the opportunity of decision of the problems d–PDF and/or d–GPDF for an arbitrary d–dimensional global transition function, defined in an arbitrary finite state alphabet A (d ≥ 1)?

In consideration of the specificity of the decomposition problem, first of all, from the applied standpoint, in case of the positive decision of the given rather interesting question it would be greatly desirable to determine a constructive solving algorithm. In the given direction the following a rather interesting result takes place [54–56,79,88,90,640]**.**

Theorem 161. For an arbitrary d–dimension global transition function determined in a finite state alphabet A there is a constructive solving algorithm for both the d–PDF, and the d–GPDF (d ≥ 1).

Actually, it is enough simple to make sure, that for an arbitrary global transition function, determined in an arbitrary finite alphabet, there is a *constructive* algorithm of ascertainment of an opportunity of decision of the ***d–PDF,*** and the ***d–GPDF (d ≥ 1)*** for the function. The essence of this algorithm is represented enough in detail, for example in [88] and in a few words can be described as follows.

In view of conditions $n_j < n$ for case of the ***d–PDF*** and $n_j \le n$ for the ***d–GPDF*** $(j = 1..m)$ an allowable representation ***(31)*** for an arbitrary ***GTF*** $\tau^{(n)}$ defined in a finite state alphabet ***A*** will consist of global functions out of a quite concrete finite set, we shall say, ***G.*** Therefore, in case of ***d–PDF*** at once suggests itself simple, but laborious enough algorithm consisting in direct search of all possible compositions in the form ***(31)*** out of functions of the set ***G.*** At that, inasmuch as, generally speaking, a composition of two ***GTF*** is noncommutative, then the above search should be exhaustive. Similar algorithm has been programmed in the above procedure ***PDF.*** Whereas in case of the ***d–GPDF*** the situation get complicated by the fact that a composition ***(31)*** generally speaking generates a global transition function with a neighbourhood template whose size is more than n for a verifiable function $\tau^{(n)}$. So additionally arises the problem of revealing of obtained results of compositions for which some of extreme elements of neighbourhood indexes would be insignificant, for example $(\forall x_1)(\sigma^{(n+1)}(x_1, ..., x_n, x_{n+1}) = \sigma^{(n)}(x_1, ..., x_n))$. Taking into account the above remarks, a decision algorithm for the ***d–PDF*** and the ***d–GPDF*** $(d=1,2)$ for connected neighbourhood templates has been programmed; in ***2***–dimension case for connected rectangular neighbourhood templates. Meantime, available computing resources impose the certain restrictions on sizes of a neighbourhood template and a state alphabet of an analyzed global transition function, above all, for dimensionality greater than one. For example, only exhaustive search of all possible compositions from the simplest global transition functions $\tau^{(2)}$ which could generate a ***1***–dimensional necessary global transition function $\tau^{(n)}$ defined in a state alphabet $A=\{0,1, ..., a-1\}$ will demand about $n(n-1)a^{a^2+n}/2$ averaged operations. Naturally, the value will essentially rise under the condition of use in such compositions in the form ***(31)*** of global transition functions $\tau^{(n_j)}$ for $n_j \le n$ $(j = 1..m)$.

Hence, the existing theoretical possibility of a constructive decision of both the ***d–PDF*** and the ***d–GPDF*** is limited by opportunity of modern computing resources. Naturally, the suggested solving algorithm is a rather bulky, however such programmed algorithm allows to receive ***(****under the condition of their existence****)*** all possible representations of any required global transition function $\tau^{(n)}$ of dimensionality $d=1,2$ that is defined in an arbitrary state alphabet ***A*** in the form of composition of more simple global transition functions, namely:

$$\tau^{(n)} = \tau^{(n_1)} \tau^{(n_2)} \tau^{(n_3)} \ldots \tau^{(n_k)} \qquad (n \ge 3;\ n_j < n;\ j=1..k) \qquad (32)$$

where for the function $\tau^{(n)}$ with neighbourhood template of size $n \ge 3$ as the global transition functions making up its *decomposition* **(32)** any combinations of functions $\tau^{(n_j)}$ can act only under the condition of the same state alphabet and $n_j < n$ $(j=1..k)$; i.e. the exhaustive constructive decision of the problem takes place. At that, labor expenditures of the constructive algorithm can be at times enough essentially reduced, if to restrict oneself to receiving of any allowable representation ***(32)***, or local transition function $\sigma^{(n)}$ corresponding to a tested global function $\tau^{(n)}$ has a certain special kind. In spite of the solvability of the ***d–PDF*** there is a series of the more special problems whose decision is known and is not so bulky. We shall start with consideration of similar most simple cases of decompositions of global transition functions.

Above all, we shall mark an important enough, especially from *applied* standpoint, case of special presentation of a global transition function $\tau^{(n)}$ for the limited number of constituent functions in representation ***(32)***, i.e. under the condition of $k \le k^* = const$. For the given special case the decomposition problem is algorithmically resolvable, and proof of that is simply enough [53]. At the same time, the resolving algorithm is constructive and enough simply implemented already on ***PC*** [8,15]. Similarly it is possible to consider the resolvability of the ***d–PDF*** and for a number of other special cases of the above representation ***(32)*** for global transition functions. In this connection it is interesting enough to consider the decomposition problem not only concerning all set of global transition functions, but also concerning a series of its subsets.

Let Q is a set of all such d–dimensional global transition functions $\tau^{(n)}$ determined in an arbitrary finite state alphabet A that for any function $\tau^{(n)} \in Q$ the following relation $(\forall c \in C(A,d,\phi))(|c| \le |c\tau^{(n)}|)$ takes place, where $|c'|$ – the maximal diameter of a configuration c'. Then in class Q of global functions the ***d–PDF*** is algorithmically solvable, allowing a very simple and fast enough resolving constructive algorithm [8,54].

<u>Theorem 162</u>. In class Q of d–dimensional global functions the d–PDF (d≥1) is solvable, possessing a simple constructive solving algorithm.

The decomposition problem of a global transition function $\tau^{(n)}$, when for its composition ***(32)*** the relation $\Sigma_j n_j = n-k+1;\ n_j < n\ (j = 1..k)$ takes place, represents a large enough interest from applied and theoretical standpoints. For this special case of representation the problem also is algorithmically solvable, and the proof of its solvability is *constructive*; at that, the solving algorithm has program realization, complexity of which depends on kind of a tested global transition function and/or of constraints imposed on functions making up composition ***(32)*** [15,56].

The so-called problem of the ***elementary*** representation of an arbitrary global transition function that consists in disclosure of functions with minimal sizes of neighbourhood templates making up its presentation ***(32)*** is interesting enough, having a simple but laborious constructive resolving algorithm. Owing to practical importance of the ***d–PDF*** and the ***d–GPDF*** a receiving of essentially constructive solving algorithms allowing to establish for a global transition function the impossibility of a decision of the above problems or to give exhaustive solutions in the form of required concrete decompositions ***(32)*** appears important enough. Thus, at level of solvability both the ***d–PDF*** and the ***d–GPDF*** ***(d≥1)*** are equivalent while at level of a possibility to receive a solution the second problem is by far more preferable. The general problem of algorithmic solvability of the ***d–PDF (d≥1)*** is generalization of a more particular problem of decomposition, namely:

Whether exists for a global transition function $\tau^{(n)}$ a positive decision of the d–PDF under the condition of belonging of functions $\tau^{(n_j)}$ which compose its representation (32) to some basic subset S of the set of all global functions defined in the same d–dimension and an alphabet A?

In this direction the following basic result having a series of important enough appendices takes place [72,617,618,640–643].

<u>Theorem 163</u>. Concerning an arbitrary basic subset W of the set of all d–dimensional global transition functions defined in a finite alphabet A both the d–PDF, and the d–GPDF are algorithmically solvable.

To a certain extent the problem of uniqueness of representation of an arbitrary global transition function $\tau^{(n)}$ in the form of a composition of finite number of the more simple functions is *intermediate* between the general problem and particular problem of algorithmic solvability.

Of the results of research of the ***d–PDF*** we know of existence of global functions for which this problem has the negative decision along with functions permitting at least two various nontrivial positive decisions. In this connection there is an interesting enough question of existence of global transition functions permitting only one positive decision of the ***d–PDF.*** Here, it is necessary to note at once one important enough property of global transition functions making up a sole presentation in the form ***(32)*** for an arbitrary global function, namely, the negative decision of the ***d–PDF*** for each of them. Thus, it is necessary to search similar global transition functions among global transition functions having the negative decision of the ***d–PDF.*** In particular, for that it is quite enough to consider a class ***S*** of linear global transition functions whose local transition functions $\sigma^{(n)}$ are determined by the following formula under the condition $A = \{0,1, ..., a-1\}$, namely:

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = \sum_{j=1}^{n} b_j x_j \quad (mod\ a); \quad x_j, b_j \in A, \quad a - a\ prime\ number \quad (j = 1\ ..\ n)$$

It is shown that in the class of similar linear global transition functions exist the functions allowing only single *positive* decision of the ***d–PDF.*** Furthermore, the fulfilled analysis has shown, that among already all classical ***1–***dimensional binary global functions with neighbourhood index $X = \{0,1,2\}$ exist functions having sole representations ***(32)*** *(with numbers* ***1,2,8,15,16,18,19,24,36,55,64,66,72,85,90,126,127)*** together with sole representations consisting of degrees of the same functions *(with numbers* ***1,15,85,90,127)*** [9,72]. Therefore, among ***d–***dimensional global transition functions defined in a finite state alphabet ***A,*** it is possible to distinguish at least *four* noncrossing sets of global transition functions concerning the possible decision of the ***d–PDF*** $(d \geq 1)$, namely:

- ***global functions not having a positive decision of the d–PDF;***
- ***global functions having the positive decisions of the d–PDF;***
- ***global functions having a sole positive decision of the d–PDF;***
- ***global functions having a sole positive decision of the d–PDF that consists of a degree of some more simple function.***

Therefore, an interesting enough question of dependence of the above four types of representation ***(32)*** of global functions on a chosen set of global functions arises. So, the enlargement of classical ***1–***dimensional binary global functions with neighbourhood index $X = \{0,1,2\}$ up to all binary global functions of the same type violates the oneness of global functions representations with numbers ***1, 15, 85, 90, 127,*** ensuring for them double representations in the form ***(32).*** Concerning the class of ***1–***dimensional linear global transition functions $\tau^{(n)}$ along with results represented above there are some interesting enough which have a lot of rather useful applications in investigations of dynamics of classical ***HS–***models. In particular, the above results can be rather useful in case of researches of representations of ***GTF,*** namely [88,640–643]:

An arbitrary global transition function $\tau^{(n)}$ defined by local function $\sigma^{(n)}(x_1, ..., x_n) = \sum_j x_j \pmod a$; $x_j \in A=\{0,1, ..., a-1\}$ has a presentation in the form $\tau^{(n)}=\left[\tau^{(a)}\right]^{(a^k-1)/(a-1)}$ if and only if n is presented as a^k, where k is a positive integer, a is a prime number and $\tau^{(a)}$ is a linear global transition function with connected neighbourhood template of size a. Each linear global transition function $\tau^{(n)}$ with local function $\sigma^{(n)}(x_1, ..., x_n) = \sum_j x_j \pmod a$ has presentation in the form $\tau^{(n)}=\tau_1^{(n/2+1)}\tau_2^{(n/2)}$ where a is an arbitrary integer, n is an even number and linear global transition functions τ_1 and τ_2 forming the similar representation have neighbourhood indices $X1=\{0, n/2\}$ and $X2=\{0,1, ..., n/2-1\}$ accordingly. For another values of n a linear function $\tau^{(n)}$ has the negative decision of the PDF in the form of functions composition of the same kind. For any integers $n\geq3$ & $a\geq2$ the class (15) contains N(n,a) linear functions allowing compositions from more simple functions of the same class, where N(n,a) is a function increasing in variables n and a.

With the representability problem of global transition functions $\tau^{(n)}$ in the form of composition ***(32)***, arises the question of interrelation of the nonconstructability types of global functions $\tau^{(n)}$ and global functions forming their decompositions. So, closure of the set ***C(A,d,∞)*** relative to an arbitrary global mapping, defined by global transition function $\tau^{(n)}$ plays considerable enough part in the question of possessing the nonconstructability such as ***NCF, NCF–1, NCF–2*** and ***NCF–3.***

The certain interest the question of influence on ***non–closure*** of the set ***C(A,d,∞)*** relative to a mapping defined by a global transition function $\tau^{(n)}$, of the existence of similar property for global transition functions $\tau^{(n_j)}$ ***(j = 1..k),*** that make up the above decomposition, represents. The possibility of the nonconstructability such as ***NCF*** to do configurations $c^{\infty}{}_j \in C(A,d,\infty)$ for which the relation $c^{\infty}{}_j\,\tau^{(n_j)} =$ $(j \geq 2)$ is possible by the nonconstructible ones, lays at the heart of receiving an answer to this question; i.e. arises a possibility to inhibit arising of non–closure of the set ***C(A,d,∞)*** concerning a function $\tau^{(n)}$ as a whole. In view of the told, an example of the composition ***(generally speaking 32)*** basing on this possibility has been found, allowing to formulate the following result [79,88,90,567,617,618,640–643].

<u>Theorem 164</u>. In order that a global transition function representable in the form (32) determine a global mapping concerning which the set

C(A,d,∞) is non–closed is required the existence of at least one global transition function $\tau^{(n_j)}$ (j = 1..m) of the presentation (32) that defines a global mapping concerning which the set C(A,d,∞) is non–closed.

With the question uniqueness of solution of the ***d–PDF*** for some types of global transition functions, the absence of the *positive decision* of the ***d–PDF (d ≥ 1)*** as a whole is closely connected. Hence, each example of global transition function having the *single solution* of the ***d–PDF (d≥1)*** proves negativity of the solution of the ***d–PDF*** as a whole. Interesting enough examples of such type of the global transition functions can be found in our works [5,53–56,79,88,90,536,567,617,618,640–643].

Let's consider one more algebraic approach to research of the ***d–PDF*** and the ***d–GPDF*** that represents rather significant interest for research of dynamic properties of classical structures ***d–HS (d≥1)*** as a whole. In the given context some earlier presented results on the decomposition problem have been overproved, and also some new results have been received thanks to the given approach. The possibility of presentation of a local transition function $\sigma^{(n)}$, defined in an alphabet ***A*** in the form of a polynomial of the following general kind underlies this approach:

$$\sigma^{(n)}(x_1, x_2, x_3, ..., x_n) = \sum_{j=1}^{j=m} b_j Y_j \pmod a \qquad (33)$$

where $b_j \in A$ and Y_j is a variable of the set $\{x_1, ..., x_n\}$ ***(j=1..m)*** or product of degrees of these variables. One of the basic results relative to the ***a–***valued logics ***(a > 2)*** says: ***An arbitrary function of a–valued logic can be represented in the polynomial form (33) if and only if a is a prime.***

The given result appears a rather useful not only for representation of ***1–***dimensional local transition functions, defined in a state alphabet ***A,*** by means of the above polynomials of the kind ***(33).*** The result can be quite successfully used and for ***d–****dimensional* local transition functions $\sigma^{(n)}$, presenting in a special way the neighbourhood index of classical structures ***d–HS (d ≥ 1).*** At that, in our opinion, the way of polynomial representation of the local transition functions will find wide enough application in researches on classical ***HS–***models and their numerous appendices. This reception was used for the further researches of both the general problem and the generalized problem of decomposition in classical structures ***d–HS.*** As a theoretical basis of the given reception the following basic result serves [5,53–56,79,88,90,567,617,618,640].

<u>Theorem 165</u>. An arbitrary local transition function $\sigma^{(n)}$, defined in a finite state alphabet A can be represented in the form of a polynomial

over S from n variables of a degree not higher than n*(a–1) if and only if an algebraical system S = <A; +; x> is the field.

If a number ***a*** is prime, the field ***M*=<A; +; x>*** also will be prime. As a prime field ***M**** is *isomorphic* to a ring of classes of residues of a ring of integers modulo ***a*** then we can limit oneself only to the representative polynomials modulo ***a.*** It is necessary to mark that at $a=p^k$ (***p*** *– a prime number and* ***k>a*** *is an integer)* an algebraic system ***M*=<A;+;x>*** also can be reformed into a field, defining peculiar operations of multiplication and/or addition, but in such case the representative polynomials over the field ***M**** will be inconvenient enough for research of properties of local transition functions. Therefore, in impossibility of representation of local transition functions by *polynomials* modulo ***a*** in a whole series of cases it is expedient to apply a little bit other approaches. Besides, one interesting enough example of algebraic system ***Q*** for polynomial presentations of ***a***–valued logic functions in case of composite number ***a*** is given by theorem ***74*** and enough in detail discussed in our works [5,84]. The problem of polynomial presentability of the local transition functions has a constructive decision for cases of both *prime* number ***a,*** and ***composite*** number ***a*** in the environment of such algebraic system, supposing effective enough computer realization basing on the simple algorithm whose description can be found, for example, in [5,56,640].

The question of polynomial presentability of local transition functions above a field ***A*** is enough closely connected to the question about the reducibility of the given polynomials over the field ***A,*** i.e. a possibility of representation in the form of a *product* of finite number of the more simple polynomials over the field ***A,*** namely:

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = \prod_1^k F_j(x_{j_1}, x_{j_2}, ..., x_{j_n}); \quad j_n \in \{1, 2, ..., n\} \quad (j = 1..p; p \le n)$$

In more general *posing* the given question is reduced to a possibility of representation of a polynomial, adequate to a local transition function $\sigma^{(n)}$ defined in a state alphabet ***A,*** in the form of a certain function ***F*** of more simple polynomials or polynomials of the definite type over the field ***A.*** This question is of interest as from the standpoint of research of a whole series of properties of local transition functions in classical structures and practical realization of computational and other certain discrete systems of various purpose on their base [536,640–643].

The factoring problem of the representative polynomials modulo ***a*** is directly connected to questions of research of dynamics of the *classical* ***HS***–models by *algebraic* methods. So, if for a certain local function the polynomial representing it admits decomposition of the next kind:

$$P(x_1, ..., x_n) = (x_j - b)^k P_1(x_1, ..., x_n) \pmod{a};\ j \in \{1,...,n\};\ b \in A\backslash\{0\};\ k \geq 1$$

then the appropriate global transition function of a classical structure ***1–HS*** will possess the nonconstructability such as ***NCF (NCF–3)*** and/or ***NCF–1.*** Thus, the *reducibility* problem of the representing polynomials modulo ***a*** seems as a rather important means of the dynamics analysis of classical structures ***d–HS (d ≥ 1).*** But, practical use of this approach is rather difficult because for an any numerical field ***A*** the structure of polynomials irreducible over the ***A*** is rather various, and any general methods establishing *reducibility* or *irreducibility* of polynomials from ***n > 1*** variables do not exist. Thus, for revealing of reducibility enough frequently it is necessary to use skilful enough receptions, or to solve problems of dynamics of classical ***HS***–models by means of other more acceptable methods [5,15,53–56,79,82,88,90,536,617,618,640–643].

Among set of all polynomials over a field ***A*** it is possible to single out a series of interesting enough classes. The polynomials in *normal* form of which all additive monomials have the same degree concerning the totality of variables play a peculiar part. These polynomials are called the ***homogeneous*** polynomials or ***forms.*** Owing to the fact that forms can be enough essentially simplified with the help of replacement of variables, it is possible to carry out the research enough effectively of properties of local transition functions corresponding to them [53–56].

The so–called symmetric polynomials constitute one more interesting enough class. A polynomial from ***n>1*** variables over a field ***A*** is called *symmetric* polynomial if it does not vary at any *reversible* permutations of its essential variables. It is well–known that the set of all symmetric polynomials from ***n*** variables forms a subring of ring of all *polynomials* from ***n*** variables over a field ***A.*** For purposes of our research a special interest represents the case when polynomial ***PP*** over a field ***A*** can be presented by a complex function in the form of a polynomial from the symmetric polynomials. In addition, the symmetric polynomials over a field ***A*** that are called the *elementary* polynomials play a special part:

$$p_k(x_1, x_2, ..., x_n) = \sum_1^r R_j(k, x_1, x_2, ..., x_n); \qquad r = C_n^k/k!$$

$$p_1(x_1, x_2, ..., x_n) = \sum_{j=1}^n x_j; \qquad p_n(x_1, x_2, ..., x_n) = \prod_{j=1}^n x_j$$

where $R_j(k,x_1,x_2,x_3, ..., x_n)$ are various arrangements *(within symmetry)* on ***k*** from elements of the set $\{x_1,x_2, ..., x_n\}$. In particular, the *elementary* polynomials $\{P_1, P_n\}$ have the kind, presented above. The basic result here can be formulated as follows:

An arbitrary symmetric polynomial over a field A can be presented by only single way in the form of a certain polynomial from elementary symmetric polynomials P_k (k=1..n).

For a practical decision of this problem it is possible to use the known method of indefinite coefficients, applying it to elementary symmetric polynomials onto which each symmetric polynomial will decompose. The theory of symmetric polynomials basing on the above base result has numerous appendices in various areas of the polynomials theory. The reasons determining the role of *symmetric* polynomials lay deeply enough and are discovered only at research of a series of properties of automorphisms of algebraic fields. In context of the ***HS***–problematics the *symmetric polynomials* defining *symmetric* local transition functions of the classical ***HS***–models also represent heightened interest since the models of such type under the certain conditions possess for example the property of universal or essential reproducibility in Moore's sense of finite configurations. Besides, the ***HS***–models with symmetric local transition functions $\sigma^{(n)}$ represent special both theoretical and applied interest in bio–medical sciences, physics, modelling, mathematics, and computing sciences, and also in a lot of other appendices [5,88,90,536]. The class of elementary symmetric polynomials from ***n*** variables over a field ***A*** is enough closely connected to the decomposition problem of global transition functions $\tau^{(n)}$ in the classical ***HS***-models. Let's denote the class of global transition functions $\tau^{(n)}$ of classical structures ***1–HS,*** whose local transition functions $\sigma^{(n)}$ are representable by elementary symmetric polynomials, through $\Psi(n, a)$. It is easy to make sure, that any global transition function $\tau_j^{(n)} \in \Psi(n,a)$ excepting the first (P_1) and last (P_n) has at any rate the general presentation of the following kind:

$$\tau_j^{(n)} = \tau_1^{(n-j+1)} \tau_j^{(j)}; \quad \tau_j^{(n)} \in \psi(j,a); \quad \tau_1^{(n-j+1)} \in \psi(n-j+1,a)$$

$$\sigma_1^{(n-j+1)}(x_1, \ldots, x_{n-j+1}) = \sum_{1}^{n-j+1} x_k \ (mod\ a); \ \sigma_j^{(j)}(x_1, \ldots, x_j) = \prod_{1}^{j} x_k \ (mod\ a) \quad (1 < j < n)$$

The question of representability of the first global transition function $\tau_1^{(n)} \in \Psi(n, a)$ was enough in detail discussed in section ***7.1.*** Inasmuch as the product is some kind of analogue of operation of addition, the results for function $\tau_1^{(n)} \in \Psi(n, a)$ are enough easily disseminated onto function $\tau_n^{(n)} \in \Psi(n, a)$. At that, for an integer $n \geq 2$ the global transition functions $\tau_1^{(n)}$ and $\tau_n^{(n)}$ are called ***basic*** functions of the set ***E(n, a)*** of all *symmetric* global transition functions $\tau^{(n)}$ from not more ***n*** variables defined in a certain state alphabet ***A***. So, an arbitrary global transition function $\tau_j^{(n)} \in E(n,a)$, excepting *basic* functions can be presented in the form of a composition of two *basic* functions $\tau_1^{(n-j+1)}\tau_j^{(j)}$ $(1 < j < n)$. On the other hand, not any *basic* function of the set ***E(n,a)*** is representable in the form of a *composition* of more simple *basic* functions. In addition, the presented arguments allow to formulate the following interesting enough result [5,53–56,79,82,88,90,536,567,617,618,640–643].

Theorem 166. Every global transition function $\tau^{(n)} \in E(n, a)$, except the basic functions, is representable in the form of the composition of two more simple basic functions of the following kind $\tau_1^{(n-j+1)}\tau_j^{(j)}$ $(1<j<n)$. Meantime, by far not each basic function of the set E(n, a) has similar representations in the form of composition of more simple functions.

