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V.Z. Aladjev, M.L. Shishakov, V.A. Vaganov

Selected problems in the theory of classical cellular automata

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In the book we present certain results of the work we have done in the theory of *Classical Cellular Automata* (***CA***). At present, these results form an essential constituent of the ***CA*** problematics. In particular, we have studied such problems as the *nonconstructability* problem in the ***CA,*** the *decomposition* problem of global transition functions in the ***CA,*** extremal constructive possibilities, the parallel formal grammars and languages defined by ***CA,*** complexity of finite configurations and global transition functions in the ***CA,*** simulation problem in classical ***CA,*** etc. At present, the ***CA*** problematics is a rather well developed independent field of the mathematical cybernetics that has a rather considerable field of various appendices. In addition, with the equal right the ***CA*** problematics can be considered as a component of such fields as discrete parallel dynamical systems, discrete mathematics, cybernetics, complex systems and some others. In our viewpoint, the book will represent an indubitable interest for students, post–graduates and persons working for doctor's degree of the appropriate faculties of universities, above all, of naturally scientific level along with teachers in subjects such as mathematical and physical modelling, discrete mathematics, automata theory, computer science, cybernetics, theoretical biology, computer technique, and a lot of others. In recent years, the classical ***CA*** models are one of the most promising simulating environments for various highly parallel discrete processes, objects and phenomena admitting reversible dynamics, that is enough important from a physical point of view, in the first place.

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For contacts: *aladjev@europe.com*, *aladjev@yandex.ru*, *aladjev@mail.ru*

Introduction

First of all, a few words about the terminology used below. Today, the problems of *Cellular Automata (**CA, CA-**models)* is rather well advanced, being quite independent field of the modern mathematical cybernetics, having own terminology and axiomatics at existence of broad enough domain of various appendices. In addition, it is necessary to note that at assimilation of this problems in the Soviet Union in Russian-lingual terminology, whose basis for the first time have been laid by us at *1970*, for the concept «*Cellular automata*» the term «*Homogeneous structures*» *(**HS; HS–**models)* has been determined which nowadays is the generally accepted term together with a whole series of other notions, definitions and denotations [1-6]. Therefore, during the present monograph along with this term its well-known *Russian*-lingual equivalent «*Homogeneous structures**; HS***» can be used too.

*Cellular Automaton (**CA**)* – a parallel information processing system that consists of intercommunicating identical *Mealy* automata *(elementary automata)*. We can interpret ***CA****s* as a theoretical basis of artificial high parallel information processing systems. From the logical standpoint a ***CA*** is an infinite automaton with specifical internal structure. So, the ***CA*** theory can be considered as structural and dynamical theory of the infinite automata. At that, ***CA*** models can serve as an excellent basis for modeling of many discrete processes, representing interesting enough independent objects for research too. Recently, the undoubted interest to the ***CA*** problems *(above all in applied aspect)* has arisen anew, and in this direction many remarkable results have been obtained. Further, by ***CA*** we mean both cellular automata and a separate cellular automaton, depending on the context.

So, the ***CA–***axiomatics provides three fundamental properties such as ***homogeneity, localness*** and ***parallelism*** of functioning. If in a similar computing model we shall with each elementary automaton associate a separate microprocessor then it is possible to unrestrictedly increase the sizes of similar computing system without any essential increase of its temporal and constructive expenses, required for each new expansion of the computing space, and also without any overheads connected to coordination of functioning of an arbitrary supplementary quantity of elementary microprocessors. Similar high–parallel computing models admit practical realizations consisting of rather large number of rather elementary microprocessors which are limited not so much by certain

architectural reasons as by a lot of especially economic and technologic reasons defined by a modern level of development of microelectronic technology, however with the great potentialities in the future, first of all, in light of rather intensive works in field of nanotechnology. Along with it the ***CA*** models can be used for problems solving of information transformation, such as encryption, encoding and data compression [7].

The above three such features as *high homogeneity*, *high parallelism* and *locality* of interactions are provided by the ***CA***–axiomatics itself, while such property important from the physical standpoint as ***reversibility*** of dynamics is given by program way. In light of the listed properties even classical ***CA*** are high–abstract models of the real physical world, which function in a space and time. Therefore, they in many respects better than many others formal architectures can be mapped onto a lot of physical realities in their modern understanding. Moreover the ***CA***–concept itself is enough well adapted to solution of various problems of modelling in such areas as mathematics, cybernetics, development biology, theoretical physics, computing sciences, discrete synergetics, dynamic systems theory, robotics, etc. Told and numerous examples available for today lead us to the conclusion that the ***CA*** can represent a rather serious interest as a new perspective environment of modelling and research of many discrete processes and phenomena, determined by the above properties; in addition, raising the ***CA***–problematics onto a new interdisciplinary level and, on the other hand, as an interesting enough independent formal mathematical object of researches [7,21,82].

The base modern tendencies of elaboration of perspective architecture of high parallel computer facilities, a problem of modelling of discrete parallel processes, discrete mathematics and synergetics, theory of the parallel discrete dynamical systems, problems of artificial intellect and robotics, parallel information processing and algorithms, physical and biological modelling, along with a lot of other important prerequisites in various areas of modern natural sciences define at the latest years a new ascent of the interest to the formal cellular models of various type which possess high parallel manner of acting; the cellular automata are some of major models of such type.

During time which has passed after appearance of the first monographs and the collected papers which have been devoted to various theoretic and applied aspects of the ***CA*** problems, the certain progress has been reached in this direction, that is connected, above all, with successes of theoretical character along with essential enough expansion of field of

appendices of the ***CA*** models, mainly, in computer science, cybernetics, physics, modelling, developmental biology and substantial growth of number of researchers in this direction. In addition, in the USA, Japan, Germany, the Great Britain, Hungary, Estonia, etc., a series of works summarizing the results of progress in those or other directions of the ***CA*** problems including its numerous appendices in various fields has appeared. Our monographs at the substantial level have presented the reviews of the basic results received by the ***Tallinn Research Group*** on the ***CA*** problematics and its application [1,7,8,9,12,14,92,278].

From the very outset of our researches on the ***CA*** problemcs, above all, with application accent onto mathematical developmental biology the informal *Tallinn Research Group (**TRG**)* consisting of the researchers of a few leading scientifical centres of the former ***USSR*** has gradually been formed up. At that, the ***TRG*** staff was not strictly permanent and was being changed in broad enough bounds depending on the researched problems. In works [10-14] the analysis of the ***TRG*** activity instructive to some degree for research of the dynamics of development of the ***CA*** problematics as an independent scientific direction as a whole had been represented. Ibidem the basic directions of our researches can be found along with main received results.

Today, *cellular automata (homogeneous structures)* are being investigated from many standpoints and interrelations of objects of such type with already existing problems are being discovered constantly. On purpose of general acquaintance with extensive ***CA*** problematics as a whole and with its separate basic directions specifically, we recommend to address oneself to interesting and versatile surveys of such researchers as ***V.Z. Aladjev, V. Cimagalli, K. Culik, D. Hiebeler, A. Lindenmayer, A. Smith, P. Sarkar, M. Mitchell, T. Toffoli, R. Vollmar, S. Wolfram,*** et al. [7,8]. A series of books and monographs of the authors such as ***V.Z. Aladjev, A. Adamatzky, E. Codd, A. Ilachinskii, M. Garzon, M. Duff, P. Kendall, T. Toffoli, B. Voorhees, M. Sipper, O. Martin, K. Preston, V. Kudrjvcev, N. Margolus, R. Vollmar, B. Voorhees, S. Wolfram*** and some others contain a rather interesting historical excursus in the ***CA*** problems**;** in addition, unfortunately, hitherto a common standpoint onto historical aspect in this question is absent [7,21,24,102,106,278,286].

In view of that, here is a rather opportune moment to briefly emphasize once again our standpoint on historical aspect of the ***CA–***problematics, namely**:** a brief historical excursus presented below make it one's aim to define the basic stages of becoming of the ***CA–***problematics, having

digressed from numerous particulars. Having started own researches on the ***CA*-**problematics in ***1969*,** we on base of analysis of large number of publications and direct dialogue with many leading researchers in this direction have a quite certain information concerning the objective development of its basic directions, above all, of theoretical character. That allows us with sufficient degree of objectivity to note the pivotal stages of its development**;** at that, many details of historical character concerning the ***CA*–**problematics the reader can find, for example, in a whole series of works presented in the reference list [7,21,24,102,286].

From the theoretical standpoint the ***CA*** concept *(homogeneous structures)* has been introduced at the end of the forties of the past century by ***John von Neumann*** on ***S. Ulam's*** advice with the purpose of determination of more realistic and well formalized model for research of behaviour of complex evolutionary systems, including self**–**reproduction of alive organisms. Whereas *S. Ulam* has used ***CA*–**like models, in particular, for researches of the growth problem of crystals and certain other discrete systems growing in conformity with recurrent rules. The structures that have been investigated by him and his colleagues were**,** mainly**,** ***1–*** and **2–**dimensional, however higher dimensions have been considered too. In addition, questions of universal computability together with certain other theoretical questions of behaviour of cellular structures of such type also were kept in view. A little bit later also ***A. Church*** started to investigate the similar structures in connection with works in the field of infinite abstract automata and mathematical logic [7,21,24,82,286].

The ***J. von Neumann's CA–***model has received the further development in works of him direct followers whose results along with the finished and edited work of the first one have been published by ***A.W. Burks*** in his excellent works [15,16]**,** which in many respects have determined development of researches in the given direction for several subsequent years. In process of researches on the ***CA–***problematics ***A.W. Burks*** has organized at the university of Michigan the research team ***«The Logic of Computer Group»*,** of which a whole series of the first**–**class experts on the ***CA–***problematics has come out afterwards ***(T. Toffoli, J. Holland, R. Laing,*** *and others).*

Meanwhile, considering historical aspect of the ***CA–***problematics, we should not forget an important contribution to the given problematics which was made by pioneer works ***Konrad Zuse*** *(Germany)* and with which the world scientific community has been familiarized enough late and even frequently without his mention in this historical aspect.

At that, ***K. Zuse*** not only has created the first programmable computers *(1935–1941)*, has invented the first high–level programming language *(1945)*, but was also the first who has introduced idea of «***Rechnender Raum***» *(Computable Spaces)*, or else – ***Cellular Automata*** *(**Homogeneous Structures**)* in the modern terminology [7-19]**.** Furthermore, ***K. Zuse*** has supposed that physical processes in point of fact are calculations, while our universe is a certain «*cellular automaton*» [17]. In the late seventies of the last century such view on the universe was innovative, while now the idea of the *computing universe* horrify nobody, finding logical place in modern theories of some researchers working in the field of *quantum* mechanics [7,21]. Unfortunately, even at present the ***K. Zuse's*** ideas are unfamiliar to rather meticulous researchers in this field. For exclusion of any speculative historical aspects existing occasionally today, in the following historical researches it is necessary to pay the most steadfast attention on this essential circumstance. Namely therefore, only many years later the similar ideas have been republished, popularized and redeveloped in researches of other researchers such as ***S. Wolfram, T. Toffoli, E. Fredkin,*** et al. [7]. In addition, the itself ***CA*** concept has been entered by ***John von Neumann.*** Perhaps, ***John Neumann,*** being familiar with ***K. Zuse*** ideas, could use cellular automata not only for simulation of process of self–reproducting automata, but also for building of high parallel computing models.

From more practical standpoint and game experiment the ***CA*** models has notified about itself in the late sixties of the last century, when ***J.H. Conway*** has presented the now known game «***Life***»**.** The given game became rather popular and has attracted attention to cellular automata of both numerous scientists from different fields and amateurs [7,21]. At present, the game, probably, is the most known ***CA*** model**;** in addition, it possesses the ability to self–reproduction and universal computing. Modelling a work of an arbitrary Turing machine by means of spatial–temporal dynamics of such ***CA*** model, ***J. Conway*** has proved ability of the model to universal computability. Later a rather simple manner of realization of any boolean function in configurations of the «***Life***» has been suggested [7]. Thus, even such very simple ***CA*** model turned out equivalent to the universal Turing machine. Furthermore, to the given ***CA*** model the significant interest exists and till now does not disappear above all to its various computer implementations [7,24,102,106,286].

Thus, early ideas and researches of such first–rate mathematicians and cyberneticians as ***K. Zuse, John von Neumann, S. Ulam*** and ***A. Church***

along with their certain direct followers we can ascribe with complete reason to the ***first*** *stage* of formation of the ***CA–***problematics as a whole. The necessity for a good formalized media for modeling of processes of biological development and above all of self**–**reproduction process was being as one of the base prerequisites that stimulated the ***CA–***concept beginning. Thereupon, ***John Neumann*** and a whole series of his direct followers have investigated a series of questions of computational and constructive opportunities of the first ***CA–***models. The above works at the end of the *fifties* of the last century have attracted to the problems a lot of researchers [7]. In addition**,** homogeneous structures were being rediscovered not once and under various names, namely: in electrical engineering they are known as *iterative networks***,** in pure mathematics they are known as a section of *topological dynamics***,** in biologal sciences as *cellular structures***,** etc.

As ***second*** stage in formation of the ***CA–***problematics it is quite possible to consider publication of the widely known works of ***E.F. Moore*** and ***J. Myhill*** on the *nonconstructability* problem in classical ***CA–***models which along with solution of some mathematical problems in a certain sense became accelerators of activity, attracting a rather steadfast attention to the given problematics of a lot of mathematicians and researchers from other fields [7]. In particular, we have familiarized oneself with the ***CA–*** problematics in *1969* owing to Russian translation of the excellent work edited by ***R. Bellman,*** that contained well**-**known articles of ***E.F. Moore, S. Ulam*** and ***J. Myhill*** [20]. Scientific groups on the ***CA–***problematics in the *USA***,** *Germany***,** *Japan***,** *Hungary***,** *Italy***,** *France***,** and *USSR (**TRG**, 1969)* are formed up. At that, the further development and popularization of the ***CA–***problematics can be connected with names of researchers such as ***E.F. Codd, S. Cole, E.F. Moore, J. Myhill, H. Yamada, S. Amoroso, E. Banks, J. Buttler, V.Z. Aladjev, J. Holland, G.T. Herman, A.R. Smith, T. Yaku, A. Maruoka, Y. Kobuchi, G. Hedlund, M. Kimura, H. Nishio, T.J. Ostrand, A. Waksman*** and a whole series of others whose works in the sixties **–** the seventies of the last century have attracted attention to the given problematics from the theoretical standpoint**;** they have solved and formulated a lot of interesting enough problems [7]. In the future, mathematicians, physicists, and biologists began to use the ***CA*** with the purpose of research of own specific problems. In particular, in the early sixties **–** the late seventies of the last century the numerous researchers have prepared entry of the ***CA*****-**problematics into the current stage of its development being characterized by join of earlier disconnected ideas

and methods on the general conceptual and methodological platforms, along with a rather essential expansion of fields of its application.

We can attribute the beginning of the ***third period*** to the early eighties of the last century when to ***CA–***problematics the special interest again has been renewed in connection with rather active researches on the problem of artificial intellect, physical modeling, elaboration of a new perspective architecture of high–parallel computer systems, and other important motivations. So, in our opinion namely since works of such researchers as ***Bennet C., Grassberger P., Boghosian B., Crutchfield J., Chopard B., Culik II K., Gács P., Green D., Gutowitz H., Langton C.G., Martin O., Ibarra O., Kobuchi Y., Margolus M., Mazoyer J., Toffoli T., Wolfram S., Aladjev V.Z., Bandman O.L.,*** etc. a new splash of interest to the ***CA*** as an environment above all of physical modelling began [7]. The fine selection of references, including references on the Soviet and Russian–language authors can be found in not less excellent book [21].

At present, ***CA–***problematics are being widely studied from extremely various standpoints, and interrelations of such homogeneous structures with existing problems are constantly sought and discovered. A series of rather large teams of researchers in many countries and first of all in the *USA*, *Germany*, the *Great Britain*, *Italy*, *France*, *Japan*, *Australia* deals with the problematics. Active enough scientific activity in this direction was carried out and in *Estonia* within of the ***TRG*** group whose a whole series of results has received an international recognition and has made up essential enough part of the modern ***CA–***problematics.

The modern standpoint on the ***CA (HS)*** theory has been formed under the influence of works of researchers such as ***Adamatzky A.I., Aladjev V.Z., Amoroso S., Arbib M., Bagnoli F., Bandini S., Bandman O., Bays C., Banks E.R., Barca D., Barzdin J., Binder P., Boghosian B., Burks A. W., Butler J., Cattaneo G., Chate H., Chowdhury D., Church A., Cole S., Codd E.F., Crutchfield J.P., Culik K.II, Das A.K., Durand B., Durret R., Fokas A.S., Fredkin E., Gács P., Gardner M., Gerhardt M., Griffeath D., Golze U., Grassberger P., Green D., Gutowitz H.A,, Hedlund G., Honda N., Hemmerling A., Holland J., Ibarra O.H., Ikaunieks E., Ilachinskii A., Jen E., Kaneko K., Kari J., Kimura M., Kobuchi Y., Langton C., Legendi T., Lieblein E., Lindenmayer A., Maneville P., Margolus N., Martin O., Maruoka A., Mazoyer J., Mitchell M., Moore E.F., Morita K., Myhill J., Nasu M., Neumann J., Nishio H., Ostrand T., Pedersen J., Podkolzin A., Richardson D., Sarkar P., Sato T., Shereshevsky M., Sipper M., Smith A.R., Sutner K., Takahashi H., Thatcher J., Toffoli T., Toom A., Tseitlin***

G.E., Varshavsky V.I., Vichniac G., Vollmar R., Voorhees B., Waksman A., Weimar J., Willson S.J., Wolfram S., Wuensche A.A., Yaku T., Kari J. along with other numerous researchers from many countries.

Along with our works in the ***CA*** theory, it is necessary to note a whole series of other Soviet researchers who have received in the given field both fundamental and considerable enough results at the sixties – the eighties of the last century. Here they: ***Adamatzky A.I.*** *(identification of* ***CA)***, ***Bandman O.L.*** *(asynchronous* ***CA)***, ***Blishun A.F.*** *(growth of patterns)*, ***Bliumin S.L.*** *(growth of patterns)*, ***Bolotov A.A.*** *(simulation among classes of* ***CA)***, ***Varshavsky V.I.*** *(synchronization of* ***CA****, simulation of anysotropic* ***CA*** *on the isotropic ones)*, ***Georgadze A.***, ***Matevosian A.***, ***Mandzhgaladze P.*** *(growth of the configurations; universal stochastic and deterministic* ***CA****,* ***CA*** *and parallel grammars)*, ***Dobrushin R.L.***, ***Vasil'ev N.***, ***Stavskaya O.N.***, ***Mitiushin L.***, ***Leontovich A.***, ***Toom A.L.***, *(probabilistic* ***CA)***, ***Ikaunieks E.*** *(nonconstructible configurations)*, ***Koganov A.V.*** *(universal* ***CA****, stationary configurations, simulation of* ***CA)***, ***Kolotov A.T.*** *(models of excitable media)*, ***Levenshtein V.*** *(synchronization in* ***CA)***, ***Levin L.A.*** and ***Kurdiumov G.L.*** *(stochastic* ***CA)***, ***Makarevskii A.I.*** *(implementation of boolean functions in* ***CA)***, ***Petrov E.I.*** *(synchronization of* ***2D–CA)***, ***Podkolzin A.S.*** *(simulation of the* ***CA****; asymptotic of the global dynamic; universal* ***CA)***, ***Pospelov D.A.*** *(homogeneous structures and distributed AI in* ***CA)***, ***Prangishvili I.V.*** *(****CA*** *architectures of high–parallel processors)*, ***Reshod'ko L.S.*** *(****CA–****models of the excitable media)*, ***Revin O.M.*** *(simulation of anisotropic* ***CA*** *on the isotropic* ***CA)***, ***Solntzev S.*** *(growth of patterns)*, ***Tzetlin M.L.*** *(collectives of automata, games in the* ***CA)***, ***Tzeitlin G.E.*** *(algebras of shift registers)*, ***Scherbakov E.S.*** *(universal algebras of parallel substitutions)*, and a whole series of others.

It is supposed that the ***CA–***models can play extremely important part as both conceptual and applied models of *spatially–distributed* dynamic systems among which first of all an especial interest the computational, physical and biological cellular systems present. In the given direction already takes place a rather essential activity of a lot of the researchers who have received quite encouraging results [7,21]. At last, theoretical results of the above–mentioned and of a lot of other researchers have initiated a modern mathematical ***CA*** theory evolved to the current time into an independent branch of the abstract automata theory that has a rather numerous interesting appendices in various areas of science and technics, in particular, in fields such as physics, developmental biology, parallel information processing, creation of perspective architecture of high–efficiency computer systems, computing sciences and informatics, which are linked to mathematical and computer modelling, etc., and by

substantially raising the ***CA*** concept onto a new interdisciplinary level. Our concise enough standpoint on the main stages of development and formation of the ***CA*** theory is given above**;** for today there is a number of the reviews devoted to this question, for example [22], many works on the ***CA–***problematics in varying degree concern this question also [7, 9,13,21,24-29,102,106]. Furthermore, it should be noted that the matter to a certain extent has subjective character, and that needs to be meant.

Meanwhile, the separate researchers in a gust of certain euphoria try to represent the ***CA-***approach as an universal remedy of the solution of all problems and knowledge of outward things, identifying it with a «***New kind***» of science of universal character. In this connection it is necessary to mark the vast and pretentious book of ***S. Wolfram*** [23]**,** whose title has rather advertising and commercial, than scientific**–**based character.

This book contains many results that have been obtained much earlier by a whole series of other investigators on ***CA–***problematics**,** including the Soviet authors *(see references in* [7-9,21,22,24-29] *and some others)*. In addition, the priority of many fundamental results in this field belongs to other researchers. The unhealthy vanity of the author of this book does not allow him to look without bias on history of the ***CA*** problems as a whole. In general, ***S. Wolfram*** enough frivolously addresses with authorship of the results received in ***CA–***problematics, therefore there can be a impression **–** everything made in this field belongs basically to him. At that, the book contains basically results of computer modelling with very simple types of the ***CA–***models, drawing the conclusions and assumptions on their basis with rather doubtful reliability and quality. In the book we can meet an irritating density of passages in which the author takes personal credit for ideas which are «***common knowledge***» among experts in the relevant fields. Seems, such ***S. Wolfram*** passages and inferences similar to them cause utterly certain doubts in scientific decency and judiciousness of their author.

At last, we absolutely do not agree that ***Wolfram*** book presents a ***"new kind"*** of science, nevertheless his book would be more pleasant to read if he were more modest. In our opinion**,** this book represents in many respects a speculative sight both on ***CA–***problematics**,** and on science as a whole. Here we only shall note, contrary to the pursued purposes the book not only was not revelation for the researches working in the ***CA*** problematics but also to a certain extent has caused a little bit deformed representation about the researches domain that is perspective enough from many points of view. With relatively detailed points of view that

concerns the book, the reader can familiarize in works [24-28] and some others. Meanwhile, in spite of the told above relative to the book, it can represent the certain interest, taking into consideration the marked and some other certain remarks. In our opinion, the book doesn't introduce of anything essentially new in the cellular automata theory above all in its mathematical component.

At last, we will make one essential enough remark concerning of place of ***CA*-**problematics in scientific structure. By a certain contraposition to standpoint on the ***CA*–**problematics that is declared by the above book [23] our vision of the given question is being presented as follows. Our experience of investigations in the ***CA*-**problematics both on theoretical, and especially applied level speaks entirely about another, namely**:**

*(**1**) **CA**–models (cellular automata**,** homogeneous structures) represent one of special classes of infinite abstract automata with the specific internal structure which provides extremely high–parallel level of the information processing and calculations**;** the given models form a specific class of discrete dynamic systems that function in especially parallel way on base of principle of local short-range interaction**;***

*(**2**) **CA** can serve as a quite satisfactory model of high–parallel processing just as Turing machines (Markov normal algorithms**,** Post machines**,** productions systems**,** etc.) serve as formal models of sequential calculations**;** from this point of view the **CA**–models it is possible to consider and as algebraical processing systems of finite or/and infinite words**,** defined in finite alphabets**,** on the basis of a finite set of rules of parallel substitutions**;** in particular**,** a **CA**–model can be interpreted as a certain system of parallel programming where the rules of parallel substitutions act as a parallel language of the lowest level**;***

*(**3**) the principle of local interaction of elementary automata composing a **CA**–model which in result defines their global dynamics allows to use the **CA** and as a fine media of modelling of a broad enough range of processes**,** phenomena and objects**;** furthermore**,** the phenomenon of reversibility permitted by the **CA** does their by very interesting means for physical modeling**,** and for creation of very perspective computing structures basing on the nanotechnologies**;***

*(**4**) **CA**-models represent an interesting enough independent mathematic object whose essence consists in high–parallel processing of words in finite or infinite alphabets.*

At that, it is possible to associate the ***CA*–**approach with a certain model analogue of the differential equations in partial derivatives describing those or another processes with that difference, that if the differential equations describe a process at the average, in a ***CA*–**model defined in

appropriate way, a certain researched process is really embedded and dynamics of the ***CA–***model enough evidently represents the qualitative behaviour of researched process. Thus, it is necessary to determine for elementary automata of the model the necessary properties and rules of their *local interaction* by appropriate way. The ***CA–***approach can be used for research of processes described by complex differential equations which have not of analytical solution, and for the processes that it is not possible to describe by such equations. Along with it, the ***CA*** present a rather perspective modelling media for research of those phenomena**,** processes**,** and objects for which there are no known classical means or they are complex enough.

As we already noted, as against many other modern fields of science, the theoretical component of the ***CA–***problematics is no so appreciably crossed with its second applied component, therefore, it is possible to consider the ***CA–***problematics as two independent enough directions**:** *research of the* ***CA*** *as mathematical objects and use of the* ***CA*** *for modelling***;** at that, the second direction is characterized also by the wider spectrum. The level of evolution of the 2^{nd} direction is appreciably being defined by possibilities of the modern computing systems since ***CA–***models, as a rule, are being designed on base of the *immense* number of elementary automata and, as a rule, with complex enough rules of local interaction among themselves.

The indubitable interest to them amplifies also a possibility of practical realization of high parallel computing ***CA*** on basis of modern successes of microelectronics and prospects of the information processing at the molecular level *(methods of nanotechnology)***;** while the itself ***CA–***concept provides creation of both conceptual and practical models of *spatially–distributed* dynamic systems of which namely physical systems are the most interesting and perspective. Indeed, models which in an obvious way reduce *macroscopic* processes to rigorously determined *microscopic* processes, represent especial epistemological and methodical interest for they possess the great persuasiveness and transparency. Namely, from the given standpoint the ***CA–***models of various type represent a special interest, above all, from the applied standpoint at research of a lot of processes, phenomena and objects in different fields and, first of all, in physics, computer science and developmental biology.

The *first* direction enough intensively is developed by mathematicians whereas contribution to development of the *second* direction essentially more representative circle of researchers from various theoretical and

applied fields *(physics, chemistry, biology, technics, etc.)* brings. Thus, if theoretical researches on the ***CA–***problematics in general are limited to *classical, polygenic* and *stochastic* ***CA–***models, then the results of second direction are based on essentially wider representation of classes and types of ***CA–***models. As a whole if classical ***CA–***models represent first of all the formal mathematical systems researched in the appropriate context, then their numerous generalizations represent a perspective enough environment of modeling of various processes and objects.

In the conclusion once again it is necessary to note a rather important circumstance, at discussion of the *Classical Cellular Automata (**CCA**)* we emphasize the following a rather essential moment. We considered the ***CCA–***models that are a class of *parallel discrete dynamic systems* as formal algebraic systems of processing of finite words *(configurations)* in finite alphabets without any reference, as a rule, to their microprogrammed environment, i.e. without use of their cellular organization on lowest level inherent into them, what distinguishes our approach to research of the given objects from approaches of a lot of other researchers. Also, we consider ***CCA-***models as formal mathematical object having specific inside organization without ascribing to them certain universality and generality in perception of the World. At similar approach the ***CCA*** are considered at especially formal level not allowing in full measure to use their intrinsic property of high parallelism in field of computations, and information processing as a whole.

Naturally, for solution of a lot of the applied problems in the ***CA–***media and obtaining of a series of thin results first of all of model character an approach on microprogram level is needed when a researched process, algorithm or phenomenon is directly embedded in ***CA–***media, using its parameters**:** *a dimension*, *a neighbourhood index*, *a states alphabet* and *a local transition function*. At such approach it is possible to receive solutions of a lot of important appendices with generalizations of a rather high level of theoretical character. In particular, by direct embedding of universal computing algorithms or logical elements into such objects it is possible to constructively prove existence of the universal computability, etc.

In spite of such extremely simple concept of the ***CCA,*** they by and large have a rather complex dynamics. In many cases theoretical research of their dynamics collides with essential complexity. Therefore, computer simulation of these structures that in empirical way allows to research their dynamics is a rather powerful tool. For this reason this question is quite natural for investigations of the ***CA–***problematics, considering the

fact that ***CA–***models at the formal level present the dynamical systems of high**–**parallel substitutions.

Indeed, the problem of computer modelling of the ***CA*** is solved at two main levels**:** *(**1**) simulation of the **CA** dynamics on computers of traditional architecture***,*** and *(**2**) simulation on the hardware architecture that as much as possible corresponds to the **CA** concept**;** so**–***called **CA–**oriented architecture of computing systems*. So, computer simulation of ***CA*** models plays a rather essential part at theoretical researches of their dynamics, meantime it is even more important at practical realizations of ***CA*** models of different processes. At present time, a whole series of rather interesting systems of software and hardware for help of investigations of different types of ***CA*** models has been developed**;** their characteristics can be found in the references [7,30]. In our works [9-13,24-29] a lot of programs in various program systems for different computer platforms had been presented. Among them a lot of interesting programs for simulation of ***CA*** models in the ***Mathematica*** and ***Maple*** systems has been programmed. On the basis of computer simulation many of interesting theoretical results on the ***CCA*** and their use in the fields such as mathematics, developmental biology, computer sciences, etc. had been received. However, the given matter along with applied aspects of the ***CA-***models in the present book aren`t considered, despatching the interested reader to detailed enough discussion of these aspects to the corresponding publications in lists of references [7]**;** a lot of interesting works in this direction can be found in *Internet* on the corresponding key phrases.

The problematics considered by the ***TRG*** researchers in many respects has been conditioned by own interests and tastes of the authors along with traditions of creative activity of the ***TRG*** in this field. At last, we will note that in our activity it is possible to allocate ***3*** main directions, namely**:** *(**1**)* researches of classical ***CA*** as a formal parallel algorithm of processing of configurations in finite alphabets, *(**2**)* applications of the classical and generalized ***CA*** in mathematics and computer facilities of highly parallel action and *(**3**)* developmental biology. With our results in two last directions the interested reader can familiarize in sufficient detail in [24-29,31-38] and in numerous references contained in them along with references concerning many other researchers in this field. Whereas here we consider problems concerning only classical cellular automata as formal parallel algorithms of processing of configurations in finite alphabets. We pass on to the basic ***CA*** concepts, preliminarily introducing the paramount concepts, definitions and designations.

Chapter 1. The basic concepts of classical cellular automata *(classical CA models)*

In conformity with afore-said, the *cellular automata (**CA**)* at all generality present highly formalized *models* of some abstract *Universes* developing by simple rules and consisting of rather simple identical elements. The ***CA*–**universes of such kind develop according to local and everywhere identical rules of interaction of the elements forming them *(laws)*. In this context we can consider ***CA*–**models as a certain analogue of a physical concept of «*field*». The space of ***CA*–**universe represents a regular lattice whose each cell presents a certain identical element *(elementary particle, finite automaton, element)* that receives a finite number of states. History of development of similar ***CA*–**universe acts in a discrete time scale *(**t** = **0, 1, 2, 3, ...**)* according to a finite set of instructions of change of states of both an elementary automaton at time ***t*** and finite number of its nearest elements during the previous moment of time *(**t-1**)*. In addition function σ acting on each elementary automaton and its neighbourings is named the *local transition function (**LTF**)* whereas its acting on all domain of ***CA*** defines so–called the *global transition function (**GTF**)*. At last, change of configurations of such universe under action of ***GTF*** defines ***dynamics*** its operating with time; this aspect plays the basic part in researches of its behavioural *(dynamical)* properties.

At that, states of elementary automata of ***CA*** models can be associated with various concepts such as commands of cellular microprocessors, characteristics of points of an abstract field, symbols of certain parallel formal systems, states of biological cells, etc. Whereas the histories of finite configurations in a certain ***CA*** model associate with dynamics of various sort of discrete processes, objects and phenomena, embedded in such model. Similar models can be successfully applied in the very various fields. We can interpret ***CA*** models not only as an abstraction of biological cellular systems, but also as a theoretical basis of artificial parallel systems of the information processing or as an environment of presentation of conceptual and practical models of *spatially–distributed* dynamic systems. Furthermore, from logical standpoint the ***CA*** models are infinite abstract automata with specific internal structure defining a number of important enough properties allowing to use their as a new perspective environment of modelling of different discrete processes utilizing a mode of maximal paralleling. In toto, the ***CA*** problems can be considered as ***structural*** and ***dynamical*** component of the theory of

infinite automata with a certain specific internal organization having an qualitative character along with important enough applied aspects.

In spite of such simple organization and principle of functioning, ***CA–***universes admit complex enough behaviour *(the dynamics of behavior of configurations of states of elements forming them)*, providing modelling a plenty of objects, processes, and phenomena in multifarious fields of science, technics, etc. For more objective consideration of the concept of ***CA–***universes *(models)* we need a whole series of the basic concepts and definitions, allowing at a formal level to investigate the opportunities of cellular automata as a perspective environment of modeling in series of strategically important directions of the modern natural sciences along with other applied fields. In addition, such ***CA*** models present a certain interest as an independent object of researches. These objects are considered namely in such context in the present book, i.e. as the formal systems of high–parallel processing.

In this chapter the basic concepts, definitions and designations that are connected to the concept of the classical ***CA*** models and used during all our further consideration are introduced. A rather detailed discussion of the basic concepts of the ***CA–***problematics and questions linked with them will allow the reader to understand more deeply the ***basics*** of the given field of general theory of infinite abstract automata.

First of all, we note, the basic consideration of the material is based on so–called *classical* concept of ***d–****dimensional cellular automata* ***(d-CA, d ≥ 1)*** concerning which a whole series of basic definitions is introduced and certain results concerning the important enough question of generality degree of classical concept are totalized. An axiomatic definition of an arbitrary *classical cellular automaton* ***d–CA (d ≥ 1)*** is introduced as follows *(we will use the designation* ***CA*** *both for separate cellular automaton, and for a set of such automata; in addition, the sense of this designation will easily follow from a context).*

1. A classical ***d–***dimensional cellular automaton ***(d–CA)*** is defined as an ordered tuple of the following four components, namely**:**

$$d\text{–}CA \equiv \langle Z^d, A, \tau^{(n)}, X \rangle$$

where ***A*** is a finite, non-empty set called the *states alphabet*, and it is the set of states which each of an individual finite–state automaton in the ***d–CA*** can assumes. The state alphabet ***A*** contains a *distinguished element* called the ***quiescent*** *state*, that is designated by symbol «**0**» *(in addition,*

for convenience in a number of cases the symbol «**0**» *is replaced with a symbol* «□»*);* essence of this *special* state will be elucidated a little later. Without loss of generality, we shall use the set $A = \{0, 1, 2, ..., p-1\}$ that contains p elements **–** integers from 0 up to $p-1$ as a state alphabet. In addition, the alphabet states, excluding quiescent state, allow various interpretations in a rather wide diapason [7,24,20-28,82,102,106,278,286].

Component Z^d is a set of all d**–**dimensional tuples **–** *integral coordinates* of points in Euclidean space E^d**,** i.e. Z^d is an integer lattice in E^d**,** whose elements serve for *spatial identification* of individual automata of a ***d-CA.*** It is shown [39], that such lattice does not introduce anything *essentially* new in fundamental properties of dynamics of configurations *(finite and infinite)* in the classical ***d–CA*** therefore for our purposes it is reasonably to be limited oneself to Z^d lattice and the integer lattice Z^d is completely sufficient. Thus, lattice Z^d determines *homogeneous space* of the ***d–CA*** in which it functions. So, Z^d is a set of all d**–**tuples of integers that is used to name the cells of the ***d-CA*** where Z is the set of integers, and is called ***space,*** i.e. space in which all processed elements are identical.

An automaton in Z^d lattice can be thought of as a ***name*** or ***address*** of a particular finite-state machine which occupies this *position* in the space Z^d**.** In addition, it will frequently be rather convenient to identify an j automaton located at an j cell with j**–**cell itself. Naturally, in a lot of the applied aspects of ***d–CA*** $(d \geq 1)$ their geometry plays a rather essential part *(so, question relative to geometry of the lattice gets special significance in the structural theory, when properties of* ***d–CA*** *are investigated depending on the internal organization)* however in the present book, this question isn`t not considered and the interested reader is referred to works presented in bibliography [7,24,20-28,82,102,106,278,286].

Dimensionality (d) of homogeneous space of the ***CA–***models also plays a rather essential part, differentiating all set of models into two various subsets**:** ***1–***dimensional models $(d=1)$ and d**–**dimensional models $(d \geq 2)$. Transition from ***1–***dimensional to **2–**dimensional case not only sharply changes the dynamics of the ***CA*** models which is caused by increase of dimensionality, but and increases the complexity of most of problems being solved concerning them. In particular, below we shall show that certain problems of the dynamics for case of classical ***1–CA*** and ***d–CA*** $(d \geq 2)$ are solvable and unsolvable accordingly**;** i.e. in the second case for their solution any decision algorithms are absent. In most cases the proof of unsolvability meets essential enough complexites, what can be

carried also to the ***CA*** problematics.

In this attitude ***1-CA*** present a *special* subclass of class of all ***d-CA (d≥1),*** researched effectively enough. If in the plan actually of modelling the ***1-CA*** in our opinion not have special prospects however they present a certain interest as an independent mathematical object. In addition, on the example of ***1–CA*** it is much easier to master the concept of classical ***CA*** models, what and will be widely used by us hereinafter. Because of what the ***1–CA*** are much easier even than the ***2–CA*** *(much smaller set of possible rules of transition for an individual automaton of* ***1–CA;*** *solvability of a series of problems, etc.),* a lot of types of ***1–CA*** was the most intensively investigated from theoretical standpoint**;** furthermore, the majority of both the theoretical works, and computer simulation their for purpose of research of those or other dynamic properties have been devoted just to the given class of the ***CA*** models.

