Cellular Automata: Brief historical survey

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First of all, a few words about the terminology used below. Today, the problems of *Cellular Automata (**CA, CA**–models)* is rather well advanced, being quite independent field of the modern mathematical cybernetics, having own terminology and axiomatics at existence of broad enough domain of various appendices. In addition, it is necessary to note that at assimilation of this problems in the *Soviet Union* in Russian–lingual terminology, whose basis for the first time have been laid by us at ***1970*** [1]**,** for the concept «*Cellular automata*» the term «*Homogeneous structures*» *(**HS; HS**–models)* has been determined which nowadays is the generally accepted term together with a whole series of other notions, definitions and denotations [1–3]. Whereas a rather detailed list of publications on ***CA*** problems can be found, for example, here [8]. Therefore, during the present survey along with this term its well–known Russian–lingual equivalent «*Homogeneous structures; **HS***» can be used too.

Cellular Automaton** (**CA**)* **–** a parallel information processing system that consists of infinity intercommunicating identical *Mealy* automata *(elementary automata)*. We can interpret ***CAs as a theoretical basis of artificial high parallel information processing systems. From the logical standpoint a ***CA*** is an infinite automaton with specifical internal structure. So, the ***CA*** theory can be considered as structural and dynamical theory of the infinite automata. At that, ***CA*** models can serve as an excellent basis for modeling of many discrete processes, representing interesting enough independent objects for research too. Recently, the undoubted interest to the ***CA*** problems (above all in the applied aspect) has arisen anew, and in this direction many remarkable results have been obtained. Further, by ***CA*** we mean both cellular automata and a separate cellular automaton, depending on the context.

So, the ***CA***–axiomatics provides three fundamental properties such as *homogeneity*, *localness* and *parallelism of functioning*. If in a similar computing model we shall with each elementary automaton associate a separate microprocessor then it is possible to unrestrictedly increase the sizes of similar computing system without any essential increase of its temporal and constructive expenses, required for each new expansion of the computing space, and also without any overheads connected to coordination of functioning of an arbitrary supplementary quantity of elementary microprocessors. Similar high–parallel computing models admit practical realizations consisting of rather large number of rather elementary microprocessors which are limited not so much by certain architectural reasons as by a lot of especially economic and technologic reasons defined by a modern level of development of microelectronic technology, however with the great potentialities in the future, first of all, in light of rather intensive works in field of nanotechnology. Along with it the ***CA*** models can be used for problems solving of information transformation, such as encryption, encoding and data compression [3].

The above three such features as high homogeneity, high parallelism and locality of interactions are provided by the ***CA***–axiomatics itself, while such property important from the physical standpoint as reversibility of dynamics is given by program way. In light of the listed properties even classical ***CA*** are high–abstract models of the real physical world, which function in a space and time. Therefore, they in many respects better than many others formal architectures can be mapped onto a lot of physical realities in their modern understanding. Moreover the ***CA***–concept itself is enough well adapted to solution of various problems of modelling in such areas as mathematics, cybernetics, development biology, theoretical physics, computing sciences, discrete synergetics, dynamic systems theory, robotics, etc.

Told and numerous examples available for today lead us to the conclusion that the ***CA*** can represent a rather serious interest as a new perspective environment of modelling and research of many discrete processes and phenomena, determined by the above properties; in addition, raising the ***CA–*** problematics onto a new interdisciplinary level and, on the other hand, as an interesting enough independent formal mathematical object of researches [8].