The suggested method of polynomial presentation of a local transition function $\sigma^{(n)}$ from ***n*** variables over a field ***A***={***0,1,2, ..., a–1***} in case of a prime number ***a*** is naturally extended and to the binary case ***(a = 2)***. In the further we need a series of concepts and definitions.

Definition 29. Elementary conjunction is called monotonous if it does not contain the negation of variables; the formula of the next kind:

$$P(x_1, x_2, x_3, ..., x_n) = \sum_{k=1}^{s} \Theta_k \pmod 2$$

represents a Zegalkin's polynomial, where Θ_k (k=1 .. s) are pairwise–various monotonous elementary conjunctions over the set of various binary tuples $<x_1, ..., x_n>$. At that, the greatest of ranks of elementary conjunctions entering in such polynomial will be called the degree of the Zegalkin's polynomial.

Of the theory of boolean functions follows, that any boolean function can be presented in the form of a ***Zegalkin's*** polynomial, i.e. a binary local transition function $\sigma^{(n)}$ can be *uniquely* presented by a ***Zegalkin's*** polynomial from ***n*** variables of degree not above ***n***. At the same time, for creation of the ***Zegalkin's*** polynomial realizing an arbitrary binary local transition function $\sigma^{(n)}$ a series of methods is used among which it is possible to mark the known method of indefinite coefficients that is similar to case of ***a***–valued logics. Other method having a computer realization can serves as basis for proof of a possibility of presentation of any boolean function by a ***Zegalkin's*** polynomial [53,54,640–643].

In more general posing it is necessary to keep in mind that binary ***HS–***

models are the most convenient for research within ***Zegalkin***'s algebra being a version of algebra of logic [326]. About that speaks the simple fact, that at the ***Zegalkin's*** algebra a function of algebra of logic from n variables can be uniquely presented by some *reduced* polynomial from variables of degree not higher ***1***, whereas its coefficients are elements of the binary field {***0,1***} ***(k=1..n)***. At that, the ***Zegalkin's*** algebra admits natural generalization onto case of a–valued logics, if a is a degree of a certain prime number. It allows to apply enough effectively apparatus of the theory of polynomials over finite fields to investigation of both a–valued logics, and classical structures ***d–HS (d ≥ 1)*** for case of more general types of a state alphabet ***A***. Interesting enough discussions in this direction can be found, for example, in works [88,90,536,567,640].

Using a possibility of polynomial representation of an arbitrary local transition function, defined in an alphabet $A=\{0,1,\ldots,a-1\}$ ***(a – a prime)*** we have the possibility to overprove, generalize or improve a series of earlier received results concerning the *decomposition* problem, but also essentially advance research in this direction. In particular, on basis of research of class ***G*** of local transition functions that can be represented in the form of polynomials of the following kind:

$$\sigma^{(n)}(x_1, x_2, x_3, \ldots, x_n) = g\sum_{k=1}^{k=n} x_k \pmod{a}, \quad x_j \in A;\ j = 1..n;\ g \in A \setminus \{0\}$$

over the above field ***A*** along with the class of all *binary* local transition functions, representable by the ***Zegalkin's*** polynomials it is possible to receive the following basic result [5,53–56,79,88,90,617,618,640–643].

<u>Theorem 167</u>. For prime a and n by far not each local function $\sigma^{(n)} \in G$ ***can be presented in the form of superposition of finite number of more simple functions in the same alphabet*** $A=\{0,1,\ldots,a-1\}$***. For an arbitrary prime integer n≥3 the binary local transition functions*** $\sigma^{(n)} \in G$ ***cannot be presented in the form of superposition of finite number of the more simple local transition functions*** $\sigma^{(j)} \in G$ ***in the same binary alphabet.***

Of proof of this theorem follows, on basis of *polynomial* representation of local transition functions $\sigma^{(n)}$ by polynomials modulo a, except case of composite number a, it is possible to receive constructive decisions of the decomposition problem of global transition functions, not using the above concept of basis. In addition, on basis of the given theorem the absence of a finite basis for the set of all global transition functions $\tau^{(n)}$ of classical structures ***d–HS (d≥1)*** is easily proved. In view of told, the general criterion of the decision of the decomposition problem for an arbitrary global transition function, defined in a finite alphabet ***A=***

{***0,1, ..., a–1***} ***(a –*** *a prime****)*** can be received [5,53–56,82,88,90,536,567,640].

Theorem 168. A global transition function $\tau^{(n)}$ can be presented in the form of a composition of a finite number of more simple functions in the same alphabet A only if a polynomial P_n (mod a) appropriate to its local transition function $\sigma^{(n)}$ can be represented by a superposition of polynomials of the following general kind, namely:

$$P_{n_k}(P_{n_{k-1}}(P_{n_{k-2}} \cdots (P_{n_1}) \cdots))) \pmod{a}; \qquad n_j < n,\ j = 1..k$$

However, a method of decision of the *decomposition* problem on basis of this criterion contains, at times, insuperable complexities for rather big number of variables, though in simple cases can turn out a rather effective tool [5,15,53-56]. As the suggested method of research of ***HS–*** models is based on basis of polynomial presentation of local transition functions $\sigma^{(n)}$ over a field ***A,*** later under alphabets A_p and A_c we shall understand the set ***A*** = {***0,1, ..., a–1***} with composite and prime number ***a*** accordingly. The told concerning this algebraical approach concerns an alphabet A_p. Now we present a series of interesting enough results concerning both the ***d–PDF,*** and the ***d–GPDF*** that have been received on basis of *polynomial* representations of local transition functions $\sigma^{(n)}$ over a field A_p [54–56,79,88,90,536,567,617,618,640–643].

Theorem 169. For an arbitrary global transition function defined in an alphabet A_p the d–GPDF has the negative decision generally speaking except the trivial cases.

Thus, removal of restrictions on global transition functions entering in decomposition ***(31)*** of a global function $\tau^{(n)}$ does not change generally speaking negativity of decision of the decomposition problem of such type. As generally the ***d–PDF*** and the ***d–GPDF*** are nonequivalent, the following result represents particular interest and, first of all, from the standpoint of theoretical researches of ***HS–***models [53-56,82,88,90,536].

Theorem 170. If for a d-dimensional global transition function $\tau^{(n)}$ the d–PDF and the d–GPDF are equivalent then for such global transition function they are algorithmically solvable too.

Consideration of the decomposition problem in all community for the global transition functions defined in an alphabet A_p allows to receive the following important enough result [5,54–56,79,88,90,536,567,640].

Theorem 171. For each d–dimensional (d≥1) global transition function $\tau^{(n)}$, determined in an arbitrary state alphabet A_p, the d–PDF and the

d–GPDF are equivalent and algorithmically solvable.

The result of theorem ***171*** allows to receive answers to some questions put up in work [53]. Moreover, results of theorems ***170*** and ***171*** reveal, that the structure of a state alphabet ***A*** has rather essential significance for the question of equivalence of the ***d–PDF*** and the ***d–GPDF*** ***(d ≥ 1).*** Namely, for case of an alphabet A_p the equivalence and solvability of both problems takes place, while in general case of a state alphabet A_c these problems generally speaking are nonequivalent and question of their solvability has been considered on other base above. Further, the following important enough result allows to substantially make clear an interrelation of solution of the ***d–PDF*** and the ***d–GPDF*** for general case of a state alphabet ***A***={***0,1,2, ..., a–1***} ***(a –*** *a prime****);*** it gives a simple enough constructive decision of these **2** problems for global transition functions of the classical ***HS***–models [5,54–56,79,82,88,90,536,567,640].

<u>*Theorem 172*</u>***. For each d–dimensional global transition function*** $\tau^{(n)}$ ***defined in an alphabet*** A_p ***the d–PDF and the d–GPDF have positive solutions if and only if the global function*** $\tau^{(n)}$ ***can be presented in the form of composition*** $\tau^{(n)} = \tau^{(m)}\tau^{(q)}$ ***(m, q < n; m+q–1 = n; n > d+1) of two more simple d–dimensional global transition functions determined in the same state alphabet*** A_p***.***

The represented approaches to the representability problem of global transition functions in the form of a composition of the more simple functions allows in a lot of cases to solve the ***d–PDF*** more acceptably, than by means of the procedure of exhaustive search. However, such approach despite of its labouriousness allows to solve the ***d–PDF,*** and the ***d–GPDF*** at the most general case.

Questions of the general decomposition problem of abstract automata are directly connected to the optimization problem, that in case of the classical ***HS***–models has two basic aspects, namely**:** ***(1) decomposition of a global transition function into finite number of the most simple functions,*** and ***(2) decomposition of a global transition function into minimal number of more simple functions.*** It is simple to make sure in presence of a constructive solving algorithm for the ***PDF/GPDF*** for an arbitrary ***d***–dimensionality, neighbourhood index and a state alphabet on basis of the procedure of exhaustive search. The previous proofs of this fact illustrate only an admissibility of the used apparatus. At that, both aspects of the above minimization problem have the constructive positive decision, generally speaking. Besides, on basis of theorem ***172*** it is possible to receive the following rather interesting result [55,88].

Theorem 173. ***For an arbitrary integer $n > d+1$ there are d–dimensional global transition functions, determined in an alphabet A_p, for which the d–PDF and d–GPDF are equivalent and have negative decision.***

This result represents one more proof of negativity of the solution of both the ***d–PDF*** and the ***d–GPDF*** generally. In addition, on basis of the proof of theorem ***173*** it is possible to solve rather interesting question of estimation of number of global transition functions $\tau^{(n)}$ defined in a state alphabet A_p for which the ***d–PDF*** and the ***d–GPDF*** have positive decisions. The following base result representing independent interest too summarizes the research of this question [5,54–56,71–75,88,90,536].

Theorem 174. ***For nearly all d–dimensional global transition functions determined in an arbitrary state alphabet A_p both the d–PDF, and the d–GPDF ($d \geq 1$) have the negative decision.***

Consequently, we receive rather unexpected result, namely:

Quota of all d–dimensional global transition functions, determined in an arbitrary state alphabet A_p which admit positive decisions of the d–PDF, and the d–GPDF ($d \geq 1$) equals zero.

Thus, the study of the ***d–PDF*** instead of proof of existence of *negativity* of its decision turned into search of its rare enough positive decisions. Hence, relative to the *composition* operation the set of all ***d***-dimensional global transition functions defined in an alphabet A_p, appears enough *legibly* differentiated and has good enough prerequisites for definition of an appropriate *hierarchy* of *complexity* of global transition functions.

7.4. The complexity problem of global transition functions in the classical HS–models

Of our previous results of research of the ***d–PDF*** and the ***d–GPDF*** it is possible to establish enough easy that among all ***d***–dimensional global transition functions $\tau^{(n)}$ ($n \geq d+1$) which are defined in a state alphabet A_p a hierarchy of complexity of global transition functions $\tau^{(n)}$ can be introduced concerning the decomposition problem as follows.

Definition 30. ***A global transition function $\tau^{(n)}$ belongs to a s–level of complexity [$s < n$; designation: $\tau^{(n)} \in L(s)$] if and only if for this global transition function there are representations in the following form:***

$$\tau^{(n)} = \tau_1^{(n_1^p)} \tau_2^{(n_2^p)} \tau_3^{(n_3^p)} \ldots \tau_{k_p}^{(n_{k_p}^p)}; \quad n > d+1; \quad s = min\left\{ max\left\{ n_1^p, \ldots, n_{k_p}^p \right\} | p = 1..m \right\}$$

i.e. for the function $\tau^{(n)}$ the PDF has the positive decision. If the PDF for a certain global transition function $\tau^{(n)}$ has the negative decision, then such global function is ascribed to a complexity class L(n).

On account of the above results, definitions and proof of the theorem ***174*** it is possible to receive the following *asymptotic* relations having a series of important enough applications in the ***HS***–problematics [3,54–56,73,90,567,617,618,640–643], namely:

$$(\forall s \geq 2)(\#L(s) > 0); \quad \lim_{s \to \infty} \#L(s) / a^{a^s} \geq 1 \quad \text{(a is a prime number)}$$

where #G is cardinality of a finite set G.

In addition, on basis of theorem ***171*** and the complexity concept of ***d***–dimensional global transition functions concerning the ***d–PDF*** and the ***d–GPDF (d ≥ 1)*** the following important enough result on *solvability* of ***complexity*** levels of global transition functions of *classical* ***HS***–models [54–56,73] which represents, first of all, theoretical interest at research of *algorithmic* properties of dynamics of classical structures ***d–HS (d≥1)*** as the conceptual models of the spatially-distributed dynamic systems takes place [54–56,79,88,90,536,567,617,618,640–643].

<u>Theorem 175</u>. The determination problem of belonging of an arbitrary d–dimensional global transition function $\tau^{(n)}$, defined in an alphabet A, to a s–level of complexity (s ≤ n) is algorithmically solvable.

On basis of the introduced complexity concept of the global transition functions concerning the ***d–PDF (d–GPDF)*** we can receive interesting enough characteristics of global transition functions $\tau^{(n)}$. Of the above results follows, that we essentially used algebraic properties of a state alphabet A_p, since a local transition function can be *uniquely* presented by a polynomial modulo ***a*** of the maximal degree ***n(a–1)*** over the field A_p, and vice versa [10,54-56,88,90]. While in case of a state alphabet A_c by far not each local transition function, defined in a state alphabet of such type, can be presented in the polynomial form of the above kind. Namely, the following basic result takes place [5,54–56,73–75,88,567].

<u>Theorem 176</u>. For an arbitrary state alphabet A_c={0,1,...,a-1} the quota (W) of local transition functions $\sigma^{(n)}$, which are determined in a state alphabet of such type and admit polynomial representations modulo a, satisfies the following relation, namely:

$$\frac{1}{a^{a^n-4^n}} \le W \le \frac{1}{a^{a^n-(a-2)^n}}$$

Of the given result follows, that for case of a *composite* integer ***a*** nearly all local transition functions, defined in a state alphabet A_c cannot be represented in polynomial form modulo ***a*** for large enough values of parameters ***n*** and/or ***a*****.** In this connection the following question can be formulated, namely [53]**:**

Whether it is possible to define such algebraic system within which a polynomial representation for a local transition function defined in a state alphabet A_c could be determined?

As a result of the fulfilled analysis, one interesting enough example of such algebraical system has been proposed**;** within this system nearly all local transition functions defined in an alphabet A_c, can be *uniquely* represented by polynomials modulo ***a*** ***(****theorem* ***74)*** [5,84]**.** On basis of proofs of theorems ***74, 170, 171*** the following interesting enough result relative to the ***d-PDF*** and the ***d-GPDF*** in case of general state alphabet A_c of classical structures ***d–HS*** has been received**;** the result represents rather essential theoretical interest for the ***HS*****–**problematics as a whole along with the certain appendices [5,54-56,73-75,79,88,536,567,617,640]**.**

Theorem 177. Concerning nearly all global transition functions which are defined in an alphabet A_c and whose appropriate local transition functions $\sigma^{(n)}$ admit polynomial representations in the form (17), the d–PDF and d–GPDF are equivalent and algorithmically solvable.

Thus, on basis of theorems ***74*** and ***177*** the above results of research of the ***d-PDF*** and the ***d-GPDF*** are spread onto nearly all global transition functions defined in an alphabet A_c**.** However, in spite of it we cannot so far spread the results received in this direction onto general case of a state alphabet ***A*** which needs additional researches, except approach basing on the above exhaustive search.

Along with the above algebraic method which is based on polynomial representations of local transition functions**,** to their formal researches the methods and results of the algebraic theory of many-valued logics, for example, ***Post's*** iterative algebras can be enough successfully used [327]**.** In this respect it is interesting to determine and research a class of some nonconventional algebraic systems within which satisfactory presentations of local transition functions $\sigma^{(n)}$ defined in an arbitrary state alphabet ***A*** are possible. In particular, as elements of an alphabet

A, it is possible to choose certain objects investigated in the algebraic theory of numbers (*circular integers*, *divisors*, *etc.*) along with usage of results and methods of the advanced algebraic theory of numbers [54–56,79,88,90,328,536,567,617,618,640–643]. At last, research of different sets of global transition functions, closed concerning the composition operation represents essential enough interest from many standpoints. In this connexion a number of similar sets interesting from the applied standpoints and in context of dynamical, and extremal possibilities of homogeneous structures had been researched [640–643].

Along with representation of global transition functions that is based on the composition operation, a series of other representations rather useful in many theoretical researches of classical ***HS***–models and their applied aspects is interesting too. In particular, one example of similar representation is considered in our works [53-56,536] where for global transition functions a new operation of ⊕–addition is defined, relative to which the set of all global transition functions $\tau^{(n)}$ $(n \geq 2)$ constitutes a finite *additive* group with a null element. Definition of this operation allows to expand opportunities of representation of global transition functions through the more simple functions since usual composition operation does not give an opportunity to receive *positive* decisions of the ***d–PDF/d–GPDF*** as a whole concerning rather wide class of global transition functions. A series of interesting enough examples of use of ⊕–operation along with some other operations over global transition functions can be found in [5,54,90]. The decomposition problem takes place also for other interesting types of ***HS***–models. In particular, for the ***HSoS*** (*see section* **1.2**) the problem similarly to classical structures has negative decision as a whole [90]. The more detailed discussion of the ***d–PDF/d–GPDF*** can be found, for example, in [88,90,536,567,640].

On that the discussion of base results, received on the decomposition problem of global transition functions in the classical ***HS***–models that solve this problem and define a whole series of interesting questions for the further researches in this direction is being completed. At the same time, we once again shall note that negativity of decision of the *decomposition* problem along with its algorithmic solvability is proved simply enough whereas the used approaches for these purposes carry for problems of research of ***HS***-models more general character. Finally, the results that base on solution of the global decomposition problem compose a rather essential part of basic apparatus for research of ***d-HS*** $(d \geq 1)$ and are used sufficiently widely.

Chapter 8. Some applied aspects of HS–problematics

The sphere of appendices of ***HS***–models at all their generality now is sufficient extensive and needs a *special* consideration that lays beyond the present monograph. So, here we only in brief shall mark the most advanced fields of appendices**;** in addition with the more detailing we shall touch applied aspects of the ***HS***–problematics ***(****with the emphasis on some results received by us****)*** in such fields as mathematics, biological and computational sciences. At that, at present the ***HS***–concept along with properly independent interest with different degree of intensity is used as an important mathematical object of researches in a rather broad range of appendices, namely**:** cybernetics and synergetics, pure and applied mathematics, pattern recognition and signal processing, coding theory, cryptography, theory of computing, mathematical and physical modeling, theoretical and mathematical biology, information processing, artificial intellect, computer sciences, urbanistics, geology, logistics, etc. At that various ***HS***–objects can simulate the most general phenomenological aspects of the real world along with direct physical laws and processes at microscopic level [536,545]**.** ***HS***–models present a certain kind of the formal recursive worlds, whereas itself recursion is the *fundamental* concept in mathematics, physics, computer science, biology, art and even in such field as linguistics. For this reason today the such models are enough claimed, attracting the increasing number of researchers from the very various fields.

In many cases the ***HS***–models represent an alternative approach to the analysis of dynamics of the complex systems instead of the *differential* equations. As the spatial differentiation onto elementary automata is immanent property of ***HS***–models, they are simply irreplaceable there where the differential equations are ineffective or cannot be applied at all. In a lot cases there is no other way to find out *dynamics* of an initial configuration of a certain dynamic system, than to simply simulate its behaviour by means of an appropriate ***HS***–model [88,90,536,567,640]**.**

Such fundamental properties as locality and homogeneity that a priori is provided at level of ***HS***–axiomatics along with reversibility property of dynamics at programmable level allow to consider the *homogeneous structures* ***(cellular automata)*** as a perspective enough environment of physical modelling. That and a series of other essential circumstances carry over the ***HS***–problematics to the serious enough *interdisciplinary* level, turning the problematics into a certain conceptual environment of modeling, description and research of phenomena, processes, and objects from various naturally–scientific fields and some other fields. At the same time, possibilities of use of homogeneous structures as a nontrivial environment of physical modelling allow to consider them much more widely than simply independent objects for researches. In this connection they can be considered as a new perspective *conceptual* approach to the organization of the reversible computing processes in context of research of the general theory of computing and creation of new perspective means of computer facilities [90,545,567,640]. Now, it is quite possible to ascertain that the ***HS***–concept presents a *perspective* enough environment of modeling for realization of the *conceptual* and applied aspects of spatially–distributed dynamic systems of which the physical and biological systems along with various systems of parallel information processing are the most essential prototypes. Full enough presentation in the field of applied aspects of the ***HS***–problematics can be received in works [1-5,8,9,15,23,26-28,32,33,53-56,58,62,64–67,71–75, 90,146,149,150,156,157,161,166,167,171,178,184-187,201,209,213,255,273, 285,308,318,333,354,360,373,374,378,384–388,392,397–405,409–414,416, 417,536,567,640], and in numerous original sources quoted in them.

8.1. The certain aspects of use of homogeneous structures in pure mathematics

After very brief discussion of certain mathematical appendices of the ***HS***–*concept* we shall dwell on its application for research of interesting enough problems from combinatorial analysis, theory of numbers and discrete mathematics though the mathematical appendices presented in the given section are by far not exhaustive. An interesting enough survey of appendices of the ***HS***–concept in mathematics, including our certain results, can be found, for example, in [54,79,88,90,536,567,640].

8.1.1. Solution of the H. Steinhaus's combinatory problem

Polish mathematician ***H. Steinhaus*** more ***75*** years ago has formulated an interesting enough combinatory problem named «*pluses–minuses*», whose essence in our terminology is reduced to the following [5,54-56, 88,90]. Let $c(k) = p(1,1)p(1,2)p(1,3) \ldots p(1,k)$ will be the first string of the binary elements $p(1,j) \in \{0,1\}$; $j=1 \,..\, k$. Furthermore, values k are chosen of the set $M=\{3+4t, 4+4t \mid t=0,1,2,3,4, ...\}$ only. Then elements of the j–th string of length $(k-j+1)$ are calculated in terms of elements of the $(j-1)$–th string of length $(k-j+2)$ according to the next simple recurrent rule:

$$p(j,i) = p(j-1,i) + p(j-1,i+1) + 1 \pmod 2; \qquad (i=1 .. k-j+1;\ j=2 .. k)$$

It is simple to make sure that as a result a triangular figure $T(k)$ which will consist of $N=k(k+1)/2$ symbols $\{0,1\}$ will be obtained. Inasmuch as N are even numbers for values $k \in M$, we can formulate the following rather interesting question, namely:

Whether it is possible to determine for an arbitrary allowable value $k \in M$ the figures $T(k)$ that will contain the same quantity $m=k(k+1)/4$ of each of symbols «0» and «1»?

In case of the positive answer we shall speak, that similar string $c(k)$ is the solution of the ***Steinhaus*** problem for the given integer k; further for brevity the term «*S–problem*» is being used. A series of professional mathematicians along with a lot of amateurs was involved in solution of the S–problem which have received a number of interesting enough results. Meanwhile, solution of the general S–problem remained open. And only on basis of a whole series of results on classical structures **2–HS** together with *computer* modeling it was possible to receive a series of new rather interesting results along with the exhaustive decision of the general S–problem [5,8,81,640]. For the further we need a series of the basic definitions, notions and denotations.

Definition 31. Solution $S(k)$ of the S–problem for an arbitrary integer $k \in M=\{3+4t, 4+4t \mid t=0,1,2, ...\}$ is named the derivative solution if it can be presented in the form of concatenation $D(k)=S(k_1)S(k_2)S(k_3)...S(k_n)$ of solutions for values $k_j < k$ at $\sum_j k_j = k$ $(j=1..n)$ [denotation: $D(k)$]. Let $S(k)$ is a set of various solutions of the S–problem for some value k. It is easy to make sure that $S(3)=\{000, 011, 110, 101\}$ and $S(4)=\{1101, 1011, 0011, 1100, 1010, 0101\}$; these two sets of solutions are named the basic sets. A derivative solution $D(k)$ is named basic solution if in its $D(k)$–representation the defining relationship $S(k_j) \in S(3) \cup S(4)$ $(j=1..n)$ takes place {we will use designation: $B(k)$}.