Into each point of Z^d lattice a copy of the *Moore* automaton is placed, whose states alphabet is ***A.*** It is known, the *Moore* automaton presents a finite automaton whose output at the time ***t*** depends only on its *internal* state at the same time ***t*** and does not depend on values of its inputs. A state of such automaton at the time $t > 0$ is some function of its inputs in moment *(**t–1**);* at that, output of the automaton at the time ***t*** is identical to its internal state. In this case each point of Z^d lattice will determine a ***name*** *(coordinate)* of elementary Moore automaton placed in this given point. For convenience below we shall identify points of Z^d lattice with elementary automata located in them. Thus, two terms «*automaton z*» or «*automaton with coordinate* $z \in Z^d$» we shall assume as identical ones.

Further we assume, the component ***X,*** named as *neighbourhood index* of ***d–CA,*** is an ordered tuple of ***n*** elements from Z^d lattice**,** that serves for definition of the automata neighboring for any elementary automaton of ***d–CA*** $(d \geq 1)$**,** i.e. those its automata with which the given automaton is directly interlinked by information channels, i.e. communicates. So, in an elementary example of ***2–CA*** we can present Z^2 lattice in the form of cellular paper in each cell of which a copy of certain Moore automaton is located. Then, $X_N = \{(0, 0), (0, 1), (1, 0), (0, -1), (-1, 0)\}$ and $X_M = \{(i, j)\}$ $(i,j \in \{0, 1, -1\})$ are called by neighbourhood indexes of ***J. von Neumann*** and ***E.F. Moore*** accordingly**.** These neighbourhood indexes ***X*** long ago became classical, and are widely used in researches of theoretical and applied aspects ***d–CA,*** while the ***neighbourhood templates*** ***(NT)*** defined by them have a rather transparent geometrical image *(**fig. 1**)*. The given

example is rather transparent, not demanding any special explanations.

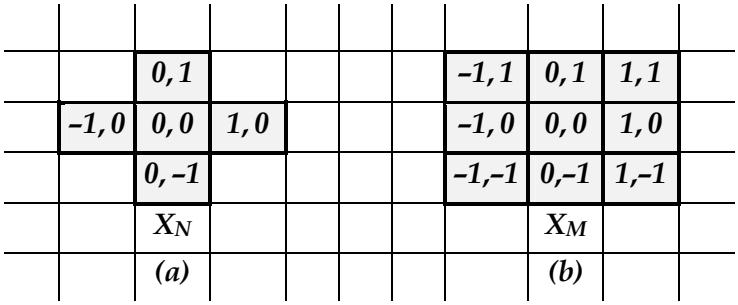
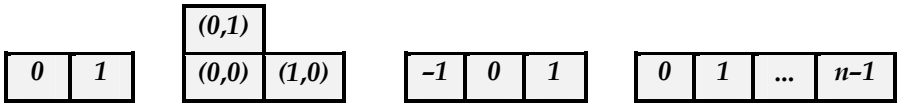


Fig. 1. Neighbourhood templates of J. Neumann (a) and E.F. Moore (b)

In one**–**dimensional case both these types of neighbourhood indexes ***X*** coincide. So, the reader can enough easily generalize two**-**dimensional neighbourhood indexes X_N and X_M onto general ***d–***dimensional case *(**d** **≥ 3**)*. Generally, the neighbourhood template of a ***CA*** model is arbitrary**;** it can accept a rather exotic form determined by applied aspects of the ***CA–***model. The following neighbourhood indexes *(with neighbourhood templates corresponding to them)* in aggregate with *neighbourhood indices* of ***J. von Neumann*** and ***E.F. Moore*** we can ascribe to the most frequently used ones, namely**:**



X_1={0,1} X_2={(0,0), (0,1), (1,0)} X_3={–1, 0, 1} X_4={0, 1, 2, ..., n–1}

Neighbourhood indexes X_1 and X_2 are elementary for ***1–CA*** and ***2–CA*** accordingly. While case of a ***d–***dimensional classical ***CA*** an elementary neighbourhood index ***X*** assumes the next form**:**

$$X=\{(0,0,.....,0),(1,0,0,.....,0),(0,1,0,.....,0),.....,(0,0,....,0,1)\}$$
$$\underbrace{\quad\quad}_{d}\quad\underbrace{\quad\quad}_{d}\quad\underbrace{\quad\quad}_{d}\qquad\underbrace{\quad\quad}_{d}$$
$$\underbrace{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}_{d+1}$$

i.e. an automaton z_o of elementary ***NT*** is central and from it along each axis of E^d lattice is spread out strictly one elementary automaton of the ***d–CA*** ***(d ≥ 1)***. But, in spite of universality of the simplest neighbourhood indexes *(a classical **d–CA** can be modeled by means of certain **d–CA** but with elementary neighbourhood indexes)* for certain applications and researches more complex indexes *(for example, for case of **1–CA –** neighbourhood index **X = {0, 1, ..., n-1}**)* with special geometry of ***NT*** are used. In a number of cases of practically important cases the mentioned approach allows to

simplify essentially the process of embedding into classical ***CA*** models of concrete processes, objects, phenomena and algorithms. At that, this approach is rather effectively applied at theoretical investigations of the ***d–CA*** $(d \geq 1)$ which are based, in particular, on computer modelling.

So, *neighborhood index* of ***d–CA*** is ***n***-tuple of different ***d***-tuples of *integers*; it is used to define the neighbors of any cell, i.e., those cells from which the cell directly receive information. Then, ***n*** neighbours of a cell z are cells $z+\alpha_0, z+\alpha_1, ..., z+\alpha_{n-1}$, where $X=\{\alpha_0, \alpha_1, ..., \alpha_{n-1}\}$. The neighborhood index ***X*** describes uniform interconnection pattern among elementary automata in ***d–CA.*** It represents positions relative to a cell z of all cells whose automata are directly connected to cell ***z.*** If index ***X*** contains the point $0^n=\{0,...,0\}$, every automaton is in its own neighborhood template, where neighborhood template contains all neighbours of cell. Without loss of generality, we shall as a rule assume that ***X*** contains the point 0^n which defines *central* automat of an arbitrary neighborhood template.

It is proved, by and large, the dynamics of ***d-CA*** $(d \geq 1)$ don`t depend on a choice of an automaton of ***NT*** as central. Thence, ***d–CA*** with strongly expressed gradient of information transfer, caused by choice of central automaton of ***NT,*** don`t change dynamic and computing possibilities of ***CA*** models in the time attitude, but influences onto their constructive characteristics, i.e. onto characteristics dependent on geometry of space of a model. In constructive attitude, generally, such ***CA*** models will be distinguich. The interesting examples of such sort with considerations in this context can be found, in particular, in [1,4-9,12,13,24-28,82].

Among all neighbourhood templates ***(NT)*** distinguish *disconnected* and *connected* ones; this parameter generally speaking essentially influences the dynamics of ***d–CA.*** A ***NT*** is called as a *connected* if area occupied by it is connected in topological sense; otherwise, ***NT*** is called *disconnected.* So, for example, two ***1–CA*** with neighbourhood indexes ***X*** and ***X'*** that have the connected and disconnected ***NT*** accordingly, even for case of *identical* local transition functions can cause rather essential distinctions in their dynamics. A detailed analysis of both types of neighbourhood templates in context of their influence on dynamics of ***CA*** models can be found in [4]. Further, we shall deal as a rule with *connected* ***NT,*** keeping in mind the circumstance what an arbitrary disconnected ***NT*** always it is possible to replace with certain connected equivalent ***NT*** of the same maximal size, having included in appropriate ***NT*** *insignificant* elements. Meantime, trends of the majority of the major dynamical characteristics of ***d–CA*** $(d \geq 1)$ with the connected and disconnected ***NT*** are kept under

the condition of identity of their local transition functions. So, property of universal reproducibility of configurations in the *Moore* sense is kept with change only of some numerical characteristics of the reproduction process of configurations [1,4-9,43,82,102,106].

2. Thus, three first components of an arbitrary ***d–CA*** ***(d ≥ 1)***, namely, the *states* alphabet ***A***, the *space* $\boldsymbol{Z^d}$, and *neighbourhood index* ***X*** form so–called *homogeneous space*. Homogeneous space is static part of the ***d–CA*** ***(d ≥ 1)*** that describes a physical structure of ***d–CA***, however it does not specify interactions that will take place among elementary automata in $\boldsymbol{Z^d}$, i.e., strictly speaking these three components don`t determine the dynamics of ***CA*** models.

In order to define and to study the operating *(dynamics)* of a ***d–CA***, it is necessary to have means for a describing of the current state of entire homogeneous space at any given time $\boldsymbol{t > 0}$. A state of the entire space is called a *configuration* ***(CF)*** of the space and it is simply the complete set of current states of each of elementary automata in the ***d–CA***. So, a configuration is any mapping $\boldsymbol{CF: Z^d \to A}$; ***C(A, d)*** denotes the set of all configurations with respect to $\boldsymbol{Z^d}$ and ***A***, i.e. $\boldsymbol{C(A,d) = \{CF \mid CF: Z^d \to A\}}$. By special symbol «$\boldsymbol{\square^d}$» is denoted completely zero configuration ***(CF)***; $\boldsymbol{\square^d: Z^d \to 0}$, i.e. when all elementary automata in ***d–CA*** ***(d ≥ 1)*** are in the quiescent state «***0***». Identifying both states {«***0***», «□»}, we shall use the second of them for designation of infinite areas of $\boldsymbol{Z^d}$ space filled with automata only in the quiescent state «***0***». This state has numerous and natural enough interpretations from the applied standpoint, above all. It is necessary to have in mind, that all results represented in this book and formulated relative to quiescent state «***0***» are valid for general case of quiescent state $\boldsymbol{h \in A}$, i.e. for all classical ***d–CA*** ***(d ≥ 1)*** *(moreover, under the designation* ***"CF"*** *we will understand both a separate configuration, and their set if from a context there is no an ambiguity)*.

The set ***C(A,d)*** of configurations is nonuniform relative to dynamics of functioning of ***d–CA*** because of presence of the selected quiescent state, therefore we determine two its basic subsets of configurations ***– finite*** configurations $\boldsymbol{C(A,d,\phi)}$ and ***infinite*** configurations $\boldsymbol{C(A,d,\infty)}$. A ***CF*** of a classical ***d–CA*** is called ***finite***, if it contains finite number of elementary automata in the states different from *quiescent* state ***"0"***, otherwise it is called ***infinite***. On fig. ***2*** two simple examples of ***1–***dimensional *finite* & *infinite* ***CF*** in alphabet $\boldsymbol{A=\{\square, 0, 1, 2, 3\}}$ ***('0' ≡ '□')*** are presented; in addition, «□» denotes an infinite chain of states «***0***» to the left or/and to the right.

a.1	...	□	2	1	3	2	0	1	3	0	1	2	3	□	...
a.2	...	1	2	1	0	1	2	2	1	0	3	2	1	3	...

Fig. 2. Examples of finite (a.1) and infinite (a.2) configurations.

So, under *finite* configuration $c \in C(A,d)$ we shall understand a ***CF*** which contains only finite number of elementary automata in states different from quiescent state «***0***»; otherwise, the configuration is considered the *infinite*. Formally, the given definition is formulated as follows.

Let $c(z)$ be the current state of an automaton z located in z point of Z^d. ***Support*** of c configuration *(denoted by* $[c]$*)* is the set of all z points such that $c(z) \neq 0$; i.e., the *support* is a nonquiescent part of the c configuration. Configurations with *finite support* are of particular interest; the set of all such *finite* configurations is denoted as $C(A,d,\phi)$ while $C(A,d,\infty)$ denotes the set of all configurations with *infinite* support and $C(A,d) = C(A,d,\phi) \cup C(A,d,\infty)$; in addition $C(A,d,\phi) \cap C(A,d,\infty) = \varnothing$ *(*$\varnothing$ *denotes the empty set)*, whereas *dimensionality* d of configurations is defined by dimensionality of relevant classical ***d–CA*** $(d \geq 1)$. Here and further we shall use standard set–theoretical and logic designations.

Special denotation is used to represent the sets of all ***1–***dimensional ***CF*** with finite and infinite support in a ***1–CA,*** namely: $C(A,1,\phi)$ & $C(A,1,\infty)$ accordingly; $C(A) = C(A,1,\phi) \cup C(A,1,\infty)$. At that, any finite configuration $c^* \in C(A,1,\phi)$ is represented as $\square x_1 x_2 \ldots x_m \square$, where at least x_1 or x_m isn't equal to *'0'* and □ indicates a string of unbounded length of elementary automata in state «***0***», i.e.,

$$c^* = \square x_1 x_2 \ldots x_m \square \equiv \ldots 00\, x_1 x_2 \ldots x_m 00 \ldots; \quad x_j \in A;\ x_1 \neq 0 \text{ or } x_m \neq 0;\ j = 1..m$$

Taking into account the specificity of classical ***d–CA*** that is caused by presence of *quiescent* state «***0***» along with a lot of other rather important reasons we shall ascribe the completely null configuration $c_o = '\square'$ to the set $C(A,d,\phi)$. This approach allows to receive a lot of interesting enough results concerning dynamics of the classical ***d–CA*** $(d \geq 1)$. In particular, that concerns the problems of nonconstructability and reversibility that are considered below. ***Diameter*** of a finite ***d–***dimensional configuration c is defined as distance between its two extreme elementary automata in the non–quiescent states of $A \setminus \{0\}$ and will be denoted as $|c|$. For ***1–***dimensional case the diameter will be associated with ***length*** of a finite configuration defined analogously. The diameter concept plays a rather essential part in certain numeric estimations of ***d–CA*** dynamics.

Along with configuration of all Z^d lattice, a configuration c_b of a finite ***d–***dimensional hypercube *(**block**)* $b \subset Z^d$ of elementary automata of the ***d–CA*** is defined too**;** a set of all such configurations we shall denote as ***C(A,d,B).*** The concept of ***block*** configurations plays a rather important part, for example, at investigation of the nonconstructability problem in classical ***d–CA*** *(**d ≥ 1**)*. This question further will be detailed. Inasmuch as further speech will go, at a great extent, about ***1–***dimensional ***CA*** *(**1–CA**)* for designation of ***1–***dimensional finite, block and infinite ***CF*** the designations $c` = \square c_1c_2c_3c_4 \ldots c_k\square$, $c_b = b_1b_2 \ldots b_p$ and $c_\infty = \infty c_1c_2c_3c_4 \ldots c_k\infty$ will be used accordingly, where $c_1, c_k \in A\backslash\{0\}$, and $c_j, b_q \in A$; $j = 2..(k–1)$; $q = 1..p$. In addition, length of a finite configuration $c`$ equals k, and is designated as $|c`|, |c`| = k$; length of a block c_b configuration is defined analogously.

As will be shown below, the differences between ***d–***dimensional finite, finite block and infinite configurations are of fundamental character as from the theoretical, and the applied standpoints. Indeed, for example, from standpoint of dynamical modelling in the environment of cellular automata of many phenomena of developmental biology in our opinion it is more natural to consider the dynamics of finite configurations than the dynamics of block configurations, and besides in the encirclement of other elementary cellular automata in states other than quiescent. Albeit in some cases such modelling is entirely acceptable.

There are fundamental differences between ***d–***dimensional finite, finite block and infinite configurations above all at considerations of dynamic properties of behaviour of configurations of the above types. Thus, the ***NCF*** nonconstructability of a finite block configuration causes the ***NCF*** nonconstructability of the finite configuration that contains such finite block configuration. On the other hand, if a certain finite configuration in a classical ***d-CA*** model is ***NCF-1*** nonconstructible then the finite block configuration appropriate it, may well be constructible in the same ***d-CA*** model. So, in a classical binary ***1–CA*** model with neighbourhood index $X = \{0, 1\}$ with local transition function defined by parallel substitutions $xy \rightarrow x + y \pmod 2$, $x, y \in \{0, 1\}$, the finite configuration $\square 1011\square$ is ***NCF–1,*** while the finite block configuration ***1011*** is constructible and even self**–**reproducing in the *Moore* sense. Below, other differences between the configurations of the above types are considered. The differentiation of configurations into sets of finite and block**–**finite configurations makes sense only for the classical ***d–CA*** *(**d ≥ 1**)* models, considered in this book.

The above differentiation of finite configurations allows us to research in more detail the dynamic properties of classical ***d–CA*** ***(d ≥ 1)*** models.

Sets of ***1–***dimensional finite**,** block and infinite configurations we shall designate as $C(A,1,\phi)$, $C(A,1,B)$ and $C(A,1,\infty)$ accordingly. Thus, directly from definitions of sets $C(A,1,\phi)$ and $C(A,1,B)$ follows, the first is closed concerning the global transformation $\tau^{(n)}$ determined below, while the second is closed concerning the concatenation. Moreover, the following two relations are obvious, namely**:**

$$(\forall c_b \in C(A,1,B))(c = \square c_b \square \in C(A,1,\phi)) \;\&\; (\forall c^* \in C(A,1,\phi))([c^*] \in C(A,1,B))$$

Zero configuration $c =$ «□» may be represented as a finite configuration of zero size. In this context with good reason we can ascribe it to the set $C(A,d,\phi)$**.** This assumption is expedient owing to many rather important reasons being discussed below in detail. We go on to the description of principle of acting of classical ***CA*** models.

3. The operating of ***d–CA*** ***(d ≥ 1)*** occures in discrete time ***t=0, 1, 2, ...*** and is specified by a ***local transition function (LTF)*** $\sigma^{(n)}$ that sets a state of each elementary automaton in the current moment $t > 0$ on the basis of states of its neighbor automata *(according to a neighbourhood index* ***X****)* in previous moment ***(t-1)***. In other words, the local transition function $\sigma^{(n)}$ is an arbitrary mapping $\sigma^{(n)} : A^n \to A$**;** further for ***LTF*** $\sigma^{(n)}$ of ***CA*** models we shall use the following basic designations, namely**:**

$$\sigma^{(n)}(a_1, a_2, ..., a_n) = a^*_1; \qquad a_j, a^*_1 \in A \quad (j = 1 .. n) \qquad (1)$$

$$a_1 a_2 ... a_n \Rightarrow a^*_1 \text{ – a set of parallel substitutions} \qquad (2)$$

where a_j **–** states of any *z***–**automaton of ***d–CA*** ***(d ≥ 1)*** and its neighbours *(relative to neighbourhood index* $X = \{x_1, x_2, x_3, ..., x_n\}$*)* at moment ***(t–1),*** and a^*_1 **–** a state of the same *z***-**automaton at the following moment $t>0$**.** At that, the considered *z***–**automaton of ***d–CA*** ***(d ≥ 1)*** is assumed the central concerning its neighbourhood template. In particular, arbitrariness of choice of the central automaton of a neighbourhood template allows to speak about admissibility of ***1–***side universal classical ***1–CA*** model. The detailed explanation of question of arbitrariness of choice of the central automaton can be found in works [1,24-28,31-38]. Whereas in each case as the *central* one the most acceptable automaton of the neighbourhood template is chosen.

Formula presentation of a ***LTF*** when calculation of the subsequent state of an arbitrary *z***–**automaton in a ***d–CA*** is made on the basis of formula

*(**1**)* is the most convenient. So, in many interesting cases such approach is possible, however in a lot of cases the use of local transition function $\sigma^{(n)}$ in the form of set of parallel substitutions *(**2**)* is necessarily required.

The set of parallel substitutions *(**2**)* defines a program *(parallel algorithm)* of functioning of classical ***CA*** models**;** parallel substitutions *(**2**)* present the lowlevel parallel programming language in environment of classical ***CA*** models.

In particular, if at moment t the current configuration c^* of ***1–CA*** with alphabet $A = \{0,1,2,3,4,5\}$, neighbourhood index $X = \{0,1,2,3\}$ & ***LTF*** $\sigma^{(4)}$ defined by parallel substitution $1023 \Rightarrow 2$ {*or by formula* $\sigma^{(4)}(1, 0, 2, 3) = 2$} has kind (φ) then in the next moment $t + 1$ it passes in configuration φ^***,** that contains the distinguished ***S*** automaton in a new state received on basis of ***LTF*** $\sigma^{(4)}$ *(parallel rules of substitution)* of such ***CA*** model**:**

t	...	1	3	0	1	0	2	3	5	2	4	3	... : φ
					⇓								
$t+1$	...				2								... : φ^*

S – *distinguished elementary automaton of* ***1–CA***

Formula representation of ***LTF*** $\sigma^{(n)}$ is especially preferable at computer realization of ***d–CA,*** whereas parallel substitutions are irreplaceable on stage of programming of certain concrete ***CA*** models. The questions of formula representations *(**1**)* of parallel substitutions *(**2**)* are considered enough in detail in [1,2,31-38]. Meanwhile, by far not all local transition functions $\sigma^{(n)}$ can be represented in a formula form, ensuring the work with ***CA*** models only at a level of systems *(**2**)* of parallel substitutions determining them local transition functions $\sigma^{(n)}$**.**

In this book the ***CA*** models ***LTF*** $\sigma^{(n)}$ of which satisfy the determinative condition $\sigma^{(n)} : 0^n \Rightarrow 0$ or $\sigma^{(n)}(0,...,0) = 0$ are considered, i.e. models with restriction on speed of information transfer in them *(a certain analogue of light speed limit according to the modern physical standpoint)*. The sumption plays a rather essential part at researches of dynamical properties of the ***d–CA*** $(d \geq 1)$ and well meets the requirements of use of the models as a basis of modelling of parallel dynamical systems of various nature and assignment.

The above determinative condition not only introduces a restriction on speed of information distribution in ***CA*** models, but also defines space *(some formal vacuum)* in which development dynamics of the researched discrete objects, processes and phenomena occurs. At that, an arbitrary

element of alphabet ***A*** of ***CA*** model can be chosen as an quiescent state, however according to a whole series of reasons for this purpose we will use element {***'0'*** | ***'□'***} as the most acceptable.

CA satisfying the above determinative condition, we shall name ***stable*** ones, otherwise ***– unstable.*** In context of research of ***CA*** models as an independent mathematical object, a certain interest the unstable models represent too. Meanwhile, unstable ***CA*** can represent interest also from the standpoint of research in them of the models based on the concept of instant information transfer upon any distances. Examples of use of unstable ***CA*** models for us for the present are not known, however the works in this direction are being presented to us as interesting enough.

At definition of classical ***d–CA*** that are basic subject of consideration of the present book we shall constantly appeal to the above determinative condition $\sigma^{(n)}(0, 0, ..., 0) = 0$ for their local transition functions $\sigma^{(n)}$; this condition defines a rather important class of ***CA*** models. Meanwhile, it is necessary to have in mind, the determinative condition for ***LTF*** of an arbitrary classical ***d–CA*** $(d \geq 1)$, generally, can has the following kind**:**

$$(\exists h \in A)(\sigma^{(n)}(h, h, ..., h) = h)$$

i.e., an arbitrary element of states alphabet ***A*** *(or their subset)* of ***CA*** can be as an quiescent state. The majority of the results, represented below, which are conditioned by presence for ***CA*** of the most typical condition $\sigma^{(n)}(0, ..., 0) = 0$, are naturally spread and on ***d–CA*** $(d \geq 1)$ of the general class of automata whose ***LTF*** satisfy the above generalized *determinative* condition.

From definition of classical ***CA*** follows that instant transition *(in one step of global transition function)* of a *finite* ***CF*** to *infinite* ***CF*** *(infinite growth of finite configurations is possible only potentially)* is impossible**,** at the same time instant transition of an *infinite* ***CF*** to *finite* ***CF*** *(theoretically allowable by axiomatics of classical* ***CA*** *models)* is deprived, in the certain degree, of any natural sense. Though in individual cases it has a certain meaning [49]**.** Thus, dynamics of finite configurations of the set $C(A,d,\phi)$ presents the basic interest for the purpose of investigation of both theoretical and applied aspects of ***CA*** theory in all their extensiveness and generality of spheres of application [7,24,40-43,82,102,106,278,286].

It is easy to estimate the number of classical ***d–CA*** $(d \geq 1)$ with alphabet $A=\{0, 1, ..., a-1\}$ and neighbourhood index $X=\{0,1,2, ..., n-1\}$**,** depending on the determinative condition defining this class of ***d–CA.*** On basis of simple enough combinatorial considerations we determine the number

of all classical ***d–CA*** ***(d ≥ 1)*** [24-28,41,42] from which follows that share of all classical ***CA*** concerning all possible ***d–CA,*** including *unstable* ones**,** doesn`t depend on dimensionality and size of neighbourhood template, but depends only on cardinality of alphabet ***A*** and is estimated as ***≈0.63.***

Now, let's estimate the number ***N*** of all classical models with alphabet ***A={0,1, ..., a–1}*** and neighbourhood index ***X={0,1, ..., n–1}*****,** depending on the above determinative condition defining the given class of ***d–CA.*** On the basis of simple enough combinatorial considerations we determine the number ***N*** of all classical ***d–CA*** ***(d ≥ 1)*** models as follows [43,82,102]**:**

$$N=\sum_{j=1}^{a}C_a^j(a-1)^{a-j}a^{a^n-a}=(a-1)^a a^{a^n-a}\sum_{j=1}^{a}C_a^j(a-1)^{-j}=a^{a^n}\left(1-\frac{1}{a}\right)^a\sum_{j=1}^{a}C_a^j(a-1)^{-j}=$$

$$a^{a^n}\left(1-\frac{1}{a}\right)^a\left[\sum_{j=1}^{a}C_a^j(a-1)^{-j}-1\right]=a^{a^n}\left(1-\frac{1}{a}\right)^a\left[\left(1+\frac{1}{a-1}\right)^a-1\right]=a^{a^n}\left(1-\frac{(a-1)^a}{a^a}\right)$$

or on the basis of more simple approach we can easily receive quantity of ***d–CA*** models relating to the «***classical***» type, namely**:**

$$N=a^{a^n}-(a-1)^a a^{a^n-a}$$

Thus, share ***(Δ)*** of all classical ***CA*** models concerning all possible ***d–CA,*** including unstable ones, does not depend on dimensionality and size of neighbourhood template, and is defined by the following relation**:**

$$\Delta=\frac{N}{a^{a^n}}=1-\left(\frac{a-1}{a}\right)^a$$

depending only on cardinality of an alphabet ***A*** of a ***d–CA*** ***(d ≥ 1)*** model under the following condition, namely**:**

$$\lim_{a\to\infty}\left[1-\left(\frac{a-1}{a}\right)^a\right]=1-\frac{1}{e}\approx 0.63212$$

i.e. share of all classical ***d-CA*** models for arbitrary neighbourhood index and dimension makes up the majority lying in the range [***0.63 .. 0.75***]**.** In addition, it is necessary to have in mind, that the case when all states of the alphabet ***A*** are quiescent states, is singular in a certain sense relative to a lot of results received on classical ***CA*** models for which alphabet ***A*** contains the states distinct from quiescent states too. Whereas in other cases many rather important results that have been obtained for the case of a single quiescent state have been spread to the classical ***CA*** models also in their generalized understanding.

So, among the ***d–CA*** having more than one quiescent state**,** i.e. cellular automata relating to the «*classical*» type, the basic results concerning the self–reproducing finite configurations in the *Moore* sense are valid also.

It is possible to show that the classical ***1-CA*** with neighbourhood index $X=\{0,1\}$, alphabet $A=\{0,1,...,a-1\}$ and symmetric local transition function $\sigma^{(2)}$ which obey the condition $(\forall b \in A)(\sigma^{(2)}(b,b) = b)$ have all finite ***CF*** as self–reproducing in the *Moore* sense. As an example, the ***LTF*** $\sigma^{(2)}(x, y) = (a+1)(x+y)/2 \pmod a$ of the generalized class of linear ***1–CA*** can serves. Can be shown that the next result takes place**:** ***For prime*** $a \geq 3$ ***and*** $n \geq 3$ ***the classical 1–CA with an alphabet*** $A = \{0,1,...,a-1\}$ ***and symmetric LTF*** $\sigma^{(n)}$ ***which satisfy condition*** $(\forall b \in A)(\sigma^{(n)}(b, ..., b) = b)$ ***there are classical 1–CA that possess the universal reproducibility in the Moore sense; but by far not all classical CA with such local transition functions possess the universal reproducibility.***

The result confirms also the fact of existence of classical ***d–CA*** for which the pure linearity of their ***LTF*** $\sigma^{(n)}$ is not compulsory**,** i.e. the more wide class of classical cellular automata possessing of *universal reproducibility* of finite configurations in the *Moore* sense takes place. The classical ***CA*** possessing property of *universal reproducibility* in certain respects define extremal properties of cellular automata. For empirical research of the *universal reproducibility* a number of procedures has been programmed in the systems ***Mathematica*** and ***Maple,*** that have allowed to research the phenomenon from various standpoints [29,44-52,82,102,106].

Meanwhile, the existence of several quiescent states in the majority of appendices doesn`t find adequate interpretation. Therefore, further we *(without loss of generality, and by certain conceptual reasons)* will consider only classical ***d–CA*** with a single quiescent state as the most typical and widely used. The interesting enough philosophical comprehensions of classical ***CA*** models of such type, though in a lot of cases and not fully indisputable ones, can be found in [24,40-43,82,102].

Naturally, it is possible to investigate also and unstable ***d–CA*** for which not exist specially chosen quiescent states; i.e. the automata that are not satisfying specified *determinative* condition $\sigma^{(n)}(h, h, ..., h) = h$**.** However, such cellular automata present speculative enough character and in our opinion don`t represent any serious interest from practical standpoint. A discussion of this question can be found in [40-43,82,102,106].

4. So, dynamics of a classical ***d-CA*** is completely determined in terms of ***LTF,*** i.e. local interactions of automata of neighbourhood template of an elementary *z*–automaton, whereas itself ***LTF*** $\sigma^{(n)}$ is the typical example of local algorithm that runs by especially *parallel* manner on the basis of configuration of states of elementary automata of local neighbourhood

determined by neighborhood index X of the current z-automaton of Z^d lattice of a certain classical CA model. Thus, simultaneous applying of local transition function to neighbourhood of every z–automaton of all Z^d homogeneous lattice defines the global transition function $\tau^{(n)}$ *(GTF)* which transforms the current configuration $c \in C(A,d)$ into the following configuration $c\tau^{(n)} \in C(A,d)$. Formal definition of the configuration $c\tau^{(n)}$ can be represented as follows.

Let $C(A, d)$ denotes the set of all configurations with respect to Z^d and A. If $s[z]$ denotes the current state of an elementary z–automaton, then formally *GTF* $\tau^{(n)}$ with neighbourhood index $X = \{x_1, x_2, x_3, x_4, ..., x_n\}$ is determined by the following formal condition, namely:

$$c\tau^{(n)} = c^{**} \leftrightarrow (\forall z \in Z^d)(s^{**}[z] = \sigma^{(n)}(s[z+x_1], s[z+x_2], ..., s[z+x_n]))$$

From this definition immediately follows, that every *LTF* $\sigma^{(n)}$ defines a unique global transition function $\tau^{(n)}$, and $\tau^{(n)}$ cannot be defined by two different local transition functions $\sigma^{(n)}$. In other words, exists biunique correspondence between set of all global transition functions $\tau^{(n)}$, and the set of all local transition functions $\sigma^{(n)}$ for the given states alphabet A, dimensionality d of space Z^d and neighbourhood index X. Thus, it is possible to speak about *GTF* $\tau^{(n)}$ defined by *LTF* $\sigma^{(n)}$, and vice versa. It is proved [1] that an arbitrary *GTF* in classical *d-CA* is *primitive recursive function*. The given result defines not only place of *GTF* $\tau^{(n)}$ in hierarchy of all recursive functions, but also along with other components defines simplicity of mathematical objects such as cellular automata *d-CA* $(d \geq 1)$. Meantime, such simple *CA* models admit complex enough dynamics of finite and infinite configurations, including of universal computability.

It turned out that the family of global transition functions of classical *d–CA* represents excellent tool for solution of a rather broad range of the modelling problems in a mode of maximal paralleling. In addition, the global parallel transformations defined by classical *CA* models, in our opinion, can be used effectively enough and widely similarly to other well–known mathematical transformations *(Fourier, Laplace, etc.)*.

The *fourth* component of *d–CA* $(d \geq 1)$ can now be determined. For A, Z^d and X, a set of admissible transformations T is any nonempty subset of the complete set of all global transition functions $\tau^{(n)}$ which are defined by *3* parameters A, Z^d and X. In addition, if set T contains single global transition function $\tau^{(n)}$, then object $d\text{–}CA = \langle Z^d, A, \tau^{(n)}, X \rangle$ is said to be

monogenic or *classical* ***d–CA (d≥1)***. The operating of an arbitrary classical ***d–CA (d≥1)*** is particularly simple**:** if $c = c_o$ is an initial configuration of homogeneous space Z^d at the time $t = 0$**,** configuration at the time $t = m$ is $c^* = c_o\tau^{(n)m}$**,** the result of applying of a global transition function $\tau^{(n)}$ to configuration c_o of the homogeneous space m times.

Let $<c_o>[\tau^{(n)}]$ designates *configurations sequence* generated by some ***GTF*** $\tau^{(n)}$ from an initial ***CF*** c_o**.** Then for a finite ***CF*** $c_o \in C(A,d,\phi)$ the sequence represents a history of configuration c_o in a classical ***d–CA (d≥1)*** which plays the basic part in researches of dynamic properties of ***CA*** models. Under *dynamics* is understood functioning of either type of ***d-CA (d≥1)*** consisting in change in the course of time of configurations of a model ***CA*** as function of its initial configuration and ***LTF (GTF)***. So, dynamics of a classical $d\text{–}CA = <Z^d, A, \tau^{(n)}, X>$ {*configurations sequence* $<c_o>[\tau^{(n)}]$**;** *the history of development of objects***,** *embedded in the model*} is determined by quite uniquely the above base components d, Z^d, A, X and $\tau^{(n)}$ $\{\sigma^{(n)}\}$.

A configuration $c^{-1} \in C(A, d)$ is a ***direct predecessor*** for a configuration $c \in C(A,d)$ if $c^{-1}\tau^{(n)} = c$**.** A configuration $c \in C(A,d)$ can has a single direct predecessor, their finite or infinite number, or have no of predecessors. At that, direct predecessors for *block***,** *finite* and *infinite* configurations in classical ***d–CA (d≥1)*** also are quite naturally determined [1,4,5,7,8,12].

At present, a lot of software means have been created for the computer analysis of the presence of predecessors in a block configuration[43,82]. In particular, we have programmed procedures for this purpose in the ***Mathematica*** [49,82]. So, for ***1–***dimensional ***CA*** models this problem is resolved by a procedure whose call **HistPredecessors[*x1, Ltf, n*]** returns the list of predecessors for a block ***x1*** configuration specified in string format relative to the local transition function ***Ltf*** specified by means of the list of parallel substitutions to a ***n*** depth. If a block ***x1*** configuration has no predecessors then the procedure call returns the empty list, i.e. {} with printing of the corresponding message. It should be kept in mind that the procedure is oriented to both the block configurations and local transition functions given in the alphabet $A = \{0, 1, 2, ..., a–1\}$ for $a <= 10$**,** which is quite enough for an experimental research of most dynamical aspects of the classical ***1-***dimension ***CA*** models. The following fragment presents the source code of the procedure with examples of its use [49]. The procedure turned out to be a rather convenient mean for computer research of the reproducibility problem of finite block configurations in

classical ***1–***dimensional ***CA*** models.

```
In[4142]:= HistPredecessors[x1_String, Ltf_List, n_Integer] :=
                                                    Module[{Pr, vs = {}, y, u, h, w},
                                                  w[s_] := "The block configuration <" <> x1 <>
                                          "> has no predecessors on the " <> ToString[s] <> "–th level";
                                              Pr[x_, Ltf] := Module[{a = StringLength[x], b, d, g, gs = {},
                                                                c = StringLength[Ltf[[1]][[1]]], j, k, t},
                                                 b = Map[#[[1]] &, Gather[Ltf, #1[[2]] == #2[[2]] &], {2}];
                                                 g = b[[ToExpression[StringTake[x, {1}]] + 1]];
                                                       Do[d = b[[ToExpression[StringTake[x, {t}]] + 1]];
                                             For[j = 1, j <= Length[g], j++, For[k = 1, k <= Length[d], k++,
                                           If[StringTake[g[[j]], {-c + 1, -1}] == StringTake[d[[k]], {1, c - 1}],
                                                  AppendTo[gs, g[[j]] <> StringTake[d[[k]], {-1}]], Null]]];
                                                   g = gs; gs = {}, {t, 2, a}]; If[ g == {}, {}, g]];  y = Pr[x1, Ltf];
                                                     If[n == 1, Return[If[y == {}, Print[w[1]]; y, y]], Null];
                                               Do[If[y == {}, Print[w[h]]; Return[y], u = Map[Pr[#, Ltf] &, y]];
                                                                  y = DeleteDuplicates[Flatten[u]], {h, n}]; y]
In[4143]:= Ltf := {"000" → "0", "001" → "1", "100" → "1", "101" → "0",
                    "010" → "1", "011" → "0", "110" → "0", "111" → "1"}
In[4144]:= HistPredecessors["0111100101001", Ltf, 1]
Out[4144]= {"000100110001100", "101001011100001", "011111101010111",
            "110010000111010"}
In[4145]:= Ltf1 = {"00" → "0", "01" → "1", "10" → "1", "11" → "0"};
In[4146]:= HistPredecessors["0111100101001", Ltf1, 1]
Out[4146]= {"00101000110001", "11010111001110"}
In[4147]:= Ltf2 := {"000" → "0", "001" → "1", "100" → "0", "101" → "0",
                    "010" → "1", "011" → "0", "110" → "1", "111" → "1"};
In[4148]:= HistPredecessors["011101110101", Ltf2, 3]
           The block configuration <011101110101> has no
           predecessors on the 1–th level
Out[4148]= {}
In[4149]:= Ltf3 := {"000" → "0", "001" → "1", "100" → "0", "101" → "0",
                    "010" → "0", "011" → "0", "110" → "1", "111" → "1"};
In[4150]:= HistPredecessors["01110110", Ltf3, 6]
           The block configuration <01110110> has no
           predecessors on the 1–th level
Out[4150]= {}
```

The algorithm of this procedure is relatively easy to expand to the case of **2–**dimension classical models, however its software implementation requires several large computational resources. For an arbitrary finite block configuration this procedure allows to resolve the question of its ***NCF*** nonconstructability. The procedure makes it possible to obtain, for an arbitrary block configuration, the history of its predecessors to any depth. Investigations on evaluating of the **γ** number of predecessors of an arbitrary block configuration at the ***p*** level showed that already for rather small ***p*** values, the set of predecessors of a block configuration at the ***p–***th level becomes cumbersome, and even completely immense. To a large extent, the **γ** value is determined both by a configuration and the existence of the ***NCF*** nonconstructability for a ***1–CA*** model, that defines the *balance* degree of predecessors *(see nonconstructability definition of the Aladyev–Kimura–Maruoka)*. For instance, for estimation of the **γ** number of predecessors of a block configuration at the ***p*** level for ***1–***dimensional linear classical ***CA*** models**,** i.e. classical ***1–CA*** models not possessing the ***NCF*** nonconstructability, and whose local transition $\sigma^{(n)}$ functions are determined as follows**:**

$$\sigma^{(n)}(x_1,\dots,x_n)=\sum_{1}^{n} b_k x_k \pmod a \quad x_k, b_k \in A=\{0,1,\dots,a-1\};\ (k=1..n),\ a-\text{prime}$$

we have an estimation ***γ(p) ≤ HistPredecessors["1", g, p]^(p + 1),*** where ***g –*** local transition function of the linear ***1–CA*** model with alphabet ***A =*** {*0,1,2,…,a–1*}. For example, for linear classical ***1–CA*** with neighborhood index ***X =*** {*0,1,2,3*} there is estimation ***γ(6)=****2097152*. In addition, a rather small modification of the **HistPredecessors** procedure allows to obtain all predecessors chains to the required depth for a block configuration. The ***γ(p)*** value for classical ***CA*** models without ***NCF*** nonconstructability is a certain kind of median of the predecessors balance at the ***p–***th level. As a whole, this and a number of similar procedures are useful enough for computer research of the *reverse* dynamics of finite configurations in classical ***1–***dimensional ***CA*** models, and at a more advanced procedure extension for the classic ***2–CA*** models too [24,43,82,102,106,286]. Note that this book contains a number of procedures programmed in ***Maple*** and ***Mathmatics,*** for understanding of which requires familiarity with these systems, for example, within the framework of [24,44,46,47,50-52].