The base modern tendencies of elaboration of perspective architecture of high parallel computer facilities, a problem of modelling of discrete parallel processes, discrete mathematics and synergetics, theory of the parallel discrete dynamical systems, problems of artificial intellect and robotics, parallel information processing and algorithms, physical and biological modelling, along with a lot of other important prerequisites in various areas of modern natural sciences define at the latest years a new ascent of the interest to the formal cellular models of various type which possess high parallel manner of acting; the cellular automata are some of major models of such type. During time which has passed after appearance of the first monographs and the collected papers which have been devoted to various theoretic and applied aspects of the ***CA*** problems, the certain progress has been reached in this direction, that is connected, above all, with successes of theoretical character along with essential enough expansion of field of appendices of the ***CA*** models, mainly, in computer science, cybernetics, physics, modelling, developmental biology and substantial growth of number of researchers in this direction. In addition, in the *USA, Japan, Germany, the Great Britain, Hungary, Estonia*, etc., a series of works summarizing the results of progress in those or other directions of the ***CA*** problems including its numerous appendices in various fields has appeared. Our monographs at the substantial level have presented the reviews of the basic results received by the *Tallinn Research Group* on the ***CA*** problematics and its application [1–7]. From the very outset of our researches on the ***CA*** problemcs, above all, with application accent onto mathematical developmental biology the informal *Tallinn Research Group (**TRG**)* consisting of the researchers of a few leading scientifical centres of the former *USSR* has gradually been formed up. At that, the ***TRG*** staff was not strictly permanent and was being changed in broad enough bounds depending on the researched problems. In works [1–7] the analysis of the ***TRG*** activity instructive to some degree for research of the dynamics of development of the ***CA*** problematics as an independent scientific direction as a whole had been represented. Ibidem the basic directions of our researches can be found along with main received results.

Today, ***cellular automata*** *(homogeneous structures)* are being investigated from many standpoints and interrelations of objects of such type with already existing problems are being discovered constantly. On purpose of general acquaintance with extensive ***CA*** problematics as a whole and with its separate basic directions specifically, we recommend to address oneself to interesting and versatile surveys of such researchers as ***V.Z. Aladjev, V. Cimagalli, K. Culik, D. Hiebeler, A. Lindenmayer, A. Smith, P. Sarkar, M. Mitchell, T. Toffoli, R. Vollmar, S. Wolfram,*** et al. [8]. A series of books and monographs of the authors such as ***V.Z. Aladjev, A. Adamatzky, E. Codd, A. Ilachinskii, M. Garzon, M. Duff, P. Kendall, T. Toffoli, B. Voorhees, M. Sipper, O. Martin, K. Preston, V. Kudrjvcev, N. Margolus, R. Vollmar, B. Voorhees, S. Wolfram*** and some others contain a rather interesting historical excursus in the ***CA*** problems; in addition, unfortunately, hitherto a common standpoint onto historical aspect in this question is absent [8]. In view of that, here is a rather opportune moment to briefly emphasize once again our standpoint on historical aspect of the ***CA–***problematics, namely: a brief historical excursus presented below make it one's aim to define the basic stages of becoming of the ***CA–***problematics, having digressed from numerous particulars.

Having started own researches on the ***CA***–problematics in ***1969,*** we on base of analysis of large number of publications and direct dialogue with many leading researchers in this direction have a quite certain information concerning the objective development of its basic directions, above all, of theoretical character. That allows us with sufficient degree of objectivity to note the pivotal stages of its development; at that, many details of historical character concerning the ***CA***–problematics the reader can find, for example, in a whole series of works presented in the reference list [8].

From the theoretical standpoint the ***CA*** concept *(homogeneous structures)* has been introduced at the end of the forties of the past century by ***John von Neumann*** on ***S. Ulam's*** advice with the purpose of determination of more realistic and well formalized model for research of behaviour of complex evolutionary systems, including self–reproduction of alive organisms. Whereas ***S. Ulam*** has used ***CA***–like models, in particular, for researches of the growth problem of crystals and certain other discrete systems growing in conformity with recurrent rules. The structures that have been investigated by him and his colleagues were, mainly, 1– and 2–dimensional, however higher dimensions have been considered too. In addition, questions of universal computability together with certain other theoretical questions of behaviour of cellular structures of such type also were kept in view. A little bit later also ***A. Church*** started to investigate the similar structures in connection with works in the field of infinite abstract automata and mathematical logic [8]. The ***J. von Neumann's CA***–model has received the further development in works of him direct followers whose results along with the finished and edited work of the first one have been published by ***A.W. Burks*** in his excellent works [8], which in many respects have determined development of researches in the given direction for several subsequent years. In process of researches on the ***CA***–problematics ***A.W. Burks*** has organized at the university of Michigan the research team «*The Logic of Computer Group*», of which a whole series of the first–class experts on the ***CA***–problematics has come out afterwards *(**T. Toffoli, J. Holland, R. Laing,** and others).*