Sets of *derivative* and *basic* solutions (*together with their elements*) of the S–problem for each integer k are denoted $D(k)$ and $B(k)$ accordingly. We can mark, the *basic* solutions represent an especial interest because they consist of elementary base solutions and to some extent illustrate one of interesting examples of the self–complication phenomenon. The thorough research in terms of the classical structure **2–HS** defined by a special way has allowed to receive a few rather interesting properties of solutions of the ***S***–problem that reveal their internal structure while rather general result in the given direction the following *basic* theorem represents [5,9,54–56,79,81,88,90,536,545,617,618,640–643].

<u>Theorem 178</u>. Let S(k), D(k) and B(k) – sets of all, derivative and basic solutions of the S–problem for some integer k∈M accordingly; then for each admissible integer k>2 the set S(k) is nonempty and for arbitrary admissible integer k > 10 the relation #S(k) > #B(k) takes place, where #R is cardinality of an arbitrary finite set R.

Thus, the given theorem gives the full decision of the ***S***–problem, that has been formulated more ***75*** years ago. Use of the combined methods ***(****theoretical analysis of the appropriate structures* ***2–HS*** *along with computer modelling****)*** has allowed to receive a series of interesting estimations for all types of solutions of the above ***S***–problem [5,6,54–56,79,88,90,536].

<u>Theorem 179</u>. For each integer k∈{3+4t, 4+4t | t=0,1,2, ...} the following determinative relations take place, namely:

$$\#S(k) \succ 2^{k-r(k)} \ \ for\ \ r(k) \le [k/2]; \quad \#B(k) \ge \begin{cases} 2^{3t-2}, & if\ \ k \in \{3+4t \mid t=1,2,3,\ldots\} \\ 2^{3t}, & if\ \ k \in \{4+4t \mid t=1,2,3,\ldots\} \end{cases}$$

Similar results and for derivative solutions D(k) of the S–problem are valid too.

It is necessary to note, the ***S***–problem can be essentially generalized as follows. Instead of two symbols {***0,1***}, an alphabet ***A***={***0,1,2,..., a–1***} that is typical for classical ***HS***–models, is used whereas elements of a string ***c(k)*** are chosen of the alphabet ***A***. At that, integers ***k*** are chosen of the set ***M***={***3+4t, 4+4t | t=0,1,2,3,4,...***} only. Then elements of the ***j***–th string of length ***(k–j+1)*** are calculated in terms of elements of the string ***(j–1)*** of length ***(k–j+2)*** according to the following simple recurrent rule:

$$p(j,i) = p(j-1,i) + p(j-1,i+1) + 1 \pmod a; \qquad (i=1\,..\,k-j+1;\ j=2\,..\,k) \qquad (34)$$

It is simple to make sure that as a result a triangular figure ***T(k)*** which will consist of ***N=k(k+1)/2a*** symbols of an alphabet ***A*** will be obtained. Inasmuch as ***N*** are integers for values ***k∈M*** then we can formulate the following rather interesting question, namely:

Whether it is possible to determine for an arbitrary admissible value k∈M the figures T(k) that will contain the same number m = k(k+1)/2a of each of symbols of an arbitrary alphabet A={0,1,2,..., a–1}?

The ***S***–problem in the given posing is named the *generalized*. It is quite reasonable to assume that the generalized ***S***–problem can receive the wider interpretation, namely: an neighbourhood index $X=\{0,1, ..., n-1\}$ for it is supposed arbitrary. The allowable integers k in this posing are chosen of the set $M^*=\{n+t(n-1) \mid t=0,1,2,3, ...\}$, and step-like figures $R(k)$ that contain $L=[(n-1)t^2 + (3n-1)t + 2(n+1)]/2$ symbols of the alphabet A are considered. At the made assumptions the ***general S***–problem will be reduced to a question of existence for each allowable integer k of a figure $R(k)$ containing equal number L/a of entries of each symbol of above general alphabet A. Meanwhile, one generalization of methods of the decision of the classical S–problem allows to formulate the next result being basic in the given direction [5,9,54–56,79,81,88,90,536,545].

<u>Theorem 180</u>. For an arbitrary alphabet $A=\{0,1,2,...,a-1\}$ and allowable integer $k \geq 2a$ the generalized S–problem has at least 2a solutions. The number GW(k) of solutions of the generalized S–problem for $A=\{0,1,2\}$ and allowable integers $k \in \{2+3t, 3+3t \mid t=1,2,3, ...\}$ equals $GW(k) > 2^{k-1}$. For each allowable integer $k \in M^*$, neighbourhood index $X=\{0,1,..., n-1\}$ and the alphabet A the general S–problem has at least 2k solutions.

In conclusion of the item it is necessary to note the results of theorems ***178-180*** can be generalized as well to cases of the higher *dimensionality* and recurrent rules ***(34)*** of the more general kind. In addition, a whole series of interesting enough results of researches in the given direction can be found in our works [54–56,79,88,90,545,617,618,640–643].

8.1.2. Solution of the S. Ulam's problem from theory of numbers

Heuristic research of the growth problem already for case of two and three dimensions discloses all variety of the growing figures which it is enough difficultly to satisfactorily characterize by formal methods. Therefore, with the purpose of simplification of research of the given problem ***S. Ulam*** has tried to introduce the corresponding definitions in one–dimensional case with hope, that certain basic properties of so–called *sequences of uniquely defined sums* ***(SUDS)*** will help to make clear picture in this direction [1,3–5,90,123,128,220,225]. However, the given problem has received the big enough popularity and in one's time has attracted attention of a lot of researchers not only from the standpoint of a formal problem of growth, but, first of all, in connection with the theory of numbers. For the last case the problem represents even more interest. The basic essence of the given problem is simple enough and can be presented as follows, previously introducing requisite notions.

On the set ***M***={***1,2,3,4, ...***} of positive integers a simple *binary* operation is defined as follows **φ:** ***x+y ⇒ z,*** where ***x,y,z∈M.*** Elements ***z*** make up a subset ***M*⊂M.*** The following restrictions are put on the operation **φ:** ***(1)*** starting with integers ***a*** and ***b (a<b)*** all subsequent elements ***z=x+y*** are received as sum of any two previous elements ***x,y∈M*** of the earlier received sequence, however we not include in them those sums which can be received by more than one way**;** ***(2)*** *themselves* numbers are not added, and at addition operation the extreme element of a generated numerical segment ***(a,b)*** of the ***SUDS*** should participate. A numerical sequence received thus will be named the ***SUDS(a,b).*** In particular the first twelve elements of the ***SUDS(1, 2)*** the following natural numbers make up, namely**: *1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28.***

Pairs of the *adjacent* elements in the ***SUDS(a,b)*** which differ by a value ***p=p(a,b)*** are named the ***twins.*** Further any sets of twins pairs we shall denote ***T(p).*** So, ***T(a + b)*** is a set of twins pairs of the kind ***p(a,b) = a + b.*** The original posing of the ***S. Ulam's*** problem consists in the definition of cardinality of the set ***T(2)*** for the ***SUDS(1, 2),*** i.e. *pairs* of the adjacent elements of the set ***M**** that differ by value **2.** Thereupon, ***S. Ulam*** has put forward the hypothesis about infinity of the set ***T(2).*** Inasmuch as we investigated the given problem in more general posing, we need a series of additional notions and definitions.

In addition to the sequence ***SUDS(a, b)*** we shall consider the sequence ***SUDS1(a,b)*** which differs from the ***SUDS(a,b)*** only by absence for it of obligatory participation in binary **φ**–operation of the extreme element of already generated numeric segment ***(a,b)*** for the ***SUDS.*** At that, the both presented variants of the ***SUDS*** along with independent interest in the theory of numbers have a series of rather interesting biological interpretations connected to the growth problem, formalized for the elementary one–dimensional case. To tell the truth, this formalization is tense enough concerning natural processes of growth, consequently any interpretation of the received results is relative in many respects. Concerning this problem we have investigated a series of questions of behaviour of the ***SUDS;*** they can be formulated as follows [1,3,5,640]**:**

- ***determination of partial densities of a SUDS, since a given element;***
- ***growth degree of elements of the SUDS, since a given element;***
- ***change of partial densities of twins pairs concerning the SUDS;***
- ***change of distance between the nearest twins pairs in the SUDS;***
- ***rating of number of twins pairs in a given segment of the SUDS.***

At that, all enumerated questions concern both sequences such as the ***SUDS(a, b),*** and the ***SUDS1(a, b)*** for any positive integers ***a*** and ***b.*** The basic results received in this direction can be characterized as follows. First of all, we have established that any sequence ***SUDS1(a, b)*** should possess the infinite set of twins pairs at least of one of the next types, namely**:** ***T(a), T(b)*** or ***T(a + b).*** It is shown that if a_k is ***k***–th element of the ***SUDS1(a, b)*** then ***k***–th element of the sequence ***SUDS1(da, db)*** will be a number da_k. This property is valid also for sequences such as the ***SUDS(a, b).*** Entirely other picture takes place for *sequences* ***SUDS(a, b),*** where practically exhaustive decisions have been received for a whole series of variants of the ***generalized*** problem of ***S. Ulam.*** For example, sequence ***SUDS(1, b)*** for ***b≥5*** possesses the infinite sets ***T(b)*** and ***T(b+1)*** of twins pairs, whereas its elements a_k are calculated according to the simple recurrent formulas, namely**:**

$$a_k = \begin{cases} b+k-2, & if \quad k \in \{3, 4, ..., b+2\} \\ 4*b-2, & if \quad k = b+3 \\ (k-b+1)*b+[(k-b-3)/2]-2, & otherwise \end{cases}$$

The density of this sequence concerning the set ***N*** equals **ρ = *2/(2b+1).*** The sequence ***SUDS(a, b)*** for ***a>1*** and ***b/a – [b/a] > 0*** has the infinite set ***T(a)*** of twins pairs, while its density relative to the set ***N*** equals **ρ=*1/a.*** Elements of the given sequence, since number ***k ≥ 3,*** are calculated by the simple recurrent formula $a_k = b+(k-2)a$. In our books [9,15,54] it is possible to find a series of other interesting examples of the ***SUDS(a,b)*** for which it is possible to establish explicit functional relations of the kind $a_k = F(k,a,b)$ and to find out a lot of other interesting behavioural properties of numeric sequences of such type.

For description of behaviour of the ***SUDS(1,2),*** i.e. the ***Ulam's*** classical problem and consequently of the ***SUDS(a, 2a)*** we shall act as follows. Along with the sets ***K*** and ***A(k)*** of numbers ***k*** and values of elements a_k of the sequence accordingly, we determine the set ***P*** of differences of the kind $\Delta_k = a_{k+1} - a_k$ of changes of values of elements of the ***SUDS.*** It turned out that *structure* of the above set ***P*** is much more convenient for research. Namely since the number ***k=14,*** in the set ***P*** quite definite regularity is being already traced.

An element $\Delta_k \in P$ is named the ***jump,*** if $\Delta_k \neq \{2, 8\}$. An element $\Delta_k \in P$ is named the ***growing jump*** if it has the maximal value among all jumps Δ_k ***(j<k).*** The growing jumps are not limited from above whereas their

values grow with growth of ***k***–value. An interval out of four elements $<2,8,2,\Delta_k>$ (Δ_k – *an jump not necessarily the growing jump*) is named the ***basic*** interval. It is possible to show, that since the number ***k = 14***, the set ***P*** will consist only of ***basic*** intervals adjoining to each other, i.e. ***P*** can be represented in the form of the following sequence, namely:

$$P = \{<2,8,2,\Delta_1>, <2,8,2,\Delta_2>, <2,8,2,\Delta_3>, <2,8,2,\Delta_4>, ..., <2,8,2,\Delta_j>\}$$

Hence, research of the set ***P*** is reduced to research of behaviour of the subset ***P1***=$\{\Delta_k\}$ of its jumps. It is possible to show, that distribution of the growing jumps in the set ***P*** submits to the quite definite *regularity*, allowing to determine the following functional relation $a_k = F(k,d)$ for the ***SUDS(d,2d)*** [5,8,9,15,54,88,90], allowing to receive the full decision of the ***Ulam's*** classical problem.

<u>Theorem 181</u>. The SUDS(1,2) has infinite set T(2) of twins pairs, while its density concerning the set N of integers is defined by the relation:

$$\lim_{k \to \infty} \frac{4*(2^{k+2}-4)+14}{(12+P_o)*(2^{k+2}-4)+72*P_o*5^{2k-10}} = 0$$

In particular, for experimental research of the ***SUDS*** of various types a special simulation program, which has allowed to receive a lot of very interesting *empirical* results has been developed [9,320]. For particular, it has been shown that partial densities of the ***SUDS(1, 2)*** *monotonically* tend to a limit in compliance with the empirical formula $r(k)=14/k^4+m$ $(0 \le m < 0.002)$, whereas rate of convergence to the given limit is defined by empirical formula $\Delta(k) = 1150*k^{-2.31}$. A whole series of some other estimations and discussions in this direction can be found in [90,545].

The results received by us on the ***Ulam's*** generalized problem along with pure mathematical interest are of interest as well for research of formal models of growth in elementary ***1***–dimensional cases, and also from the standpoint of the applied theory of complexity of computing algorithms and applied aspects of the ***HS***–problematics as a whole.

Research of a number of other types of numerical sequences, that in a certain measure have to a certain extent even outlying enough formal analogies with process of growth and other biological phenomena in one-dimensional case, presents the quite certain interest. At present in this direction we investigate a series of interesting enough numerical *sequences* that with a certain degree of formalization can be associated with some biological phenomena in the one–dimensional case.

In conclusion of the given section we present one more example of use of ***HS*** concept for obtaining of rather interesting nontrivial arithmetic properties of the *generalized Pascal triangle* ***(GPT)*** & *Fibonacci* numbers, more precisely on the basis of specific types of classic ***HS***–models [53]. Further the ***GPT*** concept is introduced as follows.

Consider an arbitrary string of integers $P_o, P_1, P_2, ..., P_k$, which for $k = 0$ degenerates into a single P_o–number. We will constitute from it a new numbers string $B_o, B_1, ..., B_{k+1}$ by the following simple recurrent rule:

$$B_o = P_o;\ B_j = P_{j-1} + P_j;\ B_{k+1} = P_k\ (1 \le j \le k)$$

About this new string will say that it is derived from the previous one by the ***Pascal's*** law etc. If we let $P_o = 1$, we will obtain the well–known arithmetic Pascal triangle. We now choose the classical structure ***1–HS*** with neighborhood index of the form $X = \{0,1, ..., n-1\}$ and ***A***–alphabet of internal states that coincides with the set of all *non-negative* integers, and the ***LTF*** of the following linear form, namely:

$$\sigma^{(n)}(x_1, x_2, x_3, ..., x_n) = \sum_{j=1}^{n} x_j, \quad x_j \in A \quad (j = 1..n)$$

Then the rows of ***GPT*** are defined as final configurations generated in such structure from the *initial* configuration $P_o = \square 1 \square$. Let the members of each ***GPT*** are numbered from left to right, beginning from zero, and the number being on the ***p***–th place of ***k***–th row is denoted as $T_k^p(n)$; at that, $T_k^p(n)$ is defined for arbitrary integers $k \ge 0, n \ge 2$ and $p = 0..k$. Now let ***S(k, n)*** denotes the sum of ***k***–th row of the ***GPT***. Then on the above assumption, we have the following rather interesting result [53]:

Properties of the generalized GPT are defined by the next relations:

$$S(k,n)*S(k,m) = S(k,n*m); \quad S(k,n)*S(t,n) = S(k+t,n); \quad S^m(k,n) = S(k,n^m);$$
$$S(k,n)\,/\,S(t,n) = S(k-t,n); \quad r = k*(n-1)+1; \quad T_n^p(n) = T_k^{r-p}(n);$$
$$\sum_{p=0}^{r} T_k^p(n) = n^k; \quad \sum_{p=0}^{r} T_k^p(n)^2 = T_{2k}^p(n); \quad \sum_{p=0}^{r} (-1)^p\, T_k^p(n) = 0$$

The more detailed study allows to obtain other interesting properties of the ***GPT***, and generalize the results on the general ***d***–dimensional case. Inasmuch as Pascal triangle and Fibonacci numbers have a very close relationship with the binomial coefficients, factorials, a number of important mathematical formulas and tables, therefore such studies present a certain interest also in organization of parallel computations based on ***HS***–models. At that, the linearity of ***LTF*** of the structure that generates ***GPT*** allows to try to relate purely mathematical properties of such ***HS***-models with their property of universal reproducibility of finite configurations in the Moore sense [641,644].

8.2. Certain applied aspects of homogeneous structures in biological sciences

The latest ten years are characterized by utterly intensive penetration of the newest mathematical concepts and approaches into biomedical sciences. Above all, that is connected to the further becoming of both the theoretical biology and the mathematical biology, along with mass use of the modern computing means, allowing with a rather *high* level of visualization to investigate various biomedical models. In addition, the most intensive, interesting, and perspective attempts are made for research of evolutionary biological systems. One of the most complex and intriguing fields of the modern biology ***– biology of development*** of alive systems has been subjected to the mathematical analysis.

Considering in the section the questions of *discrete modeling* of biology of development on basis of the ***HS***–concept, it is necessary to have in mind at the same time, that this apparatus can be enough successfully used for creation and research of a lot of other biologically motivated formal models. Moreover, it is necessary to have in mind, that the ***HS***–concept has essentially more general character, involving *fundamental* medical and biologic *problematics* along with a number of biophysical directions into the modelling environment. At that, the more detailed discussion of the similar questions can be found in scientific literature quoted here, and also in Internet by the appropriate key phrases.

The detailed enough consideration of *applied* aspects of ***HS***–models by way of an environment of research of *biologically* motivated problems can be found in our works [5,90,567,545] whereas here only their brief sketch is submitted. In particular, with the basic prerequisites of such model approach to biology of development together with its historical stages the reader can familiarize oneself in [1–5,10,123,134,162,163,193, 194,264,329–335,537,545], in numerous sources, quoted in them, along with well-known journals such as «*Mathematical Biology*», «*Biophysics*», «*Mathematical Biosciences*», «*Journal of Theoretical Biology*», etc. At that, our interests in this respect were in the sphere of modelling of biology of development from the cybernetic standpoint with an emphasis on discrete aspect of modelling. In this connection, the approach on basis of two *base* concepts, namely, of ***HS***–*models* and ***Lindenmayer's*** *systems* **(*L*–*systems*)** has attracted our especial attention. This theme was being developed by us most actively [1,3–5,10,23,26,27,31,33,46,79,88,90,114, 136,163,171,201,203,233,251,264,333,536,545,567,617,618,640–643].

Growth and *regeneration* of an organism are carried out via continuous process of self–reproduction of cells in an organism, mainly, while, the *differentiation* of cells during the growth is considerably more complex for comprehension inasmuch as in the general opinion of the modern biologists all cells contain the same set of genetic rules – new cells will be genotypically identical to the predecessors. In the given connection the following important question arises, namely:

How cells become different from each other, developing into carefully produced and stable spatial forms?

Moreover, all development is strictly controllable from within in such a manner, that various parts of a developing organism develop in the certain proportions relative to each other and an organism as a whole. In addition, the organism during own development and even after in rather significant limits is capable to eliminating the damages caused by the external reasons, i.e. an organism in the certain limits is capable to ***regeneration.*** Naturally, the development uses strict mechanisms of the control, regulation and adaptation. Meanwhile, to date we do not know the best answer to all these questions, except as solution of the similar problems for artificial systems, i.e. use of the certain modelling principle for research of the development. In addition, it is necessary to have in mind, the studying of a development phenomenon has led a series of researchers to rather important conclusion that an *organism* cannot be considered as a certain artificial machine – an ***automaton.***

Therefore from the standpoint of biology, cybernetics and the general systems theory it is rather important to try to elucidate the following important gnoseological question, namely:

Whether can an artificial machine develop similarly to alive systems in general, and how we can secure that?

The essence of the given question enough in detail is discussed in [90]. Biological development includes two important phenomena: *growth* of an organism along with *differentiation* of *cells* making up the organism. It is known that *growth* is simple increase of an organism size, mainly, due to controlled self–reproduction of its cells whereas *differentiation* is much more complex phenomenon, therefore it is enough expedient to distinguish at least *two* its types – *spatial* and *phenotypic* differentiation, which ***M. Apter*** has named the *functional differentiation* [330].

So, in a growing tissue it is possible to differentiate change of the *form* and *configuration* of *intercellular connection* (*spatial differentiation)* along with increase at differentiation of separate types of its cells (*phenotypic* *differentiation***).** It is necessary to mark, that for *spatial* differentiation in biology of development the settled term «*morphogenesis*» exists but for the purposes of our modeling the first term as a more natural is used. Certainly, the phenotypic differentiation takes place and in the spatial differentiation, however, with the purpose of the greater transparency of modelling problems this will not be taken into account by us under the condition of the predominating role of *spatial* differentiation [1,4,5, 27,31,33,46,54–56,79,88,90,536,545,567,617,618,640–643]**.**

But a ***developing*** organism is characterized not only by a possibility of achievement of complex *spatial* and *phenotypic* differentiation, but also in the greater or smaller extent by presence of ability to *regulation* and ***regeneration*** during the development and the subsequent functioning. At that, the *regulation* is understood as an opportunity of an organism to develop into a normal person even in case of occurrence of *obstacles* during the development, for example, if *removal* or *reorganization* of its cells take place, excepting critical cases, lethal for the organism. While by way of the *regeneration* we understand a possibility of an organism to restore in the certain limits any infringement that the organism has received during his own full development.

In spite of importance of understanding of processes of the biological development including *spatial* and *phenotypic differentiation***,** *regulation* and *regeneration***,** along with phenomenon of self-reproduction the first attempts to achieve success in the given direction can be attributed to the *first* stage of the model approach that is characterized by modeling of *separate* phenomena of the *general* development as a whole. The *base* role of the *first stage* of *biological* modeling can be characterized by that satisfactory formalization that has been given to a whole series of very complex phenomena of the *general* process of biological development, and later on was being corrected on basis of the analysis of numerous formal models [1,3–5,8,23,26,27,31,33,123,136,162,330-338,343-351,536]**.** The subsequent analysis of a lot of the models has allowed to look in a new fashion to some important *regulator* mechanisms of development. However we had a whole series of models, non–linked by the general theoretical base. Naturally, the given position did not assist formation of the unified apparatus of modelling in biology of development.

Meanwhile, already within of the first stage two formal apparatus of modelling of some phenomena of development have arisen, namely**:**

cellular automata and Lindenmayer's parallel grammars

Cellular automata that have received subsequently Russian synonym

«*Homogeneous structures*» ***(HS)*** and have been introduced into Russian terminology at our submission along with present generally accepted terminology in this field [1,3,5,8,19,20,119], for the first time have been really used by ***John von Neumann*** for research of the *self*–reproduction problem [124,128], whereas parallel developing grammar for the first time have been introduced by ***A. Lindenmayer*** [3,163,251,264,336–338] for modeling of processes of *morphogenesis* of plants and subsequently has received the name «***L-systems***». Now ***HS*** and ***L***-systems represent the most general and popular apparatus of *discrete modeling* in biology of development whereas the mathematical theory of formal means of modelling is very well advanced and allows to investigate by formal methods at cellular level such phenomena of development as *growth*, *self–reproduction*, *differentiation*, *regulation* and *regeneration* [3,5,163,251, 264,336-338,536,567]. Along with these problems the ***HS***–models allow to study satisfactorily such questions of development as complexity of developing systems; processes controlling the growth, regulation and regeneration; the necessary and sufficient conditions of regeneration and regulation, stability of the development, etc. [1,3–5,23,26,27,31,33, 54,90,536]. Meantime, the apparatus of ***HS***–models causes also a whole series of rather essential *complexities* at studying of some biologically–motivated phenomena in their environment. The basic difficulties are connected to large enough sensitivity of ***HS***–models to such important factor as dimensionality, and with serious restrictions on opportunity of cell fission within an arbitrary modelled developing organism, i.e. with existence of rather rigid system of coordinates in the ***HS***–models.