The finding problem of ***LTF*** whose appropriate ***GTF*** generate a certain history *(dynamics)* of configurations of ***d–CA*** is similar to an inductive problem of finding of the laws underlying an observable phenomenon. This analogy underlies modelling in ***CA*** environment of various natural

and artificial systems, first of all, of cellular nature, and also at a lot of other motivations caused by research problems. In the general case this problem of full and exact description of dynamics even of rather simple classical ***CA*** models is one of the most complex in the ***CA*** problematics, and numerous attempts existing in this direction still are insufficiently effective. In addition, a lot of problems of full description of dynamics of similar models requires very significant computing resources.

We can make sure that in spite of all simplicity of mathematical object such as classical ***CA*** models their dynamics has complex character, and its research presupposes generally speaking significant efforts, and in a whole series of cases also nonconventional approaches. For this reason in this direction there are relatively a few results received by theoretical methods whereas a rather considerable part of them has been obtained by means of empirical approach, including computer modelling [7,42].

A rather interesting and instructive example of research of classical ***CA*** models by means of computer simulation is known game «***Life***» which presents a classical binary ***2–CA*** with the *Moore* neighbourhood index. Now this type of ***2–CA*** is investigated enough intensively from various standpoints [7,24-28,31-38,40-43] whereas in works [53–61] game «***Life***» has been considered in various interesting contexts as a classical binary ***2-CA*** with the *Moore* neighbourhood index. Along with ***2-CA*** of various purpose the reader can familiarize with rather interesting programmed implementations of games similar to ***CA*** models more thoroughly, for example, in the appropriate works in references [7,9,24-28,44-52]. In the same place it is possible to find interesting discussions of the matter.

Thus, the concept of the classical ***d–CA*** is being intuitively represented rather simple and in this connection exists a question concerning *degree* of its *generality* i.e. as far as widely this concept admits the expansions which not exceed the limits of some studied phenomenon or limits of this equivalence criterion *(a certain kind of property of stability of concept)*. With this purpose the detailed analysis of a series of expansions of the *classical* concept of ***d–CA*** relative to their *dynamic* properties had shown that in spite of a rather strict criteria of equivalence of dynamics of two ***d–CA*** *(which were based on the comparative analysis)* the classical concept of ***d-CA*** possesses sufficient degree of generality that allows to consider this concept as one of basic, forming a certain basis of the ***CA*** concept in all its generality. We considered only generative power of the classical ***d–CA*** *(**d≥1**)* and have proved that a whole series of *widenings* of classical concept of ***CA*** reveal that even with respect to a rather narrow concept

of *equivalence* of two ***d–CA,*** the concept of classical ***CA*** models possesses a quite sufficient degree of generality [12,13,24-28,31-38,43,82,102,286].

From definition of classical ***d–CA*** ***(d ≥ 1)*** we can simply make sure, that these objects presents formal parallel algorithms of processing of finite ***CF*** of the set ***C(A, d, ϕ)*** by means of global transition functions that may be considered as functions everywhere defined on the set ***C(A, d, ϕ).*** Of the above follows, that the concept of classical ***d–CA*** $= <Z^d, A, \tau^{(n)}, X>$ possesses an quite acceptable degree of generality for many important applications *(in spite of all its simplicity)***;** it represents a rather significant interest as a independent mathematical object being a rather important component of theoretical and applied models of parallel processing of information and computations.

5. So, if three components Z^d***, A, X*** of cellular automata ***d–CA*** ***(d ≥ 1)*** are rather simple and transparent whereas a ***GTF*** $\tau^{(n)}$ is primitive recursive function [4,5]. Hence, such simple objects as the classical ***d–CA*** possess considerable enough degree of generality and quite complex dynamics allowing to model a rather extensive class of objects, phenomena and processes having a place in a lot of fields of science and technics. Along with it, these objects present appreciable interest for investigation as an independent formal model of parallel processing. Meanwhile, within of classical ***d–CA*** the special subclasses of cellular automata with specific characteristics such as ***CA*** with refractority, memory and certain others allowing to more effectively simulate a lot of interesting enough objects and processes are chosen. Some of these types of ***CA*** are considered in [24-28,40-43], other interesting types can be found in bibliography [7].

Meanwhile, complexity and variety of the real world are not inscribed in no way in Procrustean bed of the ***CA*** concept without its any serious complicating, by influencing seriously attractiveness of its simplicity in its primordial concept. In our opinion, for today, the cellular automata are of interest in two basic natural**–**science directions, namely**:**

(1) *A modelling environment and embedding in it of various processes, objects and phenomena (first of all of those, which difficulty or impossibly to describe by other means, in particular, by means of the differential equations in partial derivatives); that is to say, for today in this direction the greatest number of researches are done;*

(2) *An independent mathematical object for researches (high–parallel dynamic discrete systems; formal high–parallel calculators similarly to the Turing and Post machines, Markov systems of substitutions, etc for sequential calculations;*

words processing systems with parallel rules of substitutions).

We introduced the certain types of cellular automata which, however, were being investigated by us essentially less actively than classical ***CA*** models. Today with different degree of intensity a lot of extensions and generalizations of classical ***CA*** models defined above is used. However not each extension of classical concept of ***CA*** models leaves us within of chosen criteria of equivalence. Specifically, the polygenic deterministic, nondeterministic and stochastic ***CA*** models are essential generalizations of the classical concept, whereas widely–used extensions – ***CA*** models with the *Margolus* neighbourhood index, memory, refractority, etc. In [12,24-28,40-43] we dwelled the above types of ***CA*** models representing a rather significant interest from many points of view. In particular, ***CA*** with the *Margolus* neighborhood are oriented, mainly, on modelling of physical applications, first of all, because of their possibility to program the reversibility of the processes embedded to them. A lot of interesting enough results on the above ***CA*** extensions has been obtained [7,40-43]. Below, from the listed types of ***CA*** models, we will consider so–called polygenical ***d–CA*** when instead of a single function more than one ***GTF*** $\tau^{(n)}$ is used, and at a discrete moment $t > 0$ to the current configuration $c^{**} \in C(A, d)$ is applied one of allowable global transition functions.

At last, the ***CA*** theory is a rather advanced independent field of modern cybernetics which has considerable domain of applications in different branches of science and engineering. The architecture of the theory and its applications from our standpoint has been presented in works [9,12, 27,40-43]. The architecture takes into account our previous attempts in this direction along with the most basic recent applied and theoretical results in the ***CA*** theory. We hope that the presented architecture will be described in detail and will be verified in the relevant scope, since its analysis can be useful in choice of subsequent directions for researches in this field.

Naturally, the presented standpoint on principal architecture of the ***CA*** problematics, including its theoretical and applied aspects along with the basic components of the apparatus of researches in the given field, appreciably has a subjective character, allowing meanwhile at certain presumptions to receive an quite definite common picture about a state of the given problematics as a whole.

CA models well enough reflect specifical features of the systems basing exclusively on local interaction of elements and providing computing universality on the assumption of maximal parallelism of functioning.

At the same time, the applied aspects of modelling have been widely investigated from the theoretical point of view. The applied aspects of the ***CA*** problematics are rather extensive, covering of modern natural sciences fuelds such as modelling of dynamics of liquids & gases, many physical, chemical, biological and geological processes, processing of images**;** computing sciences, artificial intellect, robotics, modelling of climatic processes, social processes, etc. For this reason, we undertook attempt to define an architecture of ***CA*** problematics from the our point of view. The offered architecture carries appreciably subjective nature and does not pretend to exhaustive completeness. At the same time, in the architecture certain remarks and offers received after discussion of the given question along materials of some our previous publications have been taken into account. Note, questions concerning the presented architecture presenting the certain gnosiological interest are considered in detail in our works [9,12,13,27,40-43,82,102,106].

Today, the ***CA*** theory presents a rather advanced subsection of abstract automata theory with own problematics and own methods of research along with numerous appendices. In works [9,12,27,40-43] we present a brief sketch of the basic methods and approaches which make up a ***core*** of the apparatus of research of various aspects of the ***CA–***problematics**;** at thain additiont, the basic accent was done on the classical ***CA*** models being the major object of our research. Naturally, being one of types of dynamic systems and formal systems of words processing, ***CA*** models suppose use of the much wider spectrum of research means from many fields. Our experience of investigations in the ***CA-***problematics both on theoretical, and especially applied level once again allow us to focus on the fact that**:** ***CA–***models *(cellular automata)* represent a especial class of infinite abstract automata with a certain specific internal structure that provides immensely high**–**parallel level of the information processing and calculations**;** these models form a specific class of discrete dynamic systems that function in especially parallel way on base of principle of local short**–**range interaction. At the same time, any attempts to present ***CA*** models as some models of the universe or something like this seem to us empty speculation and a kind of pseudo**–**scientific vulgarization. Should not multiply the essences, attributing to the ***CA*** models what they do not possess. With questions of understanding of the location of the ***CA*** models and their epistemological interpretation in the system of modern scientifical knowledge the interested reader can familiarize in the extended bibliography [7,24-28,82,43,102,106,278,286].

Chapter 2. Nonconstructability problem in classical Cellular Automata *(CA)*

2.1. Preliminary information on the problematics

Above all, hereinafter as the concepts «***nonconstructible configuration (NCF)***» and «*nonconstructability*» *(at that, this term in a certain degree can be associated with the term «non–constructivity»)* as a rule is understand the block configurations such as «***Garden–of–Eden***» and existence in ***d–CA*** ***(d≥1)*** of configurations of such type accordingly, i.e. the concept of *nonconstructability* determines one of fundamental characteristics of ***CA*** models that consists in presence for them of configurations that can`t be generated in moment ***t > 0*** from an arbitrary configuration in moment ***t=0.*** The *nonconstructability* problem has more wide comprehension that is briefly characterized as follows.

First of all, concerning the classical ***d–CA*** ***(d≥1)*** we deal with two sets of essentially different configurations**:** ***finite*** configurations ***C(A,d,ϕ)*** and ***infinite*** configurations ***C(A,d,∞);*** in the aggregate these sets constitute the set ***C(A,d)*** of all configurations, i.e. ***C(A,d) = C(A,d,ϕ)∪C(A,d,∞).*** The generally accepted nonconstructability concept directly concerns to the impossibility of generating from any configuration ***c∈C(A,d)*** by means of global transition function of a classical ***d–CA*** ***(d≥1)*** of a configuration containing a certain block configuration.

Meantime, the principal difference of *finite* and *infinite* configurations in case of classical ***d–CA*** ***(d≥1)*** allows quite naturally to differentiate the above nonconstructability concept what provides more detailed study of dynamics of classical ***CA*** models along with receiving of a number of results which bear fundamental character.

In particular, along with nonconstructible block configurations an quite appropriately makes sense investigation of nonconstructability of finite configurations relative as the set ***C(A,d)*** as a whole and the set ***C(A,d,ϕ).*** This approach allows naturally to introduce **2** new nonconstructability concept, namely ***NCF–1*** and ***NCF–2*** that are not equivalent as between themselves and to the standard concept ***NCF.*** Along with the generally accepted nonconstructability concept**,** certain other important enough nonconstructability concepts will be defined and considered**,** including the above**–**mentioned.

Generally speaking, the ***reversibility*** is a multiaspect enough concept.

For classical ***d–CA*** ***(d≥1)*** being a subclass of parallel discrete dynamical systems the research of the reversibility of dynamics *(trajectories)* of the finite configurations seems both interesting and natural. It is natural to assume that a configuration $c \in C(A,d,\phi)$ has reversible dynamics if for it a c^p configuration *(direct or indirect predecessor)* is sole, where $p \in \{-1, -2, -3, ...\}$ and $c^p\tau^{(n)} = c^{p+1}$, $c^p \equiv c$. However under such condition we have **2** alternatives**:** ***(1)*** c^p should belong only to the set $C(A,d,\phi)$, or ***(2)*** the set $C(A,d,\infty)$**.** Therefore in view of that below we shall define a number of concepts of reversibility, what allows to consider this concept relative to classical ***CA*** models more comprehensively. So, we shall define the concepts of formal and real reversibility by reason of two types of the nonconstructability for classical ***CA*** models *(**NCF** and **NCF–1**)*.

Questions of nonconstructability are fundamental in the mathematical theory of ***CA*** models and their numerous appendices especially at use their as conceptual and applied models of spatially–distributed discrete dynamic systems from which real physical systems are most preferable prototypes. Exactly because of this reason the given problematics opens questions of consideration of theoretical aspects of classical ***CA*** models.

The *nonconstructability* problem presents a rather serious gnoseological interest in case of embedding in ***CA*** models of cosmological objects and phenomena. That can be associated with various aspects of problem of reachability of those or other conditions or aggregations at formation of special cosmological objects. In patricular, reversibility of basic physical processes and phenomena can be as an analogue of absence of certain types of nonconstructability in classical ***CA*** models [7,24-28,40-43]. This problematics becomes more and more actual both in view of formation of modern existential physical theories, and in connexion with a whole series of attempts of interpretation of various phenomena of abnormal character from the traditional standpoint.

In this connection the problem of central importance is characterization of global behaviour *(dynamics)* of ***CA*** models as effect of local transition function ***(LTF)***. Since study of behaviour of configurations in ***CA*** plays a basic part at investigation of ***CA*** dynamics**,** it is extremely interesting to find conditions of existence of *nonconstructible* configurations ***(NCF)*****,** i.e. certain constructive limitations of ***CA*** models. Along with that, the *nonconstructability* problem presents rather considerable gnoseological interest. The problem takes place both for *monogenic***,** and for *polygenic* ***d–CA*** and also to some extent for finite structures [7,24-28,40-43]. At the

same time, along with aforesaid the nonconstructability problem can be considered as a rather important component of own research apparatus concerning dynamics of classical ***CA*** models. We considered a series of features of this problematics in a class of finite ***CA*** models [40-43] while to the finite ***CA*** models large enough attention was devoted, first of all, by the Japanese research school [7,62-64].

Whereas for polygenic ***d–CA*** this problem is known as the *completeness* problem**:** ***Whether can a configuration from C(A,d,ϕ) be generated from the given initial primitive configuration by means of a finite sequence of global transition functions of a polygenic CA model?*** A series of the researchers was engaged in research of the completeness problem, and they have received a lot of rather interesting results**.** For solution of the problem the technics for the first time suggested by ***Yamada–Amoroso*** [65] along with use of graph theory by ***Nasu–Honda*** [66] has been used. So, ***H. Yamada*** and ***S. Amorozo*** have proved**:** ***Exists a binary finite CF c* that can't be generated from the primitive configuration*** c_p ***by means of application of any finite sequence of global transition functions*** $\tau_j^{(n)}$ ***of a binary polygenic 1–CA with neighbourhood template of size n = 2.*** Omitting a series of intermediate results, the final solution of the given problem has been received by ***A. Maruoka*** and ***M. Kimura*** which have proven the next rather important result in the general case, i.e. without restriction on size of neighbourhood template [7,67,68,82,102,106,286].

<u>Theorem 1</u>. An arbitrary finite configuration c* can be generated from a primitive configuration $c_p \in C(A,d,\phi)$ ***by certain finite sequence of global transition functions*** $\tau_j^{(n)}$ ***of an appropriate polygenic d–CA (d≥1) given in the same finite alphabet A of inner states.***

So, theorem ***1*** gives exhaustive solution of the *completeness* problem for *polygenic* ***CA*** models. However, along with this problem the problem of monotonous generation of finite configurations is considered also [7,65-68]**.** In this way, completeness problem to a certain extent characterizes constructive opportunities of the polygenic ***CA*** models, and its positive solution confirms rather wide possibilities of cellular automata of this type for generating by them of finite configurations. Actually, on basis of result of theorem ***1*** we had shown**:** ***An arbitrary finite configuration*** $c \in C(A,d,\phi)$ ***can be generated from a nonzero configuration*** $c_o \in C(A,d,\phi)$ ***by means of application to it of a finite sequence of global transition functions of a polygenic d–CA (d≥1).***

Proof of this assertion is rather simple. Thereby, our assertion about full

constructibility for case of polygenic ***CA*** models easily follows from the afore–said and theorem ***1.*** In spite of direct connection of completeness problem with other questions of ***CA*** dynamics, in more details it here is not considered**;** however, separate results on it will be presented below in a context of other questions of ***CA*** problematics. With more detailed information the interested can be familiarized in the above works, and in appropriate bibliography [7,24-28]. Entirely other picture takes place for case of classical ***CA*** models. In the network of this problematics the problems of surjectivity and injectivity of global mappings induced by global transition functions are researched too. Detailed enough analysis of results in this direction is presented in works [7,12,24-28,40-43,69-81]**;** certain from them are represented below.

The first researches on the nonconstructability problem *(in Russian but already good settled terminology)* go back to known works of ***E.F. Moore*** and ***J. Myhill*** that have performed a whole series of interesting enough researches and formed this direction [7,75,76]. In a certain sense we can note that properly speaking, the *mathematical theory* of cellular automata has grown of the above problematics that till now keeps and urgency, and appeal. In the present chapter the most considerable results**,** and a modern situation of the nonconstructability problem in the classical ***CA*** models along with discussion of the further ways of researches in this direction are represented.

First of all, with the purpose of more profound coverage of all types of nonconstructability, the four classes of ***NCF*** are entered, and relations between them are established, expanding the results received today in this direction. In addition, a series of criteria of existence in classical ***CA*** models of various types of nonconstructability is established. Certain of these criteria are more convenient for theoretical qualitative researches, whileereas others allow to receive more comprehensible estimations for the basic numeric characteristics of classical cellular automata.

Within the given problematics special attention is given to algorithmic aspect of the nonconstructability problem along with its interrelations with other questions of dynamics of classical ***CA*** models. In the further representation of the ***NCF*** problematics if the opposite is not stipulated, the basic discussion will be carried out for case of classical structures ***1–CA*** though the majority of results here are being generalized to case of the classical cellular automata of supreme ***d*** dimensionalities too ***(d ≥ 2)***.

Results presented in the chapter solve the nonconstructability problem as a whole, while single–purpose questions considered in this direction

allow to research the problem in details. In addition the received results on the ***NCF*** problematics allow to form an effective enough apparatus of research of dynamics of the classical ***CA*** models.

2.2. The nonconstructability types for the *CA* models

Presentation of all types of *nonconstructability* in classical ***CA*** models we shall begin with the concept going back to ***E.F. Moore*** and ***J. Myhill,*** on whose basis a lot of basic results on dynamics of ***CA*** models had been received which appreciably stimulated theoretical researches in the ***CA*** problematics as a whole [7,24,75,76,82,102,106,278,286].

Definition 1*. A configuration $c_b \in C(A,d,W)$ of a finite d–dimensional W hypercube (block) of elementary automata in d–CA is nonconstructible configuration (NCF) if and only if doesn`t exist configuration $c \in C(A,d)$ such that $c_b \subset c\tau^{(n)}$ $(d \geq 1)$. The NCF nonconstructability concerning the finite configurations is equivalent to existence of such configurations c from the set $C(A,d,\phi)$ for which there are no any predecessors from the set $C(A,d,\phi) \cup C(A,d,\infty) = C(A,d)$, i.e. $(\forall c^* \in C(A,d))(c^*\tau^{(n)} \neq c)$, where $\tau^{(n)}$ is the global transition function of the appropriate d–CA $(d \geq 1)$ model. A classical d-CA model possesses the NCF nonconstructability only if for it exists at least one pair of such finite configurations $c1,c2$ $(c1 \neq c2)$ that the relation $c1\tau^{(n)} = c2\tau^{(n)}$ takes place where $\tau^{(n)}$ is the global transition function of this d–CA $(d \geq 1)$ model.*

From this definition follows, that a configuration c_b of a finite block of individual automata will be for a ***d–CA*** model as ***NCF*** if and only if it cannot be as a subconfiguration of a certain configuration of the model at the time moment $t > 0$. We shall name similar nonconstructability as ***block nonconstructability*** *(or **NCF–type**)*. If a block configuration c_b is ***constructible,*** then it obviously will have ***c–***predecessors $\{c_b \subset c\tau^{(n)}\}$ as from the set $C(A, d, \phi)$, and from the set $C(A, d, \infty)$. The given concept of nonconstructability is the strongest *(to a certain degree it can be called the `absolute`)*. Meantime, earlier it has provoked a series of discussions and misunderstandings, therefore the *nonconstructability* concept in classical ***CA*** models was analysed in detail and differentiated by in terms of the essence of classical ***CA*** models. As a whole, definition ***1*** represents the generalized concept of the ***NCF*** nonconstructability as at the level of the block, and finite configuration, in a natural way identifying both these concepts. In addition, the 2^{nd} approach to the ***NCF*** nonconstructability

concept is presented to us more preferable from standpoint of research of various aspects of dynamics of classical ***CA*** models. Same treats also the mutual erasability concept that is the cornerstone of one of criteria of existence of the ***NCF*** nonconstructability in ***CA*** models [24-28,40-43].

In view of differentiation of the set $C(A,d)$ into non–overlapping subsets $C(A,d,\phi)$ and $C(A,d,\infty)$ we can quite naturally differentiate the common nonconstructability problem of finite configurations in the classical ***CA*** models concerning these subsets what is visually illustrated by the next table which isn't demanding any special explanations.

Predecessors existence for configuration $c\in C(A,d,\phi)$		
$C(A,d,\infty)$	$C(A,d,\phi)$	*Nonconctructability type*
–	–	***NCF***
+	–	***NCF–1***
–	+	***NCF–2***
+	+	***ACCF***

It must be kept in mind that the nonconstructability of ***NCF–3*** type that very closely adjoins the absolute ***NCF*** nonconstructability *(in contrast to the absolute constructability* ***– ACCF****)* hasn't found reflection in the table**;** the ***NCF–3*** type is determined below**,** playing a certain part in dynamics research of the classical ***CA*** models. The above table exhausts the basic nonconstructability types of finite configurations in classical ***CA*** models whereas the nonconstructability question of infinite configurations goes out of the frame our consideration, first of all, due to their insufficiently elaborated processing, interpretation and formation principles. In the mean time, the infinite configurations, for example, ***1–***dimensional can be researched in connexion with a possibility of representation by them of numerical or others well interpreted objects [24,43,82-87,102,106].

Choosing a certain set $C^*\subset C(A)$ as allowable configurations, we have a possibility to define so–called ***relative*** nonconstructability in contrast to *absolute nonconstructability (definitions* ***1*** *and* ***4****)* which allows not only to research essentially more in detail the nonconstructability *essence* in ***CA,*** but also to receive powerful enough tools of research of many dynamic properties of classical ***CA*** models.

<u>Definition 2</u>. A configuration B_w of a finite d-dimensional hypercube of elementary automata in a d-CA model we will call the nonconstructible relatively to a set S (NCF_S) only if doesn`t exist such configuration $c^*\in S\subseteq C(A,d)$ ***that the following relation*** $B_w\subset c^*\tau^{(n)}$ ***takes place*** $(d\geq 1)$.

Obviously, in ev ent of identity $S \equiv C(A, d)$ the concepts of *absolute* and *relative* nonconstructability coincide. Otherwise, each ***NCF*** will be in the same ***CA*** model as NCF_S relative to a preassigned set $S \subset C(A,d)$, but not vice versa. Thus, the concept of relative nonconstructability in classical ***CA*** models has a series of interesting interpretations of both theoretical, and applied character, stimulating its subsequent researches that now are enough being activated [7,24-28,40-43,82-87,102,106,278,286].

Strictly speaking, a reason of differentiation of the nonconstructability concept in classical ***CA*** models is being defined by differentiation of the set of all configurations $C(A,d)$ into two noncrossing subsets of the *finite* $C(A,d,\phi)$, and *infinite* $C(A,d,\infty)$ configurations which concerning parallel mappings $\tau^{(n)}$ *(global transition functions)* are non–equivalent along with the used definitions of configurations [$C(A,d,\phi) \cup C(A,d,\infty) = C(A,d)$ *and* $C(A,d,\phi) \cap C(A,d,\infty) = \varnothing$, *where* $\varnothing$ *is the empty set*].

So, if the set $C(A,d,\phi)$ is closed relatively to a mapping $\tau^{(n)}$ of a classical ***d–CA*** $(d \geq 1)$, then the set $C(A, d, \infty)$, generally speaking, is non–closed. That is caused by existing of a special ***quiescent*** state which satisfies the condition $\sigma^{(n)}(0, 0, ..., 0) = 0$ $\{0 \in A\}$ for local transition function of a ***d–CA*** $(d \geq 1)$. Therefore, classical ***CA*** model of such type can serve as a certain formal analogue of physical reality and during further consideration *(if the opposite was not stipulated)* we shall consider the classical ***CA*** models, mainly, considering briefly and some other interesting types.

Use of sets of ***finite, block*** and ***infinite*** configurations allows to advance appreciably both differentiation, and specification properly of concept of nonconstructability in classical ***d–CA*** relative to the previous state of this question. Enough simply we can make sure, the *nonconstructability* concept such as ***NCF*** relates to the ***block*** configurations, allowing us to consider two, generally, nonequivalent classes of nonconstructability: *(1) block* nonconstructability and *(2) configuration* nonconstructability in classical ***CA*** models.

Indeed, let for a ***1–CA*** exists a block configuration c^*_b, being for it ***NCF.*** Then in conformity with definition ***1*** a configuration $\square c_b \square = c \in C(A, d, \phi)$ also will be ***NCF;*** whereas the opposite is generally incorrect, about it a simple example testifies. We consider classical ***1–CA*** with alphabet $A = \{0,1,2\}$, neighborhood index $X = \{0,1\}$, and global transition function $\tau^{(2)}$, which at local level is determined by parallel substitutions of the kind**:**

$$00 \to 0 \quad 01 \to 1 \quad 02 \to 1 \quad 10 \to 1 \quad 11 \to 2 \quad 12 \to 1 \quad 20 \to 2 \quad 21 \to 1 \quad 22 \to 1$$

Obviously, at condition c_0=□*11*□ we obtain $c_0\tau^{(2)}$ = □*121*□, i.e. the block configuration c_b = *2* is not ***NCF*** in such ***CA*** model. While configuration c_1=□2□ will be ***NCF*** in the ***CA*** model. So, the *block* nonconstructability of type ***NCF*** provokes the *configuration* nonconstructability whereas the opposite is, on the whole, incorrect. Therefore, it is possible to define a new type of nonconstructability which arises on the border of the *block* and *configuration* nonconstructability, allowing its qualitative extension. The ***NCF–3*** nonconstructible configurations are defined as follows.

Definition 3*. A configuration* c^* = □c_b□∈$C(A,d,\phi)$ *is nonconstructible of type NCF–3 if and only if the block configuration* c_b *of d–dimensional hypercube W of elementary automata in a d–CA (*$d \geq 1$*) is constructible but configuration* c^* *is nonconstructible, where* □ *– edging of the block configuration* c_b *by infinite number of states «0» (in other words, on all other elementary automata of the classical d–CA model, i.e. outside of the W block).*

Obviously, a configuration $c \in C(A,d,\phi)$ being ***NCF–3*** is as well the ***NCF*** however it cannot be neither ***NCF–1,*** nor ***NCF–2.*** At that, it is simple to show, exists not less $N=(a-2)^{a-1}[(a-1)^{a^n-a} - (a-2)^{a^n-a}]$ of classical ***1–CA*** models that possess the ***NCF–3*** nonconstructability. At the heart of that the simple enough combinatorial considerations lay [24,42,82,102,106].

In view of the above remarks, additionally to *2* considered types of the nonconstructability *(**NCF** and **NCF–3**)* we determine *2* important types of nonconstructability in classical ***d–CA.*** These types are conditioned, first of all, by a feature of ***CA*** models that allows to ascribe their to the special class mentioned above and which naturally allows to introduce for the set $C(A,d)$ of all configurations its differentiation into *noncrossing* subsets of *finite* $C(A,d,\phi)$ and *infinite* $C(A,d,\infty)$ configurations.

Definition 4*. A finite configuration* $c^* \in C(A,d,\phi)$ *is nonconstructible of NCF–1 type for a classical d–CA if and only if* $(\exists c' \in C(A,d,\infty))(c'\tau^{(n)}=c^*)$ *and* $(\forall c \in C(A,d,\phi))(c\tau^{(n)} \neq c^*)$*. On the other hand, a finite configuration* $c^* \in C(A,d,\phi)$ *is nonconstructible of NCF–2 type for a classical d–CA, if and only if* $(\exists c' \in C(A,d,\phi))(c'\tau^{(n)} = c^*)$ *and* $(\forall c^\infty \in C(A,d,\infty))(c^\infty\tau^{(n)} \neq c^*)$.

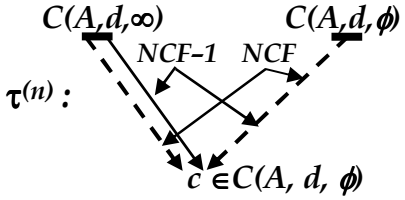
Obviously, if in a ***d–CA*** *(**d**≥**1**)* exist ***NCF–1*** then $(\exists c \in C(A,d,\infty))(c\tau^{(n)} = \square)$. At that, in a series of cases the distinctions *(prima facie frequently hardly perceptible)* between ***five*** types of nonconstructability determined by us introduce rather essential qualitative influence on dynamics of classical

CA models. With a view of more precise concentration of our attention on the above four concepts of nonconstructability by the words of more clear terminology it is possible to establish, that**:**

*(**1**) The block nonconstructability (nonconstructability of **NCF** type) is being characterized by impossibility of generating of a block configuration c_b from a **c** configuration, i.e. $(\forall c \in C(A,d))(c_b \not\subset c\tau^{(n)})$; naturally, that any configuration containing such block configuration will be nonconstructible of **NCF** type;*

*(**2**) Nonconstructability of **NCF–1** {**NCF–2**} type is determined by existence of a configuration $c^* \in C(A,d,\phi)$ that has predecessors (i.e. configurations c' such that $c'\tau^{(n)} = c^*$) only from the set $C(A,d,\infty)$ $\{C(A,d,\phi)\}$;*

*(**3**) At last, nonconstructability of **NCF–3** type is determined by the condition that a finite configuration $c = \square c_b \square \in C(A, d, \phi)$ constructed on the basis of block constructible configuration c_b is not constructible **CF**, i.e. the **c** configuration has not predecessors from the set $C(A, d)$ of infinite and finite configurations. It is easy to be convinced: **If a classical d–CA ($d \geq 1$) model possesses the nonconstructability of NCF or/and NCF–1 type then NCF and NCF–1 respectively will be infinite sets of configurations*** [24,41]. Here it makes sense to clear more in details difference between nonconstructability of finite configurations of the ***NCF*** and ***NCF–1*** types. Graphically the basic distinction between ***NCF*** and ***NCF–1*** configurations can be represented by the following diagram, namely**:**



where**:** ——— **–** existence of predecessors c^{-1} for a finite **c** configuration and **------ –** absence of predecessors c^{-1} for case of the **c** configuration of ***NCF*** type. So, of this diagram the difference of ***NCF*** nonconstructability from ***NCF–1*** nonconstructability is rather clear. Besides, ***NCF*** represents so-called *absolute* nonconstructability when a finite configuration has no predecessors from the set $C(A,d)$**.** Therefore, for a number of reasons the research of the ***NCF*** and ***NCF–1*** nonconstructability is represented to us more preferable.

Meantime, for case of classical ***1–CA*** models it is rather simple to obtain estimate of existence of ***NCF-1*** configurations in them. We will consider

the classical ***1–CA*** models with alphabet $A=\{0,1,\ldots,a-1\}$, neighbourhood index $X=\{0,1,\ldots,n-1\}$ and ***LTF*** $\sigma^{(n)}$ defined as follows:

$$\sigma^{(n)}(0,0,\ldots,0)=0;\ \sigma^{(n)}(0,0,\ldots,x_n)\in A\setminus\{0\};\ \sigma^{(n)}(x_1,0,\ldots,0)\in A\setminus\{0\};\ x_1,x_n\in A\setminus\{0\}$$

Obviously, that for such ***1–CA*** models the next dependence takes place $(\forall c\in C(A,d,\phi))(|c\tau^{(n)}|>|c|)$, where $|h|$ – diameter of a h configuration.

Proceeding from the given dependence it is simple to prove that among models of this group there are ***CA*** models possessing the configurations of type ***NCF*** and ***NCF–1;*** number of ***CA*** models of this group is equal to $(a-1)^{2a-2}*a^{a^n-2a+1}$, whereas density of this group concerning all ***1–CA*** models is defined as $\Delta\approx e^{-2}$, i.e. quota of this group relative to all ***1–CA*** models with dimensionality increasing of ***A*** alphabet tends to a limit.

A simple enough modification of this reception *(for example, by means of choice of neighbourhood template in form of* ***d–****dimensional hypercube)* works also for case of dimensionality $d>2$. Taking into account the aforesaid, further we will consider ***4*** main nonconstructability types ***NCF***, ***NCF–1***, ***NCF–2*** and ***NCF–3*** in classical ***CA*** models.

In work [42] the existence problem of ***NCF–1*** for elementary types of ***1–CA*** models with neighbourhood index $X=\{0,1\}$ has been considered. In this direction a lot of interesting enough results have been received, in particular, ***1–CA*** with ***LTF*** $\sigma^{(2)}(x,y)=x^*=x+y\ (mod\ 2)$ $\{x,y\in B=\{0,1\}\}$ not possesses ***NCF*** and possesses finite configurations of ***NCF–1*** type which contain odd number of states `***1***`; at that, quota of ***NCF-1*** configurations relative to the set $C(B,1,\phi)$ of all binary finite configurations equals ***1/2.*** Whereas for so-called ***"linear" 1–CA,*** i.e. models with a neighbourhood index $X=\{0,1,2,\ldots,n-1\}$, a states alphabet $A=\{0,1,2,\ldots,a-1\}$, and ***LTF*** $\sigma^{(n)}$:

$$\sigma^{(n)}(x_o,x_1,\ldots,x_{n-1})=x^*_o=\sum_{j=0}^{n-1}b_jx_j\ \ (mod\ a);\ \ b_j,x_j,x^*_o\in A;\ j=0..n-1$$

has been shown, for such ***1–CA*** models the next relation takes place:

$$\bigcup_j\{c_j\tau^{(n)k}\mid k=0,1,2,3,\ldots\}=C(A,1,\phi)\setminus\{c_j\};\qquad c_j\tau^{(n)0}\equiv c_j,$$

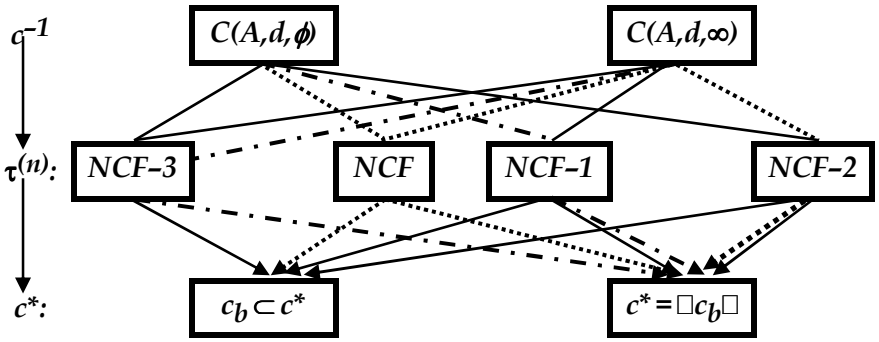
$$where\ \{c_j\}-a\ set\ of\ all\ finite\ configurations\ of\ NCF\text{-}1\ type$$

Hence, in ***1–CA*** of such type the set of all finite configurations is being generated from a set of configurations of ***NCF–1*** type, including ***NCF–1*** themselves. In other words, a set ***NCF–1*** is that basis from which the set $C(A,1,\phi)$ is generated. This result can be relatively simply generalized to

general case of *linear* classical ***d–CA*** ***(d≥1)***. Moreover, the above result is generalized to ***d–CA*** for which the relation $(\forall c \in C(A, d, \phi)(|c\tau^{(n)}| > |c|)$ takes place where $|h|$ is maximal diameter of a finite h configuration.

In addition, presence in ***CA***-models of *nonconstructible* configurations of ***NCF-3*** type determines to a certain extent a rather unexpected result**:** ***at presence of a constructive core (nonzero part) for a finite configuration, the configuration can be absolutely nonconstructible.*** So**,** presence for a ***CA***-model of ***NCF-3*** nonconstructability necessarily entails presence for it of ***NCF*** nonconstructability too**;** whereas converse assertion generally, speaking, incorrectly. So, a binary ***1–CA*** with neighbourhood index $X = \{0, 1\}$ and ***LTF*** $\sigma^{(2)}(x, y) = x^*y$ possesses the ***NCF*** nonconstructability *(for instance*, ***CF 101****)* whereas in the model the ***NCF–3*** nonconstructability is absent. In addition, it is possible to show, in class of simple ***CA*** models $<Z^1, A, \tau^{(2)}, X=\{0,1\}>$**,** the models exist that have ***NCF-3*** of minimal size $m = a–1$ *(where* ***a*** *is cardinality of* ***A****)* [41]. Other interesting estimations and characteristics for ***1–CA*** of similar type can be found in [1,9,13,41,82-87].