Meanwhile, considering historical aspect of the ***CA***–problematics, we should not forget an important contribution to the given problematics which was made by pioneer works ***Konrad Zuse*** *(Germany)* and with which the world scientific community has been familiarized enough late and even frequently without his mention in this historical aspect. At that, ***K. Zuse*** not only has created the first programmable computers *(1935–1941)*, has invented the first high–level programming language *(1945)*, but was also the first who has introduced idea of «*Rechnender Raum*» *(Computable Spaces)*, or else – *Cellular Automata (Homogeneous Structures)* in the modern terminology [1–7]. Furthermore, ***K. Zuse*** has supposed that physical processes in point of fact are calculations, while our universe is a certain «cellular automaton» [8]. In the late seventies of the last century such view on the universe was innovative, while now the idea of the computing universe horrify nobody, finding logical place in modern theories of some researchers working in the field of quantum mechanics [8]. Unfortunately, even at present the ***K. Zuse'***s ideas are unfamiliar to rather meticulous researchers in this field. For exclusion of any speculative historical aspects existing occasionally today, in the following historical researches it is necessary to pay the most steadfast attention on this essential circumstance. Namely therefore, only many years later the similar ideas have been republished, popularized and redeveloped in researches of other researchers such as ***S. Wolfram, T. Toffoli, E. Fredkin,*** et al. [8,9]. In addition, the itself ***CA*** concept has been entered by ***John von Neumann.*** Perhaps, ***John Neumann,*** being familiar with ***K. Zuse*** ideas, could use cellular automata not only for simulation of process of self–reproducting automata, but also for building of high parallel computing models.

From more practical standpoint and game experiment the ***CA*** models has notified about itself in the late sixties of the last century, when ***J.H. Conway*** has presented the now known game «*Life*». The given game became rather popular and has attracted attention to cellular automata of both numerous scientists from different fields and amateurs [8]. At present, the game, probably, is the most known

CA model; in addition, it possesses the ability to self–reproduction and universal computing. Modelling a work of an arbitrary *Turing* machine by means of spatial–temporal dynamics of such ***CA*** model, ***J. Conway*** has proved ability of the model to universal computability. Later a rather simple manner of realization of any boolean function in configurations of the «*Life*» has been suggested [8]. Thus, even such very simple ***CA*** model turned out equivalent to the universal *Turing* machine. Furthermore, to the given ***CA*** model the significant interest exists and till now does not disappear above all to its various computer implementations [8]. Thus, early ideas and researches of such first–rate mathematicians and cyberneticians as ***K. Zuse, John von Neumann, S. Ulam*** and ***A. Church*** along with their certain direct followers we can ascribe with complete reason to the ***first*** stage of formation of the ***CA***–problematics as a whole.

The necessity for a good formalized media for modeling of processes of biological development and above all of self–reproduction process was being as one of the base prerequisites that stimulated the ***CA***–concept beginning. Thereupon, ***John Neumann*** and a whole series of his direct followers have investigated a series of questions of computational and constructive opportunities of the first ***CA***–models. The above works at the end of the fifties of the last century have attracted to the problems a lot of researchers [8]. In addition, homogeneous structures were being rediscovered not once and under various names, namely: in electrical engineering they are known as iterative networks, in pure mathematics they are known as a section of topological dynamics, in biologal sciences as cellular structures, etc.