Taking into account essential complexities of modelling in ***HS*** of some biological phenomena and processes, ***A. Lindenmayer*** has introduced the systems known nowadays as the ***L–systems*** [3–5,9,162,163,251,260, 336]. Within of ***L****-systems* for modeling of morphogenesis and growing structures ***A. Lindenmayer*** has offered the *branching* algorithms, while a series of investigators for modeling of development and growth has introduced the graphic generative systems [337]. On basis of ***L****-systems* subsequently a lot of interesting enough growing algorithms has been realized; good review can be found in fine work [338]. Lately on basis of ***L***-systems the increasing number of models of both actually growth and of growth accompanying the general *phenomenon* of development is being worked up. Meanwhile, in spite of the large *preferability* of the ***L***–systems as an environment of modeling in biology of development, the ***HS***–models represent rather interesting means of research of many processes and phenomena of development as they well enough meet the cellular nature of the biological systems and allow to create rather effective models of development which are qualitatively visualized by means of modern computing means. It is possible to ascertain with all definiteness, that the ***HS***–models and the ***L***–systems well supplement each other, stimulating creation of the *modern* apparatus inheriting the best features of both specified systems of modeling of processes in the *development* biology. Definition of ***L***–systems along with more detailed discussion of questions of their applicability for solution of problems of biologic modeling can be found, for example, in [5,90,536,567,640].

In particular, it is shown that ***L***–systems enough essentially expand ***1***–dimensional ***HS***–models in sense of sets of words generated by them. In addition, from the standpoint of biological adequacy the ***L***–systems receive satisfactory enough interpretations, perfectly showing itself at the description of a lot of biological processes; they are now the most developed, perhaps, in the mathematical plan and represent adequate apparatus in the biological attitude for many problems of the discrete modeling in the development biology [1,3-5,536,567]. Concerning the apparatus, strictly speaking, the ***L***–systems are more abstract than the ***HS***–models because they are not linked rigidly with co–ordinates and, as a matter of fact, they are one of types of parallel formal grammars that now are intensively researched [5,264]. In addition, it is necessary to have in mind, the ***HS***–models can be considered as a certain type of parallel formal grammars too (*τ_n–grammars; chapter* ***5***).

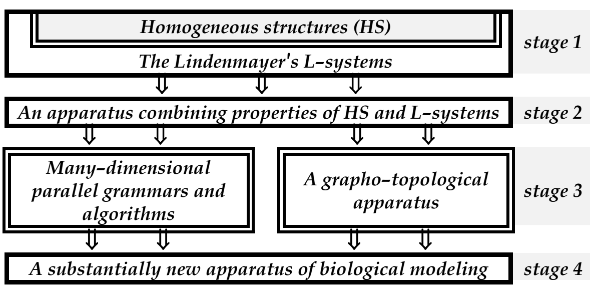


Fig. 16. A prospective scheme of development of the perspective apparatus of biological modelling.

In works [5,88,90,567] the ***HS***–models and the ***L***–systems are analyzed in detail in a context of their potential possibilities for modeling in the development *biology*, disclosing a series of defects of both systems. So, the defects intrinsic to both systems suppose the urgent necessity of continuation of works on creation of a mathematical apparatus, most adequate to problems of biological modeling. In this direction rather intensive investigations with use of both interdisciplinary approaches and attracting of a lot of the known experts working in adjacent areas are conducted [536,567,640]**.** A detailed enough discussion of possible ways of the making up of a new apparatus for *discrete* modeling in the development biology can be found in [5,31,90,567]**.** Whereas on fig. ***16*** the basic *predictable* stages of the making and the further development of discrete model approach in the development biology modeling and in a series of some other fields of mathematical biology are presented.

In addition, further we accentuate attention on the discrete modelling apparatus, therefore a whole series of interesting enough continuous models of development had remained beyond scope of consideration [1,4,5,536]**.** Further the most interesting approaches and models of the base aspects of the development biology are discussed within the ***2****nd* stage of the modelling which has received the brightest development recently. With other aspects of modelling of the development biology the reader can familiarize oneself enough in detail in literature quoted above and numerous sources contained in it. We shall start discussion with the modeling problem of such general phenomenon for all alive substance as the *self–reproduction***.** The given problem made up a *corner* stone of the basic discussions about possibilities of machines and alive systems, and was one of the base catalysts of stimulation of researches on abstract automata for their opportunities as the analogues or even substitutes of the mature alive systems, including the man. Naturally, the actuality of the given problematics has rather great and extremely multifold character, including and such extremely complex in every respect the question, as the problem of a life origin and its destination in the general system of the universe.

8.2.1. Formal discrete models of self–reproduction

The mentioned phenomenon of self–reproduction is the most typical feature of animate nature and it is no wonder that the first attempts of cybernetic modelling have touched this process. To detailed research of a possibility to embody in a machine the process of *self–reproduction* by the first, perhaps, has proceeded ***J. von Neumann*** who has offered a conceptual approach to the solution of the given problem [124]**.** The results received further on molecular genetics detect startling *analogies* between elements of the ***Neumann's*** self–reproducing automaton and processes in an alive biological cell [536]. A series of researchers have considered the self–reproduction problem from purely mathematical standpoint. So, ***J. Myhill*** [128] and ***A.R. Smith*** [131] have investigated the problem from the recursive–theoretic standpoint, whereas ***S. Ulam*** has formulated the existence problem of an universal self-reproducing matrix system [220]. In work [16] we have proved absence of *universal* self–reproducing matrix systems of large enough rank. In our opinion, these mathematical models existing for today, allow to investigate on a formal level only the most general phenomenon of self–reproduction at the same time they are not able to add *anything new* for the problem of cybernetical modelling of the development biology. In that they are similar to mechanical models of self–reproduction [1,3–5,88,536,567].

In view of the restrictions of application of *mechanical* and *mathematical* models ***J. Neumann*** at ***Ulam's*** suggestion has addressed to the cellular automata (*homogeneous structures*; ***HS–models***). Thus, concept of the ***HS*** has allowed ***J. von Neumann*** to construct an interesting cellular model which gives an opportunity to investigate process of self-reproduction by formal methods of logic and the theory of abstract automata. Great many researchers, working on the ***Neumann's*** cellular model, later on have considerably improved and have simplified it; the fine review of the results in this direction can be found in [1,3–5,46,128,235]. A whole series of self–reproduction models combining mechanical and cellular approaches has been investigated too. So, ***R. Laing*** has offered a *mixed automaton* capable of modelling of structure of any machine, including itself [339]. ***R. Laing*** has presented one more rather interesting class of *kinematic* molecular machines that possess the ability to both universal calculations, and reproducing of own descriptions and copies [90,340]. He has shown, that structurally–simplified molecular machines can be embedded in classical ***HS***–models, allowing to form in a sense bridges between kinematic models and models, implemented on basis of the ***HS***–concept. At last, ***R. Laing*** has given interesting enough discussion of the ***Neumann's*** cellular model from the biological standpoint [341]. Similar questions of interesting biological interpretation of the cellular and automaton models in the biology of development can be found in works [1,5,88,90,332,536,567,640] and in references contained in them. With the purpose of protection against trivial cases of *self–reproduction* ***J. Neumann*** has demanded for fulfillment of a condition of universal computability for the self–reproducing configurations in ***HS***–models. However, ***G. Herman*** has shown, that the given requirement does not protect from cases of trivial self–reproduction [88,353]. This important circumstance speaks about necessity of more attentive approach to the choice of requirements to configurations of ***HS***–models which should be interpreted as certain biological processes and phenomena whereas to the ***HS*** as a formal environment of modeling of biological processes and phenomena. Therefore, the following important enough question seems a rather actual, namely**:**

Whether exists other quite satisfactory measure of complexity for the self–reproducing configurations in HS–models that is not based on the concept of universal computability?

Certain interesting grapho-topological approaches in this direction are submitted in works [1,3–5,8–10,88,90,536]**,** and also in connection with the complexity problem of finite configurations in ***HS***–models ***(****ch.* **4*)*.** In spite of the certain successes in clearing up a question of the *concept* of complexity of finite configurations in ***HS***–models, we should well present the real problems put forward by the necessity of satisfactory biological interpretations of the self–reproduction phenomenon [5,88, 90,567,640]**.** So far that is open and important enough problem, whose decision in the near future, in all probability, is not foreknown.

As against ***J. Neumann E.F. Moore*** in the research of self–reproduction on base of ***HS***–models does not bind oneself by *universal computability* [274]**. *E.F. Moore's*** definitions cover only the most general essence of the *reproduction* process, allowing to concentrate our attention only on it ***(****section* **3.2*)*.** In the given direction a lot of rather original models has been received but all of them are of interest only from the most *general* standpoint onto the complicated process of self–reproduction**;** at that, these models can't receive satisfactory enough *biological* interpretation. Indeed**,** research of similar ***HS***–models demonstrates**,** on the one hand, that in them in a certain sense an analogue of the ***Malthus's*** law takes place, i.e. quantity of descendants of a certain self–reproducing finite configuration ***(****abstract organism****)*** can't be increasing faster than ***«vital»*** *space* available for them – active area of space of a model. On the other hand, in the given ***HS***–models the phenomena directly inherent in the modelled process of reproduction take place which however have not any satisfactory biological analogues. For example, we found a rather interesting phenomenon, inherent in ***HS***–models of self–reproduction in the ***Moore's*** sense of finite configurations [1,3–5,54–56,79,88,90,545]**.**

Let**, *c1*** and ***c2 (c1 ≠ c2)*** are complex enough finite configurations of the same size in classical structures ***2–HS*.** Further, we shall consider self–reproduction of finite configurations in the Moore's sense. It is shown that there are classical structures ***2–HS*** for which the self–reproducing configurations ***c1*** and ***c2*** are capable to reproducing $M_1(t)$ and $M_2(t)$ own copies accordingly during time ***t;*** at that, the same configurations ***c2*** and ***c1*** are capable to reproducing during the same time ***t*** $N_1(t)$ and $N_2(t)$ copies accordingly of configurations ***c1*** and ***c2*** at presence of the following determinative relation, namely**:**

$$\lim_{t \to \infty} \frac{M_1(t)}{M_2(t)} = \lim_{t \to \infty} \frac{N_1(t)}{N_2(t)} = 1$$

Furthermore, existence of classical ***HS***–models for which on basis of a finite configuration in aggregate can be generated all set of finite *block* configurations not only gives a rather interesting example of classical structures possessing the property of universal reproducibility in the Moore's sense but also brings an attention to the question concerning the adequacy of biologic interpretations of such extremely important process as self-reproduction. Similar moments should be kept in mind at biologic interpretations of ***HS***–models for their adequacy to various modeled processes. Multi–aspect discussion of a whole series of other questions of simulation of biologic phenomenon of self–reproduction along with use for these purposes of possibilities of *classical* structures can be found in [54,79,88,90,640] and in references contained in them.

The above models of self–reproduction in a sense, in our opinion, are similar to copying of the genetic information in a cell nucleus, but not to real self–reproduction of organisms [88,536]. Therefore in the given direction rather serious researches still are in prospect. Depending on analysis of *self–reproduction* models existing for today and approaches to modeling of the given phenomenon follows, that they represent the certain interest from standpoint of self–reproduction of robots but not alive organisms. But that is our private standpoint at this question.

8.2.2. Modelling of processes of the growth in homogeneous structures of various types

The phenomenon of *growth* in that or another extent is inherent in any evolutionary system. From biological standpoint the growth, perhaps, is one of the most simple components of the general development, but a series of open questions exists and here. Growth is one of immanent properties of alive because for survival of any species the individuals making up it should reach a quite definite weight, without which the performance of all necessary vital functions by them is impossible. An individual has the finite sizes, achieved in process which is named the ***growth.*** The generally accepted standpoint – the *sizes* of organisms are fixed genetically. Now there is not a common opinion about influence on process of growth of various factors such as metabolic, ecological, thermodynamic, etc., along with degree of abstraction from the partial phenomena. Without penetrating into deep essence of the process of the growth as one of base components of the general development, at a formal level we shall consider only ***3*** basic problems characterizing the growth as the independent biological phenomenon representing indubitable interest from many standpoints.

One of the basic problems of development – ***how can be reproduced a certain organism, using possibly least instructions?*** That is important enough from the standpoint of understanding of development in alive systems as the zygote should be somewhat simpler, than an organism to which it gives a life. Second problem touches the ***restrictions of the sizes of an organism,*** growing in various conditions, if such process is completely caused by a genotype of cells, self–reproducing during the growth. At last, the third range of questions touches the investigations of such *growth*, when a ***spatial differentiation*** during continuous self–reproduction of an initial set of instructions without influence of some external influence can take place.

For answers to that and other questions the various formal models of growth have been suggested**;** at that, part of them is being discussed a little below with emphasis on a discrete approach. The current variety of models of growth is explained by prevalence of mechanisms of the restriction of growth of a developing organism that are widely spread as well as itself process of self–reproduction. In addition, researches of mechanisms of *regulation* of growth are urgent also for comprehension of the *morphogenesis* phenomenon since the growth can be considered as ***1***–dimensional analogue of the morphogenesis [3-5]**.** The reader can familiarize oneself with the widely enough presented problematics of *continuous* models of growth in the collective monograph [4] and very extensive bibliography quoted in it, and also in the bibliography [536]**.**

The certain simplest models of growth were investigated by means of computer modeling by ***S. Ulam*** and his colleagues which were among the first initiators of research of the *growth* phenomenon by the *discrete* apparatus, however much earlier this problem was being investigated by a series of researchers ***(A. Thompson, L. Bertalanfi,*** *etc.****)*** with use of the continuous apparatus of the modeling [4,225]. The discrete growth models studied by group of ***S. Ulam*** are most suitable for description of certain abiotic systems similar to crystal structures, simple plants or simple organic molecules, than for real complex biological systems. In spite of that the work with similar models has allowed to make clear a series of questions of the growth of figures at various restrictions such as logical, geometrical and certain others [536,545,617,618,640–643].

At working with discrete growth models of ***S. Ulam*** we have used the apparatus of classical structures ***2–HS,*** that has allowed to receive a lot of new interesting properties of discrete process of growth according to various *recurrent* rules, allowing to research the given phenomenon by formal means [1,3–5,54–56,90]. The further development of the ***HS–*** concept as a basis of *discrete* modeling of the growth phenomenon has been received by ***J. Buttler*** and ***S. Ntafos*** [309,567]; the more detailed information on the given question can be found in works [88,90,536].

From the standpoint of research of process of growth the indubitable interest the problem of excitations spread in the ***HSR*** presents (*section* ***1.2***). On basis of such class of structures a series of interesting models of *excitable* environments has been proposed; part of them can be used for researches of processes of self-organizing in systems of the cellular nature of various type [1,3–5,8,9,79,90,167,198,203]. For the purpose of modelling of phenomenon of discrete process of growth ***M. Apter*** has used the classical Turing machines and *propositional* calculus [330,345]. A series of interesting enough questions connected to the growing in ***HS–***models of spatial forms of various geometry has been considered in the book [158]. We have marked earlier, that the classical structures well enough simulate processes of *growth* on basis of relatively simple generative rules and restrictions. However, such rules are *insufficiently* complex to model natural growth and a series of other phenomena of the development of alive systems. So, often we are forced to use types of structures distinct from classical for tasks of discrete modelling.

While the results already on polygenic structures show that they can be enough successfully used for modeling of rather complex growing real systems which simulate some natural phenomena of growth. So, in work [329] the polygenic structures ***2–HS*** are successfully used for modeling of process of growth of inflorescences. On basis of results of such modelling an interesting comparative analysis of classifications of growth of inflorescences on classical botanical base and on basis of structures ***2–HS*** is submitted; furthermore, the competitiveness of ***HS–***

models relative to ***L***–systems on a series of problems of modelling of growth and morphogenesis of plants has been shown. Meantime, the structures ***HS*** of such type are insufficiently simple in order to supply the researcher with the convenient and visual apparatus of modelling of phenomena which themselves are complex enough. However, use of polygenic ***HS***–models together with computer simulation, perhaps, can essentially improve the given situation.

For modeling of phenomena of biological development the structures ***HS*** with storage ***(HSS)*** have been suggested [1] which enough simply realize networks of growing automata of ***M. Apter,*** allowing to model processes of growth of rather complex spatially–differentiated figures. With the purpose of definition of *complexity* of such figures of growth a complexity concept basing on the grapho–topological approach has been introduced**;** the given concept has been discussed from a whole series of biologically–motivated points of view [1-5,54-56,345,536,545]**.**

Very interesting problems of optimization arise in connexion with the questions of *restriction* of process of growth. Indeed, the real biological organisms do not grow with no limits, but completely supervise own growth during all development and vital functions. In this connexion ***D. Gajski*** and ***H. Yamada*** have investigated the rules of growth in the ***HS***–models, that allow to grow figures of the given limited size [191]**.** The chief task here is reduced to revealing of the greatest possible size of the *passive* configurations generated by the classical structures ***d–HS*** from some simple initial finite configurations. The interesting enough results concerning the lower estimations of the sizes of such maximal passive configurations in terms of various key parameters of the ***d–HS*** along with rather interesting discussions of biological interpretations of the results received in the given direction can be found in [191] too. Rather interesting questions of growing of chains of finite automata of the given length can be found and in interesting enough work [195]**.**

The works marked in this direction enough closely adjoin our results on the *limited growth problem* ***(LGP)*** considered in section ***1.2.*** The ***LGP*** concerns a class of *minimax* problems in the ***HS***–problematics, being of the certain interest from the standpoint of developing cellular systems of the various nature. Indeed, the process of growth in real biological systems is limited, is strictly controllable from within, and depends on *genetic* and of some external factors. Moreover, the ***LGP*** has rather big cognitive significance, allowing to estimate in a sense the quantity of information required for growth of complex multicellular organisms.

In more detail with the ***LGP*** and interpretation of the received results it is possible to familiarize oneself in section **1.2** and in works [3–5]. In view of more applied aspects it is necessary to mark utility of the ***LGP*** for research of questions of information connection of the intercellular interactions of developing systems along with formation of the certain considerations about character of the genetic code [5,79,88,90,536,567]. As distinct from the above discrete ***HS***–models studying the ***LGP*** and explaining mechanisms of management by the process of *restriction* on basis of the ***HS***–concept, exists a series of other ***HS***–models explaining this phenomenon from other standpoints such as similarity principle, thermodynamic laws, adaptation to external environment, mechanical stability, energetic expediency, etc. Similar diversity of interpretations is undoubtedly necessary and allows to carry out *multifold* research of the problems of growth and development as a whole. Along with that it in a certain extent can be considered as a biological analogue of the principle of complementarity [3,5,54–56,79,88,90,536,545,567,617,640].

Here it is necessary to mention about a problem arising in ***HS***–models, and, at first sight, not connected with their biological applied aspects. That is the *general synchronization problem* ***(GSP)*** of processes in the ***HS;*** its partial cases are known problems of synchronization of a network of automata [196], the limited growth [3–5,191] and the French flag [3–5]. Indeed, ***HS***–models are systems of parallel information processing and consequently for them problems of synchronization of processes arise considerably more sharply, than for sequential systems. At that, it is necessary to note, that the approaches to solutions of the ***GSP*** can be rather useful at the solution of synchronization problems in many models realizable in ***HS***-environment. In addition from the standpoint of *biological* modeling on basis of the ***HS***–concept, first of all, the *classes* of qualitative aspects of problems of synchronization of the processes managing those or other phenomena in the development biology and in other biologically motivated considerations are of interest. Now, on base of the ***HS***–approach a series of the models describing important enough behavioural aspects of biological systems of a various level of the organization – from *cellular* and *tissue* level up to *kainogenetic* level are created [3,5,54,79,88,90,536]. The review [332] covers a wide range of models in theoretical biology and mathematical biophysics, being of interest, first of all, to theoretical biologists. In the work the cellular automata models in biology and medicine are discussed in detail. At last, it is necessary to note that use of ***HS***–models namely in biological sciences seems to us one of the most perspective directions.

8.2.3. HS–models of differentiation, regulation and regeneration in the developmental biology

In the previous items of the section from the cybernetical standpoint it has been shown that certain features of development such as growth and self–reproduction can be inherent in artificial systems too. Below, we shall try to briefly consider the questions of discrete modeling and more complex phenomena of development – *differentiation*, *regulation* and *regeneration*. The *differentiation of* ***cells*** represents one of the major problems in the modern *development* biology. In spite of huge number of works, devoted to those or other features of cellular differentiation, today we not possess the general theory of *differentiation* – the *majority* of theoretical considerations and hypotheses concern only molecular mechanisms of cellular differentiation. In particular, today there is not *quantitative* theory of *differentiation* and the approaches to construction of this theory are even not clear. In great degree, it is conditioned by both an ambiguity of the differentiation concept, and absence of exact criteria for a receiving of its numerical estimations [3–5,88]. At that the *forming* ***(morphogenesis)*** along with *spatial differentiation* is an essential enough feature of the biological development.

At present, in our opinion, any persuasive enough experimental data for creation of dynamic models directly commensurable with the real biological phenomena or development are absent. Thus, the modeling is directed at illustration of an opportunity of realization of different phenomena of development at a level of rather general assumptions. At that, the ***HS***–models being created in the given direction illustrate such opportunities as formation of hierarchical structures; control by processes of growth, regeneration and regulation, etc. Today, similar models are intensively investigated not only from the biological point of view, but also within a new scientific discipline – *discrete synergetics* [4,3–5,90,155,536]. Along with that, certain control algorithms used in these models can appear useful enough to parallel computing systems [11,12,54]. Concrete elaborations of similar type by present time exist; they enough naturally use a series of control algorithms, used in some discrete models of development on basis of ***HS***–models [9,15]. While a rather detailed discussion on the model approach to researches of the differentiation problem can be found in [1,3,5,9,88,90,536,545,567]. We pass now to question of formalization of the problem of development and regulation of a biological structure.

The *central* development problem is reduced to the following question ***How of an egg that seems the completely undifferentiated and simple in the structural attitude a rather complex multicellular organism can next develop?*** In this respect, the assumption has been expressed, that such egg contains only the development program, instead of complete specification of all organism that of him should develop. For example, in ***C. Waddington's*** posing the formation problem of spatial structure consists in determination of the immediate reasons of separation of a homogeneous cellular area into *separate* parts that are located in space in strictly determined order [331,640–643].

Along with that, we inevitably should come to a conclusion, that the *differentiation* of cells at highly organized alive essences is direct result of activity of extremely complex regulator mechanisms. First of all, for us, apparently, the *effective* enough acting models are really necessary, whose purpose should be to help with formalization of this problem and, apparently, to discover a certain key to understanding of the *base* approaches to the problem decision in languages of the exact science. The discrete development models, discussed lower, pursue the given goal. Ability to restoration of an *axial* structure can be tracked in many biological systems; some its most typical features are discussed in [17, 26,27,31,567]. The experimental approach to this problem has allowed to formulate a lot of concepts interesting and simplifying the problem; among them it is necessary to mark such principles as dominance and gradients [1,3-5,331]. The first serious attempt of creation of a working model capable to development and regulation of an *axial* structure has been undertaken by ***S. Rose*** [347]. In further, a series of the interesting enough models has been suggested, whose comprehensive review can be found in monographs [1,3–5] and works [31,33,54-56,88,90,331,348].

However, the most known formal model of differentiation, regulation and regeneration is the *French flag's problem* ***(FFP)*** that has been offered by ***L. Volpert*** [331]. In the most elementary form the given problem is formulated as follows, namely:

There is an one–dimensional connected system out of 3m cells, each of which admits the states «red», «white» and «blue»; it is necessary to determine rules of functioning of such cellular system whose the final state is the configuration of French flag (CFF) which to some extent is stable to external influences and damages.

For solution of the ***FFP*** in its classical posing a series of mathematical and automaton models has been offered, and their analysis from the biological standpoint has been carried out [1,3–5,23,26,27,31,33,46]. In particular, discussions of the ***FFP*** formulation as some formal model of *differentiation*, *regulation* and *regeneration* of *axial biological* structures on concrete biological objects have been carried out [3–5,54,79,88,90].

For the solution and research of the ***FFP*** the ***HS***–models of a few types were used, putting before modeling a whole series of tasks. Above all, the question relative to the minimal complexity of a model capable to differentiation, regulation and regeneration interested us. It is shown that at modeling of the ***FFP*** even on basis of *polygenic* structures ***1–HS***, an algorithm deciding the problem should be algorithm over alphabet ***A*** whose elements are symbols composing the ***CFF*** [26,27]. In addition additional states of the model should admit a *reasonable* interpretation in the corresponding biological categories [54–56,79,88,90,545,640].