Four types of the nonconstructible configurations *(****NCF****,* ***NCF–1****,* ***NCF–2****,* ***NCF–3****)* introduced above are pairwise nonequivalent and allow more in detail to investigate the nonconstructability problem in the classical ***d–CA*** models *(****d*** **≥** ***1****)*. So, the ***NCF–1*** nonconstructability allows the more rigorously to investigate the question of dynamics reversibility of finite configurations in the classical ***d–CA*** models *(****d*** **≥** ***1****)*. The diagram below illustrates interconnections of the above four types *(****NCF****,* ***NCF–1****,* ***NCF-2*** *and* ***NCF–3****)* of the nonconstructability *(fig. 3)*.



where $c^{-1}\tau^{(n)} = c^*$***;*** c^{-1} ***– a predecessor for a finite*** c^****configuration.***

Fig. 3. Diagram illustrating the essence of basic 4 types of (NCF, NCF-1, NCF-2, NCF-3) nonconstructability in the classical d–CA models (d ≥ 1)

Thus, on fig. ***3*** the essence of concepts determined by nonconstructible block configurations and finite c^* configurations that are determined by

absence for them of predecessors from the sets $C(A,d,\phi)$ or/and $C(A,d,\infty)$ is schematically submitted. Thus, decomposition of the diagram *(fig. 3)* onto more detailed components *(fig. 4)* allows to make the interrelation picture between introduced **4** fundamental types of nonconstructability considerably more transparent.

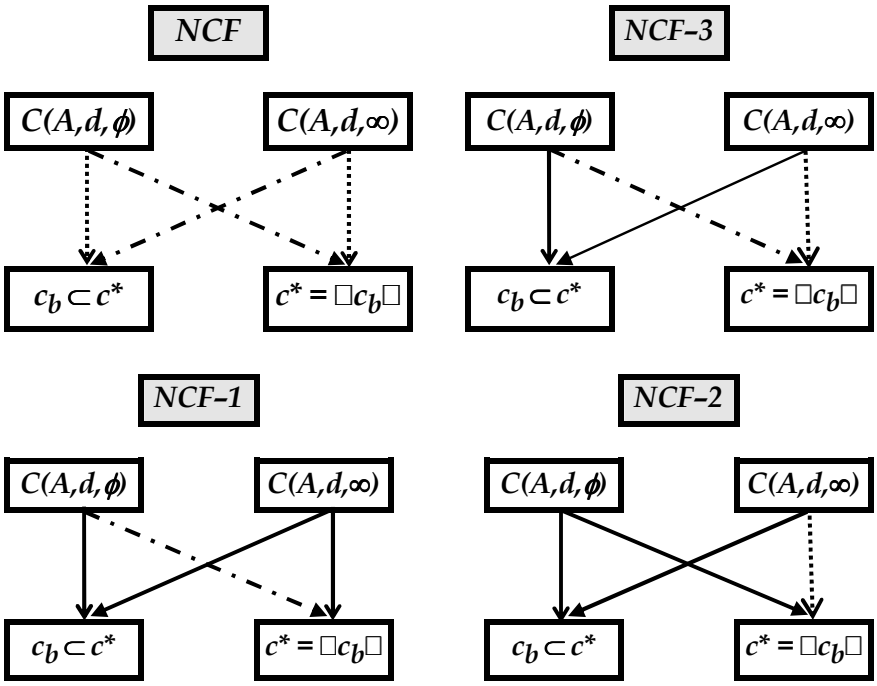


Fig. 4. Diagrams representing possibilities of existence of predecessors in sets $C(A,d,\phi)$, and $C(A,d,\infty)$ for configurations $\{c_b \mid c = \square c_b \square\}$ relative to the above four types of the nonconstructability.

Moreover, the continuous *(dotted, dashed)* marking of lines on figs. **3** and **4** designates accordingly the admissibility *(impossibility)* of the existence of predecessors c^{-1} for any configuration c^* in classical ***d–CA*** $(d \geq 1)$. In addition, if ***NCF (NCF–3)*** – the absolute nonconstructability relative to the set $C(d,A) = C(d,A,\phi) \cup C(d,A,\infty)$, whereas ***NCF–1,*** and ***NCF–2*** will be nonconstructability relative to sets $C(d, A, \phi)$ and $C(d, A, \infty)$ accordingly. It is obvious, at constructability of a finite configuration $c^* = \square\, c_b \,\square$, the nonconstructability of a block configuration c_b is impossible. Therefore, ***NCF, NCF–1, NCF–2*** and ***NCF–3*** cover four most interesting occasions of existence of nonconstructible configurations in classical ***CA*** models. The following basic result expressed by theorem **2** reveals the relations between the above four types of nonconstructability in classical ***d–CA.***

Theorem 2. An arbitrary classical d–CA (d ≥ 1) has at least one type of nonconstructability NCF (and, possibly, NCF–3), NCF–1 or NCF–2. The non–empty sets of NCF, NCF–1, NCF–2 and NCF–3 for a classical d-CA model are infinite. A global transition function of a classical d–CA can possess the nonconstructability types according to the table 2 which is presented below. If for a classical d–CA the set C(A, d, ∞) is non–closed relative to the mapping determined by its global transition function $\tau^{(n)}$ then d–CA will possess the nonconstructability of type NCF–1 and/or NCF while converse proposition in general case is incorrect. At that, a classical d–CA that not possess NCF–1 and NCF (NCF–3) will possess NCF–2; an arbitrary classical d–CA (d ≥ 1) which not possesses NCF–2 will possess NCF-1 and/or NCF. There are classical d–CA (d≥1) models for which all constructible finite configurations, at the same time, are absolutely constructive, i.e. simultaneously have predecessors as from the set C(A,d,ϕ), and the set C(A,d,∞). So, there are classical d–CA (d ≥1) for which constructive finite configuration has at least one predecessor from the set C(A,d,∞). If for a classical d-CA (d≥1) model exists the NCF nonconstructability at the absence of such configurations c ∈ C(A, d, ∞) that $c\tau^{(n)}$=□ ($\tau^{(n)}$ is global transition function of the model), then it will be possess the NCF and NCF-2 nonconstructability. There are d-CA (d≥1) models that not possess the NCF nonconstructability at existence only NCF–1 and ACCF provided that each finite configuration has at least |A|–1 predecessors from the set C(A,d,∞), |g| is a g set cardinality. A classical d–CA (d ≥ 1) model possessing the NCF configurations allows existence of combinations of the finite ACCF, NCF, NCF–1 and NCF–2 configurations according to the following table.

Admissible combinations of the nonconstructability types				*Possibility of a combination*
NCF	*ACCF*	*NCF–1*	*NCF–2*	
+	+	+	+	*Yes*
+	+	+	–	*Yes*
+	+	–	–	*Yes*
+	–	–	+	*Yes*
+	–	–	–	*Yes*
–	+	+	–	*Yes*

On the other hand, for a classical d–CA (d ≥ 1) all finite configurations can`t be absolute constructive; among them can be configurations NCF, ACCF, NCF–1 and NCF–2 in various allowable combinations. There are classical d-CA (d≥1) models for which an arbitrary finite configuration is absolute constructive or NCF–1 nonconstructible.

In particular, a classical model can possess the ***NCF*** nonconstructability in case of closure of the set $C(A,d,\infty)$ relative to the above mapping $\tau^{(n)}$. A classical ***d–CA*** $(d \geq 1)$ model can possess an arbitrary combination of nonconstructabilities of {***NCF-1, NCF***} type. As a rather simple example, three binary classical ***1–CA*** models whose local transition functions are determined as follows:

$$\sigma 1^{(3)}(x,y,z) = If(4x+2y+z \in \{4, 5\}, x, z); \quad \sigma 3^{(3)}(x,y,z) = x+y+z \ (mod\ 2);$$
$$\sigma 2^{(3)}(x,y,z) = If(4x+2y+z \in \{1, 4\}, 1, 0); \quad x,y,z \in B = \{0, 1\}$$

can serve. Obviously, is easily proved the incorrectness of the converse proposition; indeed, the binary local transition functions $\sigma 1^{(3)}$, $\sigma 3^{(3)}$ and $\sigma 2^{(3)}$ define classical ***1–CA*** which possess ***NCF*** without ***NCF–1,*** possess ***NCF–1*** without ***NCF,*** and possess both ***NCF*** and ***NCF–1*** accordingly. In turn, there are classical ***d–CA*** $(d \geq 1)$ which do not possess both the ***NCF*** and ***NCF–1,*** possessing the ***NCF–2*** nonconstructability.

If ***NCF*** ***(NCF–3)*** is an arbitrary *absolutely* nonconstructible configuration relative the set $C(A,d,\phi) \cup C(A,d,\infty)$, the configurations ***NCF-1*** and ***NCF-2*** are *relatively* nonconstructible configurations relative the sets $C(A, d, \phi)$, and $C(A, d, \infty)$ accordingly. In the table 2 the mark «**+** *(–)*» identifies the existence *(absence)* of appropriate type of *nonconstructible* configurations in classical ***d–CA*** $(d \geq 1)$, defining the admissible combinations of types. The detailed discussion of these questions from various points of view can be found in our works [24,40-43,82-87,102,106].

Table 2

Admissible types of nonconstructability for the classical structures d–CA $(d \geq 1)$				***Possibility of combinations***
NCF	***NCF–1***	***NCF–2***	***NCF–3***	
+	**+**	**+**	**+**	***Yes***
+	**+**	**+**	**–**	***Yes***
+	**+**	**–**	**+**	***Yes***
+	**+**	**–**	**–**	***Yes***
+	**–**	**+**	**+**	***Yes***
+	**–**	**+**	**–**	***Yes***
+	**–**	**–**	**+**	***Yes***
+	**–**	**–**	**–**	***Yes***
–	**+**	**–**	**+**	***No***
–	**+**	**–**	**–**	***No***
–	**–**	**+**	**+**	***No***
–	**+**	**–**	**–**	***Yes***

–	–	+	–	*Yes*
–	+	+	+	*No*
–	+	+	–	*No*
–	–	–	+	*No*
–	–	–	–	*No*

In particular, from the table **1** follows, the classical ***d–CA (d ≥ 1)*** models have not less than one type of nonconstructible configurations, namely**:** ***NCF, NCF–1, NCF–2*** and/or ***NCF–3.*** In particular, ***128*** binary classical ***1–CA*** with the *Moore* neighbourhood index relative the *nonconstructible* configurations of the above types are differentiated as follows**:**

Type	*at least NCF*	*NCF–1 without NCF*	*only NCF–2*
Quantity	***113 ≈ 88.3%***	***10 ≈ 7.8%***	***5 ≈ 3.9%***

Proof of the theorem **2** along with a lot of interesting enough examples of classical ***d–CA (d ≥ 1)*** models can be found in works [82-87]. Wherein, the result presented below allows also to consider the concept of *closure (nonclosure)* of the set ***C(A, d, ∞)*** wide enough.

<u>Proposal 1</u>. For a classical d–CA (d ≥ 1) model there are configurations $c^* \in C(A,d,\infty)$ ***which satisfy the relation*** $c^*\tau^{(n)} = c \in C(A,d,\phi)$ ***if and only if for the model there are configurations*** $c_o^{\infty} \in C(A, d, \infty)$ ***which satisfy the relation*** $c_o^{\infty}\tau^{(n)} = \square$***, excepting the case of the trivial model whose LTF*** $\sigma^{(n)}$ ***satisfies the following relation, namely:***

$$(\forall \langle x_1 x_2 \ldots x_n \rangle \mid x_j \in A;\ j = 1..n)(\sigma^{(n)}(x_1, x_2, \ldots, x_n) = 0)$$

Furthermore, completely null configuration «□» is referred by us to the set C(A, d, ϕ) of all finite configurations defined in a states alphabet A.

Therefore, the existence for a non–trivial ***d***–dimensional mapping $\tau^{(n)}$ of configurations $c^{\infty} \in C(A,d,\infty)$ such that relation $c^{\infty}\tau^{(n)} = \square$ takes place is equivalent to the existence for the mapping of configurations c_o^{∞} such that $c_o^{\infty}\tau^{(n)} = c \in C(A,d,\phi)$**.** So, in more general case as *closure (non-closure)* of the set ***C(A,d,∞)*** of all infinite configurations relative to ***GTF*** $\tau^{(n)}$ of a classical ***d–CA (d ≥ 1)*** throughout the book we will understand *existence (absence)* in the set ***C(A, d, ∞)*** of infinite configurations $c^{\infty} \in C(A, d, \infty)$ for which the relation in the form $c^{\infty}\tau^{(n)} = c \in C(A, d, \phi)$ takes place.

We a little bit more in details illustrate the nonconstructability concept of ***NCF–3*** type. Formally, a finite configuration $c^* = \square c_b \square \{c_b = x_1 x_2 \ldots x_p;$

$x_1, x_p \in B\setminus\{0\}$; $x_j \in A$; $j=2..p-1$, $B=\{0,1\}\}$, being the nonconstructible ***CF*** of ***NCF–3*** type, is defined by the following condition, namely:

$$(\forall c \in C(A,d,\phi) \cup C(A,d,\infty))(c\tau^{(n)} \neq c^*) \;\&\; (\exists c \in C(A,d,\phi) \cup C(A,d,\infty))(c_b \sqsubset c\tau^{(n)}))$$

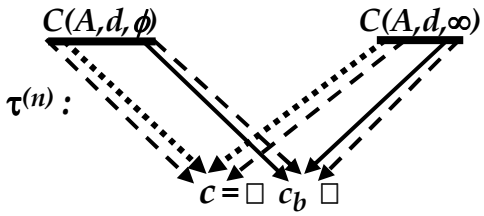
Binary classical ***1–CA*** with neighbourhood index $X=\{0,1,2\}$, whose ***LTF*** $\sigma^{(n)}$ is defined by the formula of the next kind, is considered as a rather simple but not trivial example, namely:

$$\sigma^{(3)}(x,y,z) = If(4x+2y+z \in \{0,1\},\ z,\ If(4x+2y+z \in \{2,3\},\ y,\ x+y+z+1 \ (mod\ 2)));$$
$$x,y,z \in B = \{0,1\}$$

Such representation of local transition functions in binary ***1-CA*** models, using natural numbering of tuples $<x, y, z>$, is presented rather evident, compact and convenient.

Immediate examination shows, that this binary ***1–CA*** possesses a block-nonconstructible configuration $c^* = <0010100>$ [87]. On the other hand, the block configuration $c_b = <101>$ – a core of the previous configuration c^* – is a constructible configuration, what easily follows from existence for c_b of predecessor of simple kind, namely: $c^{-1} = <01001>$. So, in case of the *block* constructability of a configuration c_b, configuration $c^* = \square c_b \square$ can be configuration–nonconstructible ***CF***, i.e. to not have predecessors from the set $C(A,d,\phi) \cup C(A,d,\infty)$ of finite and infinite configurations. It is necessary to note, the both *concepts* of configuration nonconstructability and block nonconstructability are essentially different and appreciably caused by classical type of the ***d–CA*** models, which allows naturally to differentiate the set ***C(A,d)***.

Graphically basic distinction between a configuration–nonconstructible configuration $c = \square c_b \square$ and block–nonconstructible configuration c_b can be presented by the following diagram, namely:



where: ——— – existence of predecessors c^{-1}, and ..... – absence of the predecessors c^{-1} for case of ***NCF–3***; -------- – absence of predecessors c^{-1} for case of ***NCF***. Of this diagram the difference of nonconstructability of

NCF-3 type from nonconstructability of ***NCF*** type is rather clear. In the *first* case the nonconstructability**,** called ***configurational,*** relates to finite configurations, whereas in the *second* case the ***block*** nonconstructability takes place.

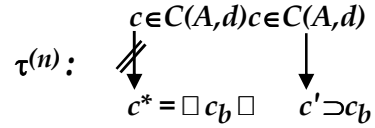
Thus, it is possible to show ***(the generalized criterion): A classical d–CA (d≥1) model possesses nonconstructability of NCF type and, probably, NCF–3 if and only if for it exist finite g configurations which not have predecessors*** g^{-1} ***from the set*** $C(A,d,\phi)\cup C(A,d,\infty)$**.** The existence problem of configurations of the given type for an arbitrary classical ***d–CA (d≥1)*** is solvable if $d=1$**,** and unsolvable if $d\geq 2$**.** Proof of this result is based on nonsolvability of known «*domino*» problem [24,43,82,102,106].

Therefore, the ***NCF–3*** nonconstructability can be considered as a *special* subclass of the general ***NCF*** nonconstructability which in certain cases presents an quite certain interest both in theoretical, and in the applied considerations of classical ***CA*** models. Above all it concerns the cases of research of ***CA*** models as formal parallel systems of processing of finite words in finite alphabets, and also at modelling on formal level of some processes, including processes of computing character. Results relative this type of nonconstructability present a certain interest as compound components of own apparatus of researches of *dynamics* of classical ***CA*** models and a lot of their abstract appendices [24-28,40-43,82-87,102].

Certainly, the ***NCF–3*** nonconstructability can be considered as a special case of ***NCF*** nonconstructability, defining existence of nonconstructible configurations of a special kind interesting from many standpoints. At that, concept ***NCF–3*** lays at the turn of the *block* nonconstructability and the *configurational* nonconstructability, belonging to both of them. So, if nonconstructability of types ***NCF–1***, ***NCF–2*** is caused by differentiation of the set $C(A,d)$ onto **2** non–overlapping subsets $C(A,d,\phi)$**,** and $C(A,d,\infty)$ of finite and infinite configurations, then distinction in the set of ***NCF*** of a separate subset of ***NCF–3*** is caused by differentiation of the ***absolute*** *nonconstructability* according to the configurations kind, most natural to ***CA*** axiomatics of classical models which defines their global dynamics. So, configuration $c^*=\square c_b\square\ \{c_b=x_1x_2\ldots x_p;\ x_1,x_p\in A\setminus\{0\};\ x_j\in A;\ j=2..p-1\}$**,** being ***NCF–3,*** is defined by the following relation, namely**:**

$$(\forall c\in C(A,d))(c\tau^{(n)}\neq c^*)\ \&\ (\exists c\in C(A,d))(c_b\subset c'=c\tau^{(n)})$$

Essence of an arbitrary configuration c^* of ***NCF–3*** type the next a rather simple diagram well enough illustrates, namely**:**



Thus, via definitions of four types of nonconstructability ***(NCF, NCF–1, NCF–2*** *and* ***NCF–3)*** we cover the given fundamental concept as a whole. The *block* nonconstructability ***(NCF–****type)* determines the strongest and leading component of nonconstructability concept as a whole**: *A block configuration c_b is nonconstructible only if not exists a configuration $c \in C(A,d)$ such, that $c_b \subset c\tau^{(n)}$.*** Whereas other base nonconstructability types ***(NCF–1, NCF–2, NCF–3)*** have relative character and are defined only by distinctions of definitions of the sets of *block*, *finite*, and *infinite* configurations relative to global mappings $\tau^{(n)}$ of classical ***CA*** models.

The nonconstructability concepts of {***NCF, NCF–3***}, and {***NCF–1, NCF–2***} are considered with respect the sets ***C(A,d)*** and ***C(A,d,ϕ)*** of infinite and finite configurations accordingly, being based on natural differentiation of the set ***C(A,d)*** of all possible configurations of a classical ***CA*** model.

Meantime, other interesting definitions of ***relative*** nonconstructability are possible too. In particular, ***U. Golze*** [88] has determined concepts of sets of *recursive* (C^r) and *rational* (C^q) configurations, which satisfy the relation $C(A,d,\phi) \subset C^q \subset C^r \subset C(A,d)$, and has investigated the kinds of nonconstructability relative to the sets C^q and C^r for classical ***d–CA*** ***(d = 1,2)***. ***U. Golze*** has shown that if in classical ***1–CA*** model a configuration $c \in C(A,1,\phi)$ has a predecessor c^* then it necessarily has also predecessor from the set C^q; configurations $c \in C(A,2,\phi)$ exist that have predecessors only from the set $C(A,2) \setminus C^r$. From these results the non–equivalence of classical ***1–CA*** and ***d–CA*** ***(d ≥ 2)*** relative to nonconstructability concept which had been entered and investigated by ***U. Golze*** follows.

Let's consider now a little bit more in details a question of existence of possible combinations of types of *nonconstructability* for classical ***1–CA.*** First of all, on the basis of the theorem ***6*** [9,10] along with the aforesaid the bottom ***(****combinations* ***9..16*** *of nonconstructability types)* of the table ***2*** identifies opportunity of the specified combinations in case of absence of ***NCF*** nonconstructability for a classical ***1–CA.*** Proof of this part of the theorem ***2*** is relatively simple, not demanding any special elucidations. While for research of admissibility of combinations ***1..8*** *(Table* ***2****)* certain concrete examples of simple classical ***1-CA*** models are considered, then for them the existence of appropriate nonconstructability types has been

established [85-87]. This proof is rather easily generalized to the general ***d–***dimensional *(**d ≥ 2**)* case of classical ***CA*** models. The detailed proof of the above assertions can be found in our works [7,24,41-43,85-87,102].

From the theorem ***2,*** in particular, follows that a ***d–CA (d ≥ 1)*** model not possess property of *absolute constructability*. Below, under designations ***NCF, NCF-1, NCF-2*** and ***NCF-3*** we shall understand both the concrete non-constructible configurations of appropriate type, and the sets of all such configurations relative to the given ***GTF*** $\tau^{(n)}$ and ***A–***alphabet of a classical ***d–CA (d ≥ 1)***. In view of told, it is quite possible to represent in a sense the upper boundary of existence of nonconstructability types in classical ***CA*** models *(Theorem* ***7*** [1,5,25,27]*)*.

Theorem 3. For a classical d–CA (d ≥ 1) model the next relations hold:

$$NCF\text{–}3 \subset NCF \subset C(A,d,\phi),\ NCF\text{–}1 \subset C(A,d,\phi);$$

$$NCF \cup NCF\text{–}1 \cup NCF\text{–}2 \cup NCF\text{–}3 \subset C(A,d,\phi).$$

Let $G = C(A,d,\phi) \setminus \{\square\}$***, then exist d–CA (d ≥ 1) models for which relation*** $G \cup \{\square\} = NCF\text{–}2$ ***or*** $G = NCF$ ***takes place, excepting the case*** $G = NCF\text{–}1$***.***

So, the result of the theorem ***3*** gives one more argument in favour of an essential distinction between the nonconstructability types ***NCF, NCF-1, NCF-2*** and ***NCF-3.*** Relative to nonconstructability problem the question about quotas of classical ***CA*** models possessing by the nonconstructible configurations of the different types represents undoubted interest. In work [75] ***E.F. Moore*** has put forward the hypothesis, that the share of ***d–CA (d ≥ 1)*** models possessing ***NCF*** configurations approach to ***1*** with growth of cardinality of ***A–***alphabet of the models. Therefore, in books [1,5] have been represented asymptotical estimations of share of ***d–CA (d ≥ 1)*** possessing ***NCF,*** and ***NCF–1*** in the absence of ***NCF*** for them.

On basis of simple enough combinatorial approaches utilizing the basic criteria of existence of the nonconstructability of types ***NCF*** and ***NCF–1*** together with some other considerations a lot of rather useful relations has been received in this direction [8]. In addition it is necessary to have in mind, that obtaining of optimal lower bounds was not pursued, but they can be useful at some quantitative analysis of classical ***CA*** models.

In particular, it was shown**:** ***Share of classical d–CA models possessing the nonconstructability types NCF and/or NCF–1 is more than (e–1)/e, irrespective of size of the neighbourhood template of the models (d ≥ 1); in addition, the quota approaches to one with growth of cardinality of the states alphabet of elementary automata.*** In [1] and in a whole series of other works the systematic investigation of it and the questions that

are connected to it had been initiated. So, ***E. Ikaunieks,*** using a simple stochastical procedure has shown that «*almost all*» classical ***d–CA*** $(d \geq 1)$ possess ***NCF*** [89]. Using the concept of **γ–**configurations [2] introduced and considered by us in the following chapter, we managed to receive an asymptotic estimation of number of classical ***d–CA*** models that not possess ***NCF.*** This result not only completely has closed the problem of *E. Moore* but also has shown degree of generality of nonconstructability concept of the ***NCF*** type. Absolutely other picture takes place in case of other types of nonconstructability. So, with growth of cardinality of ***A–***alphabet the quota of ***1–CA*** models, possessing ***NCF–1*** quickly enough decreases. Therefore, by degree of generality the ***NCF*** concept seems to us the most representative relative to nonconstructability types ***NCF–1, NCF–2*** and ***NCF–3.***

Meantime, despite absence of exact estimations of number of ***1-CA*** $(d \geq 1)$ models possessing those or other *nonconstructability* types, the received estimations allow to drawn quite certain conclusions about generality degree of the specified nonconstructability types in classical ***1–CA*** and in ***d-CA*** $(d \geq 2)$ as a whole. Proof of the relation presented below is based on the following a rather useful lemma [8,12,13,24-28,40-43,82,102,106].

Lemma 1. Number of the d-CA $(d \geq 1)$ ***models without NCF (NCF-3) with alphabet*** $A=\{0,1,...,a-1\}$ ***and neighbourhood template from n elementary automata is less than the following value N, namely:***

$$N = \prod_{j=0}^{a-1} C_{a^{n-1}}^{a^{n-1}(a-j)} = \frac{1}{(a^{n-1}!)^a} \prod_{j=0}^{a-1} \frac{[a^{n-1}(a-j)]!}{[a^{n-1}(a-j-1]!} = \frac{(a^n)!}{(a^{n-1}!)^a}$$

Hence, of simple enough combinatory considerations and the result of lemma ***1,*** quota $\Delta(a,n)$ of homogeneous structures ***d–CA*** $(d \geq 1)$ that not possess the nonconstructability of types ***NCF*** & ***NCF-3*** can be presented by the following a rather useful asymptotical relation [40].

Theorem 4. Share of classical d–CA $(d \geq 1)$ ***models with states alphabet*** $A=\{0,1,3,4,...,a-1\}$ ***and with neighbourhood template from n elementary automata without the nonconstructability of NCF (NCF–3) type can be determined by means of the following asymptotical relation, namely:***

$$\Delta(a,n) \approx \frac{1}{\sqrt{(2\pi)^{a-1} a^{a(n-1)-n}}}$$

Thus, the received estimation directly not depend on dimension of a ***d–CA*** model, confirming the fact, that with growth of values ***a*** and ***n*** the quota of models which not possess ***NCF (NCF–3)*** and with property of

dynamics *reversibility* of ***d-CA*** becomes arbitrarily small, i.e. ***d-CA (d≥1)*** models with the above feature with growth ***a*** and ***n*** become more and more «exotic» on general background of all abundance of ***d–CA (d ≥ 1)*** models. The technique of obtaining of other relations of theorem ***12*** [45] is simple enough and can be found in [7,24–28]. Ibidem the discussion of different types of relations similar to the mentioned ones along with a lot of interesting enough questions for the further research has been represented. The related estimations can be found in [24,40-43,82-87].

A lot of interesting enough results concerning the existence of ***NCF*** and ***NCF-1*** in classical ***d-CA (d≥1)*** models with states alphabet $A=\{0,1,...,a-1\}$ and an arbitrary neighbourhood index ***X***, whose ***GTF*** functions satisfy the condition $(\forall g \in C(A, d, \phi)(|g\tau^{(n)}| > |g|)$ have been received, where $|g|$ is maximal diameter of a finite configuration ***g;*** i.e. models of such type produce configurations sequences strictly increasing in diameter from a finite configuration that is different from fully null configuration $c^* =$ «□». A lot of results in this direction can be found in [40-43,82-87].

Classical ***1–CA*** models with alphabet $B=\{0,1\}$ and neighbourhood index $X = \{0, 1, 2\}$ has been considered as an example of distribution of models relative to the nonconstructability types represented above. Obviously, quantity of such models is equal ***128.*** For simplicity we shall enumerate these models by appropriate discriminating numbers**;** i.e. from ***0*** up to ***127.*** For example, the model defined by $LTF\ \sigma^{(3)}(x_o,x_1,x_2)=\sum_j x_j\ (mod\ 2)$; $x_j \in B = \{0, 1\}$, $j = 0..2$ will have the *discriminating number* ***105.*** On basis of the analysis [82-87] we have shown, that among classical binary models ***1–CA*** of the above type exist ***113*** models that possess the ***NCF*** *(possibly, additionally* ***NCF–3*** *or/and* ***NCF–1*** *too);* their discriminating numbers are

0..14, 16..23, 24..29, 31..44, 46..50, 52..59, 61.. 74, 76..84, 87..88, 91..100, 103..104, 107..119, 121..127.

Models of this subset are researched enough in detail. In particular, we had studied the model with discriminating number ***29,*** which presents a certain interpretating character and whose local transition function is defined as follows**:** $\sigma^{(3)}(x_o, x_1, x_2) = If(4x+2y+x \in \{3,4,5,7\}, 1, 0)$**.** It became clear that the model has the single block nonconstructible configuration ***"1100"*** of minimal size ***4.*** Evidently, a block configuration that contains subconfiguration ***1100*** is nonconstructible too. Obviously, for a classical ***d-CA*** model possessing ***NCF-1,*** there must exist infinite configurations $h \neq$ □ such that $h\tau =$ □, where τ **–** a global transition function of the model and □ **–** infinite zero configuration. For the model with discriminating

number ***29*** this condition is wrong, such model not possess the ***NCF–1*** nonconstructability. For computer study of minimal size of ***NCF*** for the above ***1–CA*** model the procedure ***MinNCF*** in ***Mathematica*** system was programmed. Whereas the ***NcfAll*** procedure allows to research number of nonconstructible block configurations of *length* ***n*** and their *density* ***(ρ)*** relative to all block configurations of the same length ***n,*** i.e. procedure call returns the list {***n, ρ***} whose the first element determines number of nonconstructible block configurations of length ***n,*** whereas the second element defines their density relative to all block configurations of the same length ***n.*** Furthermore, source codes of the above both procedures can be found in [27,29,49]. In particular, the package ***MathToolBox*** [49] contains a number of tools for computer study in ***Mathematica*** system of various aspects of classical ***CA*** models among *1171* tools in general.

The ***1-CA*** model with discriminating number ***120*** has a certain cognitive and applied character, and whose local transition function is defined by the following parallel substitutions as follows**:**

$$000 \to 0,\ 001 \to 1,\ 010 \to 1,\ 011 \to 1,\ 100 \to 1,\ 101 \to 0,\ 110 \to 0,\ 111 \to 0$$

Dynamics of the model is characterized by chaotic behavior, generating complicated, in many respects casual configurations from rather simple initial configurations; local transition function of the model is used as a generator of pseudorandom numbers. The local transition function was offered for use as a shifrator of sequences in cryptography. Meanwhile, ***M. Sipper*** and ***M. Tomassini*** showed, that in the first case the function badly passes a test for criterion of a consent of Pearson in comparison with other pseudorandom sequences which were received by means of other cellular automata [7,90,91]. On assumption of the criterion of the nonconstructability basing on the mutually–erasable configurations, it is simple to be convinced that the above ***1–CA*** model not possess the ***NCF*** nonconstructability. In addition, this model possesses the simplest finite configuration $w = \square 1 \square$ as a ***NCF–1*** configuration.

Other interesting example of this group of binary ***1–CA*** models is the model with discriminating number ***120*** whose local transition function $\sigma^{(3)}$ is defined as follows**:** $\sigma^{(3)}(x_0, x_1, x_2) = \mathbf{If}[4x + 2y + x \in \{0, 5, 6, 7\}, 0, 1]$**.** Analysis of dynamics of finite configurations in such model allowed to reveal: the ***1–CA*** not possess nonconstructability of ***NCF*** type, possesses nonconstructability of ***NCF–1*** type, possessing the property of essential self–reproducibility in the *Moore* sense of finite configurations. At that, the detailed analysis of both constructive opportunities and dynamical

features of the classical binary ***1–CA*** models with *Moore* neighbourhood index $X = \{0, 1, 2\}$ can be found in our works [24,40-43,82-87,102,106].

Note, that the above results regarding the classical binary ***1–CA*** models with *Moore* neighbourhood index in a great extent can be generalized to more general cases of classical ***d-CA*** models. In particular, for obtaining a rather complex dynamics we should pay the attention to the models possessing the types of nonconstructability ***NCF*** and/or ***NCF–1.*** Thus, absence for classical ***CA*** models of ***NCF*** nonconstructability *(especially in the aggregate with* ***NCF–1****)* can serve as a certain kind of *"filter"* in case of selection of models with complex dynamics of finite configurations presenting the certain applied interest [7,24-28,40-43,82-87,102,106,286].

Four types of non–constructible configurations introduced above have been considered concerning the sets $C(A,d)$, $C(A,d,\phi)$, $C(A,d,\infty)$, basing on concepts of *block*, *finite* and *infinite* configurations, that characterize so-called *absolute* concept of nonconstructability in classical ***CA*** models. A series of interesting enough interpretations in theoretical and applied aspects of ***CA*** models has a concept such as *relative* nonconstructability when constructability is considered relative to a certain subset C^* of set $C(A,d)$, for example, the most known cases $C^* \equiv C(A,d,\phi)$, $C^* \equiv C(A,d,\infty)$ or $C^* \subset C(A,d)$ – a *finite* nonempty or *infinite* set of configurations. In our opinion, it is necessary to define so-called *relative* nonconstructability in view of a lot of rather interesting interpretations in both theoretical and applied aspects of dynamics of classical ***CA*** models. Let's introduce the more general concept of the *relative* nonconstructability in classical ***CA*** models regardless of a basic set of configurations.

<u>Definition 5</u>. A configuration $c \in C(A,d)$ is nonconstructible relative to a set of configurations $B \subset C(A,d)$ and GTF $\tau^{(n)}$ of a classical d–CA ($d \geq 1$) model if and only if the relation $(\forall c^* \in B)(c^*\tau^{(n)} \neq c)$ takes place.

Meanwhile, the further specification of the relative nonconstructability is possible similarly to the considered above. So, the concept of relative nonconstructability in the more wide sense than it was done in case of classical ***CA*** models, has a number of rather interesting interpretations which stimulate its further research [43]. First of all, important enough question about interrelation of *absolute* and *relative nonconstructabilities* in classical ***CA*** models arises. The next result gives the partial answer to this question, interesting from many points of view [24,40–43,82,102].

<u>Theorem 5</u>. If a classical d–CA model not possess NCF, then it also not possess NCF–3; such model with respect to a set $B = C(A,d) \setminus C$ possesses

NCF-1 and/or NCF-3, and can possess NCF-2 and/or NCF-3, and NCF-3, if some set C will be strict subset accordingly of sets C(A,d,ϕ), C(A,d,∞) & C∩C(A,d,ϕ) ≠ Ø, C∩C(A,d,∞) ≠ Ø. If a classical d–CA model (d≥1) not possess the NCF nonconstructability at the same time it possesses the NCF–1 nonconstructability, then for this model nonconstructible finite configurations relative to the set C(A,d,ϕ) exist. If a classical d–CA not possess the nonconstructability of types NCF and NCF-1 then for it any finite configuration is nonconstructible relative to the set C(A,d,∞). If a classical d-CA (d≥1) possesses the NCF nonconstructability, then for it the nonconstructible finite configurations as concerning the set C(A,d,ϕ) and the set C(A,d,∞) will exist, being absolutely nonconstructible ones. There are no classical d–CA (d ≥ 1) models possessing only the NCF–1 nonconstructability, whereas there are d–CA (d ≥ 1) models possessing only NCF–1 and ACCF.

Thus, the classical ***d–CA (d ≥ 1)*** models can possess the ***NCF*** or ***NCF–2*** nonconstructability only *(in particular, trivial models)*, while the classical ***d–CA (d ≥ 1)*** models can`t possess the ***NCF–1*** nonconstructability only. Of the theorem ***5*** follows that types of nonconstructability ***NCF, NCF-1, NCF-2*** and ***NCF-3*** generally are not equivalent and at a level of relative nonconstructability in classical ***d–CA*** models. In the same time the ***NCF*** nonconstructability to a certain extent can be considered as an absolute nonconstructability of finite configurations that don't have predecessors from the set ***C(A,d).*** Once again, we note that here, mainly, classical ***CA*** models are considered. Whereas with our results of research of relative nonconstructability for non–deterministic, polygenic and asynchronous ***CA*** models, including some of their other types the interested reader can familiarize in works [24-28,40-43,82-87,102,106,278].

2.3. Existence criteria of the basic nonconstructability types in classical *CA* models

The first nonconstructability criterion in classical ***CA*** models ascends to ***E.F. Moore*** and ***J. Myhill*** [75,76] and is based on concept of the *mutually erasable configurations (**MEC**)*. In slightly generalized kind, equivalent to the initial concept of ***MEC***, the concept is introduced as follows.

***Definition 6**. Two different configurations $c_1, c_2 \in C(A,d,\phi)$ $(c_1 \neq c_2)$ form a pair of MEC concerning global transition function $\tau^{(n)}$ in a d-CA ($d \geq 1$) model if and only if the following relation $c_1\tau^{(n)} = c_2\tau^{(n)}$ takes place.*

***The configurations** $c1, c2 \in C(A,d,\infty)$ **differing only with finite number of states are called the finite–differing (FDcf). If for 2 FDcf** $c1, c2 \in C(A,d,\infty)$ **the relation** $c1\tau^{(n)} = c2\tau^{(n)}$ **occures where** $\tau^{(n)}$ **is global transition function of a certain d–CA model then such configurations are called the finitely mutually–erasable and denoted as** MEC_∞.*

It is easy to be convinced that this definition is equivalent to definition of ***MEC*** of ***E.F. Moore***, but it is more convenient for definite theoretical qualitative investigation of dynamics of classical ***CA*** models. Using the ***MEC*** concept, ***E.F. Moore*** and ***J. Myhill*** have received [75,76] a criterion of existence in ***d–CA*** models *(not necessarily classical)* of nonconstructible configurations of ***NCF*** type that has been generalized to case of ***NCF–3*** nonconstructability [5,24,40-43,82,102,106], namely.