As ***second*** stage in formation of the ***CA***–problematics it is quite possible to consider publication of the widely known works of ***E.F. Moore*** and ***J. Myhill*** on the nonconstructability problem in classical ***CA***–models which along with solution of some mathematical problems in a certain sense became accelerators of activity, attracting a rather steadfast attention to the given problematics of a lot of mathematicians and researchers from other fields [8]. In particular, we have familiarized oneself with the ***CA***–problematics in ***1969*** owing to *Russian* translation of the excellent work edited by ***R. Bellman,*** that contained well–known articles of ***E.F. Moore, S. Ulam*** and ***J. Myhill*** [8]. Scientific groups on the ***CA***–problematics in the *USA, Germany, Japan, Hungary, Italy, France*, and *USSR (ESSR*, ***TRG,*** *1969)* are formed up. At that, the further development and popularization of the ***CA***–problematics can be connected with names of researchers such as ***E.F. Codd, S. Cole, E.F. Moore, J. Myhill, H. Yamada, S. Amoroso, E. Banks, J. Buttler, V.Z. Aladjev, J. Holland, G.T. Herman, A.R. Smith, T. Yaku, A. Maruoka, Y. Kobuchi, G. Hedlund, M. Kimura, H. Nishio, T.J. Ostrand, A. Waksman*** and a whole series of others whose works in the *sixties* – the *seventies* of the last century have attracted attention to the given problematics from the theoretical standpoint; they have solved and formulated a lot of interesting enough problems [8]. In the future, mathematicians, physicists, and biologists began to use the ***CA*** with the purpose of research of own specific problems. In particular, in the early *sixties* – the late *seventies* of the last century the numerous researchers have prepared entry of the ***CA***–problematics into the current stage of its development being characterized by join of earlier disconnected ideas and methods on the general conceptual and methodological platforms, along with a rather essential expansion of fields of its application.

We can attribute the beginning of the third period to the early eighties of the last century when to ***CA***–problematics the special interest again has been renewed in connection with rather active researches on the problem of artificial intellect, physical modeling, elaboration of a new perspective architecture of high–parallel computer systems, and other important motivations. So, in our opinion namely since works of such researchers as ***Bennet C., Grassberger P., Boghosian B., Crutchfield J., Chopard B., Culik II K., Gács P., Green D., Gutowitz H., Langton C.G., Martin O., Ibarra O., Kobuchi Y., Margolus M., Mazoyer J., Toffoli T., Wolfram S., Aladjev V.Z., Bandman O.L.,*** etc. a new splash of interest to the ***CA*** as an environment above all of physical modelling began. The fine selection of references, including references on the Soviet and Russian–language authors can be found in [8] and in Internet. At present, ***CA***–problematics are being widely studied from extremely various standpoints, and interrelations of such homogeneous structures with existing problems are constantly sought and discovered. A series of rather large teams of researchers in many countries and first of all in the *USA, Germany, the Great Britain, Italy, France, Japan, Australia* deals with the problematics. Active enough scientific activity in this direction was carried out and in *Estonia* within of the ***TRG*** group whose a whole series of results has received an international recognition and has made up essential enough part of the modern ***CA***–problematics.

The modern standpoint on the ***CA*** *(**HS**)* theory has been formed under the influence of works of researchers such as ***Adamatzky A.I., Aladjev V.Z., Amoroso S., Arbib M., Bagnoli F., Bandini S., Bandman O., Bays C., Banks E.R., Barca D., Barzdin J., Binder P., Boghosian B., Burks A. W., Butler J., Cattaneo G., Chate H., Chowdhury D., Church A., Cole S., Codd E.F., Crutchfield J.P., Culik K.II, Das A.K., Durand B., Durret R., Fokas A.S., Fredkin E., Gács P., Gardner M., Gerhardt M., Griffeath D., Golze U., Grassberger P., Green D., Gutowitz H.A,, Hedlund G., Honda N., Hemmerling A., Holland J., Ibarra O.H., Ikaunieks E., Ilachinskii A., Jen E., Kaneko K., Kari J., Kimura M., Kobuchi Y., Langton C., Legendi T., Lieblein E., Lindenmayer A., Maneville P., Margolus N., Martin O., Maruoka A., Mazoyer J., Mitchell M., Moore E.F., Morita K., Myhill J., Nasu M., Neumann J., Nishio H., Ostrand T., Pedersen J., Podkolzin A., Sato T., Richardson D., Sarkar P., Shereshevsky M., Sipper M., Smith A.R., Sutner K., Takahashi H., Thatcher J., Toffoli T., Toom A., Tseitlin G.E., Varshavsky V.I., Vichniac G., Vollmar R., Voorhees B., Waksman A., Weimar J., Willson S.J., Wolfram S., Wuensche A.A., Yaku T.*** along with other numerous researchers from many countries.