So, the second question is the revealing of those sufficient conditions that would promote a solution of the ***FFP*** along with their satisfactory biological interpretation. From this standpoint, a series of models has been investigated on the basis of the ***HS***–concept [3-5,23,26,27,31,33,46, 330,331]. In particular, one of these models is able to a perfect enough regulation and slightly resembles the known model of ***M. Arbib*** [331], however it is more simple and is free from a few defects of his model. Moreover, the basic properties of our model are absence of a gradient and thresholds along with presence in it of polarity, spontaneous self–limiting reactions and *bilateral* stream of control information [331,345]. To the given model in sense of its basic features determining solution of the ***FFP***, a model on basis of a class of structures ***1–HS**** adjoins also, allowing to decide the ***FFP*** in its *generalized* formulation. An extension of the ***FFP*** can be determined as follows. In a structure ***1–HS**** a finite configuration c_o of a length ***r*** out of states of elementary automata of the following kind is defined, namely:

$$c_o = x_1x_2x_3x_4 \dots x_r \;; \qquad x_j \in A=\{0,1,2,3, \dots, a-1\} \;\; (j = 1 \,..\, r)$$

Then, the ***generalized FFP*** is reduced to determination of a functional structure algorithm, whose complexity does not depend on number ***r*** of elementary automata of a differentiated chain, allowing to establish and support in a structure ***1–HS**** the generalized ***CFF*** of the following structural kind, namely:

$$C_f = \nabla b_{1_1} \dots b_{1_q} b_{2_1} \dots b_{2_q} \dots b_{(a-3)_1} \dots b_{(a-3)_q} b_{(a-2)_1} \dots b_{(a-2)_q} b_{(a-1)_1} \dots b_{(a-1)_k} \nabla$$

$$b_{p_j} = p;\;\; p = 1..(a-2);\;\; j = 1..q;\;\; q = [r/(a-1)];\;\; b_{(a-1)_i} = a-1;\;\; i = 1..k;\;\; k = r-(a-2)q$$

In connection with use for the solution of the generalized ***FFP*** of ***HS–***

approach we first of all would like to determine the most simple type of ***HS***–models allowing to solve the problem. In this direction the next result takes place [1,3–5,26,54–56,79,88,90,536,545,640–643], namely:

The generalized FFP defined in some finite alphabet A of general kind can't be decided by means of an one–dimensional polygenic structure determined in the same state alphabet A.

Hence, for solution of the *generalized* ***FFP*** even in the class of *polygenic* ***HS***–models we need to use an alphabet, expanded relative to its initial alphabet and, perhaps, along with some other assumptions. So, one of models on basis of a structure ***1–HS**** uses an elementary variant of the symbolical sorting allowing to solve the ***FFP*** in the ***Volpert's*** classical formulation during no more than ***t = 3*m*** steps; where ***m*** is length of a differentiated chain of automata of the structure [1]. At that, a sorting acts as one of kinds of a *logic* gradient while the model allows to make a series of interesting enough conclusions of biological nature. Along with that, the functional algorithm of a ***HS****–model which decides the generalized ***FFP*** allows to formulate the following result [5,88,90,640].

Theorem 182. There is a structure 1–HS* with alphabet A={0,1, ..., a–1} and a functional algorithm, whose complexity not depend on length r of a differentiated chain of elementary automata, and which decides the generalized FFP during no more than t = [r/2] steps for sufficiently large values r. A set of all solutions of the generalized FFP which are minimal in temporal attitude is nonrecursive.

The result presented by the theorem ***182*** is a solution of the *generalized* ***FFP*** which for today is the *best* in the time attitude. Of this theorem, in particular, follows: for sufficiently great values ***a*** and/or ***r*** *decision* time of the ***FFP*** asymptotically approaches half of length of a differentiated chain of elementary automata of a ***HS****–model. Consequently, a rather interesting question arises: ***whether exist functional algorithms of any other type that decide the given problem during the better time?*** In our opinion, any essential improvement of decision time of the *generalized* ***FFP*** defined by the theorem ***182*** not seems possible.

Along with that an interesting question of research of the generalized ***FFP*** for case of the higher dimensionalities arises, when instead of the *linear* chains the ***d***–dimensional *networks* of finite *differentiable* identical automata are considered. It is shown that the results of solution of the generalized ***FFP*** essentially depend on the kind of ***d***–dimensional ***CFF*** ***(d ≥2)*** too [88]. The above ***HS***–models solving the ***FFP,*** in a great extent allow to make clear such questions as properties of separate automata, nature of connections between them, input/output control impulses, along with a series of other prerequisites giving rise to partitioning of cellular system along axis onto segments, located in the definite order. The detailed enough analysis from the *biologic* standpoint of these and other ***HS***-models of differentiation, regulation and regeneration can be found in works [1,3-5,26,27,46,54-56,79,88,90,264,329-351,536,567,640].

The review of the basic approaches to discrete modelling of processes of biological development with all evidence shows that at present the ***HS***–concept and ***L***–systems are basic components of the apparatus of modelling in this field [3–5,88,90]. Hence, would be quite pertinent to try to carry out a certain comparative analysis of both approaches and discuss some biological considerations connected to them as well as to estimate to some extent the possible ways of the further development of the apparatus of *discrete* modeling in biological sciences. One of the possible ways a *prospective* scheme of development of the perspective apparatus of biologic modelling presents ***(fig. 16).*** Meantime, bearing in many respects the prognostic character, at the same time the given estimations represent quite certain interest for researchers in the field of mathematical and theoretical biology [4,31,54,79,88,90,536,567,640]. In conclusion it is necessary to accent our attention on that the modern advances of bioengineering allow to receive completely different look at various aspects of previous models of development. Thus, a model approach to the research of developmental biology should be revised.

8.3. Use of HS–models in computing sciences

Now, a rather significant activity of theoretical researches on creating of perspective architecture of computing systems with new properties and fields of appendices concerning the parallel computing models is observed. Work in this direction has resulted by creation of a series of interesting enough formal models of parallel calculations and parallel information processing. A whole series of interesting enough projects of highly–parallel computing systems that used those or other formal computing models of calculations has been successfully realized [5,11, 12,15,36,42,90,178,202,234,238,352–363,366–369,403]. At present, to the problematics of highly–parallel computing systems, calculations and information processing on them a whole series of specialized journals is devoted; the international and national conferences and exhibitions of different level are being held on regular base [15,35,201,202,238,308, 352,353,356,357,359–363,367,536,567], a plenty of literature of different level and direction is being issued too [368,536]. Practical elaboration of high–parallel computing systems is being presented and on already traditional *specialized* exhibitions of modern high–efficiency computer engineering [352,353,361,363,536,567]. For more detailed acquaintance with this problematics within the modern computer engineering and software it is possible to address to works [5,15,90,358,366,369,372], to catalogues of publications [368,371] and also to the above literature. A interesting enough excursus into development of architecture of high-parallel computing systems basing on the finite ***HS***-models along with their appendices can be found in [420,536,545,640–643].

Well–known cellular–like systems form an interesting enough class of *parallel computing models* ***(PCM)*** and ***HS***–models are rather typical and popular representatives of this class, forming own *subclass* of the class of all *parallel discrete dynamic systems* ***(PDDS)*** [3,5,8,9,11,15,54,128,135, 147,150,158,159,161,166,177,308,366,536,640]. In spite of an urgency of the problem of development of high-parallel principles of information processing and computations, we have not an opportunity to consider within this item even basic aspects of the given problematics owing to its extensiveness. Hence, we shall restrict consideration to only those basic applied aspects that are to some extent based on computing ***HS***–models and are in sphere of our interests, referring the more exacting reader to the corresponding literature in extended bibliography [536].

So, the *homogeneous computing structures* ***(HCS)*** represent a rather wide class of *discrete* devices having the ***HS***-conception as a theoretical base. In the network of the ***HCS*** a model of parallel operating system which enough appreciably uses a whole series of parallel control algorithms and principles *directly* from the theory of homogeneous structures can be submitted [40,45,51,52]. More precisely, the above model of parallel operating system essentially uses a series of control algorithms from a number of biological models basing on classical structures. It is shown that ***HS***–approach can serve as a mathematical base of *methodology* for parallel microprogramming. In view of above approach the methods of description of parallel microprograms, their transformations, and synthesis are based on the special *systems of parallel substitutions* ***(SPS)*** which turn out strongly equivalent to the classical ***HS***–models. As an example of application of this approach it is possible to represent the solution of the recognition problem of self–consistency of *algorithms of parallel substitutions* ***(APS)*** that are defined by the appropriate systems of parallel substitutions along with procedures of their application. In this respect, using the corresponding results on the ***HS***–problematics, we managed to prove the following rather useful result [53], namely:

The recognition problem of consistency of the APS has a constructive resolving algorithm.

The given result has allowed to receive a series of interesting enough consequences, including of especially applied character; in particular, for a designing of microprogrammed controlled systems and devices.

Artificial intellect ***(AI)*** has sucked in the basic attempts to reflect *various* neurological functions of a human brain on basis of modern computer technologies. At that, technologies used by the ***AI***, suppose processing and analysis of huge quantity of various facts, knowledges and rules of derivation; for that the wide and fast exchange with databases and/or knowledge bases is necessary that is being provided only owing to use of computing systems with parallel architecture. The carried out extensive analysis shows all advantages and very great future for the parallel computing architectures [1,8,9,15,83,87,90,121,150,159,160,201,202,308,352-373]. In this connection the parallel computing systems on basis of computing ***HS***–models represent significant interest, seriously stimulating their rather intensive researches [3,5,15,83,147,150,159,160,178,179,201,308,354,366,370,373,376,536]. The motive for such position consists in the actual possibility of achievement of ultrahigh speed of processing and calculations in combination with a low enough cost. In this attitude the so–called «***neural computers***» whose architecture and topology inherit a series of the base features of computing ***HS***–models seem rather perspective [94-96,159,354,374]. Indeed, the really existing models of such computers inspire the certain optimism, possessing a series of essential advantages before the previous architectures [536].

Standardization of means of designing of architecture of the ***HCS*** and their software is closely connected to researches of universal memory structures to which abstract ***d***–dimensional registers with periodically *defined transformations* ***(PD-****transformations****)*** relate also, playing a rather important role in designing of perspective *parallel* computing systems. It was shown that such formal constructions are equivalent to classical ***HS***–models *(section* ***6.2)*** and concepts of ***PD***–transformations and ***HS***–models very well reflect synchronous multiprocessing data structures; they are suitable at creation of the ***HCS*** along with their base software on basis of parallel control systems [11,12,15,53–56,90,325,536,567]. So, in particular, the so-called *register automata* representing mathematical model of structure of modern computers also closely adjoin to *classical* ***HS***–models [226]. It is necessary to mark, the register automata theory whose bases have been laid by ***V. Glushkov,*** quite closely coordinates with the ***HS–****problematics* and, in particular, with the theory of *iterative* automata [536]. Meantime, the classical ***HS-***models that are equivalent to the ***PD–***transformations possess concerning them rather significant advantages of constructional nature what is especially important from a whole series of the applied standpoints.

At that, with abstract models of multiprocessing, the modified *systems* of *algorithmic algebras* ***(SAA)*** oriented upon questions of formalization of structured parallel schemes of programs are closely associated. The *denotational* nature of description of process of designing of programs in terms of grammars of structural designing is combined with their explicit presentation in terms of formal ***C–***grammars, what represents as well direct applied interest [51,53,90,227,536]. At development of a new approach to information processing, that uses so–called mode of «*information paralleling*», two software packages ***PSIP*** (*parallel system of information processing)* and ***PCS*** (*parallel control system)* for *concentrated homogeneous computing systems* ***(CHCS)*** consisting of IBM/360 models with mass means of complexation have been designed and realized [8, 9,11,12,15,49–53,88]. At that, at designing of the packages the practical conclusions received at research of a series of parallel algorithms and processes in the *classical* ***HS-***models have been essentially used. So, for control the analogues of some *optimal* parallel algorithms investigated in connection with modeling of control processes in biological models of development on basis of the ***HS–***concept were used [5,11,12,52]. On base of the detailed analysis of functioning of the ***CHCS*** under control of the packages ***PSIP*** and ***PCS*** in ***IP–****mode* a computing ***HS–***model has been created, allowing to establish interesting enough analogies with real biological cellular systems [5,8,11,567]. The detailed analysis and description of approaches to realization of the ***IP-***mode on basis of the parallel packages ***PSIP*** and ***PCS*** along with questions connected to it can be found, in particular, in our scientific and technical collections [11,12,15]. Hence, the ***HS–***concept has all prerequisites to play a rather essential part in the question of unification of for today such various scientific fields of modern natural sciences. That allows to speak, the ***HS–***problematics more and more receives a new and rather important interdisciplinary character [536,567,640]. So, the above interosculation of control algorithms used in biologic ***HS–***models and computing ***HS–*** models allows to launch in a sense bridges between biologic sciences and computer sciences what will be useful for both directions.

The concepts, methods and approaches basing on the ***HS*** can be used and for solution of certain more special problems of parallel computer technique being component of realization of architecture of *perspective high-parallel* computing systems on basis of iterative networks, cellular and systolic structures. Questions of cellular computing architectures are enough in detail discussed in [5,6,137,138,156,160,169,175,242,360, 394], of systolic structures – in [178,370] whereas literature on iterative networks is so extensive that here we shall mention *only* sources of the most general character [90,230,263,308,357,360–365,367,374]. More and more actively developed both theoretical, and applied researches on *neurocomputer* architecture that make up nowadays a separate branch of the general ***HS***–concept, directly adjoin the given problematics [148, 322,342-344,347,350,354-358,536,545]. At that, practical results received in the given direction have allowed to advance enough essentially our representations about opportunities of real parallel computers for the solution of many–sided concrete problems.

The computing systems existing today with parallel architecture have called into being a whole series of rather interesting works suggesting new effective approaches to organization of high–parallel software for solution of the most frequently meeting problems [5,8,9,11,12,15,54,87, 90,94-96,121,146,160,166,175,202,234,238,308,352-374]. Along with that, the increasing attention is devoted to the questions of development of concrete *parallel* algorithms for solution of various classes of problems (*differential equations*, *mathematical and computing physics*, *linear algebra*, *and a series of others* [158,177,202,234]), along with other classes of the problems of both computing and non–computing character (*artificial intellect and robotics*, *pattern recognition and signal processing*, *forecasting*, *global and strategic planning*, *etc.*) [15,94–96,149,150,159,160,178,234,238, 308,354,356–357,366,368,372,391–392,394,567]. In the given context the appropriate researches on formal computing ***HS***–models can present a rather serious interest for theoretical basing and practical designing of perspective parallel computing architectures and their software of the various levels and purpose [5,11,15,88,90,360-372,374,377,383,420,536].

T. Toffoli by creation of a series of so-called «*cellular machines*» (***CAM*** – *Cellular Automata Machines*) has offered an interesting and perspective approach to practical implementation of computing ***HS***–models [150– 152,165,318,376,567]. Practical use of these machines turned out rather effective at modeling of complex problems of physics, at research of a lot of mathematical properties of ***HS***–models, pattern recognition, etc. [166,366,384,392,394,536]. Series of the ***CAM*** has brought out the ***HS***–problematics onto qualitatively new level, perfectly having added the formal ***HS***–models by their direct computing analogues. That allows to expand enough essentially an applied model ***HS***–problematics in a series of the important enough directions. Realization of the ***CAM*** has solved the support problem of extremely simple models of differential equations in physics such as the wave equation, Navje-Stoks equation, heat conduction equations which can be represented as extreme cases of extremely simple processes of combinatory dynamics. In addition, the ***HS***-approach provides rather extensive and continuously growing collection of typical formal models for which the embedded processes and phenomena can be researched enough conveniently and easily at an excellent illustrative level [536,545,567,617,618,640–643].

The Italian researchers of the *Institute of system analysis and information technology* have created the effective computing environment ***CAMEL*** ***(C****ellular* ***A****utomata environ****M****ent for syst****E****ms mode****L****ing****)*** that is based on a ***HS***-model with realization on multicomputer system of the transputer architecture [427,428]. System ***CAMEL*** is intended for highly effective appendices in science, technique and technologies; it can be used as a tool for modeling of complex dynamic processes, and as a computing model for parallel processing. In addition, architecture of the ***CAMEL*** admits realization on several computers with the distributed memory such as *Intel iPSC*, the *Ncube*, the *CRAY T3D* and *Connection Machine* ***5.*** More in detail with architecture of this system along with its software the reader can familiarize oneself in [427,428]. In a certain extent a ***HS***-like machine *CEPRA–8* is of interest too. In the certain attitude to the ***CAMEL*** the environment of modelling together with library ***CAME*L*** ***(C****ellular* ***A****utomata* ***M****odeling* ***E****nvironment and* ***L****ibrary****)*** for investigation of ***HS***–models adjoins, and in more broad attitude by way of tools for the distributed and parallel computations [536,545,567,617,618,640].

A whole series of researchers used genetic algorithms for definition of the ***HS***–models which are carrying out required calculations. Genetic algorithms at research of ***HS***–models are applied for formation of *rules* of *parallel substitutions* determining the mappings of configurations of neighbourhood templates in such a way that dynamics of a ***HS***–model satisfies the given criterion [536]. Search of *rules* allowing to reveal the necessary dynamics, above all, of ***d***–dimensional ***HS***–models ***(d ≥ 2)*** in a lot of the important cases is a complex enough problem. Meanwhile, it is shown that search of ***HS***–models on basis of *genetic algorithms* and *evolutionary programming* seems as a rather effective means for receipt of the required dynamics of many-dimensional ***HS***–models describing real objects, processes and phenomena [536,545,564,583–585,640].

With development of *nanoelectronics* the indubitable interest to one of the most perspective directions of modern cybernetics **–** to ***HS*–**models more and more grows. As a rule, *nanocomputers* using the ***HS*–**concept are based on **2-** or **3-**dimensional computing ***HS*-**models**.** The problems arising at transition from *traditional* computers to *nanocomputers* along with ways of their solution with use of the computing ***HS*–**models are enough actively discussed. A summary of a whole series of the basic considerations in the given direction which, having subjective enough character as a whole seems rather useful, is presented in [536,567,640]**.**

Today it is known that for achievement of high computing *efficiency* it is necessary to carry out calculations without loss of information, i.e. calculations should be reversible. ***C. Bennett*** has shown, that zero loss of energy is possible only at use of *reversible* computing blocks. Works in the given direction are carried out enough intensively though here the complex enough technical problems arise. Consequently, different computational ***HS*–**models are researched enough intensively not only from point of view of physical modelling, but also in connection with organization of the reversible calculations [376,378,430,536,567,594]**.**

Some of logic gates, sufficient for creation of a universal computer **(***for example***, *AND*)** are irreversible. Therefore up to ***1970*** it was considered that an universal calculation should be irreversible. Meanwhile, in the *seventies* of past century by works of ***C. Bennett, T. Toffoli, E. Fredkin, N. Margolus,*** and others it has been proved that universal computers can be reversible and then a series of reversible logic gates suitable for creation of an universal computer has been suggested. Essential step in this direction has been made in ***1982*** by ***C. Bennett*** who has shown, that an *universal* computer such as Turing machine can be constructed using logically and thermodynamically reversible gates. Known logic gates of ***Fredkin*** and ***Toffoli*** belong to typical *reversible* universal gates [536,593]**.** A series of interesting reversible computing ***HS*–**models that are based both on these gates, and on quite known physical principles has been suggested [15,90,150,360,366,384,388,536,567,593]**.** The given circumstance allows to reconsider a lot of traditional positions of the classical theory of calculations, giving rather serious physical ground for it, i.e. allows to bind physical and computing processes within the unified physical concept of the world.

Within of creation of the perspective computing architectures a series of directions exist**;** among them quite possibly *origination* of new logic, above all, quantum calculations along with the approaches on basis of biological principles and ***HS***–models. In accordance with opinion of a lot of leading experts the realness of prospects of the first direction is regarded very skeptically – during last ***15*** years many publications on «*quantum calculations*» has appeared but more than ***90%*** of them have theoretical character. In a greater extent rely on biological approaches**;** in addition, the given direction can give the special importance to the interdisciplinary connection between engineers and neuro–specialists. At last, other perspective approach to realization of certain computer functions consists in use of ***HS***–models, whose essence will consist in interactions of nano–scaled elements in a regular large lattice. At that, the *base* complexities at realization of quantum cellular automata have been investigated by ***D. Meyer*** as far back as at ***1996*** [536,545]**.**

Essential interest in a context of realization of computing ***HS***–models represents so-called ensemble quantum cellular automaton on basis of *nuclear magnetic resonance* ***(NMR).*** The first *real* prototypes of quantum computers of such type have been realized at ***1997 (****see* ***J. Jones*** [536]***).*** It is remarkable that quantum computers of such type in principle can work in room temperature. Itself, the idea of a *quantum computer* goes back to works of ***R. Feynman*** where he has considered all computing process from the physical standpoint [536]**.** So, he has shown, having made calculations *reversible***,** it is possible to create computing systems at a level of *nanotechnology* ***(****by modern language****)*** when quantum effects play very essential role and it is caused, above all, by *growth* of *entropy* for *irreversible* calculations.

In the middle of the eighties of past century the works of ***U. Vazirani, A. Yao, D. Deutsch*** and ***E. Bernstein*** have appeared where interesting formal models of a quantum computer have been described. Next, ***P. Shor*** at ***1994*** has created the first quantum algorithm of factorization, what has generated a lot of works in the given direction [9,536]**.** Thus, at present the «*quantum calculations*» draw the most *steadfast* attention, and the ***HS***–models in the given context due to their features such as universality, reversibility, localness, parallelism etc., can play a rather essential part**;** in particular, as models of quantum cellular processors and nanoprocessors [536,545,567,617,618,640–643]**.**

In conclusion of the item we shall briefly touch a today's interrelation between *fundamental* results in the ***HS***–problematics and their applied aspects in computing sciences. The opinion arising today about weak applicability of many theoretical results received in the ***HS***–theory, in appendices is based on following quite fallacious prerequisite**,**namely.

Complexity of structural realization by means of modern technology of the computing ***HS***–models with very large number of elementary automata leads to the situation when practical realizations are limited by relatively small number of elementary automata, i.e. we deal with finite cellular automata that are essentially distinct from the classical ***HS***–models. At that, organization of such type allows to considerably simplify both realization by means of modern technology, and control process by parallel calculations. Moreover, it is natural that degrees of paralleling of calculations and control in similar finite ***HS***–models are sufficiently far from the *maximum* possible ones, and for them the base fundamental results of the mathematical theory of *classical* ***HS***–models at present have not any serious sense, since they most fundamentally are based on the infinity of homogeneous space of ***HS***–models.

Meanwhile, at the further technology development along with spread of untraditional approach to calculations, the transition to computing ***HS***–models that will contain vast quantity of rather simple elementary automata is quite natural. Moreover, simplification of the elementary automata of the ***HS***–models allows to use the *lower* level of algorithms description, what in turn raises an allowable level of paralleling of the problems being solved on basis of computing ***HS***-models**;** whereas the *huge* number of elementary automata supposes effective realization in such models and of more complex problems of paralleling. And at the same time, growth of quantity of elementary automata of ***HS***–models essentially expands also the possibility of both use, and of satisfactory interpretation even of sufficiently abstract results from the ***HS***–theory. Meanwhile, on parallel computing systems basing on the computing ***HS***–like models it is expedient to solve by far not all problems. In our opinion, problems that can be effectively solved on *parallel* computing systems of such type will make up *narrow* enough class of all set of the problems covered with computer processing, because a wide enough spectrum of levels of paralleling takes place, that is being provided by all variety of problems that are being solved on computers. At that, it is simple enough to give a lot of computing algorithms that not allow a parallelization of even a low enough level. Meantime, the interested reader for more detailed discussion of the given question is referred, for example, to works [5,3,11,12,15,53–56,79,88,90,160,263,308,354–358, 365,372,374,383,394,536,545,567,571,617,618,640–643]**.**

One more aspect which is very important from many standpoints, but not so obvious at first sight is that influence which both the theoretical researches and the applied development in the field of the ***HS***-concept exert influence upon formation of the *parallel* approach to information processing and calculations along with formation of a parallel way of thinking as a whole. This substantial aspect is rather interesting from philosophical and gnosiological standpoints along with expediency of forming appropriate standpoints for students above all. In addition to marked aspects a whole series of other actual enough applied aspects of the ***HS***–problematics in computing sciences takes place. However, in view of their abundance and diversity within the given monograph even their brief consideration is being not presented possible. Though the interested reader can familiarize oneself with these and them–like aspects in extensive enough bibliography presented below and rather numerous references contained in it to original works corresponding to the problematics, and also in extended bibliography [536,545,640].