***Theorem 6**. An arbitrary classical d–CA ($d \geq 1$) model possesses the NCF (and, perhaps, NCF–3) nonconstructability if and only if for its global transition function $\tau^{(n)}$ the MEC pairs exist. An arbitrary classical d-CA ($d \geq 1$) model possesses the NCF (perhaps, NCF–3) nonconstructability if and only if for it there are MEC_∞ pairs.*

The second part of the theorem ***6*** is the natural generalization of its first part and its proof is based on the above ***MEC*** concept. At the same time it is necessary to note that infinite configurations whose basic substrate is □–configuration are excluded from the MEC_∞ concept. The criterion on the basis of the MEC_∞ concept has qualitative character with a light gnoseological hue unlike criterion on the basis of the ***MEC*** concept; it doesn't give a possibility to receive results of the quantitative character, in particular, concerning the ***MEC*** and ***NCF*** sizes. However, and in the second case similar results are far from optimum.

Existence criterion of ***NCF*** in classical ***d–CA*** ***(d ≥ 1)*** models remains fair also for case of *nonstable* models for which the condition $\sigma^{(n)}(x, ..., x) = x$ is not carried out, i.e. for these models an quiescent state ***"x"*** is missing. The criterion supposes research of certain questions connected with the ***MEC*** properties**;** certain of them are considered below. But, previously we need in the ***MEC*** concept strictly according to ***Moore-Myhill.*** For the greater obviousness and without loss of generality, the ***MEC*** definition according to ***Moore–Myhill*** we shall give for case of ***1–CA*** [1,4,5,8,40].

Definition 7*. Let W will be a block of m adjacent automata of a 1–CA and B will be a set of all neighbour automata for W according to index of neighbourhood X={0,1, ..., n-1}. Let now CF(P) will be a configuration of finite P block of automata of this model. Then 2 block configurations* $CF(B_1) \cup CF(W_1) \cup CF(B_2)$ *and* $CF(B_1) \cup CF(W_2) \cup CF(B_2)$ *are called a pair of MEC for global transition function* $\tau^{(n)}$ *in the 1–CA if and only if:*

$$[CF(B_1) \cup CF(W_1) \cup CF(B_2)]\tau^{(n)} \equiv [CF(B_1) \cup CF(W_2) \cup CF(B_2)]\tau^{(n)}$$

$$CF(W_1) \neq CF(W_2)$$

Under the made agreements the block W we name below as the internal block (IB) of a pair of MEC or in abbreviated form simply IB MEC.

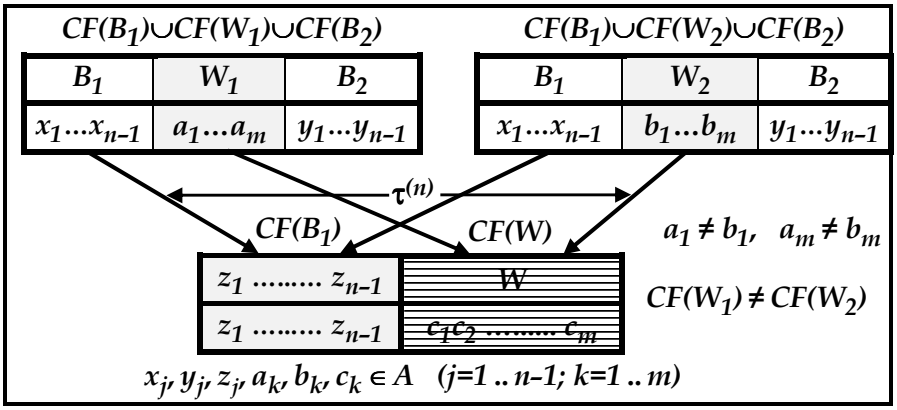


Fig. 5. Illustration of mapping of a MEC pair by means of GTF $\tau^{(n)}$ ***of a 1–CA model into the same block configuration*** $CF(W) \cup CF(B_1)$***.***

Essence of ***MEC*** for case of ***1–CA*** well illustrates fig. **5.** Of definition **7** it is simple to be convinced that the set of ***MEC*** pairs is infinite, and their structure is of interest for theoretical and applied researches. Moreover, research of ***MEC*** structure for dimensions $d \geq 2$ meets essential enough difficulties and is allowable only in the special cases, therefore the most interesting and essential results in this direction for today are received in case of ***1–CA*** of different types [1,5,7,24-28,40-43,82-87,102,106].

In one's time relative to ***MEC*** by ***E.F. Moore*** a lot of interesting enough questions has been formulated, whose solution has allowed to obtain a series of interesting results for ***1–***dimensional case. The following result illustrates variety of ***IB MEC*** types even in case of rather simple binary classical ***1–CA*** models with neighbourhood index $X = \{-1,0,1\}$ [1,3,5,7].

Theorem 7. Binary classical 1–CA models with neighbourhood index of form $X = \{-1, 0, 1\}$ ***possess by pairs of MEC with simple IB of size L only of one of the following basic types, namely:***

1) $L = \{p^{+} \mid p \geq 1\}$; ***2)*** $L = \{1^{+}, 2^{+}, 3^{+}, p^{-} \mid p \geq 4\}$; ***3)*** $L = \{1^{-}, p^{+} \mid p \geq 2\}$

4) $L = \{1^{+}, 2p^{-}, (2p+1)^{+} \mid p \geq 1\}$; ***5)*** $L = \{1^{-}, 2^{+}, p^{-} \mid p \geq 3\}$; ***6)*** $L = \{1^{+}, p^{-} \mid p \geq 2\}$

where upper index $\{+ \mid -\}$ ***defines*** {***existence*** | ***absence***} ***of MEC pairs with simple IB (which not contain other pairs of MEC).***

For research of a number of questions of the *nonconstructability* problem in the classical and the unstable ***CA*** models the following rather simple lemma can be useful enough [13,28].

Lemma 2. A d–CA $(d \geq 1)$ ***model will possess the MEC pairs if and only if the model possesses the MEC pairs whose internal blocks have both the odd and the even sizes. In addition, the internal block of minimal size can contains even or odd number of elementary automata in case of a model whose local transition function*** $\sigma^{(n)}$ ***depends on coordinates of the current elementary automata of the CA model.***

Thus, the lemma **2** concerns the classical and unstable models, mainly. It is easy to show that the lemma **2** is incorrect for models whose local transition function σ depends on coordinates of the current elementary automata. A lot of examples shows that with increase of states alphabet ***A*** and neighbourhood index ***X*** of a classical ***1–CA*** the allowable types of ***IB MEC*** expand [40-43]. Moreover, the structure of states alphabet ***A*** appreciably influences the nonconstructability concept in classical ***CA*** models. So, the nonconstructability problem in its general formulation is connected not only to a differentiation of alphabet ***A,*** but also with its cardinality. In particular, if the set ***N*** of all non–negative integers is took as an alphabet ***A*** of ***1–CA*** model then already relative to a model ***1–CA*** $\equiv \langle Z^1, A=N, \tau^{(2)}, X\rangle$ the standard meaning of the ***NCF*** nonconstructability concept is being lost [5,24,41,82-87,102,106].

Meanwhile, if the abolition of differentiation of alphabet ***A*** in context of defining of special quiescent state «***0***» uphold a criterion of existence of ***NCF,*** abolishing all other types of nonconstructability *(**NCF–1**,**NCF–2*** &

NCF–3), whereas its extension onto infinite case results in infringement of the nonconstructability concept in such ***CA*** models. Thus, structure of alphabet ***A*** of classical ***CA*** models is rather essential for definition of the nonconstructability problem***;*** some useful discussions of other cases confirming importance of structure of states alphabet ***A,*** can be found in our works [5,8,9,12,13,33,40-43,82-87,102,106].

As it was noted earlier, a lot of results concerning the *nonconstructability* problem form an effective enough part of the basic means of research of dynamics of the classical ***CA*** models, therefore various estimations of ***IB MEC*** of along with other aspects of the above problem present a special interest [5,12]. In this connexion concerning a rather important question about the minimal size of simple ***IB MEC*** that goes back to ***E.F. Moore,*** one of answers the next basic theorem gives [5,24,33,40-43,82,102,106].

Theorem 8. For integers $a \geq 3$ and $n \geq 2$ the classical 1–CA models with states alphabet $A = \{0,1,...,a-1\}$ and neighbourhood index $X = \{0,1,...,n-1\}$ exist that possess the MEC with simple internal block of minimal size $L = n$. The determination problem of the MEC of minimal size in a d–CA ($d \geq 2$) model is algorithmically unsolvable.

In connection with research of nonconstructability problem in classical ***d–CA*** ***($d \geq 1$)*** models we considered a lot of existence questions of ***MEC*** in detail. The obtained results have both the qualitative and the numeric character [5,8,9,12]. In this connection the desire to receive estimations for minimal size of simple ***IB MEC*** is a quite natural. In case of classical ***1–CA*** the following rather useful results take place [1,5,8,24]. Naturally, it is simple enough to give some examples of classical ***1-CA*** models that possess pairs of ***MEC*** with ***IB*** of minimal size ***1;*** the following theorem determines their quota [24,40-43,82,102,106].

Theorem 9. Share $\Delta(a, n)$ of classical 1–CA models which possess pairs of MEC with IB of minimal size 1, relative to all models 1–CA with an alphabet $A = \{0,1,2, ..., a-1\}$ and neighbourhood index $X = \{0,1,2, ..., n-1\}$, satisfies the following relation $\Delta(a,n) > \left(2a^n - 1\right)/a^{2n}$.

An arbitrary classical ***1-CA*** model with states alphabet $A=\{0,1,...,a-1\}$ and neighbourhood index $X = \{0, 1, ..., n-1\}$ will possess the nonconstructible configurations of ***NCF*** type of minimal size ***1,*** if and only if for its local transition function $\sigma^{(n)}$ the following rather obvious relation takes place

$$(\exists y \in A \setminus \{0\})(\forall \langle x_1, x_2, ..., x_n \rangle)(\sigma^{(n)}(x_1, x_2, ..., x_n) \neq y); \quad x_j \in A \ (j=1..n)$$

Basing on this relation, we can easily make sure that quantity of similar

models equals $\sum_{j=0}^{a-2} C_{a-1}^{j}(j+1)^{a^n-1}$, whereas their share is $\sum_{j=0}^{a-2} C_{a-1}^{j}\left(\frac{j+1}{a}\right)^{a^n-1}$. In addition, share of classical models possessing the ***NCF*** configurations of minimal size ***1*** is rather negligible concerning all classical models as a whole and the models possessing the ***NCF*** nonconstructability.

The method basing on the nonconstructability of types ***NCF*** and ***NCF–1*** is a very powerful tool in analysis of ***CA*** models. Effectiveness of such approach in considerable degree depends on quantitative knowledge of main correlations between ***γ–CF, MEC, NCF*** and ***NCF–1*** on the level of both *local* transition functions and *global* transition functions. At present the exhaustive knowledge in this topic not exist. We present the certain separate results in this direction. As a whole the next rather interesting estimations for the minimal size of ***IB*** of ***MEC*** pairs exist [5,9,24,40-43].

<u>Theorem 10</u>. If a classical 1–CA model possesses the MEC pairs with IB of minimal size W, the relation $1 \le W < a^{n-1}(a^{n-1}-1) + n - 2$ ***takes place. Number*** $G(a,n)$ ***of classical 1–CA models with alphabet*** $A = \{0,1,...,a-1\}$ ***and neighbourhood index*** $X = \{0,1,...,n-1\}$ ***that possess only MEC pairs with IB of simplest type (which not contains other pairs of MEC) in the form*** $\{<0^{n-1}|1|0^{n-1}> \mid <0^{n-1}|0|0^{n-1}>\}$ ***and*** $\{<0^{n-1}|10^{p}|0^{n-1}>|<0^{n-1}|0^{p}1|0^{n-1}>\}$ ***(where*** $p=1..n$; h^p ***– concatenation of p symbols 'h'), at least*** $(a!)^{a^{n-1}}/a^{n+1}$; ***in addition, share of classical 1–CA of this type relative to all classical 1–CA models of the same type is at least*** $\left(a!/a^{a}\right)^{a^{n-1}}/a^{n}$.

Unfortunately, we can't receive a similar estimation for case of *classical* ***d-CA,*** and that is one of consequences of *algorithmic unsolvability* of the problem of existence in any classical ***d–CA*** $(d \ge 2)$ of the *nonconstructible* configurations of ***NCF*** type. Indeed, otherwise this problem would be algorithmically solvable. Whereas, on the other hand on the basis of the theorem ***9*** the algorithmic solvability of the existence problem of ***MEC*** pairs *(as well as* ***NCF****)* for the general case of ***1–***dimensional *classical* and *nonstable* ***CA*** models is easily established [1,5]. By using an approach of ***E.F. Moore*** [75] and result of the above theorem ***9,*** it is possible to obtain the upper limit for *minimal* size of ***NCF*** in classical ***1–CA*** models. By the way such estimation seems a very rough and, practically, of little use in applications. On basis of appreciably other approach we have received considerably the best estimations for the minimal sizes of ***NCF (L)*** and ***NCF–1 (P)*** for the case of classical ***1–CA*** models with arbitrary alphabet ***A*** and neighbourhood index ***X*** [8,9,24,40-43,82,102,106].

***Theorem 11**. Minimal sizes L and P for NCF and NCF–1 configurations for a classical 1–CA model are defined by the relations $L \le (2^a-1)^{n-1}+1$ and $P \le (2^a-1)^{n-1}$ accordingly. The classical 1–CA models with alphabet A={0,1, …, a–1} and neighbourhood index X={0,1, …, n–1} which possess NCF of the minimal size L = n + 1 exist.*

This result is not based at all on the ***MEC*** concept and allows to define constructively in a ***1–CA*** not only nonconstructability existence of types ***NCF*** & ***NCF–1,*** but in many respects and their structure. Similar results concerning the minimal sizes of ***NCF*** for simplest binary ***1–CA*** models with a lot of interpretations of the *nonconstructability* concept have been presented by ***A. Adamatzky*** and ***A. Wuensche.*** In addition, in [8,9,12,24] the well–founded refutation of incorrect doubt represented in [92] that concerns estimations of the theorem ***11*** has been given. In view of told it would be rather interesting to establish concerning even the class of binary ***1–CA*** models with a neighbourhood index ***X={0,1,2,3,…,n–1}*** the distribution of sizes of minimal blocks of ***NCF,*** is more exact**:** ***Whether can form the given sizes a subset of set $N^+ = \{0,1,2,\ldots,G\}$ $(G = 3^{n-1}+1)$?***

The carried out computer *analysis* of a series of rather interesting ***1–CA*** models has allowed to hold the opinion, that this assumption is cogent enough [42]. Let`s represent now certain estimations for minimal sizes of the ***NCF*** configurations in case of classical ***1–CA*** models.

To the above questions the existence problem in classical ***d–CA (d≥1)*** of so–called *vanishing* configurations playing a very essential part in study of a lot of dynamic properties of such class of ***CA*** models, including the nonconstructability problem, adjoins.

***Definition 8**. A configuration $c \in C(A, d, \phi)\setminus\{\square\}$ is called the vanishing configuration (VCF) for a classical d-CA (d ≥ 1) model if and only if the relation $(\exists m>0)(c\tau^{(n)m}=\square)$ takes place where '□' and $\tau^{(n)}$ – are fully null configuration and global transition function of the model accordingly.*

Obviously, a ***VCF*** set in a classical ***CA*** model is empty or infinite. If in a classical ***CA*** model exist the ***VCF,*** then for the model the set $C(A,d,\phi)$ is nonclosed relative to mapping induced by global transition function of the model, ensuring presence for the model of ***NCF*** and perhaps ***NCF–1*** nonconstructibility whereas the *converse* statement generally is not true**;** a ***CA*** can possess the ***VCF*** and ***NCF*** without ***NCF–1.*** Existence problem of vanishing configurations for classical ***CA*** models represents a certain interest from standpoint of models dynamics. The theorem below gives a rather useful result.

Theorem 12**. If in a classical d–CA model there are VCF then the model will possess the NCF nonconstructability. The minimal length L(a,n) of VCF in a classical 1–CA model with alphabet A={0,1, ..., a–1} and index of neighbourhood X={0,1,...,n-1} is defined by relation $L(a,n) < a^n + n - 1$. There are classical 1–CA models with minimal size of VCF at least n–1. A classical 1–CA model with neighbourhood index X = {0, 1}, possessing VCF of minimal size L(a,2)=a–1 exists. Share of classical 1–CA models with alphabet A = {0,1, ..., a–1} and neighbourhood index X={0,1, ..., n–1} which possess the VCF of minimal size 1 along with pairs of MEC with IB of minimal size 1 relative to the models of the same kind is not less than $\left(2a^n - 1\right)/a^{2n}$. The existence problem of VCF for the 1–CA models is solvable while for d-CA (d≥2) models it is unsolvable. Share of classical d–CA (d≥1) models not possessing VCF is at least 1/e for large enough cardinality of the models alphabet. If the set C(A,d,∞) is nonclosed with respect to a global mapping $\tau^{(n)}$ then for d–CA (d≥1) model with global transition function $\tau^{(n)}$ which has no NCF (NCF–1), the NCF–1 (NCF) nonconstructability exists, excluding trivial cases. There are classical d–CA models in which any finite configuration is VCF; in d–CA models the finite configurations can be VCF and NCF simultaneously (d ≥ 1). If there are VCF for a d–CA, then there are NCF for it, but not vice versa.

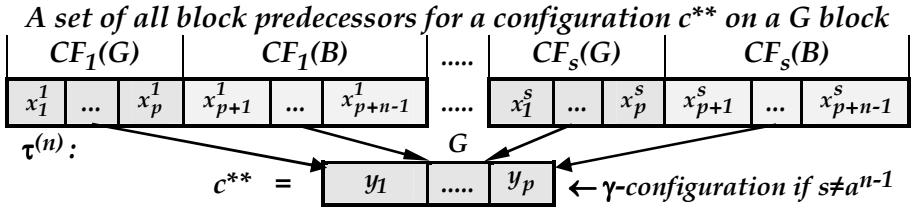
A lot of interesting examples of ***d–CA*** models, presenting certain other interrelations between ***VCF, NCF*** and ***NCF–1*** the reader can find in [24]. Meantime, the detailed researches of the nonconstructability problem in the classical ***CA*** models have suggested to us an idea not only to better understanding of insufficient efficiency of approach on the basis of the ***MEC*** concept but also have allowed to introduce the concept of so-called **γ-***configurations (**γ-CF**)* which appeared fruitful enough [93]. Our concept of **γ-*CF*** is slightly distinct from the concept of ***k-****balanced global transition functions* introduced by *A. Maruoka* and *M. Kimura* [94] irrespective of us however both concepts are completely equivalent.

This concept have been introduced by us with the purpose of research of the nonconstructability problem in classical ***CA*** models, whereas by ***A. Maruoka*** and ***M. Kimura*** for researches of parallel global mappings $\tau^{(n)}$: $C(A, d) \to C(A, d)$ defined by global functions $\tau^{(n)}$ in classical ***d–CA***. We shall introduce now actually concept of block **γ-***configurations (**γ-CF**)*.

***Definition 9**. Let G is a block of elementary automata in a d–CA model, B will be a block of elementary automata neighboring the all automata of G block according to neighbourhood index of the model and CF(P) is configuration of a P block of elementary automata. We shall say that a*

d-CA model possesses γ-configuration on a G block if and only if at any rate for one CF(G) exist $s \neq a^{n-1}$ predecessors (a and n are cardinality of states alphabet of d–CA and automata set of B block accordingly; d≥1).

A rather evident chart well illustrates essence of the above definition of **γ–**configurations for case of a ***1–CA*** model with alphabet ***A={0,1,..., a-1}***, neighbourhood index ***X={0,1,..., n–1}*** and global transition function $\tau^{(n)}$.



On the basis of the ***γ–CF*** concept it is possible to receive a new criterion of existence of ***NCF*** in ***CA*** models [93] that presents a rather significant interest for a lot of investigations in the ***CA*** problematics, above all, of dynamic aspects of ***CA*** models. In view of definition ***9*** the next theorem has been proved [24,95,102,106].

<u>Theorem 13</u>. A d-CA (d≥1) model possesses the NCF nonconstructability (perhaps, and NCF–3) if and only if in the model exist γ–configurations; in addition, the assertion concerns for nonstable, and classical models.

Theorem ***13*** gives another criterion of existence of the nonconstructible configurations *(**NCF**)* in the ***CA*** models of both classical and nonstable ones, and it is more convenient for a lot of theoretical researches in ***CA*** models. This criterion not depends on the concept of erasability *(**MEC**)* in ***CA*** models. It is necessary to note that in spite of entire equivalence of both existence criteria of the ***NCF*** nonconstructability *(Moore-Myhill and Aladjev-Maruoka-Kimura)* some distinctions exist at their specific use.

This criterion has allowed us to obtain more acceptable estimations for some numerical characteristics of ***d–CA (d ≥ 1)***. Let's give an estimation of the minimal size of ***NCF*** in classical ***d–CA*** models *(**d = 1, 2**)*, using the criterion *(Theorem **13**)* of configurations nonconstructability on the basis of the introduced concept of **γ–**configurations [24,40-43,82-87,102,106].

<u>Theorem 14</u>. If in a 2–CA models with Moore neighbourhood index and states alphabet A = {0,1, ..., a-1} exist the γ–CF on blocks of predecessors of size PxP, then for such models exist NCF of size LxL, namely:

$$L = 2(P+2)\left\lceil \frac{2\ln a}{4(P+1)\ln a - \ln s}\left(P+3+\sqrt{P^2+\frac{\ln s}{\ln a}}\right)\right\rceil - 2$$

For the case of 1–CA models with the above neighbourhood index and states alphabet A the similar estimation assumes the following form:

$$L = \left\lceil \frac{(n-1)\ln a}{(n-1)\ln a - \ln s} \right\rceil (P+n-1) - n + 1;$$

n – size of neighbourhood template of the model, s – corresponds to the definition 9, whereas a is cardinality of A alphabet and]Z[denotes an integer ≥ Z.

In addition, theorem ***14*** is true for classical and nonstable ***d–CA*** models ***(d = 1, 2)***. Using results of theorem ***14,*** it is possible to obtain clear rather essential contrast of results concerning use of the concepts of ***MEC*** and ***γ–CF.*** In particular, an interesting enough example of application of the approach for estimation of minimal sizes of ***NCF*** can be illustrated on known game ***Life*** [7,96,97]. It is simple to be convinced, that this game is nothing else than a binary classical ***2–CA*** with *Moore* neighbourhood index [5,8,24,43]. Researchs of the ***Life*** game have been done by a lot of mathematicians and programmers along with amateurs as a rule on the basis of methods of computer modelling. Here, for today, multitude of interesting and simply amusing results have been obtained [7,24,42,90, 102,106,286]. This classical binary ***2–CA*** model is one of most famous.

A.R. Smith, researching binary ***2–CA,*** appropriate to the ***Life*** game, has shown that this model possesses ***NCF*** of minimal size ***LxL*** ***(L=10^{10})*** [99]. For an obtaining of the estimation *A.R. Smith* has used the approach on the basis of ***MEC*** concept, meantime, as on the basis of ***γ-CF*** concept we could essentially improve the estimation, reducing it to foreseeable size, namely**:** ***49x49.*** It does by rather real the obtaining of ***NCF*** kind in such model using possibilities of modern computers [7,9,24]. A lot of others ***Life–***like ***2–CA*** with *Moore* neighbourhood index exists among which it is possible to note, for instance, binary ***2–CA*** *Seeds*, initially investigated by *B. Silverman*, and whose local transition function is defined as**:**

$$\sigma^{(9)}(x_o, x_1, ..., x_8) = \begin{cases} 1, & \text{if } x_o = 0 \ \& \sum_{j=1}^{j=8} x_j = 2 \\ 0, & \text{otherwise} \end{cases}$$

where as a x_j the generalized coordinates of elementary automata and their states associated with them from alphabet ***A={0,1}*** are represented. Under the generalized coordinates of automata making neighbourhood template of a ***(i, j)–***automaton are understood coordinates {***(i, j), (i+1, j), (i+1,j+1), (i,j+1), (i-1,j+1), (i-1,j), (i-1,j-1), (i,j-1), (i+1,j-1)***}. Below, current states of elementary automata are associated with their coordinates. In particular, the central automaton x_o of neighbourhood template can has

the state ***0*** or ***1.*** For receipt of the estimation of minimal size of ***NCF*** it is expedient to use the ***γ–CF*** concept which allows to obtain a reasonable estimation, namely**:** ***18x18.*** It does by rather real obtaining of ***NCF*** kind of in such model using the facilities of computers.

In addition, for obtaining of the above results the theorem ***14*** has been used, whereas theorem ***15,*** presented below, allows to obtain lower and upper bounds of the ***NCF*** of *minimal* size. In particular, the lower bound of ***NCF*** of minimal size for the above two ***CA*** games equals ***10,*** whereas upper bound is essentially top**–**heavy, that is conditioned the fact, real number of disproportion in mappings of block configurations in such ***2–CA*** under influence of global transition functions was not taken into account. So, using theorems ***14*** & ***15,*** it is possible to obtain an essential contrast of results concerning the application of concepts ***NCF*** and ***γ-CF*** as the above examples of estimation of ***NCF*** minimum sizes illustrate.

In addition, similarly we also researched universal classic binary ***2–CA*** of *E. Banks* [7,100]**,** which today is minimal by complexity**.** It is possible to show that this classical binary ***2–CA*** is suitable as an environment of realization of computing circuits of an arbitrary complexity. So, on the basis of theorem ***14*** we received estimation ***14x14*** for ***NCF*** size**;** i.e. in the *Banks* universal ***CA*** model exist ***NCF*** already on blocks of size ***14x14.*** In this connexion the next rather interesting hypothesis arises, namely**:**

<u>Hypothesis</u>. Universal classical d–CA models of minimal V complexity {V=dimension∗(size of neighbourhood template)∗(alphabet cardinality)} possess the nonconstructability of type NCF and/or NCF–1.

For today, all minimal universal ***CA*** models known to us conform with this hypothesis [5,8,9,24-28,40-43,69-71,82-87,99,100]. A rather extensive bibliography on this matter can be found in [7,24,82,102,106,278,286].

Thus, the ***Aladyev–Kimura–Maruoka γ–CF*** concept and criterion of the nonconstructability based on it allow to investigate an quite effectively a number of quantitative aspects of dynamics of classical ***d–CA*** models, whereas the ***MEC*** concept in a number of cases is more acceptable for their qualitative research. Thus**,** in many respects both concepts enough well supplement each other. In a number of cases instead of optimum or asymptotic estimates of the ***NCF*** sizes in classical ***d–CA*** models there are sufficiently simple formulas, as functions of key parameters of the models. In this context these formulas represented by theorem ***15*** were repeatedly used in particular at research of the ***NCF*** problematics [24].

In a lot of cases instead of optimum or asymptotical estimations of ***NCF***

sizes in classical ***CA*** models the simple formulas in *functional* form from *basic* parameters are quite sufficient. In this context the similar formulas repeatedly were used in research of these problems. In this respect the result of theorem ***15*** assumes the form which in case of need is a rather convenient for finding of certain *numeric* estimations for some problems connected to research of the ***NCF*** nonconstructability [24,40-43,82-87].

Theorem 15*. If in a 2–CA model with Moore neighbourhood index and alphabet A={0,1,...,a-1} exist γ–CF on blocks of predecessors of size PxP, then for such model exist NCF configurations of size LxL, namely:*

$$\left\lceil \frac{2P^2+5P+4}{P+1} \right\rceil \le L \le 8(2P^2+9P+10)\rceil \ln a\lceil a^{4P+4}-2$$

For case of 1-CA models with the above neighbourhood index and states alphabet A the similar estimation assumes the following form:

$$P \le L \le 2(P+n-1)(n-1)a^{n-1}\rceil \ln a\lceil$$

where $\rceil M\lceil$ ***is an integer not less than M.***

This theorem gives estimations for minimal sizes of ***NCF*** in the classical ***CA*** models in the form of a function from their main parameters**:** a size of predecessors of ***γ–CF,*** a neighbourhood template, and cardinality of a alphabet ***A.*** The results of theorems ***14,15*** can be useful enough both for the theoretical and for numerical researches in the ***CA*** problems as well. Furthermore, the results can be rather easily extended onto ***CA*** models with arbitrary neighbourhood indexes and higher dimensionality. They seem useful enough at researches of certain aspects of dynamics of the classical ***CA*** models [5,8,9,12,13,24-28,40-43,82-87,102,106].

Both for deeper comprehension of the nonconstructability concept and for creation on its base of effective apparatus of research of dynamics of ***CA*** models the determination of various interrelations between various characteristics of configurations ***MEC, γ–CF, NCF,NCF–1,NCF–2,NCF–3*** both quantitative and qualitative ones is extremely desirable. For today, the full picture in this question is absent, excepting a series of separate results represented below. In particular, from the theoretical standpoint a certain interest represents ascertainment of dependences between the sizes *(S)* of the minimal blocks containing ***MEC, γ–CF, NCF*** and ***NCF–1*** in classical ***CA*** models. So, for example, for ***γ-CF*** and ***NCF*** the following relation takes place**:** $S(\gamma\text{–}CF) \le S(NCF)$ [5,9,24]**,** whereas for case of ***MEC*** and ***NCF*** the picture is essentially more complex**,** namely**,** the following rather important result takes place [5,40-43,82-87,102,106].

***Theorem 16**. For an integer $n \geq 2$ the classical 1-CA models with index of neighbourhood $X = \{0,1,2, ..., n-1\}$ exist, for which the minimal blocks of NCF and MEC satisfy the following relation $S(MEC)/S(NCF) = 1/(n+1)$ or $S(MEC)/S(NCF) = n$ accordingly where minimal $S(\gamma\text{–}CF) = 1$. For each integer $n \geq 2$ the classical 1–CA exist which do not possess the NCF and NCF–3, but have NCF–1 of minimal size $L \geq n-1$. For each integer $n \geq 3$ a classical binary 1–CA exists, that possesses γ–CF of minimal size $L = n$ along with simple IB of MEC of minimal size 1.*

Determination of an upper limit for minimal sizes of ***IB MEC***, of sizes of ***γ–CF*** or nonconstructible configuration of a required type *(**NCF**, **NCF–1**, **NCF–2** and **NCF–3**)* is a rather essential question. Amid received results, in particular, for case of classical ***1–CA*** models a rather useful theorem, presented below, can be noted [1,5,8,9,24,40-43,82-87,102,106].

***Theorem 17**. For an integer $n \geq 2$ there are classical binary 1–CA models with global transition function $\tau^{(n)}$ possessing the following properties simultaneously, namely:*

- *they possess γ–configurations of minimal length $L = n$;*
- *difference of minimal sizes of γ–CF and IB MEC equals n–1;*
- *they not possess the NCF–1 nonconstructability;*
- *for an integer $k \geq 1$ their global transition functions $\tau^{(n)k}$ possess the γ–configurations of minimal size n;*
- *there are integers $t_1 = t_1(n)$ & $t_2 = t_2(n)$ for which block configurations in the form $c_b = 0^{p_1 \geq t_1} 1 0^{p_2 \geq t_2}$ are NCF nonconstructible in such models where $t_1 < t_2$ – growing functions from variable n;*
- *configurations in the form $c_p = \square 1^p \square$ $(p \geq n-1)$ are passive in similar models, i.e. $c_p \tau^{(n)} = c_p$.*

Theorem ***17*** is enough easily generalized to case of dimensionality $d > 1$ by means of special embedding of ***1-CA*** model satisfying the conditions of the theorem, into a classical ***d–CA*** $(d \geq 2)$. Of this theorem and certain other results in theis direction follows that, generally, it is impossible to obtain satisfactory quantitative relations between minimal sizes of ***γ–CF*** and ***IB MEC*** for classical ***CA*** models [1,5,24,82,102,106].

So, the above situation serves as one of principal causes of difficulties in quantitative research of the nonconstructability problem in classical ***CA*** models of general type. In addition, from theorem ***17*** and that fact that minimal size of ***γ–CF*** no more than minimal size of ***NCF*** easily follows,

that classical ***CA*** models with an arbitrary predetermined minimal size of the ***NCF*** nonconstructible configurations exist.

Meantime, the question of revealing of relations between minimal sizes of ***NCF***, ***IB MEC*** and ***γ–CF*** seems interesting enough. At that, above we nored, that minimal size of ***γ–CF*** is not more than minimal size of ***NCF***, i.e. $\min S(\gamma\text{–}CF) \le \min S(NCF)$ where $S(G)$ denotes size of G. On the other hand, we have shown that the relation $\min S(IB\ MEC) \le \min S(NCF)$ is valid [41-43,101]. So, in context of the question about relations between minimal sizes of ***γ–CF***, ***IB MEC*** and ***NCF*** it is possible to formulate the following rather useful result.

Theorem 18*. If an arbitrary classical d–CA (d ≥ 1) model possesses the NCF nonconstructability then for minimal sizes of NCF, γ-CF and IB of MEC the following relations take place:*

$L_{min}^{ncf} \ge L_{min}^{ibmec}\ \{\le|\ge\}\ L_{min}^{\gamma}$; *where* L_{min}^{ncf}, L_{min}^{ibmec}, *and* L_{min}^{γ} *are minimal sizes of the block configurations NCF, IB MEC, and γ-CF accordingly.*

Number N of classical d–CA models with alphabet A = {0, 1, ..., a–1} and neighbourhood index X = {0,1, ..., n–1} that possess the configurations of NCF type and γ–CF of minimal size 1, is defined by the next formula:

$$N(a,n) = \sum_{j=1}^{a-1} (-1)^{a+j+1} C_a^j\, j^{a^n}$$

This result has a number of applications at research of questions of the nonconstructability in classical ***d–CA (d ≥ 1)*** models whereas the result below represents a certain interest for case of classical ***1–CA*** models.

Theorem 19*. Number T(a, n) of all classical 1–CA models with alphabet A = {0,1, ..., a–1} and neighbourhood index X = {0,1, ..., n–1} that possess the γ–CF and NCF of minimal sizes 1 simultaneously along with pairs of IB MEC with IB of the simplest type (that not contain other pairs of MEC) in the form* $\{<0^{n-1}|1|0^{n-1}>, <0^{n-1}|0|0^{n-1}>\}$ & $\{<0^{n-1}|10^{p}|0^{n-1}>, <0^{n-1}|0^{p}1|0^{n-1}>\}$ *(p=1..n; g^p is concatenation of p symbols g), satisfies the following relation, namely:*

$$T(a,n) \ge \begin{cases} 1 & ,\ if\ \ a = 2 \\ \sum_{j=1}^{a-2} (-1)^{a+j} C_{a-1}^{j}\, j^{a^n - n - 1} & ,\ if\ \ a \ge 3 \end{cases}$$

At that, it is necessary to mark, the result represented in theorem ***19*** is naturally generalized to dimensionality $d > 1$. The results represented above not only more deeply uncover the nonconstructability problem

in classical ***CA*** models, in particular in ***1–CA*** but also to a certain extent expand opportunities of the apparatus of research of dynamics of this type of ***CA*** models which is based on the nonconstructability concept. So, on the basis of theorem ***18*** follows that it is rather difficult to speak about preferability of that or other criterion as a whole *(even under the condition of equivalence of both criteria)*, though the criterion on the basis of ***γ–CF*** in a lot of cases seems more preferable. Namely criterion on the basis of ***γ–CF*** determines the existence of the ***NCF*** nonconstructability in the finite ***CA*** models.

For computer study of binary classical ***1–CA*** with neighbourhood index ***X***={***-1, 0, 1***} for their classification in the context of possessing of *minimal* sizes of ***NCF*** and ***γ–CF*** the procedure ***T1_HSb*** has been programmed in ***Maple;*** its source code can be found in our library [24,48,102,106].

The procedure call ***T1_HSb()*** returns ***2*** tables whose indices determine minimal sizes of ***NCF*** and ***γ–CF*** accordingly whereas entries determine discriminating numbers of ***1–CA*** which correspond to them, namely**:**

1 = {***0***}, ***2*** = {***2, 4, 8, 12, 16, 24, 32, 34, 48, 64, 66, 68***},
3 = {***1, 3, 6, 10, 11, 14, 17, 18, 19, 20, 28, 36, 40, 42, 46, 47, 50, 55, 56, 63, 70, 72, 76, 80, 81, 84, 96, 98, 112, 116, 117, 119, 126, 127***},
4 = {***7, 9, 13, 21, 26, 27, 29, 31, 33, 35, 38, 39, 44, 49, 52, 53, 58, 59, 65, 69, 71, 74, 78, 79, 82, 83, 87, 88, 92, 93, 100, 111, 114, 115, 123, 125***},
5 = {***5, 23, 25, 41, 43, 54, 61, 62, 67, 77, 94, 95, 97, 103, 107, 108, 110, 113, 118, 121, 122, 124***}, ***6***={***57, 73, 99, 109***}, ***8*** = {***22, 104***}, ***9*** = {***37, 91***} ⇐⇐⇐***NCF***
1 = {***0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 44, 47, 48, 49, 50, 52, 55, 56, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 76, 79, 80, 81, 82, 84, 87, 88, 91, 93, 94, 95, 96, 97, 98, 100, 103, 104, 107, 109, 110, 111, 112, 115, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127***},
2 = {***23, 27, 29, 39, 46, 53, 54, 57, 58, 71, 77, 78, 83, 92, 99, 108, 114, 116***},
3 = {***43, 113***} ⇐⇐⇐***γ–CF***

So, among all binary ***1–CA*** models with neighborhood index ***X***={***-1,0,1***} that possess the ***NCF*** nonconstructability with minimal size ***9*** only two models with discriminating numbers ***37*** and ***91*** exist. In addition, ***1–CA*** with number ***104*** possesses ***γ–CF*** with minimal size ***1,*** and ***IB MEC*** with minimal size ***1,*** for example, {***<11|0|11>, <11|1|11>***} and with minimal size ***8*** of nonconstructible ***NCF*** configurations.

We have similar situation for the model with number ***37*** that possesses ***γ-CF*** with minimal size ***1,*** and ***IB MEC*** with minimal size ***2,*** for instance,

<00|00|00> and **<00|11|00>.** In the following table the columns ***1–CA, NCF, γ–CF*** and ***IB MEC*** define discriminating numbers of binary ***1–CA*** models, minimal sizes of ***NCF, γ–CF*** and ***IB MEC*** accordingly.