Along with our works in the ***CA*** theory, it is necessary to note a whole series of other *Soviet* researchers who have received in the given field both fundamental and considerable enough results at the *sixties* – the *eighties* of the last century. Here they: ***Adamatzky A.I.*** *(identification of **CA**),* ***Bandman O.L.*** *(asynchronous **CA**),* ***Blishun A.F.*** *(growth of patterns),* ***Bliumin S.L.*** *(growth of patterns),* ***Bolotov A.A.*** *(simulation among classes of **CA**),* ***Varshavsky V.I.*** *(synchronization of **CA**, simulation of anysotropic **CA** on the isotropic ones),* ***Georgadze A., Mandzhgaladze P., Matevosian A.*** *(growth of the configurations; universal stochastic and deterministic **CA**, **CA** and parallel grammars),* ***Dobrushin R.L., Vasil'ev N., Stavskaya O.N., Mitiushin L., Leontovich A., Toom A.L.,*** *(probabilistic **CA**),* ***Ikaunieks E.*** *(nonconstructible configurations),* ***Koganov A.V.*** *(universal **CA**, stationary configurations, simulation of **CA**),* ***Kolotov A.T.*** *(models of excitable media),* ***Levenshtein V.*** *(synchronization in **CA**),* ***Levin L.A.*** and ***Kurdiumov G.L.*** *(stochastic **CA**),* ***Makarevskii A.I.*** *(implementation of boolean functions in **CA**),* ***Petrov E.I.*** *(synchronization of 2D–**CA**),* ***Podkolzin A.S.*** *(simulation of the **CA**; asymptotic of the global dynamic; universal **CA**),* ***Pospelov D.A.*** *(homogeneous structures and distributed AI in **CA**),* ***Prangishvili I.V.*** *(**CA** architectures of high–parallel processors),* ***Reshod'ko L.S.*** *(**CA**–models of the excitable media),* ***Revin O.M.*** *(simulation of anisotropic **CA** on the isotropic **CA**),* ***Solntzev S.*** *(growth of patterns),* ***Tzetlin M.L.*** *(collectives of automata, games in the **CA**),* ***Tzeitlin G.E.*** *(algebras of shift registers),* ***Scherbakov E.S.*** *(universal algebras of parallel substitutions)*, and a whole series of others.

It is supposed that the ***CA***–models can play extremely important part as both conceptual and applied models of spatially–distributed dynamic systems among which first of all an especial interest the computational, physical and biological cellular systems present. In the given direction already takes place a rather essential activity of a lot of the researchers who have received quite encouraging results [8]. At last, theoretical results of the above–mentioned and of a lot of other researchers have initiated a modern mathematical ***CA*** theory evolved to the current time into an independent branch of the abstract automata theory that has a rather numerous interesting appendices in various areas of science and technics, in particular, in fields such as physics, developmental biology, parallel information processing, creation of perspective architecture of high–efficiency computer systems, computing sciences and informatics, which are linked to mathematical and computer modelling, etc., and by substantially raising the ***CA*** concept onto a new interdisciplinary level. Our concise enough standpoint on the main stages of development and formation of the ***CA*** theory is given above; for today there is a number of the reviews devoted to this question, for example [8], many works on the ***CA–***problematics in varying degree concern this question also [8]. Furthermore, it should be noted that the matter to a certain extent has subjective character, and that needs to be meant.