8.4. Some other fields of applications of HS–models

The modern opinion allows to consider the *nature* in general as certain kinds of *calculations*, i.e. we can process various objects as very simple computers, everyone submitting to an own set of laws. ***Homogeneous structures (HS)*** essentially expand this analogy, providing methods of research of the whole sets of interacting elementary automata, each of which is an independent computer ***(finite automaton).*** At embedding into such computer of the appropriate rules we can simulate different kinds of rather complex behaviour – from movement of liquids being described by physical equations, up to landscape architecture. In the present section the very brief review of applied aspects that were not reflected in the previous sections of the chapter is given. At that, here we will not do a *special* distinction between classical, nondeterministic and other classes of ***HS***–models, and the attention will be accented on the applied aspects exclusively. It is quite natural that *extensiveness* of such themes does not allow to give all exhaustive picture therefore we shall be limited only to the appendices *most* interesting in our opinion, directing the more exacting reader to bibliography containing enough many examples of appendices [536], and references to corresponding original works. Yet more of useful information in the given direction the reader can find in internet by appropriate key phrases. Meantime, here it is desirable to do the following remark too.

With the purpose of the greater *convenience* of perception of the given material we carried out its some rubrication however it is necessary to perceive this rubrication with definite degree of conditional character because at present in applied aspects of the ***HS***–problematics is more and more precisely becoming apparent the interdisciplinary character, what very seriously complicates attempts of precise differentiation of appendices of the ***HS***–concept at those or other fields. So, for example, the ***HS***–models for research of the problems of pattern recognition are successfully applied and for the works connected to the robotics that, in turn, *directly* adjoins to the problem of artificial intellect, etc. That is quite naturally and lays in a course of today's realities of the modern scientific progress when onto leading positions the researches not on separate directions are concentrated, but on the integrated problems demanding the participation of rather large collectives of experts from the various fields of knowledge. Similar *interdisciplinary* situation has caused the circumstance that a lot of ***HS***-appendices mentioned below can be ascribed with the same success to those or other headings.

HS–models in physics*.** Use of discrete lattice systems in physics has a long and productive enough history. Numerous examples lay at wide enough diapason – from the *exact* theoretical models considered in the statistical mechanics, up to the *approximate* numerical researches of the continuous models. However rather little attention was being devoted to the exact lattice models submitting to *reversible dynamics* – from any state of a dynamic system its previous state can be uniquely deduced. The given kind of ***microscopic *reversibility* is the important property of all microscopic physical dynamics. Reversible lattice systems become even more physically realistic if we shall superinduce into the systems the implementation of the localness principle of interaction and strict laws of preservation. Similar dynamic lattice systems can be enough effectively embedded into ***HS***–models. Today, ***HS***–models of different types are enough effectively used at solution of different problems of the nonlinear physics along with experimental researches in the field of high–energy physics [536,545,640–643].

The given types of discrete dynamic systems not only provide rather intriguing information-dynamic approach to modelling of *macroscopic* physics, they can be also extremely practically useful models. At that, precisely the same properties that provide the models by their *physical* realness, provide their effective implementation too. In this regard the approach basing on *various* modifications of ***HS***–models is perspective enough. With more detailed discussion of this question the interested reader can familiarize oneself, for example, in works [90,536,567,640].

It is known that an essential defect of the traditional quantum theory –

absence in the theory of the concept of «***trajectory***». This theory gives only the recipe for calculation of amplitude of probability of transition of a particle from one state into another; moreover, the wave function is defined by local equations, not being a real *physical* field. The theory does not consider processes of spatio-temporal transition; a procedure of measurement and reduction in an explicit kind is not included into the apparatus of the theory, i.e. into model of a particle. Therefore, ***G. Malinetskii*** and ***P. Kurakin*** have offered a ***HS***–model of the *simplified* world in which an artificial particle with pseudo–quantum behaviour exists. The detailed discussion of the model can be found in [536,545].

Lattice gas automata **(*LGA*)** are a special class of the ***HS***–models widely used, in particular, for simulation of movement of a viscous fluid and the wave equations. At present, the researches on the ***LGA*** represent a well developed field of the *general* research of the ***HS***–models. At that, some researchers in ***HS***–models have completely devoted themselves to this direction whereas others only heard about it. The quantum ***HS***-models become more and more important for problems of simulation of physical systems such as *quantum* lattice gases. An appreciable part of interesting enough works are devoted to the given direction [536].

Now, more and more intensively the elaboration of new mathematical methods and means of modeling of processes in physics of *elementary* particles, *nuclear* physics along with physics of the *condensed* mediums with use of methods of artificial neural networks and ***HS*** of different types and classes are being carried out. So, in experimental researches in high–energy physics in *United Institute of Nuclear Researches* (*Dubna*, *Russia*) along with neural networks the ***HS***–*models* of various types are enough widely used too [536,545,618,640–643].

At the latest years an interest to the ***HS***–concept from both theoretical and mathematical physics, and physical sciences as a whole also has essentially increased. At that, by certain researchers even an opinion, that, in our opinion, is not adequate to the reality was being expressed that the ***HS***–*problematics*, in spite of all breadth and many-sided nature of its appendices will remain at a level of good intellectual game until the serious appendices in physical modeling will not be found. While, it is supposed that the ***HS***–approach for physical modelling can play a part, up to the certain degree similar to the modern role of differential equations [54,150-152,274,378,386,640]. At that the modern progress in this direction is caused by the following two major factors – *conceptual* and *technological*. The technological factor is stipulated by appearance of computing technique which is based directly on the computing ***HS***–models *(cellular **ML**–coprocessors of **Legendi**, machines **CAM** of **T. Toffoli**, etc.)*. While the conceptual factor consists in opportunity of creation of the discrete distributed models that reflect the such most fundamental aspects of physical laws as *homogeneity*, *locality*, and also *reversibility* of the basic physical processes and laws at a microscopic level [5,8,94–96, 121,150,169,175,176,178,308,354,361,366,371,545,567,617,618,640–643]. ***E.F. Moore*** [274], ***J. Myhill*** [275], ***A. Smith*** [131], ***S. Amorozo*** [270,293], ***V.Z. Aladjev*** [3,5,8,19,43,53-56,88,90], ***T. Toffoli, N. Margolus*** [150-152, 187,268,273,318,376,378] and a lot of other researchers have made the essential enough contribution to research of the reversibility problem of dynamics of classical ***HS***-models that has predetermined possibility of use of the ***HS***–concept in theoretical and mathematical physics, and also in other more applied sections of modern physics [536]. On basis of the preliminary analysis that is based on modern physical concepts, with a sufficient degree of certainty we can believe that the ***HS***–*concept* and computing ***HS***–models can present useful enough means for the description and knowledge of the *physical* world surrounding us in all its complexity and variety.

It is simple to be convinced, ***HS***–models directly possess a series of the base features of physical model of the world – *homogeneity* and *locality*. While their ability of *reversibility* generally speaking not exists a priori (*number of reversible **HS**–models is very little; see chap.* **2**) and it should be programmed beforehand in a ***HS***-model. Moreover, the ascertainment problem of *reversibility* of *dynamics* of classical ***HS***-models is in general algorithmically unsolvable. On the other hand, there is a possibility to design the reversible ***HS***–models beforehand, probably, at the cost of losses of some their essential features – *universality*, *dimensionality*, etc., and by introduction of those or other presumptions and suppositions too (*chap.* **2**) [152,268,274,318]. So, the ***HS***–models, perhaps, simplified in the sense of their separate characteristics, however more adequate to fundamental aspects of physical processes and phenomena in their modern understanding can be used. On basis of a series of reversible ***HS***–models a lot of interesting enough physical models of turbulence, diffusion, balance, hydrodynamics, gas dynamics, wave optics, others have been constructed and investigated [15,90,147,150,151,152,157,160, 166,187,204,218,273,318,376,378]. At present the models of ***Izing's*** spin systems [384,387], lattice gas [386,388], diffusion [263], and a series of others [360,418,536] are some of the most widespread physical models on basis of the ***HS***–concept. At modeling of processes and phenomena of the enumerated sections of physics one fundamental peculiarity of ***HS***–models was being used – the simple behaviour of large number of rather elementary automata of a ***HS***–model determines any necessary complex behaviour of all model as a whole. It is well known the given feature underlies many physical processes and the phenomena.

In connection with the principal question of existence of some formal computing model basing only on *reversible* computing elements a ***HS***–model for description of physics of ideal gas which essentially reflects the physical model making up a basis of the modern kinetic theory of gases had been suggested [8,150-152,268,273,388,393,394,536]. Already creation of this model has shown two important aspects, namely:

♦ ***formal HS-models can quite simulate the certain reversible physical objects, processes and phenomena;***

♦ ***even under condition of necessity of microscopic reversibility such models allow to reflect as a matter of fact any complex behaviour, i.e. they possess the property of universal computability.***

Thus, researches of the possibility question of ***HS***–models to simulate not only phenomenological aspects of our real world but also the base physical laws and processes have led to creation of physical *computing* ***HS***–models providing one of the most *fundamental* features of modern physics – *microscopic* ***reversibility*** of processes. Appearance of the first computing ***HS***–like systems and works of ***T. Toffoli, N. Margolus, G. Vichniac*** along with a lot of others have given a powerful incentive to fast development of *new* perspective approaches to physical modeling on basis of the ***HS***–concept [150,2632,273,394,545,617,618,640–643].

Meanwhile, use of ***HS***–models in physics presumes their founded and adequate interpretations, instead of speculations of a various sort. So, on basis of the *irreversibility* property of dynamics inherent in majority of the *classical* ***HS***-models, certain authors put forward assumptions of multi–variant approach to the *future* and even to the *past*. And if in the first case we do not feel any special emotions, whereas the second not fully yields to a *sensible* comprehension if only to not have in view the circumstance what for many dynamic systems their trajectories can be determined by those or other influences, and in this sense the current trajectory is a caused choice from a set of all trajectories allowable for the given system. And as an illustration for multi–variant approach to the past a ***HS***–model for which the current configuration has a set of previous configurations ***(predecessors)*** is being cited, in particular. In this case the gravity of a substantiation of the well–known facts by the references to the ***HS***-models is being perceived enough skeptically [54, 79,88,90,263,366,376,383,384,386,387,391–393,536,545,617,618,640].

HS–models and the differential equations*.** Above all, it is well–known that the significant part of numerical mathematics deals with methods of solution of both differential equations and difference equations of a various class and type that describe various *processes* in those or other applied fields. In particular, the methods of solution of the differential equations in partial derivatives that describe many processes in solid–state physics, hydrodynamics, plasma physics, thermodynamics, gas dynamics, aerodynamics, seismology, and in a lot of other appendices make up a large enough section. The ***HS–models are discrete dynamic systems that are enough frequently used as quite acceptable *analogues* of the *differential* equations in partial derivatives that describe those or other continuous dynamic systems. The base idea here that we will be to not describe some *dynamic* system by means of complex differential equations (*in the majority not admitting an analytical solution*), modelling this system by a certain set of locally interacting elementary automata functioning according to simple rules, i.e. to pass on from research of complex system with complex equations to research of a phenomenon of origin of this complexity as a result of dynamics of the appropriate ***HS***–model, whose elementary automata operate according to simple enough rules. In particular, ***HS***–models are most advantageously used for simulation of hydrodynamical and gas–dynamic currents [536]. At that, the hydrodynamics equations well correspond to a mathematical model describing behaviour of lattice gas (*one of kinds of* ***HS–models***) at a macrolevel. On basis of the ***HS***–models the problems of modelling of currents with free boundary, distribution of thermal streams, and a lot of others have been very successfully solved [536,545,640–643].

One of the base methods for numerical solution of various differential equations in partial derivatives is difference method, which is usually named in case of solution of boundary problems by the grid method. Meanwhile the idea of the grid method in the best way corresponds to the concept of computing ***HS***–models in which the *global* behaviour of an object described by the differential equations completely is defined by ***local*** interactions of elementary automata. Thus, use of ***HS***–models allows to apply new methods of solution of the differential equations in partial derivatives, the difference equations and integro-differential equations. There is a whole series of rather interesting works devoted to the comparative analysis of results of both numerical modelling of the equations of physical processes, and of effective enough methods on basis of ***HS***–modelling of these physical processes [536,545,640].

The general principle underlying such approaches consists in creation of a computing ***HS***–model whose dynamics strictly reflects behaviour of solution of the required differential equation. It is shown that such approaches are rather perspective for solution of boundary problems with complex geometry of borders or even with boundary conditions dependent on the time since the local interaction principle used in the ***HS***–models can be enough easily adjusted to the specified boundary conditions [263]. At that, a more direct approach to use of computing ***HS***–models for solution of *boundary* problems and *difference* equations is based on the *direct* analogue of *difference operators* and *local transition functions* of the ***HS***–models [1,3,5]. In particular, such approach is very closely connected to productive enough idea of ***E.F. Banks*** relative to the use of the general ***HS***–concept for modeling of *continuous* physical spaces [132]. Meanwhile, it is easy enough to carry out the satisfactory quantitative analysis with ***HS***–models without loss of simplicity and clearness of a rule of functioning of elementary automaton of the ***HS***–models what is obviously important *advantage* of this method. At that, the definition of such rules is frequently intuitive enough process, and sometimes is difficult enough one. Meantime, in a series of works the questions of acceptability and efficiency of a simulating of differential equations of certain types by ***HS***–models are discussed [90,536]. In our opinion, as well as at any other question here it is necessary to base on results of the subsequent check onto conformity of the received model with phenomenon or process described by differential equations.

HS-models, artificial intellect, robotics, pattern recognition and some adjacent directions*.** Very high degree of *parallelism* of the information processing along with other certain fundamental properties, provided with ***HS–models create perspective enough prerequisites for their use in adaptive systems of various types. For example, the neural network systems determined by some ***HS****–models quite can become a starting point for creation of the advanced adaptive robots [381]. For decision of on–line control by *adaptive autonomous transport robots* an interesting way of adaptive control based on use of *homogeneous control* structures being by typical computing ***HS****–models has been suggested [88,381]. At present the research problem of intellectual algorithms is based on active semantic networks. It is shown that the structures of ***HS****–class well satisfy the purposes of modeling of such networks [1,3,5,54]. The computing ***HS****–models allow to organize a rather efficient control in the *distributed continuous* objects too. This problem is extremely actual at designing and development of the flexible manufacturing systems along with some other industrial objects working in conditions of the unmanned production [54-56,88,90,302,360,364,368]. Already there are rather interesting practical examples of *stochastic **HS**–models* of various technological processes [536,545,640–643].

Use of ***HS***-models for *pattern* recognition and solution of the problems connected to it seems a rather perspective. First of all, the interesting question of receiving of solution of similar problems in *real* time along with fast processing of signals and images arises. In this direction the *structural* ***HS***–models of preliminary images processing and extraction of informative attributes have been developed and investigated. So, a rather interesting high-parallel architecture of the neural computer for solution of problems of numerical solution of differential equations in partial derivatives, accumulation and storage of knowledges, and for images processing has been offered on basis of ***HS***–like models [237].

At present, there is a series of prerequisites for understanding and use of computing opportunities of neuro-like nets which allow to simulate architecture of neural ensembles in brain tissues. In the given context the homogeneous structures, whose the certain types admit modeling of rather complex self-organizing dynamic systems, of self-organizing neuro-like systems, some phenomena of the chaos theory along with a lot of other interesting enough cooperative phenomena and processes represent significant interest [5,9,54–56,90,149,153,201,333,354,371,374, 382,383,407]. In particular, from our results follows, that the approach on basis of ***HS***–models can be rather useful to simulation and research of a lot of interesting phenomena in the antichaos theory along with a lot of other chaotic phenomena and processes [88,90,536]. Along with that there are interesting attempts of use of ***HS***–models for research of reflexive systems and creation of the mathematical theory of conflicts that deserve steadfast attention [3,5,55,90]. In the given connection the research of problem of the distributed intelligence on basis of the ***HS***–concept seems interesting enough also [189,536]. Numerous problems solved by means of ***HS***–models base on the actual building of devices within such models. The approach of such type is used since works of founders of the ***HS***–problematics, being one of the basic till now. A lot of results achieved in this direction was received on basis of building in ***HS***–models of the appropriate logical gates, various devices, etc. In the works [88,135,146,175,230,589,622,626] and many others the reader can acquaint oneself with the actual building of devices in ***HS***-models. A rather interesting information in this field can be found in Internet.

HS–models in cryptography*.** Cryptology is the science about safety of communication; this science is enough precisely subdivided into two base parts, namely: *cryptography* (*science about ciphers*) and *cryptanalysis* (*science about deciphering*). ***P. Guan has offered to use in cryptosystems with open keys the generative opportunities of ***HS***–models [9,536]. At that, it is necessary to mark, the reminder concerning the perspective possibilities of ***HS***-models for solution of problems of data *compression* and *cryptography* has been given by us earlier in monographies [1,3,9]. In spite of absence of careful check and a possibility of existence of the cryptographic bottlenecks, this direction presents indubitable interest and now develops enough intensively. One of the *important* properties of ***HS***–models is what even in case of their reversibility in general it is impossible to calculate predecessors for a finite configuration, having inverted a rule of generating of descendants. The researches related to creation of cryptosystems on basis of a series of ***3***–dimensional ***HSoS***–models suggested by ***T. Toffoli*** and ***N. Margolus*** represent significant practical interest too. There are many examples of interesting enough algorithms of *enciphering* and *coding* of information, of organization of *authentification* protocols with use of certain properties of ***HS***–models. At that, rather interesting attempts of elaboration of principles of the information protection are undertaken on basis of parallel processes, adequate to formal computing ***HS***–models [53–56,88,166,219,536,567].

HS–models and physical–chemical processes*.** Within already even of the classical ***HS-models some interesting enough properties of growth and morphogenesis of various crystal structures can be investigated. The simplest structures observable in crystals are explicitly periodical and with help of appropriate ***HS***-models the properties of such crystal *formations* can be investigated in exact mathematical formulations [1,3, 5,55,85,211,225,332]. So, a lot of the interesting ***HS***–models simulating certain types of chemical reactions, whose inorganic solution reflect a certain bio–rhythmic *periodicity* has been demonstrated [5,166,263,332, 333,536]. So–called *excitable environments* and *tissues*, whose researches are carried out in many directions in biology, medicine, chemistry and a lot of other natural sciences represent important enough class of ***HS***-models [2,54,134,147,160,166,167,172,198,203,205,214,346,536,567]. On basis of stochastic ***HS***–models the researches of physical and chemical processes flowing on surface of a rigid body are carried out. Program realization of similar models allows to make effective enough analysis of elementary physical and chemical processes.

The spatially–distributed dynamic systems cover rather broad area of appendices; for example, complex technological, geochemical, social, and chemical processes, populations of animals and insects, and so on. For study and modeling of the complex dynamics of *nonlinear dynamic* systems of such type certain authors suggest use of the stochastic ***HS***–models. An interesting enough review of application of stochastic ***HS***–models as an apparatus of research and modeling of *nonlinear dynamic* systems of the above type is represented in article [422] with accent on description and analysis of opportunities of the stochastic ***HS***–models of various classes. In the same place, the summary table of *applicability* of ***HS***–models of various types for solution of those or other classes of problems is presented. In a modified kind the above table is presented by the table *7* [617]. Meantime, it is necessary to note, both tables does not provide all completeness of scope of the problems being solved by the above types of ***HS***–models and, first of all, by classical ***HS***–models. However, the extensiveness of areas of use of apparatus of ***HS***–models for researches of the spatially–distributed complex dynamic systems is being looked through even on base of these tables. A little from other standpoint certain questions of research of *dynamic* systems by means of the ***HS***-apparatus have been considered in interesting enough work [421]. In our opinion, this problematics seems rather perspective [90].

The book [566] is the first one that generalizes and summarizes results of modeling of chemical systems on basis of the ***HS***-concept. The book is practical introduction to the ***HS***–problematics with *specific* accent on its applied aspects, representing simultaneously original textbook and practical guidance on application of ***HS***-models at chemistry. With the purpose of popularization of *practical* orientation of the ***HS***-concept, in our opinion, this book will play a rather essential part both for initial courses in chemistry, biology, physics, bio–computer science, applied mathematics and as a good enough additional material for laboratory courses in general chemistry, organic chemistry, medicinal chemistry, physical chemistry, chemical engineering and other courses that deal with statistical and dynamic systems. Simultaneously the given book represents a quite decent textbook and on applied aspects of the ***HS***–models. Recommendations and models of the given book play a part of a didactic manual on organization of similar ***HS***–oriented courses for a lot of university cycles [536,545,567,640–643].

HS-models in biomedicine*.** Today ***HS–models find the more and more wide application in biomedicine in the widest sense of this concept as attractive enough facilities of simulation of different processes, objects and phenomena of wildlife. First of all, one of the major appendices of the ***HS***–concept is simulation of interacting cellular systems in biology and medicine. So, for research of intercellular interactions ***A. Deutsch*** and other researchers have defined and investigated rather interesting types of stochastic cellular automata as microscopic models of cellular interaction [536]. The mentioned phenomena, depending on complex cellular interactions, cannot be received only exceptionally on basis of an experimental analysis, however they can be essentially more easy investigated, using mathematical models, in particular, on basis of the ***HS***-approach. Furthermore, such ***HS***-models can be spread and onto a series of other rather interesting appendices in biology, medicine and immunology. Importance of the ***HS***–concept for bio–medical sciences, in particular, reflects the important fact, that Council on biology of the Ministry of Education of the Russian Federation had included themes devoted to ***HS***–models into the program of discipline «***The concept of modern natural sciences***» as the separate section.

A. Kansal, S. Torquato et al. have developed a versatile and novel **3**–dimensional cellular automaton model of brain tumor growth. They showed that macroscopic tumor behavior can be realistically modeled using microscopic parameters. Using only four parameters, the model simulates Gompertzian growth for a tumor growing over nearly three orders of magnitude in radius. The model predicts the composition & dynamics of the tumor at selected time points also in agreement with medical literature. The given model incorporates some important and novel features in the rules governing the model and in the underlying structure of the model [617,618,640–643].

A. Deutsch and ***S. Dormann*** in own monograph have done accent on a challenging application field of cellular automata: *pattern formation in biologic systems such as the growth of microorganisms*, *dynamics of cellular tissue and tumors*, *and formation of pigment cell patterns*. The monograph is divided into **3** parts, namely: the first deals with general principles, theories, and models of pattern formation; the second part examines cellular automaton modeling; the third explains various applications. In addition, the web–based Java applet «*Cellular Automaton Simulator*» enables interactive simulation of various appendices described in the monograph. The book's accessible presentation and interdisciplinary approach make it suitable for graduate and advanced undergraduate courses in mathematical biology, biomodeling, biocomputing [640].

HS–models and nanotechnology. Modern nano-technologies are as yet very far from practical realization of self–reproduction in that kind as it ***John von Neumann*** has described, however the idea of synthesis of the computing environment in the form of two–dimensional array of elementary *transistor* cells is being traced distinctly today in a series of experimental works, that are being carried out in some large research centers ***(****IBM*, *Bell Lab*, *etc.****)***. The aspiration of nanoclasters of a series of chemical elements to *self–organizing* with forming of regular structures also in many respects promotes success of the given direction. Experts of *Communications Research Laboratory* ***(****Japan****)*** that carry out researches in the given direction, directly declare, that the main purpose of their development is creation of a ***HS***–model – the large *matrix* consisting of simple identical components of nanometer scale or cells. Furthermore, for completion of all work with use of separate *molecules* as a working element the Japanese researchers plan to spend about twenty years. In addition, similar works are planned and in some other countries [536]. For research of problems of *nanotechnology* the approaches on basis of the ***HS***-concept are suggested. In this direction the discrete systems on basis of the ***HS***–models seem some of most perspective for simulation of objects of nano–world in a language of discrete environments [536]. During latest twenty years rather active works on development of the mechanisms capable to self-reproduction are being carried out. One of the most perspective directions of creation of physical *self–reproducing* automata the area of nanotechnology represents. Here, the ***HS***–models in which processes of self-reproduction are being investigated already for a long time and where a series of various interesting results whose considerable part is already suitable to practical realization have been received, will play one of the leading parts, as is presumed.