1–CA	*NCF*	*γ–CF*	*IB MEC*
37	*9*	*1*	*2*
91	*9*	*1*	*2*
22	*8*	*1*	*1*
104	*8*	*1*	*1*
57	*6*	*2*	*2*
73	*6*	*1*	*2*
99	*6*	*2*	*2*
109	*6*	*1*	*2*
23	*5*	*2*	*1*
43	*5*	*3*	*1*
108	*5*	*2*	*2*
113	*5*	*3*	*1*
7	*4*	*1*	*1*
100	*4*	*1*	*1*
125	*4*	*1*	*1*

Thus, from this table follows, that for classical binary ***1-CA*** models with neighborhood index $X = \{-1, 0, 1\}$ the following relations take place**:**

$$min_{\gamma-CF} > min_{IB\ MEC} \qquad (23, 43, 113)$$

$$min_{\gamma-CF} < min_{IB\ MEC} \qquad (37, 73, 91, 109)$$

$$min_{\gamma-CF} = min_{IB\ MEC} \qquad (7, 22, 57, 99, 100, 108, 125)$$

One more interesting enough example in the given context is the above familiar game ***Life.*** It is simple to be convinced that binary ***2–CA*** model**,** equivalent to the ***Life,*** possesses the ***γ–CF*** of minimal size ***1*** and ***IB MEC*** of minimal size ***1.*** On the other hand on the basis of the ***γ-CF*** concept we have obtained an estimation for *minimal* size of ***NCF*** in the above model namely no more than ***49x49*** [13,102]. Now, using computer tools we can receive the more precise estimation. For analysis of the above ***2–CA*** for receipt of minimal size of ***NCF*** the ***MinNCFlife*** procedure in ***Maple*** has been created [48]**.** The procedure call ***MinNCFlife(n)*** returns a set of the discovered binary ***NCF*** of size ***(n–1)x(n–1)*** with print of the appropriate warning**;** otherwise, the procedure call returns the empty set with print of appropriate warning. A lot of computer experiments allows to obtain the estimation no less than ***20x20,*** i.e. for the above model the minimal size of ***NCF*** satisfies the relation $20 \le min_{NCF} < 49$**.** In addition, the more powerful computers will allow to obtain the ***NCF*** of minimal size in the

above model. So, the above estimations can serve as a *sensible* argument in favour of impossibility to give priority to ***γ-CF*** or ***MEC*** as a the main reason of possession of the ***NCF*** nonconstructability by a model. In the final analysis, we to the full measure can consider ***γ–CF*** and ***MEC*** as the equivalent reasons of presence of the ***NCF*** in ***d–CA (d≥1)*** models.

Meanwhile, these reasons and some others quite convincingly explain the fact that exactly on the basis of ***MEC*** concept is impossible to obtain satisfactory estimations of minimal sizes of ***NCF*** and many other rather important numerical characteristics for ***d–CA (d≥1)*** models. As a whole the nonconstructability phenomenon is seemingly caused by a certain type of asymmetry of functions ***LTF (GTF)*** of the ***CA*** models [40–43]. In connexion with the aforesaid we with full ground can ascertain that the ***Moore–Myhill*** *(Theorem* ***6****)* criterion of existence of ***NCF*** in classical and nonstable ***CA*** models in many respects enough essentially yields to the equivalent ***Aladjev–Maruoka–Kimura*** *(Theorem* ***13****)* criterion that bases on the above concept of ***γ–***configurations.

For example, on the basis of ***Aladjev-Maruoka-Kimura*** criterion we can obtain the more acceptable estimations for many *classical* and *nonstable* ***CA*** models possessing the ***NCF*** nonconstructability. At the heart of the approach the calculation of number of ***CA*** models possessing ***γ–CF*** on blocks of one elementary automaton lays. The following theorem gives some of such estimations which on the basis of an experimental testing have shown quite satisfactory accordance.

<u>Theorem 20</u>. Number n(a, n) and share q(a, n) of classical and nonstable d–CA (d≥1) models with neighborhood index X = {0,1,...,n–1} and states alphabet A = {0,1, ..., a–1} which possess the nonconstructability of NCF type satisfy the following relations, namely:

$$n(a,n) \geq a^{a^n} - \frac{\Gamma\left(a^n+1\right)}{\Gamma\left(a^{n-1}+1\right)^a} \qquad q(a,n) \geq 1 - \frac{\Gamma\left(a^n+1\right)}{\Gamma\left(a^{n-1}+1\right)^a a^{a^n}}$$

where Γ is the Gamma function. In the case of the simplest binary 1–CA with neighborhood index X = {0, 1} the strict equalities exist. Number of 1–CA with alphabet A = {0, 1, ... , a–1}, neighbourhood index X = {0, 1} and symmetrical LTF without NCF equals at least Barnes G(a+2).

Values ***q(a, n)*** which have been obtained on the basis of theorem ***20*** for ***a = 2 .. 6*** and ***n = 2 .. 10*** enough evidently illustrates quick converging of quota of the above models possessing the ***NCF*** nonconstructability to ***1*** already for rather small ***n*** and ***a.*** So, with growth of values ***n*** and ***a*** both

the *classical* and *nonstable* ***CA*** models not possessing ***NCF*** become more and more exotic objects of research regardless of their dimensionality.

Of the ***γ–CF*** concept and a lot of other reasons follows [42] that a certain asymmetry of ***GTF*** lays at the heart of the nonconstructability concept seeing the ***γ–CF*** concept is nothing else than ***asymmetry*** in mappings of configurations on finite blocks of elementary automata by means of the global transition function $\tau^{(n)}$. In the case of the detailed picture of such *asymmetry* it would be possible to essentially advance in research of the nonconstructability problem, and in numerous and interesting enough contiguous questions concerning dynamics of *classical* and *nonstable* ***CA.***

We pass to discussion of other three nonconstructability types *(**NCF–1**, **NCF–2** and **NCF–3**)* in classical ***CA*** models that according to table ***1*** can combine in rather wide bounds. Having considered criteria of existence of ***NCF,*** we shall characterize the current state of the question for three other types of nonconstructability in classical ***CA*** models. Of definitions ***1, 3, NCF–3*** definition and theorem ***5*** it is simple to be convinced, that a configuration $c \in C(A,d,\phi)$ can't has more ***one*** type of nonconstructability simultaneously. So, the pair–wise crossings of sets ***NCF, NCF–1, NCF–2*** for global transition function $\tau^{(n)}$ of a classical ***CA*** model are empty. The next theorem presents a criterion of the nonconstructability existence of types ***NCF-1*** and ***NCF-2*** in classical ***d-CA*** $(d \geq 1)$ models that not possess the nonconstructability of types ***NCF*** and ***NCF–3.***

Definition 10*. Let's speak, that the set* $C(A,d,\infty)$ *is closed relative to the global transition function* $\tau^{(n)}$ *in a classical d–CA model if and only if the next relation* $(\forall c^* \in C(A,d,\infty))(c^*\tau^{(n)} \in C(A,d,\infty))$ *is carried out* $(d \geq 1)$*; otherwise, the set* $C(A,d,\infty)$ *will be considered as nonclosed. Let's speak that a configuration* $c` \in C(A,d,\infty)$ *is* $\square$*-reducible in a classical d-CA* $(d \geq 1)$ *model with global transition function* $\tau^{(n)}$ *if for it the relation* $c`\tau^{(n)} = \square$ *takes place.*

It is shown [41,43] that the nonclosure of the set $C(A,d,\infty)$ relative to the global transition function $\tau^{(n)}$ of a classical ***d–CA*** model is equivalent to existence in it of a configuration c_∞ such that $c_\infty\tau^{(n)} = \square$ $(d \geq 1)$. Whereas, basing on the last, solution of the above solvability problem is reduced to the modified ***domino*** problem which is algorithmically unsolvable, defining algorithmic unsolvability of closure of the set $C(A,d,\infty)$ relative to the global transition function $\tau^{(n)}$ of a classical ***d–CA*** model for the case $(d \geq 2)$; meantime, for dimension $d=1$ the problem is algorithmically solvable. Taking into account the above, we will identify nonclosure of

the set $C(A,d,\infty)$ relative to the global transition function $\tau^{(n)}$ of classical *d–CA* model in the future with existence of □–reducible configurations $c_\infty \in C(A,d,\infty)$ in a classical *d–CA* $(d \geq 1)$ model.

Theorem 21. ***A classical d–CA*** $(d \geq 1)$ ***model that not has NCF (NCF–3), possesses NCF–1*** {***NCF–2***} ***if and only if the set*** $C(A,d,\infty)$ ***is non–closed*** {***closed***} ***relative to mapping induced by GTF*** $\tau^{(n)}$ ***of the model. At that, a classical d–CA*** $(d \geq 1)$ ***model will possess the nonconstructability of NCF type and/or NCF–1 if the set*** $C(A,d,\infty)$ ***will be non–closed relative to mapping induced by GTF of the model. If for a classical d–CA*** $(d \geq 1)$ ***model the set*** $C(A,d,\infty)$ ***is nonclosed relative to mapping induced by the global transition function*** $\tau^{(n)}$ ***of the model along with absence of the NCF–1 (NCF) nonconstructability then the model will possess the NCF (NCF–1) nonconstructability. If for a classical structure d-CA*** $(d \geq 1)$ ***the set*** $C(A,d,\infty)$ ***is closed relative to mapping defined by global transition function*** $\tau^{(n)}$ ***then the model not possess the NCF–1 nonconstructability, irrespective of existence of the NCF nonconstructability. If a classical d–CA not possess the*** γ***–configurations and*** □***–reducible configurations, then a configuration*** $c \in C(A,d,\phi)$ ***will be possess the single predecessor from the set*** $C(A,d,\phi)$ ***at absence of predecessors from the set*** $C(A,d,\infty)$.

It is shown [7,42] that share δ of classical *d–CA* $(d \geq 1)$ models for which the set $C(A,d,\infty)$ is nonclosed relative to global transition function $\tau^{(n)}$ is more than $(e-1)/e$ on condition that $n>2$ or/and $a>2$, i.e. $\delta > 0.632$. Thus, quota of classical *CA* models which possess the nonconstructability of type *NCF* and/or *NCF-1* more than $(e-1)/e$ irrespective of parameters *d, n.* The share of classical *d–CA* $(d \geq 1)$ models that not possess the *NCF–1* and/or *NCF* less than $1/e$, i.e. less than 0.37. In addition, it was shown that quota of classical *d–CA* $(d \geq 1)$ models which not possess the *NCF–1* and/or *NCF* aspires to *1* with growth of alphabet cardinality of models [24,40-43]. Furthermore, in the assumption that null configuration c_o = □ was ascribed by us to the set $C(A,d,\phi)$ of finite configurations, we obtain possibility slightly differently to reformulate theorem *21,* representing essentially more convenient existence criterion of *NCF–1* in classical *CA* models not possessing the *NCF* and *NCF–3* nonconstructability[24,42].

Theorem 22. ***Existence for an arbitrary classical d–CA*** $(d \geq 1)$ ***model of configurations*** $c' \in C(A,d,\infty)$ ***such that relation*** $c'\tau^{(n)} = c \in C(A, d, \phi)$ ***takes place is necessarily but not sufficiently for existence in the d–CA of the NCF–1 nonconstructability. In particular, the existence of d–CA*** $(d \geq 1)$ ***models for which each finite configuration is NCF or ACCF confirms***

the second part of the previous statement. For a classical d–CA (d ≥ 1) model which not possess the NCF nonconstructability there are finite configurations of types from {NCF–1, NCF–2, ACCF} if the set C(A, d, ∞) is nonclosed relative to mapping defined by global transition function $\tau^{(n)}$ of the CA model.

The first part of the theorem **22** is proved by existence of classical ***1–CA*** models for which the set ***C(A, d, ∞)*** are nonclosed relative to mappings induced by their global transition functions $\tau^{(n)}$ and for which a finite configuration, different from ***NCF***, has predecessors from set ***C(A, d, ϕ)*** and set ***C(A, d, ∞)***, i.e. such ***CA*** models not possess the ***NCF–1*** [40-43].

Theorem **22** gives answers to a number of questions, raised in our books [1,8] and in some other works. In addition, the theorem can be used for generalization of a number of results concerning the *nonconstructability* problem in classical ***d–CA (d ≥ 1)*** models. The results received by us in this direction allows to formulate the following proposal.

<u>Proposal 2</u>. If for a classical d–CA (d≥1) model the NCF–1 exist and the NCF nonconstructability absent then a configuration c ∈ C(A, d, ϕ) has at least one predecessor from the set C(A, d, ∞), i.e. in such model the set C(A,d,ϕ) can be generated from a set G⊆NCF–1, i.e. the dynamics of an arbitrary finite configuration and the model as a whole are irreversible in our comprehension. As a predecessor of a configuration c∈C(A, d) we understand a configuration c`∈C(A,d) if such p≥1 exists, that c`$\tau^{(n)p}$ = c.

In addition, the further research of this problematics has allowed us to introduce a new concept of the ***MEC*** as certain basis of the generalized criterion of the nonconstructability in classical ***d–CA (d ≥ 1)*** models [5,8, 9,12,13,24-28,40-43,82-87,102,106], namely.

<u>Definition 11</u>. Two configurations c_1, c_2∈C(A, d) ($c_1 \neq c_2$) form a pair of the generalized mutually erasable configurations (MEC–1) concerning the global transition function $\tau^{(n)}$ of a classical d–CA (d≥1) if and only if for their the relation $c_1\tau^{(n)} = c_2\tau^{(n)} = c^{\#} \in C(A, d, \phi)$ is valid.

In addition, the ***MEC–1*** pairs analogously to ***MEC*** pairs can be made up by configurations such as ***NCF-1*** and/or ***NCF***, i.e. {***NCF-1,NCF-1***}, {***NCF, NCF***}, {***NCF, NCF–1***}. As distinct from the ***IB MEC*** for ***MEC–1*** a certain analogue in the form of in a sense *'absorption node'* $c^{\#}$ is defined whose size presents a certain interest in a whole series of numerical researches of the nonconstructability problem caused by presence of ***MEC–1*** pairs.

The expediency of definition of the given concept is caused by the fact, that pairs of ***MEC–1*** permit also infinite configurations. In particular, it was shown that minimal sizes of $|c^{\#}|$ and configurations ***NCF–1*** and ***NCF*** can be equal. If absorption node $c^{\#}$ can be a ***NCF–1*** configuration, then $c^{\#}$ cannot be as a ***NCF*** configuration. At the heart of this property the definition ***11*** and principal difference between the ***NCF*** and ***NCF–1*** nonconstructability lay.

Meanwhile, between the concepts of ***MEC*** and ***MEC–1*** a series of other differences takes place. In particular, presence for a classical ***d–CA*** $(d \geq 1)$ model of ***MEC*** pairs entails presence for the model of ***MEC–1*** pairs that are formed by infinite structures on the basis of ***MEC;*** but on the other hand, presence for a classical ***d-CA*** of ***MEC–1*** pairs that are structurally distinct from ***MEC*** not necessarily entails presence for the ***CA*** model of ***MEC*** pairs. This circumstance is caused by the fact that presence for an arbitrary classical ***d–CA*** $(d \geq 1)$ model of ***MEC*** pairs is one of ***2*** criteria of existence for the model of the ***NCF*** nonconstructability, while existence for a classical ***CA*** model of ***MEC-1*** pairs not necessarily causes presence for the model of the ***NCF*** nonconstructability.

The above ***MEC–1*** concept of mutual erasability in classical ***d–CA*** $(d \geq 1)$ models is closely related with the general nonconstructability problem what the following rather important result testifies [24-28,40-43,82,102].

Theorem 23. An arbitrary classical d–CA $(d \geq 1)$ ***model posesses at least the nonconstructability of type NCF (perhaps, NCF–3) or NCF–1 if and only if for the model exist the MEC–1 pairs. If for a d–CA*** $(d \geq 1)$ ***model the MEC–1 pairs are absent, then the model will be possess NCF–2; in addition, existence of the NCF–2 nonconstructability can fully combine with existence of the MEC–1 pairs.***

A simple example of a classical ***1–CA*** model with neighbourhood index $X = \{0,1\}$ and states alphabet $A = \{0,1,2\}$ serves as a proving of the second part of the second assertion of the theorem ***23*** [42,82,102,106].

Result of theorem ***23*** is a rather essential generalization of well–known criterion of ***Moore–Myhill*** *(theorem* ***6****)* and criterion of ***Aladjev–Kimura–Maruoka*** *(theorem* ***13****)* that is equivalent to the first, extending them onto other nonconstructability types too. As the ***NCF–3*** concept is immediate consequence of ***NCF*** nonconstructability differentiation onto block and configurational nonconstructability then determination of criterion that a finite configuration is the ***NCF–3*** seems rather useful. In this respect a criterion is represented by the following theorem [5,8,9,40-43,82,102].

Theorem 24. ***A configuration $c = \square c_b \square$ in classical 1–CA model is NCF–3 configuration if and only if the block configuration c_b in the model will be constructible, however block configuration $c`_b = 0^p c_b 0^p$ $(p \geq a^n + n - 1)$ will be as a NCF configuration.***

Result of theorem ***24*** represents in a certain sense a constructive test for belonging of a configuration $c \in C(A,\phi)$ to the ***NCF–3*** type by being thus a rather effective means in a lot of theoretical researches concerning the dynamics of classical ***d–CA*** models. Meantime, this nonconstructability type is not ascribed by us to the basic type. This solution is based on the circumstance caused by the following result [5,8,9,24,40-43,82-87,102].

Theorem 25. ***Existence in a classical d–CA ($d \geq 1$) of nonconstructability of NCF-3 type causes existence in it of the NCF nonconstructability too while the converse assertion is, generally speaking, false. In a classical d–CA ($d \geq 1$) exist configurations NCF (NCF–3) only if in it exist finite configurations not having predecessors from the set $C(A,d,\infty) \cup C(A,d,\phi)$.***

Meantime, in a lot of cases it will be convenient enough for us to group both nonconstructability types by common concept «***NCF***», separately not discriminating thus peculiar type ***NCF–3*** which in the mean time in certain cases appears useful enough. Hereinafter we will concider ***NCF*** and ***NCF-1*** as the principal nonconstructability types. The next theorem ***16*** summarizes a number of useful enough interrelations between ***MEC***, ***γ–CF***, ***NCF*** and ***NCF–1*** [5,8,9,12,13,24-28,40-43,82-87,102,106].

Theorem 26. ***Let $\tau^{(n)} = \tau^{(m)}\tau^{(p)}$ is a decomposition of a global transition function $\tau^{(n)}$ into two global transition functions $\tau^{(m)}$, $\tau^{(p)}$ of the same dimension 1 and which are determined in the same states alphabet A. If GTF $\tau^{(m)}$ not possess MEC, and $\tau^{(p)}$ possesses a set D of γ–CF then GTF $\tau^{(n)}$ possesses the same set D of γ–CF. If the GTF $\tau^{(m)}$ not possess MEC and $\tau^{(p)}$ has pairs of MEC then the GTF $\tau^{(n)}$ has the same set of NCF as the function $\tau^{(p)}$. There are transition functions $\tau^{(n)}$ having MEC with the limited size of minimal simple IB of MEC and NCF of an arbitrary given minimal size. There are functions GTF $\tau^{(n)}$ for which any pair of MEC contains the nonconstructible configurations of NCF type. Let a global transition function $\tau^{(n)}$ possesses a set M of pairs of MEC. Then functions $\tau^{(n)m}$ $(m > 1)$ have the set M of MEC identical with function $\tau^{(n)}$ if and only if at least only one configuration of each MEC pair from M is NCF for GTF $\tau^{(n)}$; otherwise, $(\forall k \geq 1)(M_k \subset M_{k+1})$ where M_k are sets***

of all MEC pairs for global transition functions $\tau^{(n)k}$ ***(*** $k \geq 1$***). A function*** $\tau^{(n)}$ ***possesses a NCF if and only if at least one from functions*** $\{\tau^{(m)}, \tau^{(p)}\}$ ***possesses NCF. If at least one function from pair*** $\{\tau^{(m)}, \tau^{(p)}\}$ ***possesses a NCF–1 then their composition*** $\tau^{(n)}=\tau^{(m)}\tau^{(p)}$ ***(*** $n=m+p-1$***) will possess NCF and/or NCF–1. Last two assertions are valid for the case of dimensions*** $d \geq 1$ ***and any finite states alphabet A of the CA models.***

For example, in view of the represented results follows, that in classical ***CA*** models there are global transition functions having different sets of ***MEC*** along with identical sets of ***NCF.***

We shall consider the set ***G*** of the binary classical structures ***1–CA*** with neighbourhood index $X = \{0,1,2\}$ as a rather simple example concerning the distribution of the above nonconstructability types. It is obvious that models quantity of this set equals ***128*** while its each model is uniquely identified by an appropriate discriminating number whose principle of calculation has been described earlier.

First of all, according to the nonconstructability criterion on the basis of ***γ–CF*** *(theorem* ***13****)* we can simply make sure that all ***1–CA*** with numbers**:**

0..14, 16..29, 31..39, 40..42, 44, 46..50, 52, 54..56, 59, 61..74, 76, 77, 79..84, 87, 88, 91, 93..98, 100, 103, 104, 107, 109..112, 115, 117..119, 121..127

possess ***NCF*** and, possibly, ***NCF–1*** *(we relinquish confirmation of it to the reader)*. The of such models is ***93.*** A number of models of this subset can support a rather complex dynamics whose features we here disregard.

The following subset is formed by the models possessing pairs of ***MEC,*** and consequently ***NCF.*** Quantity of similar models is ***11,*** their numbers are ***43, 53, 57, 58, 68, 92, 99, 108, 113, 114, 116.*** In particular, models with numbers ***53, 99, 113*** not possess ***NCF–1*** while ***CA*** models with numbers ***92, 108, 114*** moreover possess ***NCF–1.*** Three ***1-CA*** models with numbers ***15, 51, 85*** not possess ***NCF*** and ***NCF–1,*** generating identical sequences of configurations *(within shift)* and from standpoint of dynamics of especial interest do not present. At last, the ***CA*** models that not possess the ***NCF*** owing to absence for them the ***MEC*** belong to the last subset. Meantime the *discriminating* numbers of the models with certain useful comments are represented below, namely**:**

30 – configurations of the kind $\{\square(1110)^k11\square \mid k = 0, 1, 2, ...\}$ are ***NCF–1;***
45, 101 do not possess ***NCF–1,*** but any finite configuration is periodical**;**
60 – configurations of the kind $\{\square 1^{2k+1}\square \mid k = 0, 1, 2, ...\}$ are ***NCF–1;***
75, 102, 105, 106, 120 possess ***NCF–1*** at least of the simplest kind $c = 1$***;***

86, 90 possess ***NCF–1*** of the simple kind ***c* = □*11*□;**

89 – configurations of the kind **<□*1x*$_1$*... x*$_n$*111*□>** are ***NCF–1*; *x*$_j$∈*A*={*0,1*}, *j=1..n*;** the model has finite configurations only ***ACCF*** and ***NCF–1.***

Thus, from total ***128*** binary ***1–CA*** models of the considered set ***G:***

– 113 models possess ***NCF*** and, possibly, ***NCF–1;***
– three models with numbers ***15, 51, 85*** do not possess ***NCF*** and ***NCF–1;*** in addition**,** they are not of any interest from standpoint of dynamics**;**
– 12 models not possess ***NCF;*** in addition, only ***2*** of them with numbers ***45*** and ***101*** do not possess ***NCF–1*** in addition.

With the aim of the more detailed investigation of binary classical ***1–CA*** models with neighbourhood index ***X* = {*0, 1, 2*}** which possess ***NCF*** three procedures have been programmed in ***Maple.*** For computer research of ***CA*** models we programmed a number of programs implemented in the systems of computer mathematics ***Maple*** and ***Mathematica.*** Using the above three procedures ***(NcfQ, MinNCF, NfToLtf),*** implemented in the ***Mathematica*** system**,** we researched the problem of minimum size of ***NCF*** in such ***1–CA*** models with discriminating numbers from diapason ***0..127.*** The ***2–***column table below reflects the obtained result, whose the first column defines minimum size of ***NCF,*** whereas the second defines the discriminating numbers of binary ***1–CA*** models with such ***NCF*** [42]. Meantime, our experience of use of the above tools with all conspicuity showed that in the temp relation the ***Maple*** system is more preferable to such purposes provided that for some procedures identical algorithms were used [24,40-43,48,47,52,82,102,106].

MinNCF	*Binary 1–CA models with discriminating numbers*
9	***37, 91***
8	***22, 104***
6	***57, 73, 99, 109***
5	***5, 23, 25, 41, 54, 61, 62, 67, 77, 94, 95, 97, 103, 107, 108, 110, 113, 118, 121, 122, 124***
4	***7, 9, 13, 21, 26, 27, 29, 31, 33, 35, 38, 39, 44, 49, 52, 53, 58, 59, 65, 69, 71, 74, 79, 82, 83, 87, 88, 92, 93, 100, 111, 114, 115, 123, 125***
3	***1, 3, 6, 10, 11, 14, 17, 18, 19, 20, 28, 36, 40, 42, 46, 47, 50, 55, 56, 63, 70, 72, 76, 80, 81, 84, 96, 98, 112, 116, 117, 119, 126, 127***
2	***2, 4, 8, 12, 16, 24, 32, 34, 48, 64, 66, 68***
1	***0***

Hence, quota of models possessing ***NCF*** and, probably, ***NCF–1*** is ***≈ 0.88*** whereas only ***2*** models ***1–CA*** with numbers ***45*** and ***101*** not possess the basic types ***NCF*** and ***NCF–1*** of nonconstructability. Furthermore, in this direction the following rather interesting result has been proved.

Proposal 3. For a classical d–CA (d ≥ 1) model each finite configuration is periodic if and only if the model not possess the nonconstructability of type NCF and NCF–1. Thus, if a classical model not possess the NCF and NCF-1 nonconstructability then all its finite configurations will be periodic and the model can't possess the universal computability.

So, only five structures from ***128*** with numbers ***15, 45, 51, 85, 101*** possess ***full reversibility*** of finite configurations generated by them**;** where the possibility of calculation of all chain of finite predecessors for any finite configuration $c \in C(A, 1, \phi)$ is understood as this concept**,** i.e. possibility to unambiguously determine its prehistory in a ***CA*** model. Thus, they do not represent particular interest because of their limited generative possibilities from standpoint of modelling applications. So, due to such standpoint the models of the set ***G*** that possess the ***NCF*** or/and ***NCF–1*** nonconstructability can present the greatest interest only. In addition, it substantially is generalized to the more general cases of the classical ***CA*** models. Hence, we should pay the attention to models ***CA*** that possess the ***NCF*** and/or ***NCF–1*** nonconstructability for providing their a rather complex dynamics.

Concept of ***NCF*** and ***NCF–1*** nonconstructability is closely connected to types of dynamics *(state graphs)* of classical ***d–CA (d ≥ 1)*** models. In this direction, particular, the following result takes place, namely**:**

If a classical d–CA (d ≥ 1) model not possess the nonconstructability of types NCF and NCF–1 then for any configuration $c_j \in C(A,d,\phi)$ the model will generate the configurations sequences (state graphs) only of one of the following three kinds, namely:

(a) all configurations sequences $\Theta_j=\{c_j\tau^{(n)k} \mid k \geq 0; j=1..\infty\}$ are periodical;

(b) $\ldots \rightarrow c_{j-k} \rightarrow \ldots \rightarrow c_{j-2} \rightarrow c_{j-1} \rightarrow c_j \rightarrow c_{j+1} \rightarrow c_{j+2} \rightarrow \ldots \rightarrow c_{j+k} \rightarrow \ldots$ ***;***

(c) the model has configurations sequences Θ_j of types (a) and (b).

Absence of nonconstructability of type NCF for the classical CA models is necessary condition but not sufficiently for guarantee of reversibility of their dynamics relative to all finite configurations. If a classical CA model possesses the NCF–1 nonconstructability without NCF, then for a configuration $c_j \in C(A, d, \phi)$ such model generates the configurations

sequence (state graph) only of one of the following two kinds, namely:

(a) $c_j \to c_{j+1} \to \ldots \to c_{j+p} \to \ldots \bigcup_{j=1}^{\infty} \{c_j \tau^{(n)k} \mid k \geq 0\} = C(A,d,\phi);\quad c_j - NCF\text{-}1$

(b) the model has configurations sequences $\{c_j \tau^{(n)k} \mid k \geq 0;\ j = 1..\infty\}$ ***of type (a) along with periodical sequences.***

So, for case of the above binary ***1–CA*** among all ***CA*** models possessing ***NCF–1*** without ***NCF*** the models with numbers ***75,89,90,102,105*** and ***106*** belong to the ***(a)*** type while only three models with numbers ***30, 60*** and ***86*** belong to the ***(b)*** type. This fact is rather obvious, not demanding any special explanations. Now, we can easily be made sure in correctness of the following result [40-43,82,102,106]**:**

For existence in a d–CA (d ≥1) model of the NCF–1 nonconstructability without NCF is sufficient that for each finite configuration $c \in C(A,d,\phi)$ ***the d–CA generates, as a whole, a sequence of increasing configurations concerning the configurations sizes, i.e.***

$$(\forall c \in C(A,d,\phi))(\exists\{j_1 > j_2 > \ldots > j_q > \ldots\})\left(k < p \to \left|c\tau^{(n)j_p}\right| > \left|c\tau^{(n)j_k}\right|\right)\ \&$$
$$(\neg\exists(k<p))\left(\left|c\tau^{(n)k}\right| > \left|c\tau^{(n)p}\right|\right);\quad j_q - \text{ascending sequence of integers;}$$
$$|h| - \text{size of a configuration } h\ (q = 1,2,\ldots\infty)$$

If for some d–dimensional global transition function $\tau^{(n)}$***, defined in a finite alphabet A, the following relation*** $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$ ***takes place where*** $|c^*|$ ***– maximal diameter of a configuration*** c^****, then the existence problem of the NCF and/or NCF–1 nonconstructability for the global transition function*** $\tau^{(n)}$ ***is algorithmically solvable.***

Furthermore, if a classical d–CA (d ≥ 1) model does not possess the NCF nonconstructability and for it the above relation takes place, then this model possesses the NCF-1 nonconstructability, the set $C(A,d,\infty)$ ***is non-closed relative to mapping induced by GTF*** $\tau^{(n)}$ ***of the model and the set*** $C(A,d,\phi)\setminus\{\square\}$ ***can be generated by means of only of configurations such as NCF–1; in addition, the set NCF–1 is minimal generative set for the set*** $C(A,d,\phi)\setminus\{\square\}$***, i.e. the next relation takes place:***

$$\bigcup_j < c_j > \left[\tau^{(n)}\right] = C(A,d,\phi)\setminus\{\square\};\ (\forall i,j)(i \neq j \to < c_i > \left[\tau^{(n)}\right] \cap < c_j > \left[\tau^{(n)}\right] = \varnothing$$
$$\text{where } c_j \text{ are configurations forming the set NCF–1}$$

Thus, for classical ***CA*** models of the above sort a finite set of generators of ***NCF–1*** type for the set $C(A,d,\phi)\setminus\{\square\}$ absents. It comparatively easily follows of the obvious enough second relation represented above.

On the other hand, ***if a classical d–CA (d≥1) model does not possess the***

NCF-1 nonconstructability and for it the above relation takes place the model possesses the NCF nonconstructability, and the set $C(A,d,\phi)\setminus\{\square\}$ can be generated by means of only of configurations from a minimal set $K\subseteq NCF$; whereas for a model possessing the nonconstructability NCF-1 and NCF provided the above relation, the set $C(A,d,\phi)\setminus\{\square\}$ is generated by means of only of configurations from a minimal set $J\subseteq NCF\cup NCF\text{-}1$. Furthermore, the case of strict inclusion is conditioned by the fact, that the configurations such as NCF and/or NCF–1 can form MEC–1 pairs.

Among classical CA models satisfying the relation $(\forall c\in C(A,d,\phi))(|c^*| < |c^*\tau^{(n)}|)$ without the NCF nonconstructability makes sense to search models possessing the universal reproducibility of finite configurations in the Moore sense, e.g. the class of linear classical models of few other types possessing the universal reproducibility are considered below.

Basing on theorem ***21,*** estimations of number $N(a,n)$ and share $\delta(a,n)$ of the classical ***d-CA*** $(d\geq 1)$ models with a states alphabet $A=\{0,1,...,a\text{-}1\}$ and neighbourhood index $X=\{0,1,...,n-1\}$ that will possess the ***NCF–1*** and/or ***NCF*** ***(NCF–3)*** nonconstructability are expressed by the relations below:

$$N(a,n) > a^{a^n-a}\left[\sum_{j=0}^{a-1} C_{a-1}^{j}(a-1)^{a-j-1} - (a-1)^{a-1}\right] = a^{a^n-a}\left[a^{a-1} - (a-1)^{a-1}\right]$$

$$\delta(a) > N(a,n)/a^{a^n-1} = \left[1-(1-1/a)^{a-1}\right]; \quad \lim_{a\to\infty}\delta(a) > 1-1/e; \quad \delta 1(a) < 1/e$$

where $[x]$ – a least integer $\geq x$. While share $\delta(a)$ of such ***CA*** models with respect to all classical ***d–CA*** is defined by the above relation regardless of parameters d and n of a model. Hence, share $\delta 1(a)$ of classical ***d–CA*** $(d\geq 1)$ models not possessing the nonconstructability neither ***NCF–1*** nor ***NCF*** is less than $1/e$. In addition, $\delta(a)$ and $\delta 1(a)$ is enough appreciably decreased and overstated accordingly with growth of the cardinality of alphabet A depending on our numerous experimental researches.

Obviously, for existence in an arbitrary classical d–CA $(d\geq 1)$ model of the NCF–1 nonconstructability the existence for such model of infinite configurations $c\in C(A,d,\infty)$ such that $c\tau^{(n)} = \square$ is necessary, however not sufficient condition. If in a d–CA $(d\geq 1)$ model the pairs MEC exist then relative to its global transition function $\tau^{(n)}$ exist such configurations $a^\infty, b^\infty\in C(A,d,\infty)$ that $a^\infty\tau^{(n)} = b^\infty\tau^{(n)}$ $(a^\infty \neq b^\infty$ to within shift on axes of coordinates of Z^d space) while the opposite is generally incorrect that a very simple 1–CA model proves.

Theoretical and computer research of the self–reproducibility problem in the *Moore* sense of finite configurations in classical ***d-CA*** $(d\geq 1)$ models

allows us to formulate the following proposition, namely**:**

A classical d–CA model, posessing the property of self–reproducibility in the Moore sense of finite configurations, will possess the property of NCF–1 nonconstructability in the absence of NCF nonconstructibility, while the converse is generally not true. If a global transition function $\tau^{(n)}$ ***possesses NCF–1 then for it there is configuration*** $c \in C(A,d,\infty)$ ***such that*** $c\tau^{(n)} = \square$ ***whereas the converse is in general speaking not true.***

Indeed, the simple binary ***1–CA*** model, whose local transition function can be represented by means of the formula

$$\sigma^{(3)}(x, y, z) = \mathbf{If}[x = 0, x+y+z \ (mod\ 2), x+y+z+1 \ (mod\ 2)]; \quad x,y,z \in \{0, 1\}$$

will possess the ***NCF–1*** nonconstructability in the absence for it of the ***NCF*** nonconstructability does not possess, however, the property of the universal reproducibility in the *Moore* sense of finite configurations. As another example, one can suggest the simple ***1–CA*** model, whose local transition function can be represented by means of the formula

$$\sigma^{(2)}(x, y) = \mathbf{If}[x = 0, y, \mathbf{If}[xy \in \{10, 11, 21, 22\}, x+y \ (mod\ 2), 2]]; \quad x,y \in \{0, 1, 2\}$$

Each finite configuration in such model is ***NCF–1*** nonconstructable or absolutely constructive, i.e. has predecessors from the sets $C(A,d,\phi)$ and $C(A,d,\infty)$ simultaneously**,** in the absence of the ***NCF*** nonconstructability, however, it does not posess the property of universal reproducibility in the *Moore* sense of finite configurations.

Whereas the simple binary ***1–CA*** model, whose local transition function can be represented by means of the formula as follows

$$\sigma^{(3)}(x, y, z) = \mathbf{If}[x = 0, z, y+z+1 \ (mod\ 2)]; \quad x,y,z \in \{0, 1\}$$

possesses the ***NCF-1*** nonconstructability in the absence for it of the ***NCF*** nonconstructability, by possessing, in the same time, the property of the universal reproducibility in the *Moore* sense of finite configurations.

Finally, the final part of the above proposition is very clearly illustrated by an example of a simple classical ***1–CA*** model whose local transition function $\sigma^{(2)}$ is determined by parallel substitutions as follows

$$00\to0 \quad 01\to0 \quad 02\to1 \quad 10\to0 \quad 11\to2 \quad 12\to1 \quad 20\to0 \quad 21\to2 \quad 22\to1$$

It is easy to verify that for this ***1–CA*** model, each finite configuration is vanishing or the ***NCF*** nonconstructible at absence for the model of the ***NCF–1*** nonconstructability.

On the basis of analysis of ***4*** basic types of the nonconstructability and computer simulation, it is possible to establish the fact, the dynamics of

configurations $c \in C(A,d,\phi)$ in classical ***d–CA*** models is characterized by transition graphs of the following kind, namely**:**

1. ***An infinite non–periodical sequence*** ***of finite configurations from the set*** $C(A,d,\phi)$***:*** $c_o \to c_1 \to ... \; c_j \to ...$ for which there are two possibilities**:**

*(**a**)* determines an algorithm of structural organization of components of configurations depending on an initial configuration $c_o \in C(A,d,\phi)$ and a moment of time ***t;***

*(**b**)* such algorithm is absent or is complex enough for determination.

2. ***Sequence of a pure cycle;*** in addition**,** the *passive* configurations *(**PCF**)* $c_o \in C(A,d,\phi)$ defined by the relation $c_o\tau^{(n)} = c_o$ *(accurate to shift; where* $\tau^{(n)}$ *is a global transition function)* fall under the given definition too**:**

$$c_o \to c_1 \to c_2 \to .... \; c_j$$

There are ***d–CA*** models in which each configuration c_o generates a pure cycle whose period is at least **2** and depends as on kind of $c_o \in C(A,d,\phi)$ and its size [24,40-43,82,102,106].

3. ***Sequence of the mixed cycle;*** it is characterized by presence in it of the certain configuration c_j generating a pure cycle, namely**:**

$$c_o \to c_1 \to c_2 \to ... \; c_j \to ... \to c_{j+p} \to$$

It is simple to be convinced**,** that ***CA*** models whose dynamics includes transition graphs of type *(**3**)* possess the ***NCF*** nonconstructability.

Dynamics of classical ***CA*** models not possess transition graphs of other type**;** in addition, dynamics can correspond to the sets of the specified graphs or to one of them. Thereby a ***NCF*** is absolutely nonconstructible configuration with respect to the set $C(A,d,\phi) \cup C(A,d,\infty)$ whereas ***NCF–1*** and ***NCF–2*** are nonconstructible configurations with respect to the sets $C(A,d,\phi)$ and $C(A,d,\infty)$ accordingly.