Meanwhile, the separate researchers in a gust of certain euphoria try to represent the ***CA–***approach as an universal remedy of the solution of all problems and knowledge of outward things, identifying it with a «*New kind*» of science of universal character. In this connection it is necessary to mark the vast and pretentious book of ***S. Wolfram*** [9], whose title has rather advertising and commercial, than scientific–based character. This book contains many results that have been obtained much earlier by a whole series of other investigators on ***CA–***problematics, including the *Soviet* authors *(see references in* [8] *and some others).* In addition, the priority of many fundamental results in this field belongs to other researchers. The unhealthy vanity of the author of this book does not allow him to look without bias on history of the ***CA*** problems as a whole. In general, ***S. Wolfram*** enough frivolously addresses with authorship of the results received in ***CA–***problematics, therefore there can be a impression **–** everything made in this field belongs basically to him. At that, the book contains basically results of computer modelling with very simple types of the ***CA–***models, drawing the conclusions and assumptions on their basis with rather doubtful reliability and quality. In the book we can meet an irritating density of passages in which the author takes personal credit for ideas which are «*common knowledge*» among experts in the relevant fields. Seems, such ***S. Wolfram*** passages and inferences similar to them cause utterly certain doubts in scientific decency and judiciousness of their author. At last, we absolutely do not agree that ***Wolfram*** book presents a *"New kind"* of science, nevertheless his book would be more pleasant to read if he were more modest. In our opinion, this book represents in many respects a speculative sight both on ***CA–***problematics, and on science as a whole. Here we only shall note, contrary to the pursued purposes the book not only was not revelation for the researches working in the ***CA*** problematics but also to a certain extent has caused a little bit deformed representation about the researches domain that is perspective enough from many points of view. With relatively detailed points of view that concerns the book, the reader can familiarize in works [8] and some others. Meanwhile, in spite of the told above relative to the book, it can represent the certain interest, taking into consideration the marked and some other certain remarks. In our opinion, the book doesn't introduce of anything essentially new in the cellular automata theory above all in its mathematical component.

At last, we will make one essential enough remark concerning of place of ***CA–***problematics in scientific structure. By a certain contraposition to standpoint on the ***CA–***problematics that is declared by the book [3] our vision of the given question is being presented as follows. Our experience of investigations in the ***CA–***problematics both on theoretical, and especially applied level speaks entirely about another, namely:

*(**1**)* ***CA–****models (cellular automata, homogeneous structures) represent one of special classes of infinite abstract automata with the specific internal structure which provides extremely high–parallel level of the information processing and calculations; the given models form a specific class of discrete dynamic systems that function in especially parallel way on base of principle of local short–range interaction;*

(2) ***CA*** *can serve as a quite satisfactory model of high–parallel processing just as Turing machines (Markov normal algorithms, Post machines, productions systems, etc.) serve as formal models of*

*sequential calculations; from this point of view the **CA**–models it is possible to consider and as algebraical processing systems of finite or/and infinite words, defined in finite alphabets, on the basis of a finite set of rules of parallel substitutions; in particular, a **CA**–model can be interpreted as a certain system of parallel programming where the rules of parallel substitutions act as a parallel language of the lowest level;*

*(**3**) the principle of local interaction of elementary automata composing a **CA**–model which in result defines their global dynamics allows to use the **CA** and as a fine media of modelling of a broad enough range of processes, phenomena and objects; furthermore, the phenomenon of reversibility permitted by the **CA** does their by very interesting means for physical modeling, and for creation of very perspective computing structures basing on the nanotechnologies;*

*(**4**) **CA**–models represent an interesting enough independent mathematic object whose essence consists in high–parallel processing of words in finite or infinite alphabets.*