***HS–models, synergetics, chaos and catastrophe theory*.** During series of latest decades in many natural-science areas an essential qualitative leap has happened – the *stochastic property* of many complex processes has been realized and research of the large *nonlinear* systems has been started. Synergetics, catastrophe theory, chaos theory and complexity are the different languages describing behaviour of *nonlinear* dynamic systems, including their property of spasmodic transition into another state; these languages use such concepts as self–organizing, hierarchy, fractality, critical transitions, and so on.

As a whole, the ***HS***-models reveal a *pronounced* chaotic behaviour that causes interest to them on the part of the *catastrophe* theory. At that, if a problem is reduced to qualitative understanding of complex spatio–temporal dynamics of the large nonlinear systems, then in many cases it is more preferable to deal not with certain continuous environment, but with a ***HS***-like model. Sequences of *finite* configurations generated by these ***HS***–models of various types and classes, are analogous to the trajectories of dynamic systems and their research represents the same *paramount* interest from standpoint of research for systems of this type of such determining behavioural characteristics as reversibility, chaos, nonconstructability, self-reproduction, self-organizing, periodicity, etc. [88,90,536,567]. The ***HS***-models more and more actively associate with synergetic problematics in its widest understanding [536,545,640].

Synergetics – a new enough interdisciplinary scientific direction which uses *nonconventional* methods for revealing of the general regularities of a self-organizing. It is necessary to mark, that as against cybernetics the synergetics considers not controlling processes and the principles of organization, but the *emphasis* is done on spatio–temporal structure of the organization, on conditions of its occurrence, development and self–complication. In addition, process of formation of steady regular spatio–temporal structures (*and without any external influences*), named the ***self–organizing***, is one of characteristic features of dynamics of the certain types of ***HS***–models. Therefore, the apparatus of ***HS***–models is applied widely enough in *synergetics* for research of various problems of self-organizing. Numerous experiments on ***HS***–models have shown that they quite can serve as rather acceptable apparatus of research of various dynamic aspects of self–organizing. In particular, researches of wavy processes in ***HS***–models allow to speak about an opportunity of creation on basis of similar structures of logic elements of which the different computing devices are built up; researches of various type of attractor in ***HS***–models seem interesting enough [88,90,536,567,640].

Chaos – an essential component of complex behaviour of the nonlinear systems. Meanwhile, such phenomenon as *antichaos* not admitting an intuitive comprehension takes place too. The phenomenon consists in what the certain rather chaotic systems during own development can spontaneously «*crystallize*» themselves, achieving a rather high degree of orderliness. A series of researchers presumes that antichaos plays a rather important part in biological development, evolution along with a lot of other phenomena [526]. At that, there is a series of arguments in favour of what along with well investigated **3** types of behaviour of dynamic systems: ***(1)*** stationary states, ***(2)*** periodic and quasi–periodic fluctuations, and ***(3)*** chaos, the *fourth* specific type of behaviour on the border between *regular movement* and *chaos* exists too. It has been also noticed that on the given border the processes analogous to processes of information processing and evolution can take place. As contrasted to the dynamic chaos, the considered phenomenon that is sometimes called the the ***complexity*** arises in the systems consisting of very large quantity of interacting elements. Such systems often demonstrate and the fourth type of behaviour, and in the certain measure possess also adaptive properties. As one of the typical rather interesting examples evidently demonstrating a series of the general properties of dynamic systems on an edge of chaos in full measure it is possible to consider a series of types of, including some kinds of classical ***HS***–models.

In particular, as a rather simple example it is possible to note the well–known game ***'Life'*** representing classical binary structure ***2–HS.*** A set of rules of this model such that its *dynamics* is in narrow enough range between areas of stability and chaos. In this ***HS***–model the behaviour similar to «*natural*» vital processes is being observed. At that, on basis of analysis of objects being generated by the model the equivalence of game «***Life***» with universal Turing machine has been proved, thereby proving the equivalency of its processes with universal computability. The game played a rather essential part in rise of interest in ***HS***–model. Along with this model a lot of other ***HS***–models whose dynamics is in a *narrow* range between areas of stability and chaos exists. And in this respect the ***HS***–models can present most *direct* interest for research. By analogy, to the phenomenon of chaotic movement in *nonlinear systems* the term the «*deterministic chaos*» has been given. The observed chaotic behaviour is generated by own dynamics of a nonlinear deterministic system [536,546,547]. Similar behaviour is typical for a whole series of classical ***HS***–models and is investigated enough intensively [79,88,90]. In general, dynamic trajectories of ***HS***–models are rather chaotic, they can't be predicted on the large enough time intervals. The forecast of movement along such trajectories becomes more and more undefined at moving off from the *initial conditions* – the initial *finite configurations*. From standpoint of the information theory it means, that a ***HS***–model itself generates the information and *speed* of its making the more if the model is the more chaotic. And as a ***HS***–model creates the information then the *dynamic* trajectories of the model contain this information. On basis of that an interesting enough idea on use of these trajectories for recording, storage and search of information has arisen. In addition, it has been shown that information organized in this way can be stored and kept as trajectories of dynamic system, possessing the property of associativity [536,546,547]. So, ***HS***–models can appear rather effective at appendices of the specified technologies for storage and associative search, transfer and protection of information, etc.

During the latest pair of decades a especial attention is devoted to the research of system mechanisms of occurrence of catastrophes. On the platform of nonlinear dynamics in solution of this problem essential enough progress connected above all with development of the theory of self–organized criticality has been achieved. In addition, models in this theory are easier for description not in language of the differential equations but in language of a special class of ***homogeneous*** structures with external noise. In context of revealing of the mechanisms of onset of catastrophes the prospects of use of ***HS***–models in mining industry have been considered on a series of rather interesting examples [536]**.**

HS–models in logistics and urbanistics**.** A whole series of problems of logistics are rather important in many applied areas, including certain questions of *urbanistics* and *transport***.** At the same time these problems as a whole are complex enough, hence for their research the computer modeling is widely used. However, such modelling demands a plenty of processor time and memory size. Meantime, the traffic dynamics is described by the *parallel* algorithms realizable on *parallel* computers of very high efficiency**.** In this respect the ***HS***–models as one of models of *parallel* calculations represent especial interest, therefore they attracted large attention at researches of some practically important traffics.

We want to pay attention to some of most perspective and interesting approaches to simulation of problems in the field of traffic on basis of the ***HS***–models presenting significant applied interest in logistics and urbanistics, taking into account the important circumstance what such ***HS***–approaches are in a channel of the basic mathematical methods of modeling of various transport streams [536]**.** In particular, the known ***HS***–model named **«*Life*»** has received a new especially applied aspect in problems of simulation of transport streams. Here with full ground we should mention the *pioneer* work of ***K. Nagel*** and ***M. Schreckenberg*** [574]**.** On the one hand, management by the traffic in city conditions is critical for transport policy and road planning. On the other hand, the *collective* behaviour of a plenty of vehicles mutually interacting is very interesting from the *physical* standpoint. From this standpoint the ***HS***–models in which motorcar dynamics is simplified retaining only basic features seem the *persuasive* enough models for description of various traffic systems. In this connection a series of researchers has used the ***HS***–approach for modeling of a city traffic.

For example, ***B. Chopard*** with colleagues have offered a rather simple

HS*–**model for city traffic in conditions of *Geneva.** Authors investigated *universal* properties of the received model and have made comparison of *numerical* results of modeling with *analytical* ones**;** they have shown that the dynamics is well described in terms of the queues forming on crossroads. Now a series of the computer simulators that demonstrate modelling of streams of movement on basis of ***HS*–**models allowing to adjust for those or other modes of movement of municipal transport is already created [536]**.** ***A. Dupuis*** and others have offered an interesting enough **2–**dimentional ***HS*–**model for description of a city traffic [498]**.** Their approach gives a rather inexpensive and simple description of a city traffic along with support of rather complex global dynamics too. A rather interesting computer simulator of the traffic has been created in Japan on basis of the ***HS*–**concept and stochastic high–speed model ***(S. Hikaru,*** *etc.***).** For the simulator the *parallel* mode of performance on cluster computer system has been realized. So, the received numerical results have shown, that the suggested ***HS*–**model possesses relatively good efficiency in a paralleling mode [536,545,640–643]**.**

With a series of other interesting enough ***HS*–**models in this direction the interested reader can familiarize oneself in works [213,216,217,360, 385,499,500,502–504,536,621]**.** So, the book [500] deals with the modern aspects of the traffic theory and its application to real world problems. Scope of the given book is wide enough **–** from microscopic modeling and ***HS*–**modeling of traffic streams till air pollution of the city streets that is caused by intensive transport movement, including. Also many interesting questions of research of the traffic and transport problems on base of ***HS*–**models were discussed on conference [360] and on a lot of other scientific forums [536]**.** It is necessary to note the technique on basis of ***HS*–**models is used for researches and simulation of a series of problems as in city planning, urbanistics, and in a series of such more *exotic* areas as urban planning, landscape planning, geography, etc. So, an approach to simulation of growth and interaction of cities with use of *irreversible* ***HS*–**models suggested by ***G. Malinetskii*** and ***A. Temkina*** seems interesting enough. With a number of other interesting enough appendices of the approach on base of ***HS*–**models in urban planning it is possible to familiarize oneself in the extended bibliography [536]**.** Particularly, in the book containing the referred proceedings of the ***5th*** International Conference on ***CA*** for research and industry among a lot of many relatively novel applications and models the appendices such as population and growth dynamics, highway traffics, environmental appendices and collective intelligence have been considered [625,640]**.**

Moreover, a number of other appendices of ***HS***–models exist.

HS–models in ecology. A whole series of researchers used a technique on base of ***HS***–models and for simulation in ecology. Essential growth of number of the publications dealing with ***HS***-models and simulation in ecology takes place [505-514]. Meantime, the questions how various aspects of the ***HS***–approach touch its availability for ecological models arise. The given questions are enough in detail discussed, for instance, in interesting enough works [505,506,510–514]. Here major question is the question concerning the synchronism of the ***HS***–models [505]. But, in spite of all questions connected to applicability of the approach on base of ***HS***–models to researches in ecology, today we have a series of rather interesting ecological ***HS***–models [425,502,536,545]. As a whole, all available models of development of ecosystems can be subdivided in two large classes, which can be classified as follows:

Continuous *models basing on differential equations in partial derivatives for spatially-distributed systems together with ordinary differential equations for concentrated systems;*

Discrete *models basing on systems of algebraic equations that in most cases can be received by digitalization of the differential equations, while in spatial-distributed case the **HS**–models can be successfully used.*

However, in any case before use of models of each of the above types it is necessary to prove their adequacy to a modelled system; for that the known enough techniques are used, in particular, the *retrospective* analysis [536]. Today, there are a lot of interesting enough ***HS***–models for research of such ecologically motivated problems as the process of *biorestoration* of polluted soils, progressive disafforestation of drainage as a result of expansions of agricultural grounds; ***HS***–models of forest fires allow to solve important problems of forecast of their behaviour, etc. [376,567,640]. Rather interesting approaches to the system analysis and elaboration of software for the forecasting problems of ecological safety are proposed on base of the specialized ***HS***–models. Moreover, the ***HS***–approach can be applied and in the problems of investigation of structure, act and evolution of natural, and natural–anthropogenic landscapes. Interesting enough results can be noted on application of ***HS***–models and for creation of geographical information systems, in particular, of urban geography as an example of practical application of methods of spatial subdivision and analysis [536]. In the interesting enough work [575] authors visually illustrate penetration of an idea of nonlinear dynamics to which the ***HS***–models in such areas as ecology, economy and social sciences directly concerns.

<u>HS–models in volcanology</u>. Attention is also devoted to use of the ***HS***-concept even in such area prima facie far from the ***HS***–problematics as volcanology [397–399]. Forecast of lava streams was mainly limited by qualitative aspects till use of the methods of computer modeling. As a matter of fact, in the real conditions analytically investigating the lava streams, use of the differential equations was, practically, impossible, excepting very simple cases. With a view of removal of complexities connected to that an approach basing on the concept of ***HS***–modelling has been used. The approach on base of ***HS***–modelling when the space and time are considered discrete, overcomes many of those difficulties which have been connected to application of differential equations. In this direction rather considerable successes have been achieved by the *Italian* researchers. So, in work [397] a ***HS***–model of lava streams with its application to modeling of eruptions of the volcano *Etna* at ***1986–87*** and ***1991–92*** is submitted. Important problem of volcanology is ability to model the *volcanic* phenomena for a receiving of the most authentic prediction of their potential threat. In this respect the model approach has paramount value, that allows to receive rather satisfactory results allowing to make rather authentic conclusions of prognostic character. In addition, the ***HS***–model chosen for study of lava streams has been realized by program ***SCIARA*** and approved on the data of growth of a stream field of *Etna* at periods ***1986–87*** and ***1991–93.*** Even provided that the data set was homogeneous, the simulated and real fields of a stream illustrate amazingly similar conformity of forms of growth.

The further development of the ***HS***–model for research of *behaviour* of *extensive* lava streams has been presented in work [399] with reference to volcanic activity of the volcano *Etna* too. At present, there is a series of other approaches to modelling the *described* phenomena [536]. So, ***K. Ishihara*** and others, using to a certain extent similar approach on base of the ***HS***-concept basing on the *Navje-Stocs* equations, have received a series of the important numerical characteristics [398]. However, their approach is inapplicable to numerous streams or to those lava streams which are being pushed out periodically. At that, use of ***HS***–models in conjunction with their computer realization for modeling of landslips in mountainous districts seems interesting enough [401].

It is well–known that with the processes connected to *volcanic* activity, the seismological processes also are very closely connected. So, for the research of seismic processes the spring–block models of breaks along with models such as «***heap of sand***» on basis of the ***HS***–approach are rather widely used. At that, the existing hypothesis about *impossibility* of forecasting in self–organized critical systems has been carried over to seismic processes and has begotten a series of doubts concerning an opportunity of any *reliable* forecasting of strong earthquakes [536,640]. For today, this pessimistic hypothesis has been not confirmed but and not refuted. In this connexion ***I.B. Kuznecov*** and others have analysed the *inverse* problem for ***HS***-models simulating dynamics of the seismic processes, i.e. they dealt with a problem of forecast of strong events in ***HS***–models on base of solution of the inverse problem [536]. At that, it is necessary to intend, the similar researches focused on seismological appendices can have essentially broader applicability in the problems considered from standpoint of synergetics.

At last, successful application of synergetics in modern geomechanics in the certain measure is connected to use of the methodology basing on the ***HS***–models. That is caused by that important circumstance that ***HS***–models provide satisfactory simulation of the phenomena that are researched in geomechanics and whose *mechanism* is not fully known. In this situation a strict mathematical apparatus is unsuitable, namely the ***HS***–models present the most effective apparatus of research in the given area, allowing to model rather complex dynamics [536,567,640].

HS-models in economics*.** The applied nonlinear dynamics, ***HS-models theory, fuzzy logic, neural networks and certain newest mathematical theories are attracted for solution of a number of modern problems of the nonlinear economic dynamics *(economic **synergetics**)* together with systematic application of fractal analysis too. Works on use of the ***HS***–models in banking technologies are known, for example, as model of diffusion of innovations for simulation of spread of internet–banking. Now, the ***HS***–approach are more and more widely used as the tool of evolutionary simulation of economic dynamics. For elaboration of the mathematical models of a series of social and economic systems along with classical methods all more widely are used *nonclassical* methods, for example, the ***HS***–models allowing to take into account the spatial structure of systems. As one of modern conceptual approaches to the description of some nonlinear economic processes on base of the ***HS***–concept, the laws of evolution of economic structures on macrolevel and microlevel, life cycles of economic structures, the goods, and also technologies from standpoint of *evolutionary economy* are investigated.

At that, in number of modern methods of the analysis of financial and economic processes the methods on base of the ***HS***–concept more and more emphatically declare themselves [536]. In particular, interesting mathematical model and a method of analysis of a securities market, including forecasting of the quotation of shares of a series of leading Russian companies have been suggested. Such model is based on the concept of linear ***HS***–models which have a series of advantages before traditional classical models. In researches of problems of the nonlinear *economic* dynamics (*economic synergetics*) among modern mathematical means are more and more broadly used the ***HS***–models. In particular, a nonlinear approach to analysis of markets on basis of ***HS***–models is one of the most perspective directions in the area of both financial and economic modelling. On basis of the ***HS***–concept various problems of self-reproduction of social and economic systems are also considered. In addition, it turned out that ***HS***–models of a forecasting of economic dynamic series have the definite advantages compared to well–known classical methods of a forecasting [536,545,567,617,618,640–643].

HS–models in sociology. The ***HS***–concept is enough widely used also for research of automatic models of collective behaviour of a various sort [177,196,201]. In many problems of modeling of *social* phenomena the classical mathematical methods today are enough widely applied, for example, the description of dynamics of the certain process by the differential equations, however already first rather interesting models of social phenomena basing on base of the ***HS***–concept have appeared too. A similar concept is used and for creation of models of formation of *public* opinion, including modeling of *electoral* processes. In opinion of a lot of researchers the ***HS***–approach allows to create more realistic market models than in case of *traditional* approaches to research of the innovations diffusion. The base advantage of this approach consists in possibility of *empirical* estimation of factor of probability of acceptance of a novelty. At that, *other* advantage of the given approach consists in a possibility getting of estimations of necessary quantity of supporters and their spatial distribution in the beginning of marketing campaign. In particular, a rather interesting work has been devoted to questions of simulation of innovative processes by stochastic ***HS***–models [515]. On basis of the ***HS***–models the attempts of elaboration of a conceptual approach to modeling of processes of social globalization are made, in particular, definition of the general features of systems basing on the ***HS***–concept and society, estimation of an opportunity of modeling of public processes within of such models, along with definition on basis of their terminology of such widely discussed social phenomenon as

globalization. There are interesting enough attempts of application of the ***HS***–approach to modelling of spread of hearsays and epidemics.

Moreover, in contrast to traditional approaches it is suggested to give a mathematical description even to such enough poorly formalizable processes as self–organizing of the population, formation and growth of cities or even of states on the base of ***HS***-modelling. The attempts of a society modeling as a self-organizing system are undertaken on base of the ***HS***–concept. On base of the ***HS***–models certain questions about a possibility of numerical experimentations simulating the movement of unorganized group of people are considered**;** at that, on base of the ***HS***–approach the models of receiving of preforecasting information in trade etc. are created. A lot of approaches confirms ample possibilities of use of the ***HS***-approach for mathematical modelling of social-ethnic processes. On base of the ***HS***–approach interesting enough models of propagation of information have been suggested [536,545,640–643]**.**

There are rather good grounds concerning use of the ***HS***–approach for mathematical modeling of historical macroprocesses [582]**.** The certain interest the attempts of application of the ***HS***–concept to modelling of electoral process represents too. In a series of works of ***T. Brown,*** a lot of contextual models of electoral process is considered [536]**.** A series of the imitating models of propagation of a panic has been developed**;** within of the concept of *multi-agent* systems on base of the ***HS***–models their new kinds that allow to model dynamics differentially according to economic, social–psychological and role characteristics of subjects of society relative to various scenarios of infecting, convalescence and prophylaxis are created. In this connection it is necessary to mention, already at ***1997 M. Stepantsev*** **(***Moscow State University***)** was seriously engaged by elaboration of dynamics models of a crowd, having come to a conclusion about expediency of use for these purposes of a *certain* class of mathematical ***HS***–models [536,545,640–643]**.**

At present, at a stage of becoming there is the computer sociology that is based on use of possibilities of computers for solution of theoretical, empirical and practical sociological problems. The direction has arisen as a means of creation and check of sociological theories, of measuring of social phenomena, of definition of principles and laws of structure and functioning of social processes, social systems and their prognosis [536]**.** In contrast to the traditional sociological theories existing as the texts or mathematical *sociological* theories in the form of an axiomatics, theorems, mathematical formulas, in the *computer sociology* the theory is a really functioning computer system basing on the ***HS***–concept, in particular. Algorithms for mathematical modeling of social systems of a different level the hierarchies basing on principles of functioning of the ***HS***–models is a component part of such concept. So, interest to the ***HS***–models used for research of spread of ideas, emotions, of question study of influence on dynamics of a group of such formal attributes as size of a population, opportunities of establishing of contacts together with degree of a variety of elements of the population steadily grows. Today, in connection with rapid development of information network technology and *social networks* based on it the tendency of their formal research is being outlined, where under a *social* network is understood a set of persons which can enter into an interaction with each other. In contrast to a classical network and social group, the social community being formed up, for example, in *Internet* **(***social network***)** allows more active and operative research, measurement and classification. For the analysis of similar social networks it is possible to successfully use the advanced mathematical apparatus, including ***HS***–modelling [536]**.** So, simulation of behaviour of the certain classes of ***HS***–models allows to investigate some general rules of behaviour of a group of individuals in the certain conditions. At last, some mathematical ***HS***–models have been suggested as examples illustrating some questions of philosophy**.** It is possible to ascertain that today the ***HS***–method plays a more and more growing part in mathematical modelling of social processes and phenomena, including rather unexpected ones [536,545,567,617,640]**.**

HS–models and artificial life**.** *Artificial Life* **(*AL*) –** investigation of the artificial systems demonstrating elements of characteristic behaviour of natural alive systems. These artificial systems well supplement the traditional biological sciences interested in analysis of *alive* organisms, trying to simulate life–like behaviour in computers and other artificial environments. Models of ***AL*** consider an alive organism as a huge set of simple machines nonlinearly interacting with each other by definite rules, providing a life–like global dynamics. So, many conceptual and philosophical problems concerning to ***AL*****,** have direct parallels with a problematics of complex and evolutionary systems**,** cybernetics and an artificial intellect. One of the major components of a natural life is the phenomenon of «*self-organizing*» that should be inherent in the models of ***AL*** too. Generally speaking, the ***self–organizing*** is understood as a certain process as a result of which a multicomponent systems tend to achievement of a specific state, of sets of cyclic states, or a state space of small volume without any external influence. All mechanisms that determine its behaviour are *internal* in relation to the system – the self-organizing in contrast of *externally* determined organization. The ***HS***–models can be ascribed to the category of a determined and *irreversible* self-organizing – a ***HS***-model self-organize itself exclusively according to its local transition function and an initial configuration.

In the ***HS***–models the local transition functions are often considered as a modelling of a certain artificial physics in artificial topological space, while configurations of cellular activation ***(****state cycles****)*** are considered as the certain *emergent* phenomena. In particular, when the observable sequences of configurations ***(their history)*** behave similarly to certain vital phenomena ***(****movement*, *self-reproduction*, *growth*, *regeneration*, *etc.****)***, often it is said that it represents manifestation of an artificial life from an artificial substance [88,536]. Meanwhile, one obvious problem with interpretation of the above analogy here takes place – in the real world the local rules giving rise to certain physical causal relations generates everything what us surrounds, both alive matter and nonalive matter. While in the world of ***HS***–models the arguments often are given for a substantiation of an artificial life but not for strict criteria of difference of an artificial life from artificial lifeless that is also generated by such artificial physics. In the given direction the activation of researches is highly desirable at use of ***HS***–models as an environment for research of the phenomena relating to an artificial life.

HS–like games**.** So, the special place among applied aspects of the ***HS***–problematics the so-called ***HS***-like games occupy and among them the ***Conway's*** game ***'Life'*** is the most well–known. The game represents a binary classical structure ***2-HS*** and has been offered as a formal model of research of some *general* principles of development of some abstract biological associations. At present, interest to the game is so great that many thousand fans and professionals all over the world take a great interest in it; many publications in editions of various sort, including electronic are devoted to the game [150,166,168,172]. The given game is easily generalized and to the ***3***–dimensional case and more general, than binary, state alphabet [536]. During own development the «***Life***» has turned in a sense into an independent object of the researches that attract attention of a whole series of professional mathematicians, and in this direction a set of interesting enough and diverse results which have exerted quite definite influence on all further development of the ***HS***–problematics has been received too [3,54,79,90,131,146,567,640].

The *overwhelming* number of results in the «***Life***» and its modifications has been received on base of the special computer modeling programs allowing to visually represent dynamics of the game [118,536,577,640]. Meanwhile, a little of theoretic results exists relative to the given type of classical ***HS***–models. For today they are limited, as far as we know, only by our results on the nonconstructability problem in ***HS***–models of the given type and on realization in them of base logic functions [3, 54,409]. Meanwhile, it has been shown that the «***Life***» is equivalent to universal Turing machine what is caused by presence in the game of the processes, tantamount to universal calculations. At the same time, various computer analysis of such ***HS***–models of dimensions ***d***={***1,2,3***} is enough widely submitted. It is necessary to note that the majority of the received results of theoretical character has been received on basis of computer modelling. So, ***P. Rendell*** on base of such approach could embed the universal Turing machine into the above game ***'Life'*** [429].