The above arguments have been considered as ***1*** of possible approaches to classification of dynamics of classical ***CA*** models [82-87] in context of discussion of a work of ***A.W. Burks*** [103]. Later on, systematizing ***d–CA*** models concerning their behaviour, ***S. Wolfram*** has defined into them ***4*** classes in many respects similarly to our approach.

Meantime, this classification carries purely phenomenological character and not provides any recommendations for receivining on its base the required rules of behaviour of ***CA*** models, it only characterizes possible

types of dynamics as a whole. So, there are also other *phenomenological* criteria of classification of rules of dynamics of classical ***CA*** models on which we not dwell here due to certain reasons, referring the interested reader to the appropriate works presented in the extended bibliography [7]. Meantime, phenomenological criteria only externally qualitatively characterize the ***CA*** dynamics, not allowing to use them as a toolkit for direct programming of ***CA*** models with the required dynamics.

Whereas graphs of predecessors for configurations $c_o \in C(A,d,\phi)$ belong to the following basic types, namely**:**

1. An infinite non–periodical sequence of configurations from $C(A,d,\phi)$***:***

$$\ldots \leftarrow c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o$$

2. A finite non–periodical sequence of configurations from set $C(A,d,\phi)$***:***

$$c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o$$

This case occures if a configuration c_{-j} is nonconstructible of ***NCF*** type.

3. A sequence of pure cycle; at the same time, the passive configurations $c \in C(A, d, \phi)$ *(accurate to shift)* fall under this definition also, namely**:**

$$c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o \leftarrow c_{-j}$$

4. A sequence of the mixed type; it is characterized by existence in it of a configuration c_{-j} for which predecessors can be from the set $C(A,d,\phi)$ or from the set $C(A, d, \infty)$, or from both sets simultaneously, namely**:**

$$\begin{array}{lll} \leftarrow \ldots c_{-j-p} \leftarrow \ldots \leftarrow c_{-j-1} & & c_r \in C(A,d,\phi); \quad r = 0..-j-p \\ & \nwarrow & \\ & c_{-j} \leftarrow \ldots \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_o & \\ & \swarrow & \\ \leftarrow \ldots b_{-j-k} \leftarrow \ldots \leftarrow b_{-j-1} & & b_q \in C(A,d,\infty); \quad q = -j-1..-j-k \end{array}$$

The above binary ***1–CA*** models and other types of ***CA*** models can serve as examples of such states graphs in the ***CA*** dynamics. In addition, it is possible to be convinced that graphs of predecessors such as ***(4)*** starting with configuration c_{-j} admit also such subgraphs as ***(1) – (3)***. Meantime, the next an essential enough result has been received [24,40-43,82-87].

Theorem 27. The classification problem of classical d–CA $(d \geq 2)$ ***models models according to types of transition graphs of finite configurations is algorithmically unsolvable in general case.***

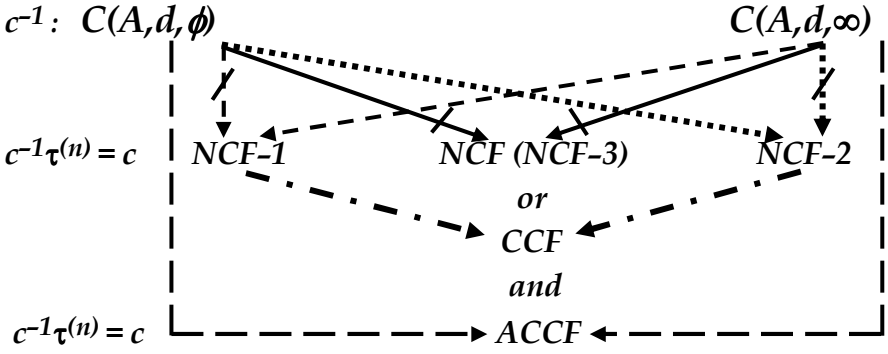
Our dynamics classification of classical ***d-CA*** models along with similar classification of ***S. Wolfram*** have mainly phenomenologic character and do practically not play any classifying part. They were received on the

basis of experiments with simple enough types of classical ***CA*** models and rather speculative experiments. Meanwhile, each classification that pretends to this name should provide certain algorithm either direct or indirect that allows to ascribe a classical ***CA*** model to this or that type. However, both mentioned classifications do not allow to do it, because, for instance in view of *unsolvability* of the existence problem of the ***NCF*** nonconstructability in ***d–CA*** $(d \geq 2)$ models we can`t differentiate a ***CA*** model already concerning type ***4.*** Impossibility of similar classification have been proved also by ***K. Culik*** and ***S. Yu*** on the basis of the problem represented below [104].

Proposal 4. The existence problem for a d–CA $(d \geq 2)$ ***model of all finite configurations as vanishing ones is algorithmically unsolvable.***

Among other ways of classification of ***CA*** models it is possible to note approach of ***C. Langton*** on base of ***λ–***parametrization which measures quota of non–zero values of ***LTF*** together with approaches of ***N. Israeli, J. Dubacq, H. Goldenfeld,*** that have offered parametrization of ***LTF*** on the basis of known complexity concept of ***A. Kolmogorov*** [7,40,82]**,** and an interesting stochastic approach of ***A.V. Lebedev*** [105]. Now, there are some other approaches to classification of ***CA*** models [7,24,82-87,102].

Along with differentiation of the nonconstructability concept relative to classical ***d-CA*** $(d \geq 1)$ models the *differentiation* question of *constructability* of finite configurations presents indubitable interest. So, a configuration $c \in C(A,d,\phi)$ in classical ***d-CA*** $(d \geq 1)$ is called as *constructible* configuration *(**CCF**)* if it has predecessors c^{-1} from the set $C(A,d,\phi)$ or the set $C(A,d,\infty)$, i.e. $c^{-1}\tau^{(n)} = c$**.** Obviously, a constructible configuration can't be as ***NCF*** *(**NCF–3**)*, but it can be ***NCF–1*** or ***NCF–2.***



On the other hand, in classical ***d–CA*** $(d \geq 1)$ a configuration $c \in C(A,d,\phi)$ is called the ***absolutely constructible*** configuration *(**ACCF**)* if and only

if it has predecessors as from set $C(A,d,\phi)$ and from set $C(A,d,\infty)$. Thus, obviously, that an absolutely constructible configuration c can't be as a *NCF (NCF–3)*, at the same time it can't be as *NCF–1* or *NCF–2* as well. The diagram presented above quite visually illustrates interrelation of all four nonconstructability types *(NCF, NCF–1, NCF–2, NCF–3)* along with constructability *(CCF, ACCF)* in the classical d*–CA* $(d \geq 1)$ models. On the basis of concept of *absolutely constructible* configurations and the theorem *21* we can prove the following rather interesting result [12,42].

***Theorem 28**. For an arbitrary classical d-CA (d≥1) model the set $C(A,d,\phi)$ can't consist only of absolutely constructible configurations as well as finite configurations such as NCF–1, while the set $C(A,d,\phi)$ can consist of the constructible configurations only of type NCF–2, for example.*

In the given context a rather interesting question about the maximal set of absolutely constructible finite configurations arises. So, the detailed enough discussion of this question can be found in [24-28,102,106]. The following result gives an answer to this question.

***Theorem 29**. For each classical d–CA $(d \geq 1)$ model one of three relations occures, namely: $CCF \subseteq C(A,d,\phi)$, $CCF \equiv ACCF$, $ACCF \subset C(A,d,\phi)$, where CCF, ACCF are the sets of all constructible and absolutely constructible finite configurations accordingly; in addition, the next relations occur: $(\exists d\text{-}CA)(C(A,d,\phi) \equiv NCF\text{-}2)$, $(\forall d\text{-}CA)(C(A,d,\phi) \supset NCF\text{-}1 \ \& \ C(A,d,\phi) \supset ACCF)$.*

The above *1–CA* with neighborhood index $X=\{0,1,2\}$ and discriminating number *116* can be considered as an interesting enough example. So, a block configuration $c^* = \langle 010 \rangle$ is a *NCF* of minimal size in this model; therefore any configuration containing this block configuration c^* will be in such *1–CA* as *NCF* while others are absolutely constructible. The detailed analysis of this *1-CA* model allows to formulate one interesting enough result, namely:

There are 1–CA models without NCF–1 for which such configurations as NCF compose «almost all» the set $C(A, 1, \phi)$, whereas the remaining configurations of the set are absolutely constructible configurations, i.e. they have predecessors both from the set $C(A,1,\phi)$ and the set $C(A,1,\infty)$. There are classical d-CA (d≥1) models only with NCF, NCF-1 and ACCF. There are classical d-CA (d≥1) models only with NCF, NCF-2 and ACCF. There are classical d–CA $(d \geq 1)$ models for which each configuration $c \in C(A,d,\phi)$ is NCF or ACCF subject to the existence of such configurations $c^* \in C(A,1,\infty)$, that the next relation $c^*\tau^{(n)}=\square$ takes place; this proves the insufficiency of existence condition of the NCF–1 nonconstructibility in

classical CA models. So, existence of vanishing infinite configurations is necessary, but insufficient condition for the existence in the classical d–CA (d≥1) model of the NCF–1 nonconstructibility.

An example [42] of ***1–CA*** model well illustrates the fact that nonclosure of the set $C(A,d,\infty)$ concerning mapping determined by a classical ***d–CA*** $(d \geq 1)$ model is the necessary but not sufficient condition for existing of the ***NCF–1*** nonconstructability in such ***CA*** models. So, on the one hand, for a classical ***d–CA*** $(d \geq 1)$ model the set $C(A,d,\phi)$ of finite configurations can`t consist only from *absolutely* constructible configurations while, on the other hand, there are classical ***CA*** models for which all constructible configurations of the set $C(A,d,\phi)$ are as well *absolutely* constructible. So, the determinative attribute of concept of classical ***CA*** models allows to *differentiate* naturally not only the *nonconstructability*, but *constructability* of finite configurations too. And in it one more essential feature of the classical ***CA*** models. Along with this feature the classical ***d-CA*** $(d \geq 1)$ are of interest, first of all, from the applied standpoint since they have such special state as a «***quiescent***» that causes a lot of natural interpretations and of this state itself, and a whole series of dynamical properties of the classical ***CA*** models determined by their availability.

As it was already marked, for existence of ***NCF–1*** nonconstructability in a ***d-CA*** model for it necessarily existence at least of one such infinite configuration $c \in C(A,d,\infty)$ that $c\tau^{(n)}=\square$, or $c\tau^{(n)}=c' \in C(A,d,\phi)$ in the more general case. However, here it is necessary to note one rather essential aspect. Among all infinite configurations concerning the classical ***1–CA*** models it is expedient to distinguish configurations of two basic types**:** *(**1**) infinite configurations into both sides* (c^{∞}), and *(**2**) configurations infinite only to the left* (c^{∞}_{-}) or *to the right* (c^{∞}_{+}).

Let for a classical ***1-CA*** model without ***NCF*** there are configurations of the kind $c^{\infty}_{+}=\square x_1 x_2 \ldots x_n \ldots x_j \infty \, (x_1, x_j \in A \setminus \{0\})$ such, that the next relation $c^{\infty}_{+}\tau^{(n)}=\square \in C(A,1,\phi)$ takes place. But then it is simple to prove existence in the model at least of two different configurations $c_p=\square x_1 x_2 \ldots x_n \ldots x_p \square$ and $c_k=\square x_1 \ldots x_k \square \, (p \neq k;\ x_1, x_p, x_k \in A \setminus \{0\})$ made up on the basis of such c^{∞}_{+} that $c_p\tau^{(n)}=c_k\tau^{(n)}=c \in C(A,1,\phi)$ takes place *(in a specific case the relation* $c_p\tau^{(n)}=c_k\tau^{(n)}=\square$ *may be also)*, where '□' belongs *(according to earlier marked agreement about structure of the set of all finite configurations)* to the finite configurations. Hence, the ***1–CA*** model will possess the ***MEC*** pairs and therefore by the ***NCF*** nonconstructability, contradicting the assumption.

Analogous situation takes place and for case of infinite configurations c_{-}^{∞}. So, for the classical ***1–CA*** models at absence of nonconstructability of the ***NCF*** type only c^{∞} configurations infinite into both sides can exist, which satisfy the next relation $c^{\infty}\tau^{(n)}=\square\in C(A,1,\phi)$. Whereas the fact of configurations existence of type c_{-}^{∞} or c_{+}^{∞} for which $c_{+}^{\infty}\tau^{(n)}=\square\in C(A,1,\phi)$ or $c_{-}^{\infty}\tau^{(n)}=\square\in C(A,1,\phi)$ for a classical ***1–CA*** model provides existence of the ***NCF*** nonconstructability for the model. In this context the following rather useful result can be formulated, namely.

Theorem 30*. A classical 1–CA model possesses the nonconstructability of NCF type if for it such configurations* $c^{*}\in C(A,1,\phi)\cup c_{+}^{\infty}\cup c_{-}^{\infty}$ *exist, that the following relation takes place, namely:* $c^{*}\tau^{(n)}=\square$.

So, concerning the nonconstructability concept, the set $C(A,1,\infty)$ of all infinite configurations of classical ***1–CA*** models is differentiated. In the certain cases this circumstance is essential enough. In addition, in a lot of cases the importance of this differentiation is not less significant than differentiation of $C(A,1)$ into the sets $C(A,1,\phi)$ and $C(A,1,\infty)$. Proof of a lot of rather interesting results relative to classical ***CA*** models above all dealing with the set $C(A,d,\infty)$ $(d\geq 1)$ of the infinite configurations rather essentially uses the following entirely obvious lemma.

Lemma 3*. For any integer* $d\geq 1$ *and alphabet* $A=\{0,1,\ldots,a-1\}$ *an arbitrary configuration in the alphabet A of d–dimensional hypercube with edge of size L defined by the following simple formula*

$$L=n\left\lceil\sqrt[d]{a^{n^d}+1}\right\rceil,\quad \text{where }]x[-\text{an integer greater than } x$$

will contain at least two identical subconfigurations on d-dimensional hypercubes with edge of n size. Whereas for the singular 1–dimensional case and an integer $n\geq 1$ *there is such integer* $m=n(a^n+1)$ *that each tuple* $P_j=<x_1x_2x_3\ldots x_m>$ *will contain at least two identical disjoint subtuples* $<y_1y_2y_3\ldots y_n>$ $(x_k, y_p\in A;\ k=1..m;\ p=1..n;\ n<m)$ *of configurations.*

Particularly, this result has been used at the proof of theorem ***30.*** Along with the above questions, there is a lot of other more special questions of global dynamics of the classical d***–CA*** $(d\geq 1)$ models connected to the nonconstructability problem. For example, an question about influence of types of ***LTF*** $\sigma^{(n)}$ onto existence of nonconstructible configurations in the classical ***CA*** models seems us rather interesting. So, investigation of the classical ***1–CA*** determined by symmetric ***LTF***, have shown [5,40-43], that the models defined by such ***LTF*** will possess such configurations

$c \in C(A, 1, \phi)$ as ***NCF (NCF–1)*** only if they will possess configurations c^R, inverse to them, as ***NCF (NCF–1)*** too. Hence, symmetry of ***LTF*** $\sigma^{(n)}$ in a classical ***1–CA*** model expands, generally speaking, the sets of ***NCF*** and ***NCF–1*** whereas for *asymmetric* ***GTF*** $\tau^{(n)}$ both configurations $c \in C(A, 1, \phi)$ and c^R can be as ***NCF*** or ***NCF–1*** separately. In addition, at more general posing the «*symmetry*» of ***LTF*** in classical ***CA*** models can be considered and concerning the separate subclasses. A lot of other special questions of dynamics of classical ***CA*** models relating to the nonconstructability problem the reader can find in our works [5,8,12,13,24-28,82-87,102].

The reversibility concept considered above of classical ***CA*** models plays a rather important part in theoretical and applied aspects, especially in case of use of the ***CA*** as models of *spatially–distributed* dynamic systems from which physical systems represent a special interest. Thus, we can imagine a ***CA*** model as an infinite automaton for a processing of some input words [*configurations from the set* $C(A, d, \phi)$] into output words of the same infinite set $C(A,d,\phi)$. In addition, each output of the automaton becomes its next input word. Thus, we can consider the classical ***d–CA*** models as *infinite autonomous automata* whose description and research of dynamics allow to successfully use both the language of diagrams of states and the graphical language of transitions. Thus, the approach on the basis of *state graphs* is effective enough facility of dynamics research of the classical ***CA*** models what was mentioned earlier. In addition, this graphic approach admits a number of modifications and interpretations responding specificity of the researched problems.

The graph approach to dynamics research of classical ***CA*** models rather widely is used, for example, in works [7,24,103,107-112]. In these terms the functioning of a classical ***CA*** model can be defined by a states graph where the current configuration of the model as a state of some infinite automaton is being understood. In turn, states graph of similar infinite automaton consist of subgraphs of certain elementary types**:**

(a) $\tau^{(n)}: c_o \to c_1 \to c_2 \to c_3 \to \ldots c_j \to \ldots \to c_k \to \ldots \to c_p \to$ (with $c_p \to c_j$)

(b) $\tau^{(n)}: c_o \to c_1 \to c_2 \to \ldots c_j \to \ldots \to c_k \to \ldots \to c_p \to \ldots$

(c) $\tau^{(n)}: \ldots \to c_{-2} \to c_{-1} \to c_o \to c_1 \to c_2 \to \ldots \to c_k \to \ldots \to c_p \to \ldots$

$$c_k \in C(A, d, \phi) \ (k = -\infty \,..\, +\infty)$$

As a result of work on the questions stated by ***A.W. Burks*** [16,112] and in connection with research of the reversibility problem for ***CA*** models

we had investigated the state graphs of the classical ***CA*** models in their connexion with the nonconstructability problem. The basic result can be formulated as follows.

Theorem 31. If a classical d–CA (d ≥ 1) model not possess NCF (NCF–3) and NCF–1 then already relative to input/output alphabet C(A,d,φ) the state graph of the d–CA model can contain only subgraphs of types (a; for j = p = 0) and/or (c); in other cases the combinations of subgraphs of types (a..c) in wide enough ranges are permitted.

A number of results of research concerning state graphs of the classical ***CA*** models represent special interest in the case of consideration of such class of parallel dynamic systems as infinite automata in the traditional meaning. For experimental study of sequences of finite configurations generated by means of classical ***1–CA*** models a procedure programmed in ***Mathematica*** was successfully used.

The procedure call **Steps1CA[*c, A, ltf, t*]** returns the result of the ***t***–fold application to a finite ***c*** configuration given in an alphabet ***A*={*0,1,...,a-1*}** a global transition function determined by the local transition function specified by the list of substitution rules ***ltf.*** Whereas the procedure call **Steps1CA[*c, A, ltf, t, h*]** with an optional fifth ***h*** argument – an arbitrary expression – returns the entire generated sequence of configurations. In order to increase the reactivity of the procedure, it processes only one particular situation – the contradiction between symbols of the ***c*** string and the elements of the ***A*** alphabet. The fragment represents the source code of the procedure with examples of its application. The procedure easily admits a number of interesting extensions, useful, in particular, for the experimental study of self–reproducibility in the *Moore* sense of finite configurations in ***1***–dimensional classical ***CA*** models.

```
In[4242]:= Steps1CA[c_String, A_List, ltf_List, t_Integer, h___] :=
                                  Module[{a = StringTrim[c, {"0" | "0"} ...], b, d,
                              g = "", p, j, m = 0, s, n = StringLength[ltf[[1]][[1]]]},
q = Complement[DeleteDuplicates[Characters[c]], Map[ToString, A]];
                    If[q == {}, s = a; b = StringRepeat["0", n – 1];
                          Do[d = b <> a <> b; p = StringLength[d];
                          For[j = 1, j <= p – n + 1, j++,
                    g = g <> StringReplace[StringTake[d, {j, j + n – 1}], ltf]];
                    a = StringTrim[g, {"0" | "0"} ...];
    If[{h} != {}, If[m == 0, Print[s]; m = 1, Null]; Print[a], 76]; g = "", t]; a,
    "String " <> c <> " contains invalid characters " <> ToString [q]]]
```

```
In[4243]:= Ltf := {"000" → "0", "001" → "1", "100" → "1", "101" → "0",
                   "010" → "1", "011" → "0", "110" → "0", "111" → "1"}
In[4244]:= Steps1CA["0000110110001000100", {0, 1}, Ltf, 6]
Out[4244]= "11101101111100111000010101"
In[4245]:= Steps1CA["0000110100", {0, 1}, Ltf, 5, g]
           "1101"
           "100011"
           "11101001"
           "1010011111"
           "110111011101"
           "10000100010011"
Out[4245]= "10000100010011"
In[4246]:= Steps1CA["03050201070", {0, 1}, ltf, 1, g]
Out[4246]= "String 300201 contains invalid characters {2, 3, 5, 7}"
```

A little bit more in details this question is discussed in [9,24,82] whereas with the above **Steps1CA** procedure the reader can familiarize in [49].

Meanwhile, a question concerning the nonconstructability problem for certain special subclasses of classical ***d–CA*** ***(d ≥ 1)*** models seems rather interesting [24-26]. In particular, it is possible to define one subclass of ***d–CA*** models in which neighbourhood index is variable and is defined by an internal state of the current elementary automaton. It is easy to be sure, the introduced ***d-CA*** models with a variable neighbourhood index and a states alphabet ***A*** of the current elementary automaton are strictly equivalent to appropriate classical ***d-CA*** models with the same alphabet. So, the ***CA*** models that are defined in above way compose a subclass of the class of all classical ***CA*** models.

Thus, depending on the current state the elementary automaton of ***CA*** models determines one's own neighbours from which it is necessary to receive the information for definition of the next state**;** i.e. behaviour of an elementary automaton of the ***CA*** model of similar type has in a sense «*intellectual*» character. In addition, the ***CA*** models of the above type are characterized by individual choice by the elementary automaton of the neighbourhood index**,** expanding *diapason* of quite interesting problems effectively represented on their base. As the further expansion of the ***CA*** of this kind it is possible to consider also the case when neighbourhood index of their elementary automata depends on history at some depth of their previous states. In the same time, similar ***CA*** models represent an quite certain interest from the theoretical standpoint in the context of the nonconstructability problem [8,9,12,13,24-28,40-43,82-87,102,106].

Meantime, a models class basing on the concept of classical ***CA*** models, whose neighbourhood index is defined by the current configuration of neighbourhood template of some constant size seems rather interesting, defining one more kind of ***CA*** models different from classical models and interesting from certain standpoints. So, in this case within of some *fixed* neighbourhood template depending on its current configuration, a neighbourhood index is determined on whose basis a calculation of the next state of its central automaton will be made. But, at such definition of modules it is possible appearance of many–valuedness at evaluation of states of elementary automata with time. Hence, for a single–valued choice of the next state it is necessary to define a certain choice function which solves this problem.

Thereby, within a geometrically annotated neighbourhood template the formative elementary automata are endowed by a little more complex function of commutation relative to the choice of a central automaton, i.e. the automaton whose state changes at the next moment depending on the evaluated neighbourhood index. Unlike the classical ***CA*** models the using to the current configuration of global transition function of a ***CA*** model determined thus can be the most conveniently programmed by a method represented in [24,48-51,82,85,102,106].

It is simple to make sure that this class of ***CA*** models *(*$\mathfrak{R}$*–class)* expands the classical ***CA*** models already at the level of generative opportunities concerning a fixed neighbourhood index within some neighbourhood template. Relative to ***CA*** models of the $\mathfrak{R}$–class it has been shown that the existence criterion of of the ***NCF*** nonconstructability which is based on the ***MEC*** in the case of classical ***CA*** models is correct and for models of the $\mathfrak{R}$–class, namely the following result takes place [40-43,82-87,102]**:**

A CA model of the $\mathfrak{R}$***–class will possess the NCF nonconstructability if only for the model there are MEC pairs in their classical conception.***

More circumstantially with questions of application of ***CA*** models from the $\mathfrak{R}$–class, and with a number of their generalizations along with a lot of their interesting properties and appendices the reader can familiarize oneself in [12,13,24-29,40-43,82-87,102,106].

To the above question the existence of the ***NCF*** nonconstructability for asynchronous ***CA*** models directly adjoins too. In particular, it appears that the criterion of the ***NCF*** nonconstructability**,** basing on the concept of mutual–erasable configurations, extends on a rather wide class of the asynchronous models. A sufficiently detailed analysis of asynchronous

CA models from this class along with a number of certain other types of models of this class has allowed to formulate the following interesting enough result, namely**:**

There is a wide enough class of asynchronous CA models for which the criterion of existence of the NCF nonconstructability basing on concept of pairs of mutually–erasable configurations (MEC) analogously to the case of classical and unstable CA models is invalid, as a whole.

Of this result follows, that such important enough concepts as ***MEC*** and ***NCF*** are inherent, mainly, in classical and unstable ***CA*** models, ***NCF–1*** concept is inherent to classical ***CA*** models only, whereas for the case of a rather wide class of asynchronous ***CA*** models these concepts lose own primary meaning. Thus, relative to interrelation of the ***NCF*** and mutual erasability the class of asynchronous ***CA*** models essentially differs from classical and unstable models, generally speaking.

Of the above results it is possible to confidently conclude the concept of nonconstructability in classical ***CA*** models has been researched enough in detail, therefore today this section of the ***CA*** theory is one of the most advanced. Meantime, this problematics contains a lot of open questions and rather perspective directions of research. In particular, it concerns the absence of satisfactory criteria of existence for classical ***CA*** models of combinations of various nonconstructability types according to table ***1*** in its complete volume.

Results represented in this section cover only some part *(though enough considerable part)* of this question, while the more detailed information relative to the nonconstructability problem can be found in proceedings quoted in the present section and in the extensive enough bibliography [7]. The further research of the problematics has allowed to introduce a new concept of the ***MEC*** as a some basis of the generalized criterion of the nonconstructability in classical ***d–CA (d≥1)*** models [8,9,24-28,84-87]. Finally, the results obtained from researches of the nonconstructibility problem, can be considered as a sufficiently effective tool for research of the dynamic properties of the classical ***CA*** models.

2.4. The nonconstructability problem for finite *CA* models and *CA* models on splitting

Along with classical ***CA*** models a rather large enough applied interest the so–called finite ***CA*** models represent as well that consist of any but *finite* number of elementary automata. This class of ***CA*** models from the theoretical standpoint is most intensively investigated by the Japanese mathematicians [62-64,71,108-110,113-115] along with a number of other researchers [7]. Our results in this direction are presented in works [8,9, 13,24-28,42,82-87]. Researches in this direction are perspective enough, having in mind numerous applied aspects of class of finite ***CA*** models, first of all, at their use as the parallel discrete models of various objects.

In the previous sections of the chapter the nonconstructability problem has been considered concerning the infinite classical ***CA*** models, but it takes place and for finite ***d–CA*** models, however with essential enough differences on which accent is being done in the present section. This problematics is represented more in details, first of all, in works of the Japanese school [62-64,71,108-110,113-115] on finite ***CA*** models and in a lot of other works [7], but here we for the first time shall try to carry out comparison concerning the nonconstructability problem of infinite and finite ***CA*** models. Thus, a finite ***CA*** model represents a finite automaton with a specific internal organization that does its as a rather convenient model in a lot of interesting appendices.

A finite ***CA*** model is similar to a certain finite automaton without inputs which processes internal states *(global configurations)* under influence of the global transition function at discrete moments t, and its output at a moment $t > 0$ corresponds to its internal state in the same moment t. As a matter of fact, a finite ***CA*** model is one of examples of the above *Moore* automata of with specific internal organization. In addition, concerning the nonconstructability problem some results for the finite ***CA*** models are presented below, here we shall present only a result that is directly connected to the *general* nonconstructability problem in the classical ***CA*** models having infinite number of elementary automata.

Theorem 32. If a global transition function $\tau^{(n)}$ stipulates the existence of NCF, NCF–1 and/or NCF–3 then there is a rather wide class of finite closed d–CA models with $\tau^{(n)}$ which possess the NCF_F, and vice versa. Among set of all NCF_F the direct and indirect analogues of the NCF–3, NCF and NCF–1 nonconstructability are directly established.

The theorem **32** is a certain spreading of the results received earlier by us to the case of the ***NCF–3*** nonconstructability [8,24-28,40-43] however, generally speaking, this theorem has not place for the finite ***CA*** models**;** above all, it concerns the ***NCF–1*** nonconstructability since it is directly linked with existence of infinite predecessors.

Number ***N*** of global configurations of a finite ***CA*** model equals $N = a^m$**,** where ***m*** **–** number of elementary ***Moore*** automata which compose the ***CA*** model, and ***a*** **–** cardinality of their alphabet ***A.*** Therefore, the global ***W*** configuration of a finite ***CA*** model is a certain mapping $W: Z_m^d \Rightarrow A;$ Z_m^d is a finite connected block of ***m*** elementary automata of the space Z^d similarly to the classical infinite ***CA*** models. At that, in consequence of finiteness of such ***CA*** model an uncertainty in the field of its *boundary* elementary automata *(according to its neighbourhood index)* arises in case of application to them of local transition function $\sigma^{(n)}$**,** demanding the appropriate boundary conditions *(a block of boundary automata together with its configuration)*. Specifically, a finite ***(mxn)–***rectangular ***2–CA*** with Neumann–Moore neighbourhood index requires definition of block of elementary automata into one layer surrounding the ***2–CA*** body *(fig. **6**)*.

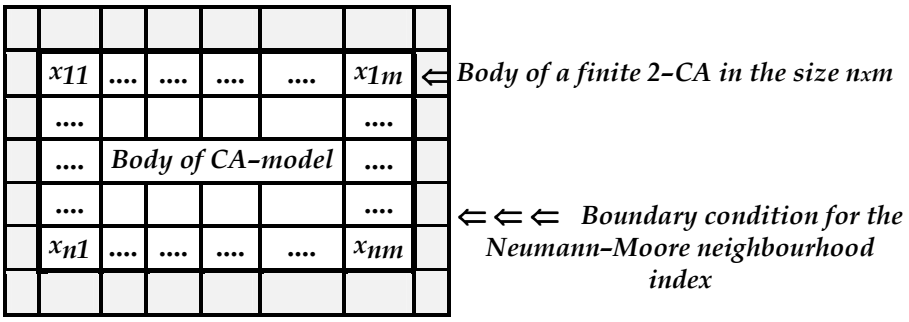


Fig. 6. An organization of a finite 2–CA model with stiff border.

The configuration of boundary automata can be both the constant**,** and the variable**,** simulating a certain interaction of a finite ***CA*** model with an environment. In particular, one of ways of modelling is creation of finite blocks simulating work of those or other real devices, including communication channels between them. Thus, ***K. Zuse*** and ***J. Neumann*** and a lot of their followers have acted in this way at researches of the first cellular models **–** of prototypes of the modern ***CA*** models [7,15-19, 69-72,81]. Above described by us a way of determination of boundary conditions we shall name «***stiff***»**.** Convolution of a finite homogeneous environment, achieved by a «***gluing***» of its opposite borders, is another

method of definition of boundary conditions. In addition obviously this approach can be illustrated by example of a finite ***1–CA*** whose left edge incorporates *(is glued)* with right edge, namely**:**

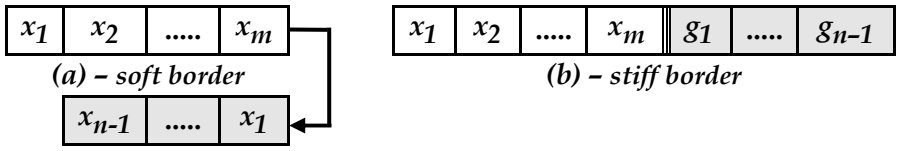


Fig. 7. Determination of soft and stiff boundary conditions for case of the finite 1–CA models.

In case of organization of soft boundary conditions *(fig. 7, a)* connection of the elementary automata of a ***CA*** model is being not interrupted and behind its ***m–***th automaton the first follows at once, i.e. cyclic scheme of communication of elementary automata of such model is organized. In case of stiff boundary conditions its boundary g_k**–**automata *(in quantity determined by its neighbourhood index **X**)* join on the right *(at the left)* from the extreme automata of the model *(fig. 7, b)*. If in case of soft border its configuration is variable, then in case of rigid border it can be both as a permanent configuration, and variable.

In view of the aforesaid, without loss of generality we can determine a finite ***d–CA*** *(**d ≥ 1**)* model as the ordered six $CA_G^m \equiv \langle Z^d, A, \tau^{(n)}, X, m, G\rangle$, for which the first four components are determined similarly to case of classical infinite ***CA*** model**;** ***m*** **–** edge size of ***d–***dimensional hypercube of elementary automata of the Z^d space *(body of the model)*, ***G*** **–** boundary conditions *(way of definition of boundary automata and their configurations)*. At that, a configuration of all elementary automata composing its body will be understood as the global state of CA_G^m**–**model. A set ***C(A,d,m)*** of global states of such finite model is named the ***complete*** if this model at the initial moment ***t = 0*** admits any possible global body configuration determined in a states alphabet ***A*** as an initial condition, irrespective of boundary conditions of a CA_G^m**–**model. Obviously, the set ***C(A, d, m)*** of global states of a CA_G^m**–**model is finite and its cardinality equals $N = a^m$**.**

If a CA_G^m**–**model does not possess the nonconstructible configurations ***(NCF)*** then, using all configurations from the set ***C(A, d, m)*** as the initial configurations, at the following moment we can receive the full set of its configurations**;** i.e. the mapping $\tau^{(n)}$**:** $C(A, d, m) \Rightarrow C(A, d, m)$ takes place**,** namely**:** global transition function of the model maps the set ***C(A, d, m)***

onto itself. Fig. ***8 (a)*** evidently enough illustrates the told. Whereas, if at the made assumptions some global configurations *(states)* of the model remain unattainable, then they are named the *nonconstructible* ***(NCF);*** in addition, the following mapping $\tau^{(n)}: C(A, d, m) \Rightarrow C \subset C(A, d, m)$ occurs, i. e. global transition function of the CA_G^m–model maps the set ***C(A,d,m)*** into itself *(fig. **8, b**)*.

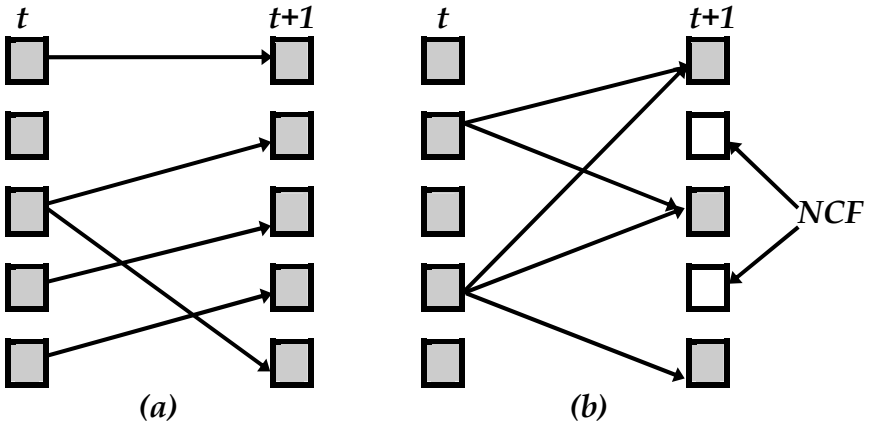


Fig. 8. Illustration of the NCF nonconstructability for finite CA models.

Below we will show, this condition, generally speaking, and determines a nonconstructability criterion for finite CA_G^m–models**:**

A finite CA_G^m–model will possess the nonconstructible configurations if and only if for the model the mapping $\tau^{(n)}: C(A,d,m) \Rightarrow J \subset C(A,d,m)$ occures where $\tau^{(n)}$ is global transition function of the model.

Thus, this criterion has a rather general character that fully corresponds to our general concept of *mutual erasability (definition **6**)*. In addition, the essential distinction in dynamics of *infinite* and *finite* classical ***CA*** models become apparent already at such level of their possibilities as existence for them of *universal* configurations, considered below. So, if the infinite ***CA*** models do not admit configurations of such type, the finite models can have a *single universal* configuration and *all universal* configurations. One of existence criteria of the ***NCF*** nonconstructability for infinite ***CA*** models consists in the existence for them of the pairs of ***MEC*** in ***Moore–Myhill*** sense *(definition **7**)*. But for case of finite ***CA*** models this criterion has not place generally speaking, being based only on our most general concept of erasability. Therefore, here it is necessary to use other certain approaches for investigations of the nonconstructability. As a whole, it

is necessary to mark, that this nonconstructability questions along with questions of reversibility of dynamics of the finite ***CA*** models are not so and simple [24,40,41,82-87,102,106].

Let's consider two classes of the finite models $AG \equiv <Z,A,\tau^{(n)},X,m,G1>$ and $VS \equiv <Z,A,\tau^{(n)},X,m,G2>$, where ***G1*** and ***G2*** represent soft and stiff boundary conditions of the models accordingly *(fig. 7)*, $A = \{0,1,...,a-1\}$, $X = \{0,1,...,n-1\}$ and ***GTF*** $\tau^{(n)}$ for the models of both classes are defined by linear local functions $\sigma^{(n)}(x_1,x_2,...,x_n) = \sum_k x_k \pmod a$, $x_k \in A$ $(k=1..n)$.