At that, it is possible to associate the ***CA***–approach with a certain model analogue of the differential equations in partial derivatives describing those or another processes with that difference, that if the differential equations describe a process at the average, in a ***CA***–model defined in appropriate way, a certain researched process is really embedded and dynamics of the ***CA***–model enough evidently represents the qualitative behaviour of researched process. Thus, it is necessary to determine for elementary automata of the model the necessary properties and rules of their local interaction by appropriate way. The ***CA***–approach can be used for research of processes described by complex differential equations which have not of analytical solution, and for the processes that it is not possible to describe by such equations. Along with it, the ***CA*** present a rather perspective modelling media for research of those phenomena, processes, and objects for which there are no known classical means or they are complex enough.

As we already noted, as against many other modern fields of science, the theoretical component of the ***CA***–problematics is no so appreciably crossed with its second applied component, therefore, it is possible to consider the ***CA***–problematics as two independent enough directions: research of the ***CA*** as mathematical objects and use of the ***CA*** for modelling; at that, the second direction is characterized also by the wider spectrum. The level of evolution of the second direction is appreciably being defined by possibilities of the modern computing systems since ***CA***–models, as a rule, are being designed on base of the immense number of elementary automata and, as a rule, with complex enough rules of local interaction among themselves.

The indubitable interest to them amplifies also a possibility of practical realization of high parallel computing ***CA*** on basis of modern successes of microelectronics and prospects of the information processing at the molecular level *(methods of nanotechnology);* while the itself ***CA***–concept provides creation of both conceptual and practical models of spatially–distributed dynamic systems of which namely physical systems are the most interesting and perspective. Indeed, models which in an obvious way reduce macroscopic processes to rigorously determined microscopic processes, represent especial epistemological and methodical interest for they possess the great persuasiveness and transparency. Namely, from the given standpoint the ***CA***–models of various type represent a special interest, above all, from the applied standpoint at research of a lot of processes, phenomena and objects in different fields and, first of all, in physics, computer science and developmental biology.

The first direction enough intensively is developed by mathematicians whereas contribution to development of the second direction essentially more representative circle of researchers from various theoretical and applied fields *(physics, chemistry, biology, technics, etc.)* brings. Thus, if theoretical researches on the ***CA***–problematics in general are limited to classical, polygenic and stochastic ***CA***–models, then the results of the second direction are based on essentially wider representation of classes and types of ***CA***–models. As a whole if classical ***CA***–models represent first of all the formal mathematical systems researched in the appropriate context, then their numerous generalizations represent a perspective enough environment of modeling of various processes and objects.

In the conclusion once again it is necessary to note a rather important circumstance, at discussion of the *Classical Cellular Automata (**CCA**)* we emphasize the following a rather essential moment. We considered the ***CCA***–models that are a class of parallel discrete dynamic systems as certain formal algebraic systems of processing of finite words *(configurations)* in finite alphabets without any reference, as a rule, to their microprogrammed environment, i.e. without use of their cellular organization on lowest level inherent into them, what distinguishes our approach to research of the given objects from approaches of a lot of other researchers. Also, we consider ***CCA***–models as formal mathematical object having specific inside organization without ascribing to them certain universality and generality in perception of the *World*. At similar approach the ***CCA*** are considered at especially formal level not allowing in full measure to use their intrinsic property of high parallelism in field of computations, and information processing as a whole.

Naturally, for solution of a lot of the applied problems in the ***CA***–environment and obtaining of a series of thin results first of all of model character an approach on microprogram level is needed when a researched process, algorithm or phenomenon is directly embedded in ***CA***–environment, using its parameters: a dimension, a neighbourhood index, a states alphabet and a local transition function. At such approach it is possible to receive solutions of a lot of important appendices with generalizations of a rather high level of theoretical character. In particular, by direct embedding of universal computing algorithms or logical elements into such objects it is possible to constructively prove existence of the universal computability, etc. In spite of such extremely simple concept of the ***CCA,*** they by and large have a rather complex dynamics. In many cases theoretical research of their dynamics collides with essential complexity. Therefore, computer simulation of these structures that in empirical way allows to research their dynamics is a rather powerful tool. For this reason this question is quite natural for investigations of the ***CA***–problematics, considering the fact that ***CA***–models at the formal level present the dynamical systems of high–parallel substitutions.