With rather interesting results relative to these simple ***HS***–models and program simulators, used for them it is possible to familiarize oneself, for example, in [5,8,88,90,146,168,172,409,429,536] and in the literature quoted in them. Of rather interesting program simulators of the game ***'Life'*** it is possible to mark, for example, such as ***3D–Life, Life, CASim, LifeLab*** and ***Plife.*** We can note the program ***FAM_Life*** possessing rich opportunities for adjustment as one of the best domestic simulators of the game «***Life***». Much interesting information in this direction can be found and in Internet, for example, by the key phrase «***Game Life***».

In the given context, it is possible to note one more type of the games suggested by ***V. Zadorozhny*** which represent classical structures ***2-HS*** with Moore's neighbourhood index and alphabet ***A***={***0,1,2***}, whose the local transition function approximately simulates the famous dialectic law of unity and fight of opposites according to the author's guess. At that, the local transition function is defined so, that populations of the elementary automata in states «***1***»,«***2***» compete among themselves but the friend without the friend they can't evolve [570–572]. Is interesting enough and the computer research of the specified ***HS***–model. A lot of interesting enough works of ***T. Toffoli, E. Fredkin, R.P. Feynman, C.H. Bennett,*** etc. that consider the universe as a certain ***HS***–model to some extent adjoin to this area too. As a whole, similar works are orientated onto the investigators who are interested in computing aspects of the physical universe or by physics of calculations. Actually, the existing theoretical models of a cellular universe is a convenient mathematical formalism, but, on the other hand, the extremely doubtful philosophic concept [536]. Meanwhile, the ***HS***–models were used by professional mathematicians and fans not only for creation of numerous games not less in detail, they were being researched as an acceptable mathematic apparatus for the scientific purposes. Due to their net architecture and ability to operate with *plenties* of discrete variables such mathematical structures have been considered as a rather effective alternative to the differential equations at simulation of complex dynamic systems.

HS–models at education*.** ***HS–models can be successfully used also at education, stimulating development of skills of modeling and creation of new technology. A lot of the learning computer games are based on the concept of ***HS***–models and students find their rather attractive. So, the concepts used at development of such learning games can be used for simulation of mathematical, scientific and social phenomena, and objects. Appropriate use of attraction of the similar games will help to activate motivations of interests of students in studying the important problems. Application of computer models for scientific experiments of a various sort should be stimulated from the first steps of student's practice as one of the most important, and effective ways of scientific research. In this attitude the ***HS***–models is a quite natural approach to teaching skills of modeling since they include both time and space. In addition, acquirement of the ***HS***–*concept* will well inculcate to students practical sense of computers along with certainty at problems solution with their use. In particular, that is caused by quite close connections between ***HS***–models and modern computer systems. A lot of modern models of computers in a definite extent is based on ***HS***–models, and one of fastest computing systems with mass parallelism is ***Connection Machine*** directly basing on the generalized ***HS***–concept.

Simplicity allows its to creatively use the ***HS***–concept with students of universities and colleges, and even high schools, learning perspective enough problematics in the fascinating game form. With examples of the approach the reader can familiarize oneself in works [408,425,426]. In particular, the paper [426] very convincingly motivates expediency of inclusion of acquaintance with the ***HS***–concept into curriculums. In our opinion, acquaintance with the ***HS***–concept seems rather useful to students of the universities first of all dealing with different problems of modelling of physical processes, phenomena and objects. At that, it concerns majority of students of natural sciences, including biological sciences. The ***HS***–apparatus is the excellent tool of modelling therefore in this quality it is of interest for many other fields too.

In addition, there are certain encouraging results in development of a mathematical model of the global education system basing on the ***HS***–approach. There is a series of successful attempts of simulation of age dynamics in the Higher School of the Russian Federation on basis of ***HS***–like models, in particular, application of models of the given type for solution of simulation questions of dynamics of the teaching staff of the Higher School of the Russian Federation [536,545,640–643].

At last, for the first time in the domestic educational literature for the universities into the course of computer science recommended by the Ministry of Education of the Russian Federation, together with Turing machines (*formal models of sequential computations*) have been included the ***HS*** as formal models of high–parallel computations [94-96,536]. At that time similar act was unusual enough decision for textbooks. In a lot of the Russian universities for candidates for the degree of master and *doctor* special courses on evolutionary cybernetics, programming, synergetics and certain others are already read with inclusion of base information on homogeneous structures ***(cellular automata)*** and their applied aspects. As a whole, all increasing quantity of universities of the Russian Federation include the given theme in curriculums of the disciplines connected to mathematical modelling, programming, and dynamic systems. At that, in secondary school for acquaintance of the schoolboys with the percolation bases also in a whole series of cases homogeneous structures «***Cellular automata***» are used. Furthermore, there is enough much interesting reports of the schoolboys at various school conferences on the problematics of cellular automata [536,545].

HS–models and music*.** The attempts of use of ***HS–models for creation of musical compositions of a various sort seem rather interesting. And indeed, as ***HS***–models generate huge quantity of the diversified finite configurations and at the assumption, that we can imagine a musical composition as being based on a certain history of a configuration and formal manipulation by its parameters, not cause surprise the fact that researchers have started to realize, that ***HS***–models can be successfully used and for creation of musical effects of a various sort. At that, ***HS***–models whose dynamics is characterized by a cyclic behaviour, a self–organizing or properties of spreading of configurations represent the greatest interest. An interesting enough discussion of significance of the given properties for musical appendices can be found in [537]. For example, ***E. Miranda*** for investigation of synthesis of sounds on basis of ***HS***–models has applied a technique of so-called «*granular synthesis*»

[538]**.** Its essence is reduced to very fast generating of the sequences of rather short sound impulses named **«*granules*»** which jointly form the large sound effects. On base of one ***HS*–**model the special generator of granules **(***Chaosynth***)** by means of which any granule is represented by a certain finite configuration of the ***HS*–**model has been created.

Firm ***NyrSound*** has produced also the program synthesizer *Chaosynth* intended for creation and reproduction of separate sounds**;** it also can be used and as a monophonic synthesizer controlled by ***MIDI.*** In this program the ***HS*–**technology and granular synthesis are applied. After a series of experiments with generator ***Chaosynth*** a special ***HS*–**model named *ChaOs* and used for simulation of behaviour of type **«*catalytic reaction*»** has been chosen. In this case ***HS*–**model allows to simulate a method of creation of most natural sounds made by acoustic devices. *ChaOs* can be imagined as a matrix of cells containing identical simple *electronic* circuits whose behaviour we can compare with behaviour of an artificial neuron or perceptron.

For disclosure of ***HS*–**models generating sequences of configurations which can be used for simulation of spread of various musical effects, the *numerous* experiments have been carried out. As a result the above experiments have resulted in creation of a system named *CAMUS* that uses two types of ***HS*–**models, namely**:** the classical **«*Life*»** and *Demon Cyclic Space* [539]**.** Roughly, *CAMUS* **–** a composition system, that uses dynamic properties of some ***HS*–**models for creation of various *musical forms***.** Among other concrete realizations it is necessary to note also on *Automatous Monk* created by ***P. Reiners*** which represents the program realization of one model ***1–HS*** for generation of musical compositions rather far from the *perfection* in reality. For more detailed acquaintance with researches and concrete progress in use of ***HS*–**models for *musical* creativity it is possible to address to the extended bibliography [536]**.**

Algorithmic musical composition **–** application of some precise algorithm to process of creation of a music. ***HS*–**models represent that type of the mathematical structures whose complex dynamics underlies a special interest from the standpoint of creation of various algorithmic musical compositions. On the other hand, computers are ideal for calculation of dynamics of ***HS*–**models and its convenient visual showing**;** similar dynamics can be mapped into sound effects too. Meanwhile, methods of detection of mappings of dynamics of ***HS*–**models into pleasant and interesting music is a rather difficult problem. In a number of works a few interesting methods for *realization* of the musical compositions are presented on base of some ***HS***–models in language ***Java,*** and concrete ***HS***–mappings resulting in the most good results are researched [536]. There are interesting enough examples illustrating the potentialities of ***HS***-models and for generation of nontrivial musical compositions, and for more best understanding of structural dynamics of ***HS***–models, by using an acoustic modality.

However not only in such kind of art as music the ***HS***–models turned out a rather interesting tool for creation of musical compositions ***(the truth, for the present not so perfect),*** but researches of their dynamics along with properties of fractals has unexpectedly led to to occurrence of a new direction in the fine arts – complexity and naturalness of the configurations of states of their elementary automata generated by the ***HS***–models turned out unusually aesthetically attractive, allowing to hope on their quite successful application in the fine arts too [536,545].

Some other appendices of HS–models. Questions of appendices of ***HS***–models which have not found reflection above or especial emphases have not been done on them are briefly marked below. In particular, a good enough analysis of appendices of ***HS***–models (*of finite and infinite ones*) to problems of the distributed intelligence, cryptography along with interacting *bio-populations* can be found in the monography. Here are considered as questions of embedding of the mentioned problems into ***HS***–models, and interesting enough conceptual discussions and reasons [161]. Interesting applied aspects of the ***HS***–problematics arise in connection with such general questions as structural theory of gas; artificial intellect, information theory and statistical physics; problems arising in modelling of microscopic physical processes, and complex *macroscopic* dynamics; a series of problems of fractal theory and chaos theory, discrete synergetics and others and in connexion with creation of special–purpose computers also [5,9,90,117-119,123-126,128,129,131, 133,149,175,182,183,192,309,328,345,348,353,371–376,377–379,567]. At a lot of cases these problems are being crossed with the above problems.

The ***O. Lafe's*** book deals with application of ***HS***–models in *multimedia* compression, data encryption & process modeling. The book includes the fundamentals of ***HS***–models, including the history and traditional applications. The digital image compression, audio compression and synthetic audio generation, ***3*** approaches for compressing video data are described. In addition, symmetric and public–key implementation of encryption on basis of global transition functions are considered.

A number of classes of homogeneous structures can be used as a base for research and modelling of conceptual phenomena of the spatially–distributed dynamical systems, including the hypothetical universes. At that, *unfortunately*, insufficiently well thought–out and professional popularization of the ***HS***–concept has a quite negative party when, in particular, in works of art in its context depict pictures of a *structure* of the Universe. Meanwhile, various *speculations* relative to the analogies of processes and phenomena observable in the Universe, and in other manifestations of the external world, with dynamic properties of ***HS***–models can inflict an irreparable harm, first of all, for the ***HS***–concept, discrediting partially or fully this perspective direction [90,536,567].

At the same time, on base of the ***HS***–concept we considered a series of rather interesting models of some phenomena of spatially–distributed dynamic systems, and certain hypothetical Universes [88,90]. At that, a lot of rather interesting development models, whose determinative characteristics has turned out invariant in relation to a time direction (*which are being not changed at inversion of the time direction*) along with certain others has been received. A whole series of rather interesting models of spatially–distributed dynamic systems was investigated on base of *asynchronous* structures of considerably more general kind. At that, similar models and their interpretations present a quite definite interest in a context of modern nonconventional concepts concerning the history of our *Universe* along with a series of certain extraordinary phenomena. Investigation in this direction seems a rather perspective from theoretical standpoint above all, demanding definite prudence.

Of the more applied aspects of the ***HS***–problematics we can note use of the ***HS***–concept and certain ***HS***–models for solution and research of problems of network planning; in particular, there is a whole series of rather interesting works on creation of a problem environment that is based on the ***HS***–approach for solution of such optimization problems as the *salesman* problem and the *scheduling* problem [536]. Interesting enough works on use of ***HS***-models in problems of reliability and self-restoration of multicomponent complex discrete dynamic systems of technical purpose can be found in [5,55,90,181,192,184,187,190,191,276, 536]. So, in particular, it is shown that in a class of non–strictly located ***HS***–models some continuously self–reproducing automata can be and self-restoring ones. Nondeterministic ***HS***–models represent significant enough interest for research of tasks of reliability and self–restoration of complex dynamic cellular systems.

At last, use of ***HS***–models as well in astronautics at the organization of interstellar flights can turn out the least practical project in the nearest future but as the most perspective project in the long–term future. So, in particular, at ***1984 R. Forward*** has introduced into this idea the new progress of the computer technics basing on computing ***HS***-models. As a result the project of interstellar space vehicle «*Staruisp*» was born. Of the more prosaic in the given area it is possible to note the researches carried out at the *Russian Academy of Cosmonautics* on use of stochastic ***HS***–models for analysis of injurious influence of factors of an extremal environment onto a space vehicle. In this case, the problem of atomic-molecular interaction and statistical modeling of process of change of physical and chemical properties of materials at their prolonged stay in a cosmic space becomes more and more actual in connection with prospects of creation of near–earth stations of various purpose, moon bases and in case of long manned space flights, for example, to Mars.

Here we in a few words shall mention the appendices of the finite ***HS***-models where the designing and manufacture of the ***VLSI*** is, perhaps, one of most important areas of their use. At that, use of ***HS***–models in ***VLSI***–technology for *generation* of pseudo-casual numerical sequences and their application in subsystems of the built–in self–checking is the most successful, perhaps. Of other perspective enough appendices of finite ***HS***–models we can note such fields as cryptosystems, graphics, codes with errors correction; testing of finite automata; development of associative memory. For today, in ***VLSI*** the binary structures ***1–HS*** are most applicable, interesting enough examples of application of the structures ***2–HS*** and hybrid ***HS***–models exist. At that, for ***VLSI*** design and testing, the linear ***HS***–models mainly have been utilized, however they have a series of essential enough limitations impeding possibility their use for achieving required results. With that end in view ***S. Das*** [622] has researched the nonlinear ***HS***–models for the purpose of their using in developing applications in ***VLSI*** design. In particular a rather efficient design of pseudo-random pattern generators which are based on the *nonlinear reversible* ***HS***-models has been presented. Furthermore the nonlinear ***HS***–models have been employed to address the issue of data services in cellular mobile network. In addition use of ***HS***-models as an efficient enough *query* processor which supposes an appropriate hardwired realization to data service has been considered. At that, it is necessary to note that at ***1975*** we proposed a developmental design of operating systems in computing environments and structures, that is based on finite classical ***HS***–models [36]. Our solution was founded on standpoint that an operating system is a certain specialized system for search of required computing resources depending on the current task. Such approach allowed to elaborate an operating system project which admits an effective enough hardwired realization. A number of accompanying problems has been considered by us in [40,50,52,57]. In addition, from the applied and theoretical standpoints concerning the finite ***HS***–models enough in details the reader can familiarize oneself in a whole series of interesting publications presented in the extended enough bibliography [536,545,618,640–643].

Completing on that a rather brief discussion of applied aspects of the ***HS***–problematics, it is necessary to note that outside of it still a whole series of interesting enough applied themes remain that represent an independent interest and whose list constantly grows. Therefore, the interested reader is recommended to constantly trace the progress of this direction of the ***HS***–problematics in corresponding publications in traditional and electronic editions. Furthermore, it would be desirable to stress that, in our deep belief, the *classical*, *polygenic*, *nondeterministic* and *stochastic* ***HS***–models along with their different modifications and generalizations can be effectively used and in other numerous fields, however one of the most interesting and perspective fields of their use is application of the ***HS***–approach as a new perspective environment of modelling of a various sort of phenomena, objects and processes, of which *physical* and *biological* appendices seem by the most perspective and interesting ones [5,88,90,392,536,567,640]. Consequently, the most steadfast attention should be devoted to the applied aspect of the ***HS***–problematics, not losing sight the theoretical researches of ***HS***–models as an important independent mathematical object.

Thus, the multiformity of the applied aspect of the ***HS***–problematics is characterized already by simple and incomplete enumeration of such areas as physics, mathematics, chaos theory, neural networks, logistics, biology, ecology, hydrodynamics, statistical physics, crystallography, epidemiology, immunology, chemistry, material engineering, storage and information search, industrial technology, seismology, sociology, powder metallurgy, volcanology, agriculture, cryptography, genetic & parallel programming, computer facilities, and many others [536,640].

So, the homogeneous structures *(cellular automata)* are interesting not only as an independent mathematical object, but also by its numerous uses. And in any case they should not be considered as a certain kind of a new universal paradigm of modern science how some researchers it try to represent. Similar standpoint is a voluntarism of clear water.

Conclusion

It is a good idea to collect and publish from time to time the different surveys of interesting and useful results from some area of research. It helps to briefly summarize the current state of knowledge and to state the most important research directions, taking into account the earlier received results. A series of surveys of our previous results on the ***HS***- problematics along with formulation of a series of unsolved problems can be found in works [3–5,7–10,88,90] and some others, while certain comments to this material can be found, in particular, in [119,138,150, 161,163,251,258,259,443,568,619] and in some other sources. In [619], in particular, a difference between our surveys of unsolved problems in the ***HS***–problematics and published by ***S. Wolfram*** (*Physica Scripta* ***T9, 1985)*** has been characterized as follows: there is a substantial *difference* between the ***Wolfram's*** paper and the reviewed one ***(V. Aladjev*** *et al*.**)**, the *authors* of the latter one concentrate themselves to the *mathematical aspects* of the topic while ***Wolfram's*** is not on such level of abstraction; **22** problems presented here deal with questions of the constructibility, hierarchical properties, modelling, complexity, decomposability, and dynamics of the classical homogeneous structures ***d–HS*** ***(d ≥ 1).***

In the offered monography both at the pithy level, and in the form of strict enough mathematical formulations and proofs a lot of results in the basic sections of mathematical theory of the *homogeneous structures* that have been received by us or earlier or generalized along with the adjusted and new results was submitted. By not covering all extensive problematics of this field of modern mathematical cybernetics wholly, meantime, the presented results lay in the course of the base modern directions, forming up an essential part of the modern researches state in this direction. The *homogeneous structures* ***(HS)*** are a bright example of generating of complex enough objects and their dynamics on basis of rather simple initial elements and prerequisites. In the given sense, homogeneous structures better answer the mathematical models used in more abstract areas of theoretical physics, discrete synergetics and mathematical development biology than to the more practical models of computing sciences basing on modern microelectronic technology. Although with the further development of technology, first of all, the *nanotechnolody* they, perhaps, can play more and more growing role in this field as formal models and prototypes of high–parallel systems of information processing.

Once again it is necessary to note that the ***HS***-concept in a great extent is an unique phenomenon – on the one hand, the ***HS***–concept is a base for formal modeling of manifold processes, phenomena and objects in broad enough spectrum of fields, and, on the other hand, the concept has equivalent technical realizations, for example, such as networks of transputers, ***CAM*** of ***T. Toffoli,*** cellular processors, systolic structures, homogeneous computing environments, etc., doing the ***HS***-concept as attractive enough facility both in theoretical and applied researches in many fields, with good enough reason raising the given concept onto a new interdisciplinary scientific level.

Hence, the ***HS*** – more than very useful abstraction since they possess a series of fundamental properties that can lead us to creation on base of reversible computing ***HS***-models of new perspective architecture of high-efficiency computer systems and the control blocks of systems of an artificial intellect of the *future* generations and also to play a part of a perspective modeling environment for the broad area of appendices.

And again, we absolutely do not agree with ***Wolfram's*** standpoint on ***HS***–problematics ***(CA***–*problematics)* as a new kind of science. Our rich experience in the ***HS***–problematics both on theoretical, and especially applied level speaks fully other, namely: ***(1) HS*** represent one of types of infinite abstract automata with specific internal structure admitting a rather high level of *parallel* information processing and computation; ***HS***–models form a *specific* class of discrete dynamic systems operating in especially parallel manner on base of principle of local interaction, and ***(2) HS*** can be considered as formal mathematic objects presenting undoubted self–dependent interest too.

Homogeneous structures (*Cellular Automata)* can be considered as a quite independent part of discrete mathematics, mathematical cybernetics, abstract infinite automata with specific internal organization, parallel discrete dynamic systems, but in any way no as a new type of science. Already name of the opus «*A New Kind of Science*» at once sets a trap, without allowing to consider it as the serious scientific edition with a pretension to a certain its significance.

Thus, on examples of computer experimentation with simple enough one–dimensional cellular automata ***S. Wolfram*** draws "*deep*" scientific conclusions which were known for a long time. Thus, in most cases he tries to present himself as their pioneer, ignoring their real authors. In particular, he ascribed to himself many results and assumptions of the well–known scientist ***K. Zuse*** and others researches mentioned in the present book. And it already at the level of plagiarism by the highest standarts without speaking about full ignoring of historical justice. If the *western* researchers still are sometimes mentioned by him, then the *Soviet* researchers are completely ignored though they received a large quantity of fundamental results on cellular automata theory and their applied aspects incommensurable on the importance with conclusions of the author of this tendentious opus.

It should be noted that our standpoint on the majority of '*fundamental*' conclusions of the opus [407] is completely consistent with numerous reviews [651] on the opus. I will give only excerpt from one review of ***S. Wolfram*** opus which is completely conformable to our opinion on it, namely: '*But he does know Goedel and Zuse and Turing. He must see that his own work is minor in comparison. Why does he desparately try to convince us otherwise? When I read Wolfram's first praise of the originality of his own ideas I just had to laugh. The tenth time was annoying. The hundredth time was boring. And that was my final feeling when I laid down this extremely repetitive book: exhaustion and boredom. In hindsight I know I could have saved my time. But at least I can warn others.*' We also consider the given opus as a certain self–advertisement of both the author, and the theory of cellular automata regardless of the well–established facts and real situation. Consequently, in our opinion, the adequacy of the name «*A New Kind of Science*» to the essence of the book of the same name is of the same level as the name of well–known computer game ***"Life"*** that is based on a rather simple **2**–dimensional binary cellular automaton to the life itself. Meanwhile, we nevertheless recommend at a leisure to acquaint oneself with the above opus at least in respect of the general acquaintance with cellular automata.

At that, the *pithy* level of statement that is illustrated by examples and separate proofs, allows to use the given monography by wide enough audience of the readers of the various spheres of potential application of methods, results and the ***HS***–problematics concept as a whole. For more full acquaintance with the modern state of the ***HS***–problematics extensive enough bibliography that, in turn, contains the references to numerous publications in this direction is represented. In our opinion the monograph will present an indubitable interest for students, post–graduates and persons working for doctor's degree of the appropriate faculties of universities and colleges, first of all, of naturally scientific level along with teachers in such disciplines as cybernetics, automata theory, mathematics, mathematical and physical modelling, computer science, theoretical biology, computer technique, and a lot of others.

References

During researches in the theory of homogeneous structures extensive enough bibliography of original sources of different level and directly in the theory, and in its numerous applications in different fields has been collected by us. Naturally, the given bibliography is not perfectly exhaustive, however it can present the certain interest for researchers in the given field, first of all, of the beginners. Meanwhile, the reader has an opportunity to supplement the presented bibliography by the materials which are absent in it. We hope, that the given bibliography will allow to outline better both the circle of researchers in the given field, and breadth of scope of the problems considered by them. First of all, it concerns Soviet and Russian researchers who have received a series of priority results of fundamental character with which English-speaking researchers are familiar insufficiently well or are not familiar entirely. Subsequently some of them have been rediscovered by other researchers. It is especially topical and for the reason, that some Soviet researchers directly stood at the beginnings of the forming up of this area of modern mathematical cybernetics. The presented bibliography is not annotated, basically, however the headings of a great many of publications give the defined enough comprehension concerning the contents of the quoted material. The more extensive bibliography of original sources on the ***HS***–theory and its numerous appendices can be found in [536], while the skilled reader is referred to Internet with appropriate key phrases. In particular, a number of our works along with works of other writers can be found in the Internet. While here a incomplete list of our works we have done in the mathematical theory of classical homogeneous structures during ***1969 – 2013,*** in truth with considerable enough pauses caused by other themes is presented.

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3. ***Aladjev V.Z.*** *Mathematical Theory of Homogeneous Structures and Their Applications*.– Tallinn: Valgus Press, 1980.
4. ***Aladjev V.Z., et al.*** *Mathematical Biology of Development*.– Moscow: Science Press, 1982, 256 p. *(in Russian with extended English summary)*
5. ***Aladjev V.Z.*** *Homogeneous Structures: Theoretical and Applied Aspects*.– Kiev: Technics Press, 1990, 272 p., ISBN 5–335–00949–7.

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