Indeed, the problem of computer modelling of the ***CA*** is solved at two main levels: *(**1**) simulation of the **CA** dynamics on computers of traditional architecture*, and *(**2**) simulation on the hardware architecture that as much as possible corresponds to the **CA** concept;* so–called ***CA***–oriented architecture of computing systems. So, computer simulation of ***CA*** models plays a rather essential part at theoretical researches of their dynamics, meantime it is even more important at practical realizations of ***CA*** models of different processes. At present time, a whole series of rather interesting systems of software and hardware for help of investigations of different types of ***CA*** models has been developed; their characteristics can be found in the references [8]. In our works a lot of programs in various program systems for different computer platforms had been presented. Among them a lot of interesting programs for simulation of ***CA*** models in the ***Mathematica*** and ***Maple*** systems has been programmed. On the basis of computer simulation many of interesting theoretical results on the ***CCA*** and their use in the fields such as mathematics, developmental biology, computer sciences, etc. had been received. However, the given matter along with applied aspects of the ***CA***–models in the present book aren`t considered, despatching the interested reader to detailed enough discussion of these aspects to the corresponding publications in lists of references [8]; a lot of interesting works in this direction can be found in Internet by the corresponding key phrases.

The problematics considered by the ***TRG*** researchers in many respects has been conditioned by own interests and tastes of the authors along with traditions of creative activity of the ***TRG*** in this field. At last, we will note that in our activity it is possible to allocate three main directions, namely: *(**1**) researches of classical **CA** as a formal parallel algorithm of processing of configurations in finite alphabets*, *(**2**) applications of the classical and generalized **CA** in mathematics and computer facilities of highly parallel action*, and *(**3**) mathematical and developmental biology*. With our results in two last directions the interested reader can familiarize in sufficient detail in [1-8] and in numerous references contained in them along with references concerning many other researchers in this field. Our interview in *Russian* on cellular automata can be found here [10].

References

1. V.Z. Aladjev. *To the Theory of the Homogeneous Structures*.- Tallinn: Estonian Academic Press, 1972. The monograph is the first Russian book in ***CA*** problems. It contains Russian terminology and a series of interesting both theoretical and certain applied results *(**CA**–approach for biological modeling above all)*. The book contains early results of the *Tallinn Research Group (**TRG**)* in the ***CA*** problems. In annual meeting of the Estonian Academy of Sciences the book was marked as one of the better works of the Academy in ***1972.***

2. V.Z. Aladjev. *Mathematical Theory of Homogeneous Structures and Their Applications*.- Tallinn: Valgus Press, 1980, 270 p. In the book the writer gives main of the work that Tallinn Research Group have done in the ***CA*** theory and its applications over the period ***1970–1980.*** In the book are discussed the topics such as: architecture of the ***CA*** theory and its applications, general aspects of the ***CA*** dynamics, some ***CA*** models, and basic, in author's opinion, problems for the following researches in ***CA*** problems.

3. V.Z. Aladiev. *Homogeneous Structures: Theoretical and Applied Aspects*.- Kiev: Technika Press, 1990 *(in Russian with extended English summary)*. In the monograph the main of the work of the Tallinn Research Group done in the mathematical ***CA*** theory and its applications in mathematical modelling, computer science, mathematical and theoretical biology, and in parallel processing and parallel algorithms during ***1969–1989*** is presented. Much of this work has been motivated by both ***CA*** models as an independent mathematical object and by the growing interest in computer science and mathematical modeling. At present, ***CA*** theory forms an self–maintained part of the modern mathematical cybernetics.

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