In consideration of the definition of soft and stiff boundary conditions for a finite ***1–CA*** by length ***m***, and also linearity of ***LTF*** $\sigma^{(n)}$ for models of both classes, a state x_k^{t+1} of an automaton x_k of models from classes ***AG*** and ***VS*** in the moment $t+1$ $(t \geq 0;\ k=1..m)$ is calculated according to the following simple formulas *(**3**, **4**)* accordingly**:**

$$\begin{cases} x_k^{t+1} = \sum\limits_{j=0}^{n-1} x_{k+j}^t \pmod a; & 1 \leq k \leq m-n+1 \\ x_k^{t+1} = \sum\limits_{j=k}^{m} x_j^t + \sum\limits_{j=1}^{n-m+k-1} x_j^t \pmod a; & m-n+1 < k \leq m \end{cases} \quad (k=1..m) \qquad (3)$$

$$\begin{cases} x_k^{t+1} = \sum\limits_{j=0}^{n-1} x_{k+j}^t \pmod a; & 1 \leq k \leq m-n+1 \\ x_k^{t+1} = \sum\limits_{j=k}^{m} x_j^t + \sum\limits_{j=1}^{n-m+k-1} g_j^t \pmod a; & m-n+1 < k \leq m \end{cases} \quad (k=1..m) \qquad (4)$$

where formulas *(**3**)* and *(**4**)* are linked with CA_G^m**–**models of the first and second class accordingly**,** which for convenience are denoted simply as $AG \equiv <a,n,m>$ and $VS \equiv <a,n,m>$ accordingly. The direct testing confirms existence for model ***AG(2,3,2)*** of four ***NCF***, namely**:** ***001, 010, 100*** and ***111.***

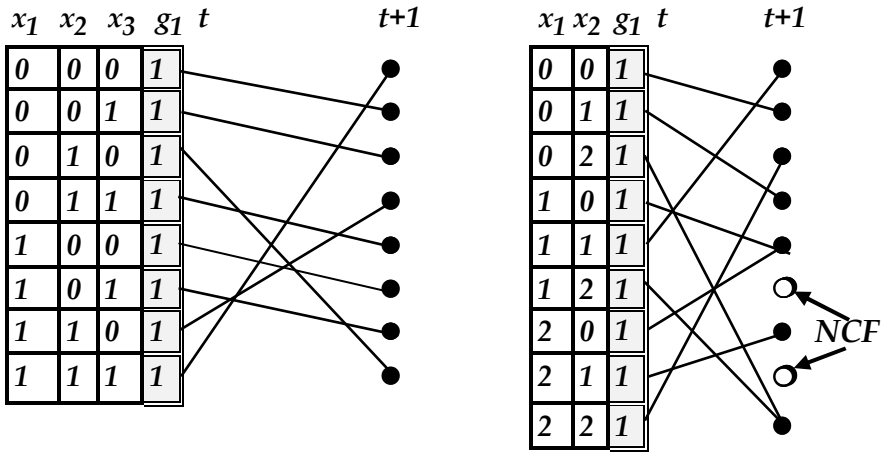
Thus, it is possible to show, in general case for models ***AG(2,m,2)*** exists exactly $N = 2^{m-1}$ $(m \geq 2)$ of ***NCF*** and with growth of value ***m*** the quota of ***NCF*** for such models tends to $q = 1/2$**.** The similar situation is valid and for the models ***AG(2, m, 2)*** if instead of an elementary automaton x_1 the automaton x_{m-1} is used as their soft border ***G1;*** i.e. executes their count executes in the order, opposite to accepted at convolution of finite CA_G^m model *(fig. **7.a**)*. However, already for models ***AG(3,m,n)*** the situation is completely different, namely**:** if for a model ***AG(3,n,n)*** exists $N = a^n - a$ of ***NCF***, then already for model ***AG(3,3,2)*** the ***NCF*** are absent. Direct check

establishes the absence of ***NCF*** for models ***AG(2, 4, 2)*** and ***AG(2, 5, 3).*** So, in class of the ***AG*** models, whose linear ***LTF*** do not support existence of the ***MEC*** pairs the criterion of ***Moore–Myhill*** of existence in the models of ***NCF*** not takes place, namely**:** ***In the absence of the MEC pairs in the Moore–Myhill sense, the models of this class can both possess and not possess the NCF nonconstructability.***

Let's consider now finite CA_G^m**–**models with stiff boundary conditions ***G2*** and linear local transition functions of the above kind**;** i.e. a class of models $VS \equiv \langle Z,A,\tau^{(n)},X,m,G2\rangle$**.** Let's analyse the dynamics of ***2*** simple models $VS1 \equiv \langle 3,2,2\rangle$, $VS2 \equiv \langle 2,3,2\rangle$ with identical boundary conditions ***G2*** *(only one automaton is in state «**1**»)* as concrete examples of models of such type.

In addition, if for the *first* model the ***LTF*** $\sigma 1^{(2)}(x,y)=x+y \pmod 2$ is used, then for the *second* model the local transition function $\sigma 2^{(2)}(x,y)$ is used, which is defined by the following simple enough formula, namely**:**

$$\sigma 2^{(2)}(x,y)=\begin{cases} y, & if \quad x=0 \\ x+y \pmod 2, & if \quad xy\in\{10,11,22\} \quad x,y\in A=\{0,1,2\} \\ x+y \pmod 2+1, & otherwise \end{cases}$$



It is easy to be convinced that local transition functions $\sigma 1^{(2)}$ and $\sigma 2^{(2)}$ completely exclude the existence for the corresponding models of ***MEC*** pairs. However, that does not guarantee absence for the finite models of the ***NCF*** nonconstructability. The schemes of transitions, represented above, of global states for the finite models are transparent enough and do not demand any special elucidations. So, in case of finite ***CA*** models with stiff boundary conditions the absence of pairs of the ***MEC*** does not

guarantee the absence of nonconstructability too. Thus, in general case of finite CA_G^m models the existence of the ***MEC*** pairs can be as sufficient condition but not necessary condition for existence for them of the ***NCF*** nonconstructability.

Moreover, the nonconstructability problem for finite ***CA*** models is very closely connected to boundary conditions type. So, if models ***AG(2,m,2)*** with soft boundary conditions ***G1*** possess the ***NCF,*** then the appropriate models ***VS(2,m,2)*** with stiff boundary conditions ***G2*** do not possess the nonconstructability. The model ***VS(2,2,2)*** with boundary condition ***g1=1*** has all global configurations as *universal configurations (**UCF**)* while with boundary condition ***g1=0*** the ***CA*** model possesses neither ***NCF*** nor ***UCF.*** Moreover, it is possible to show, that validity of the statement about the existence of $N=2^{m-1}$ ***NCF*** for models ***AG(2,m,2)*** remains in force and for models ***VS(2,m,2)*** if boundary conditions of some periodic types and of some other types are defined as their stiff boundary conditions ***G2.***

It is shown, that finite models with soft borders and variable boundary conditions can have one or all global configurations as ***UCF;*** in addition, in the first case any configuration different from the sole ***UCF*** generates all global configurations excluding the ***UCF*** one. Many very interesting properties of finite ***CA*** models were received on the basis of computer simulating in the systems ***Maple*** and ***Mathematica.*** A lot of fragments can be gathered of our programs simulating certain dynamic aspects of classical ***CA*** models [48,49,51,102,106] and from other rather interesting works [7,116-122,278,286].

For a reversible finite ***CA*** model its injective global mapping should be as well bijective. Meanwhile, if global mapping of a finite ***CA*** model is injective, it does not entail obligatory reversibility of its dynamics. So, dynamics of the finite ***CA*** models is reversible, if their global mappings are bijective. In the general case, it is enough complex to determine the reversibility of the finite ***CA*** models and more in detail it is possible to familiarize with these questions of dynamics of the finite ***CA*** models in a series of interesting enough works of ***M. Harao*** and ***S. Noguchi*** [7].

In addition, in case of linearity of global mappings $\tau^{(n)}$ the reversibility problem becomes much more accessible. Discussion of properties of the linear or additive ***CA*** models can be found in rather interesting works of ***O. Martin*** and ***K. Morita,*** and of a lot of other researchers [7,24,123]. Consequently, concerning the nonconstructability problem the infinite and finite ***CA*** models define essentially different classes of the parallel

dynamic cellular systems, stimulating the further research in the given direction that seems to us interesting enough from a lot of standpoints, above all, the applied character.

Meanwhile, with size growth of finite ***CA*** models the classical criteria of the ***NCF*** nonconstructability start to play the more and more increasing part, by basing on concepts of ***MEC*** *(in the* ***Moore–Myhill*** *sense and our)* and ***γ–CF.*** In particular, in event of models sizes exceeding the minimal sizes of ***NCF*** to the finite ***CA*** models the classical criteria of existence of the ***NCF*** nonconstructability are quite applicable.

The ***CA*** *model on splitting (****CAoS****)* is defined as the ordered tuple of five base component $CAoS \equiv <Z^d, A, m, \Psi^{(h)}, \Xi>$ where the first **2** components Z^d and ***A*** are similar to case of classical ***CA*** models**;** ***m*** **–** edge size of ***d–***dimensional hypercube into which the space Z^d of the model is broken**;** $\Psi^{(h)}$ **–** local block function of transition *(****LBF****;* $h=m^d$*);* **Ξ –** switching rules of blocks of Z^d space. Functioning of ***d–CAoS*** models is simple enough and has been considered enough in detail in [40-43]. In the same place, a certain comparative analysis of both types of models *(****CA*** *and* ***CAoS****)* has been carried out. At present, models similar to ***CAoS*** find a rather wide application, above all, for a number of interesting problems of physical modelling, by having of software and hardware on the ***CAM–***machines that are based on computing ***CA*** models [7,124-127,278,286].

Consequently, research of the nonconstructability problem for the case of ***CAoS*** models seems interesting enough. Meantime, from standpoint of the nonconstructability problem between classical ***CA*** and ***CAoS*** the certain differences exist. Above all, for ***CAoS*** models the classification of the nonconstructability, similarly to the case of classical ***CA*** models *(****NCF****,* ***NCF–1****,* ***NCF–2*** *and* ***NCF–3****)* is not valid. Indeed, according to the definition **4** a finite configuration c^* is ***NCF–1*** in a classical ***d–CA*** $(d \geq 1)$ if and only if for it there are predecessors only from the set $C(A,d,\infty)$ of the infinite configurations. That implies nonclosure of the set $C(A,d,\infty)$ relative to a global transformation $\tau^{(h)}$ of the classical ***CA*** model.

On the other hand, from definition of the ***CAoS*** model directly follows, the nonclosure of the set $C(A,d,\infty)$ concerning the global transformation $\tau^{(h)}$ defines the necessity of existence of substitutions of the following kind $(\exists x_j \neq 0)(x_1x_2x_3 \ldots x_h \Rightarrow 000 \ldots 00)$ among parallel block substitutions defining ***LBF*** $\Psi^{(h)}$ of the model. Hence, the mapping $\Psi^{(h)}: A^h \Rightarrow A^h$ will not be by biunique mapping, therefore the ***CAoS*** model should possess

the ***NCF*** nonconstructability, i.e. it will possess the most general type of nonconstructability in ***CA*** models. In addition, the existence in a ***CAoS*** model of configurations such as ***NCF–1*** with need entails the existence in it of the ***NCF*** also, i.e. that is a sufficient condition of existence in the ***CAoS*** models of the ***NCF*** nonconstructability.

Generally speaking, existence for an arbitrary ***CAoS*** model of mapping of two different states of by means of the ***LBF*** $\Psi^{(h)}$ to the same state we entirely can consider as the existence for such model of a ***MEC*** pair. So, a criterion of existence of the ***NCF*** nonconstructability for ***CAoS*** models can be formulated as follows, namely**:**

An arbitrary d–CAoS model will possess the NCF nonconstructability if and only if for the model the MEC pairs in the general sense exist, i.e. in concordance with definition 7.

It is simple to be convinced, that quantity of $CAoS \equiv <Z^d, A, m, \Psi^{(h)}, \Xi>$, which not possess the ***MEC*** pairs and hence of the ***NCF,*** equals $\left(a^{m^d}\right)!$ while their quota relative to all such models equals $\Delta = \left(a^{m^d}\right)! \Big/ a^{m^d a^{m^d}}$, i.e. enough quickly approaches to zero already for small enough values ***a, m, d.*** Thus, and in the class of ***CAoS*** models the models possessing the reversibility property to a certain extent are «*exotic*»**.** On the other hand, absence for a certain ***CAoS*** model of the *NCF* nonconstructability entails also closure of the set $C(A, d, \infty)$ relative to global transformation $\tau^{(h)}$ of the model, and hence, absence for it of ***NCF–1.*** Whereas for the classical ***CA*** models this statements generally speaking are incorrect. In addition, it is simple enough to make sure, that the following result takes place**:**

The closure problem of the set C(A, d, ∞) (d ≥ 1) concerning a global $\tau^{(h)}$ ***transformation defined by local block function*** $\Psi^{(h)}$ ***of a d–CAoS model is algorithmically solvable, whereas a set of NCF for the d–CAoS model is recursive. Nonclosure of the set C(A, d, ∞) concerning mapping that is defined by local block function*** $\Psi^{(h)}$ ***of an arbitrary d-CAoS (d≥1) model causes the presence for the model of the NCF nonconstructability, while the opposite assertion, generally speaking, is false.***

Consequently, for ***CAoS*** models the existence of ***NCF–1*** without ***NCF*** is impossible. On the other hand, for a classical ***CA*** model the types of the nonconstructability ***NCF*** and ***NCF–1*** are not equivalent, in the absence in it of ***NCF*** the model can possess ***NCF–1.*** At that, nonclosure of the set $C(A, d, \infty)$ relative to the global transformation $\tau^{(n)}$ of a classical ***CA*** is a

criterion of existence in the model of ***NCF–1*** in the case of absence in it of ***NCF*** *(theorem **19**)*. If the existence problem of ***NCF*** nonconstructability for general case of classical ***d–CA*** *(**d** ≥ **2**)* is algorithmically unsolvable, then in a class of ***d–CAoS*** *(**d** ≥ **1**)* this problem is algorithmically solvable and any constructive solution algorithm is reduced to ascertainment of existence/absence of mutual unambiguity of a mapping $\Psi^{(h)}: A^h \Rightarrow A^h$.

The one–to–oneness of local mappings $\Psi^{(h)}: A^h \Rightarrow A^h$ of ***CAoS*** models is criterion of absence of ***NCF*** for them**;** at that, the nonconstructability is determined at once on blocks of size $(m* \ldots *m)$ of elementary automata of the models, while a set of all ***NCF*** for any ***CAoS*** model is recursive.

It is shown, that in general case ***the NCF nonconstructability property relative to mutual modelling of d-CA models and d-CAoS models is not invariant*** [40-43,102]. In addition, a modelling of an irreversible model by an appropriate reversible model is quite allowable.

Of the arguments represented above follows, being based on the same nonconstructability definition of ***NCF*** type, we receive that its causal–investigatory bases for the classical ***CA*** models and ***CAoS*** models are essentially different. This difference underlies serious distinctions of a lot of fundamental dynamical properties of the classical ***CA*** models and ***CAoS*** models, and causes essentially large demand of the second for the problems of modelling of processes and phenomena which need in the reversibility property of their dynamics [12,24-28,40-43,82-87,102,106].

Once more it is expedient to note that in comparison with the classical models one of fundamental criteria of the ***NCF*** nonconstructability for the finite models and ***CAoS*** is based on the general concept of mutually erasable configurations *(definition **6**)*, instead of the ***MEC*** in the sense of ***Moore–Myhill*** *(definition **7**)*. Indeed, for classical models our concept of *mutual erasability* is based on a pair of different finite configurations that by global transition function of a model is mapped into the same finite configuration what in the full measure corresponds to absence of one–oneness for mappings of finite configurations of body of a finite model by means of its local transition function and block of fragmentation of a ***CAoS*** space by its local block function of transition.

Properties of parallel mappings such as surjectivity and injectivity that are defined by global transition functions $\tau^{(n)}$ of the ***CA*** models have the most direct attitude to the nonconstructability problem and they play a certain fundamental role at research of dynamical properties of the ***CA*** models. A number of researchers has worked in this direction, and a lot

of interesting enough results has been received [7,22,63,71,102,118,286].

In particular, within of these researches ***G.A. Hedlund*** [128] has studied the given theme for a ***1–***dimensional case of dynamic shift systems both in combinatory and topological aspects. ***M. Nasu*** has investigated the further combinatory aspects of local mappings determined by *surjective* global mappings along with local mappings defined by injective global mappings in ***1–CA*** [130]. ***A. Maruoka*** and ***M. Kimura*** have researched ***4*** new properties of parallel mappings in ***CA*** models**:** ***strong R–****surjctivity* and ***weak R–****surjctivity*, ***strong R–****injectivity* and ***weak R–****injectivity* [129]. From them the first two concepts are not equivalent to concepts known earlier, filling interspace between the bijectivity and the surjectivity. On the other hand, it has been proved that other ***2*** concepts are equivalent to the surjectivity. In addition, these concepts are characterized by the strengthened balanced conditions that are equivalent to our concept of the **γ–**configurations [24-28,73]. Basic results of ***Maruoka–Kimura*** in the direction can be easily generalized to the classical ***CA*** models with any dimension and with neighbourhood indexes of more general types [42]. With some other rather interesting results concerning the *injectivity* and *surjectivity* of global parallel mappings in the ***CA*** models the reader can familiarize in the works presented in the extended bibliography [7,24].

By completing by the present section a consideration of the basic results concerning the general nonconstructability problem in classical ***CA,*** we shall dwell on features of the problem in connexion with the *reversibility* problem of dynamics of the ***CA*** models that presents important enough both the theoretical, and the applied interest [7,24-28,82-87]. In addition, the reversibility is understood by us as unambiguity of *reverse* dynamics of finite configurations in the classical ***CA*** models.

2.5. The reversibility problem of dynamics of classical *CA* models

Reversibility of classical ***CA*** models is one of the major properties, first of all, from the standpoint of the theory of calculations and simulation of multifarious physical processes which is a closely enough connected to presence for the ***CA*** models of the ***NCF*** *nonconstructability*, first of all. Meantime, here, perhaps, certain remarks of the general character that concern the above reversibility problem as a whole that in turn is closely connected to the nonconstructability problem for ***CA*** models as a whole and for the classical ***CA*** models in particular would be quite pertinent. So, on the formal level the reversibility problem of a function ***F*** from ***n*** variables $\{x_1,x_2,...,x_n\}$ comes down to the question of possibility of one–valued restoration valued restoration for it of any tuple $<x_1,x_2,x_3,...,x_n>$ according to known kind of ***F*** function and its value $F(x_1,x_2,x_3,...,x_n)$ on the sought tuple.

Naturally, on ***n*** inputs and ***(n–k)*** $\{k = 1..n–1\}$ outputs of some algorithm provided that they belong to the same alphabet it is impossible to obtain such type of reversibility. Therefore, along with a result $F(x_1, x_2, ..., x_n)$ we should have ***(n–1)*** values of tuple $<x_1,x_2,...,x_n>$ for restoration of the missing value x_j; $j \in \{1,2,3,4,...,n\}$, i.e. we should have certain additional information, allowing on basis of kind of the function ***F*** and its value on the tuple, to restore all sought tuple. In principle, certain other ways for receiving of such additional information can be used. So, the following scheme enough evidently illustrates the given aspect.

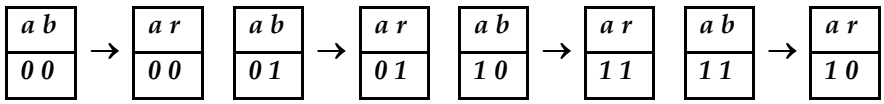
$$\overbrace{\begin{bmatrix} x_1, \\ x_2, \\ ..., \\ x_n \end{bmatrix}}^{t=0} \Bigg| \rightarrow \overbrace{\underbrace{F\left(\begin{bmatrix} x_1, \\ x_2, \\ ..., \\ x_n \end{bmatrix}\right)}_{irreversibility} , \bigcup\{x_{k_j} \mid j = 1..n-1\}; k_j \in \{1,2,...,n\}}^{t=1}$$

reversibility

As a concrete example we shall consider logic ***XOR***–function defined as follows, namely**:**

a	*b*	*a XOR b*
0	***0***	***0***
0	***1***	***1***
1	***0***	***1***
1	***1***	***0***

Obviously, this function is irreversible, because, for example, the value «*1*» for the function we can obtain on **2** various tuples **<*0, 1*>** and **<*1, 0*>.** Meanwhile, if on an output we give in addition also one of values, for example, «***a***» then we will obtain the following relations between *inputs* and *outputs* of the function, namely**:**



where ***a*** and ***r*** **–** values of ***a–***input and result ***a XOR b*** accordingly. Of the above relations obviously, that one**-**to**-**one dependence between two inputs and two outputs takes place, providing possibility of one**-**valued restoration of ***b–***input in the form of value ***a XOR r.*** A Boolean function with ***n*** outputs and ***n*** inputs is called the ***reversible*** if it maps any input tuple of values into a single output tuple.

In view of told, it is simple to notice, that any standard logical function *(excepting unary function* ***NOT****)* is nonreversible**;** i.e. by its result a tuple of source variables is not defined uniquely. Meantime, it is simple to be convinced, that for the given type of functions it is possible to provide the reversibility by a combination of their results and a value of one of inputs. So, supporting of equal quantity of inputs and outputs is a quite natural condition of reversibility providing of an arbitrary system.

We can achieve this requirement in the various ways, and now a rather large amount of various reversible logic gates is specifically offered. So, logic reversible gates ***NOT, CNOT, SWAP,*** of ***T. Toffoli*** and ***E. Fredkin*** are used most widely. As a matter of fact, in any logic gate at which the amount of inputs exceeds amount of outputs, a loss of information will be inevitable to happen as it is impossible to define the states of inputs on the basis of states of its outputs.

The reversibility problem of calculations and creation of the reversible computers is especially urgent today, when serious works on creation of a new architecture of computer equipment with orientation onto the manufactures using nanotechnology have been begun. So**,** calculations which are performed on modern computers are being done by means of the irreversible operations erasing the information.

Meanwhile, back in *1973* ***C. Bennet*** has shown that at calculations it is possible to do without an erasing of information and irreversible logical elements. Later, validity of the given position has been shown on a lot of computational models. In particular, we note here the logic gate of ***E. Fredkin*** that has three input lines and three output lines [7,131]. It does

not lose the information since a state of inputs can be defined always by state of outputs. ***E. Fredkin*** has shown that any logical device necessary for operation of a computer can be created in the form of a combination of such reversible gates. Of a series of other rather interesting reversible computing models it is possible to notice «*billiard*» computer of ***Toffoli–Fredkin*** along with certain others [7,24,40-42,82-87,102,106,278,286].

The above reversible computing models are based, mainly, on classical dynamics and electronics, however a number of researchers has offered also some other models of reversible computers basing on principles of quantum mechanics. Inherently, particles in similar models should be located so that quantum mechanics rules controlling their interaction in accuracy were similar to rules predicting values of signals on outputs of *reversible* logic gates. These models do not scatter energy and submit to laws of quantum mechanics. The interested reader can familiarize with reversible models of similar type in the lists of original sources [7,24].

The aforesaid relates, mainly, to the information processing. However a computer should not only process the information, but also remember it. So, the interrelation between storage and information processing in the best way perhaps can be described by means of the Turing machine, that in computing attitude can model any modern computer as well as solve any problem. ***C. Bennet*** has proved possibility of construction of a reversible Turing machine, i.e. such machine which does not lose any information and for the given reason during own work can consume any preset small quantity of energy. However, not all Turing machines are reversible, however it is a quite possible to construct the reversible Turing machine capable to execute any given calculation [7,24,80-87].

In addition, for creation of reversible computing models the approaches on biomolecular and chemical bases are offered. So, enzymatic reactions which are well known in genetics are reversible. Thus, it is shown that ahypothetical enzymatic Turing machine can perform computations of an arbitrary small expense of energy. With an interesting model of such reversible Turing machine and discussions from physical point of view concerning reversible calculations and reversible computers the reader can familiarize in the extended bibliography [7,24]. Here, above all, we recommend to pay the special attention on in many respects the pioneer ideas and works of such researchers as ***E. Fredkin, T. Toffoli, K. Morita, M. Margolus, R. Landauer, C. Bennett*** along with certain others.

Meanwhile, in connexion with application of ***CA*** models as the formal and perspective prototypes of computational models, the questions of

dynamics reversibility of these models also lay in a line of the marked researches. The problems of dynamics reversibility of ***CA*** models play a rather important part, first of all, from standpoint of their use in quality of simulation environment for various phenomena and processes, first of all, of physical character, and also in quality of certain prototypes of perspective computational devices that support reversible calculations and suppose use nanotechnology. In the given attitude one of the major aspects of researches on the classical ***CA*** models falls on the reversibility problem of their dynamics.

At present a number of interesting classes of the ***CA*** models possessing the general property of reversibility among which it is possible to note the above ***CAoS*** models for the first time introduced by ***N. Margolus*** [7] and widely used by him in collaboration with ***T. Toffoli*** for modelling of certain reversible processes together with the reversible ***CA*** models specially designed by ***T. Toffoli*** and researched from standpoint of the computational and constructional universality [7,71,75,111,132]. A lot of researchs has been devoted to the different reversibility questions of the ***CA*** models of various types and classes [7,12,13,24-28,40-43,82-87,102].

D. Richardson [134] has proved, that a classical ***CA*** model is reversible if and only if its global mapping defined by global transition function is injective. However the topological approach used for that does not give any constructive algorithm for providing of immediate inversion. From theoretical–automaton standpoint the approach to this problem can be found at ***K. Culik*** [104]. ***S. Amoroso, Y. Patt*** [133] and ***V. Aladjev*** [80-87] have proved existence of effective procedures deciding the reversibility problem for classical ***1-CA*** models. Whereas, ***V. Aladjev*** [82] and ***J. Kari*** [78] on the basis of various approaches have proved unsolvability of the reversibility problem for ***d–CA (d≥2)*** models. In addition, it is necessary to note that receiving of constructive algorithm of determination of the reversibility of a classical ***d-CA (d≥2)*** model is a rather complex problem even under the condition of its existence.

Meanwhile, for the lot of cases of linear and some special types of ***d–CA (d ≥ 2)*** models such constructive algorithms exist. So, ***G. Manzini*** and ***L. Margara*** have deduced a reversibility formula for linear classical ***d–CA; T. Sato*** has given an example of algorithm for definition of reversibility in the case of one special class of ***d–CA,*** whereas ***K. Sutner*** has given an example of algorithm providing definition of existence of reversibility and surjectivity of global mapping for the linear ***d–CA (d ≥ 2)*** models at quadratic time [7]. ***K. Morita*** has proved existence of a reversible ***1–CA***

which model an arbitrary ***1–CA,*** including and irreversible ones, while ***J. Dubacq*** has proved an opportunity of simulation of Turing machines by means of reversible ***1-CA*** [7]. ***T. Toffoli*** has proved an opportunity of modelling of an arbitrary ***d-CA*** model by means of reversible ***(d+1)-CA,*** having proved thus computing universality of reversible ***d-CA (d≥2)*** [75] whereas ***K. Morita*** and others have proved computing universality of reversible ***1–CA*** [135,136]. ***T. Toffoli*** and ***N. Margolus*** have presented a quite interesting review about reversible ***CA*** models [75]. More in detail with the reversibility problem for ***CA*** models the reader can familiarize in the extended bibliography [7,12,13,22,24,30,63,70,71,82,102,106,118].

Thus, even within such general concept as ***W–***modeling [41], in general it is impossible to simulate a classical ***d–CA*** $(d \geq 1)$ model by means of a reversible model of the same dimension. This result serves as a serious argument in favor of the assumption of complexity of such modelling. However beyond frames of finiteness of the alphabet of internal states, there is a possibility of simulating of classical ***d–CA*** possessing the ***NCF*** nonconstructability by means of ***CA*** models of the same dimension but without this property. Naturally, ***CA*** models with infinite alphabets of elementary automata can't be considered as classical cellular automata however they a certain extent allow to estimate those limits outside of which such modelling is possible. Interesting and instructive examples of this kind along with rather detailed discussions, the interested reader can find, for example, in [7,24,42,82-87]. In this connexion the following proposal can be formulated, namely**:**

A d–CA $(d \geq 1)$ ***model with an infinite alphabet of internal states of the elementary automaton and without the NCF nonconstructability can simulate at real time any classical d–CA model of the same dimension.***

On account of importance of research of the ***NCF*** nonconstructability of on the basis of concept of pairs of mutually**–**erasable configurations, the questions of expansion of classical concept of the cellular automata that simulate the classical cellular automata and not possess ***NCF*** represent undoubted interest. In this connexion we researched a lot of extensions of the concept of classical cellular automata [24,40-43,82-87,102,106].

However the question of modelling of a classical ***d–CA*** model by means of *reversible* ***d–CA*** still remains not up to the end the developed. So, our results concerning ***WM-***modelling and ***W-***modelling of a classical ***d-CA*** $(d \geq 2)$ definitely speak in favour of a rather essential complexity of the proof of fact itself of such modelling under the condition of operation

with ***CA*** models on formal level and on the basis of the ***Moore–Myhill*** criterion of the nonconstructability existence *(theorem **6**)* [24,40-43,82-87].

In the most widespread understanding a reversible ***CA*** model is being understood as the ***CA*** model which not loses the information with time**:** ***At a moment t > 0 the CA model is completely reversible.*** Meanwhile, in general case for definition of similar reversible ***CA*** model of any special difficulty does not exist. To this end, it is quite enough to define a local transition function $\sigma^{(n)}$ of a ***CA*** model as follows, namely**:**

$$z(t+1) = F(NT_z(t)) \# z(t-1) \qquad (5)$$

where $z(t)$ is a state of elementary *z***-**automaton of the model in moment $t \geq 0$, $NT_z(t)$ is a configuration of neighbourhood template with central elementary *z***–**automaton in moment t, F **–** a mapping $NT_z(t) \rightarrow A$, $\#$ **–** a binary operation and A **–** alphabet of the ***CA*** model. In the represented solution the number of inputs of a *logic gate* that presents an elementary automaton of the ***CA*** model equals to number of outputs, i.e. two**;** the following rather simple scheme well illustrates that, namely**:**

$$\begin{array}{l} \ldots \rightarrow \left[\begin{array}{c}\sigma_{t-1}(NT_z)\\ z(t)\end{array}\right] \rightarrow \left[\begin{array}{c}\sigma_{t}(NT_z)\\ z(t+1)\end{array}\right] \rightarrow \left[\begin{array}{c}\sigma_{t+1}(NT_z)\\ z(t+2)\end{array}\right] \rightarrow \ldots \\ \ldots \rightarrow \left[\begin{array}{c}\sigma_{t-1}(NT_z)\\ z(t)\end{array}\right] \rightarrow \left[\begin{array}{c}\sigma_{t}(NT_z)\\ z(t+1)\end{array}\right] \rightarrow \left[\begin{array}{c}\sigma_{t+1}(NT_z)\\ z(t+2)\end{array}\right] \rightarrow \ldots \end{array}$$

Therefore, we have an opportunity of one**-**valued restoring of a state of an elementary *z***–**automaton of the model in moment $t-1$ on the basis of its state in the current moment $t+1$ and its value $\sigma_t(NT_z)$**;** i.e. its state in the moment t, namely**:** $z(t-1)=z(t+1)\ \#^{-1}\ \sigma(NT_z(t))$, where $\#^{-1}$ **–** function inverse to the $\#$ function with values in A alphabet. So, the class of ***CA*** models, offered above we can entirely define as models with memory, and their elementary automaton in the certain degree is similar to the above logic gate of ***E. Fredkin*** [131] in which the states of the top level of the above scheme successfully perform the role of a control channel, i.e. $\sigma_p(NT_z)$ $(p = t+k,\ k=\pm 0, \pm 1, \pm 2, \ldots)$. Thus, local transition function $\sigma^{(n)}$ can be arbitrary, providing an opportunity of definition of the reversible ***CA*** models with wide enough set of local functions.

The binary ***1-CA*** model with *discriminating* number ***122*** *(alphabet* $B=\{0,1\}$ *and neighbourhood index* $X = \{-1, 0, 1\}$*)* is considered as an example of the above type of models. It is simple to make sure that the ***1–CA*** which is defined thus, is irreversible, because according to criterion on the basis of ***γ-CF*** *(theorem **13**)* in the model ***NCF*** exist. In this model ***NCF-1*** already of simple kind $c = \square 11 \square$ exist too.

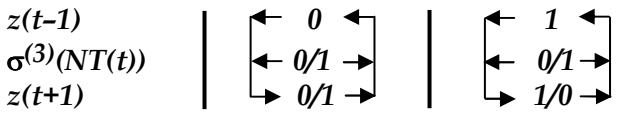
Let's determine now a new model that is constructed on the basis of the above ***1-CA*** model and whose operating is determined by the following equations for rules of transition of its elementary automata. We need to proof that the ***1-CA*** model determined thus should not possess the ***NCF*** nonconstructability, i.e. in traditional staging such model is reversible, for example, the model with function: $z(t+1) = \sigma^{(3)}(NT_z(t))$ *XOR* $z(t-1)$.

In many researches of ***CA*** models the set $A = \{0,1,2,3, ..., a-1\}$, formative a finite *commutative* ring relative to the operations of multiplication and addition modulo a, is considered as an alphabet of internal states of an elementary automaton. For our purposes, the operation of subtraction modulo a, that is inverse to addition operation in this ring, is of interest; this operation for two sets $B=\{0,1\}$ and $A=\{0,1,2\}$ is defined by operating tables of the following kind, namely:

–	0	1
0	0	1
1	1	0

–	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

In general case of set $A=\{0,1,2, ..., a-1\}$ the table of subtraction operation modulo a is also simply derived. It is simple to be convinced, the model defined thus will be reversible what the following simple scheme well enough illustrates, namely:



of which it is simple to draw a conclusion, that according to equation of functioning of an elementary automaton of such model on the basis of information about states of the elementary automaton in moments $t+1$ and t it is possible to determine unambiguously its state in the moment $t-1$, i.e. in such sense the above model is reversible.

Let's present a history of the given model during its three first steps for finite configuration c_{t-1} = □*111111*□ *(an initial state)* in moment $t-1$ and for initial configuration c_t = □*1000010*□, namely:

t–1... 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 ...
t... 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 ...
t+1... 0 0 0 0 1 0 0 1 1 0 0 1 0 0 0 0 ...
t+2... 0 0 0 1 1 0 1 1 1 1 0 1 1 0 0 0 ...
t+3... 0 0 1 1 0 0 1 1 1 1 0 0 1 1 0 0 ...

It is natural that the above reversible model is simulated in strictly real time by means of a classical ***1–CA*** with an alphabet ***A*** of the structured states and with neighbourhood index ***X*={*–1,0,1*}**, but this ***1–CA*** will not be reversible. Indeed, the modeling ***CA*** is represented by the ***A*** alphabet of states of its elementary automata and neighbourhood index ***X*={*-1,0,1*}** with the local transition function determined by parallel substitutions of the following kind, namely**:**

$$t\text{-}1\begin{bmatrix}S_{j-1}\\S^*_{j-1}\end{bmatrix}\begin{bmatrix}S_j\\S^*_j\end{bmatrix}\begin{bmatrix}S_{j+1}\\S^*_{j+1}\end{bmatrix}\rightarrow\begin{bmatrix}\sigma^{(3)}(S^*_{j-1},S^*_j,S^*_{j+1})\\ \sigma^{(3)}(S^*_{j-1},S^*_j,S^*_{j+1})\ \ XOR\ \ S_j\end{bmatrix}$$
$$t\quad S_j,S^*_j\in\{0,1\};\ \ j=0,\pm1,\pm2,...;\ \ t=0,1,2,...$$

It is simple to show, that the simulating ***1–CA*** model is classical model with neighbourhood index ***X* = {*–1,0,1*}** and the structured alphabet ***A*** of cardinality ***4;*** its first level of states defines a configuration of the model in moment ***t,*** whereas the second defines a configuration in moment ***t-1 (t=1,2,...)***. Moreover, it is supposed that in moment ***t = 0*** the second level of states of this simulating model defines a certain initial condition *(an initial configuration)* while the first level defines an initial configuration of the simulating model immediately.

For proof of irreversibility of the simulating model it is quite enough to specify for it a ***MEC*** pair that implies existence for the model of the ***NCF*** nonconstructability, consequently, absence of the reversibility property of its dynamics. On account of the aforesaid it is simple to be convinced of validity of existence for the simulating model already of ***MEC*** pairs of the following kind with ***IB*** of length ***3,*** what stipulates the existence of the ***NCF*** nonconstructability for the model, hence, and absence for its of dynamics reversibility.

$$\sigma^{(3)}:\left\langle\begin{array}{c|c|c}0\,0&0\,0\,0&1\,0\\0\,1&0\,0\,1&1\,0\end{array}\right\rangle\rightarrow\left\langle\begin{array}{c|c|c}1&1\,1\,1&1\\1&1\,1\,1&0\end{array}\right\rangle$$
$$\sigma^{(3)}:\left\langle\begin{array}{c|c|c}0\,0&0\,0\,0&1\,0\\0\,1&1\,0\,0&1\,0\end{array}\right\rangle\rightarrow\left\langle\begin{array}{c|c|c}1&1\,1\,1&1\\1&1\,1\,1&0\end{array}\right\rangle$$

In addition, a lot of other operations can be quite used as the **#** *operation (5)* analogously to the aforesaid. So, the above example of the reversible ***CA*** model is only one of possible examples of that type while receiving of reversible ***CA*** models for concrete appendices demands occasionally of complex enough research work. Now, there is a whole series of other interesting enough examples of definition of reversible ***CA*** models. We also had represented a few reversible ***CA*** models distinct from classical ones, with application to developmental biology [7,24]. Meanwhile, the

reversibility problem for the classical ***CA*** models is more many–sided and is considered below. As a whole, the problem of reversibility of ***CA*** models is not so unambiguous.

Below we shall use the more strong reversibility concept which is being understood as a possibility of one–valued restoration of all dynamics of a ***CA*** model at any moment**;** i.e. such ***reversibility,*** when it is possible to determine precisely at each moment ***t*** for any finite configuration in the ***CA*** model of its single predecessor at the previous moment ***t–1.***

Considering the classical ***CA*** models as converters of both finite**,** above all, and infinite configurations we quite can identify the configurations processed by such converters with their inner states. In case of similar interpretation a ***CA*** converter admits as a rule several inputs *(states)* for a receiving of a single output. So, the ***CA*** converter as a certain infinite automaton with ***n*** inputs and one output does not provide the property of reversibility in case ***n>1.*** From such standpoint we shall will consider further the reversibility problem for the classical models. Reversibility of ***CA*** models of such type can be considered as the global reversibility, when on the basis of a configuration $c^* \in C(A,d)$ in moment $t \geq 0$ we can *unambiguously* determine for the ***CA*** model a predecessor $c`^{*-1} \in C(A,d)$**,** using only configuration ***c**** and local transition function $\sigma^{(n)}$ or global transition function $\tau^{(n)}$ of the model, i.e. a mapping $\tau^{(n)}: C(A,d) \to C(A,d)$ should be bijective mapping. Thus, concerning the class of ***d–CA*** $(d \geq 1)$ models we can define rather naturally the *reversibility* of their dynamics as existence for a configuration $c` \in C(A,d)$ of a predecessor $c`^{-1}$ from the set ***C(A,d),*** i.e. the following relation should be carried out, namely**:**

$$(\forall c^* \in C(A,d))(E!\, c^{-1} \in C(A,d,\phi) \bigcup C(A,d,\infty))(c^{-1}\tau^{(n)} = c^*)$$

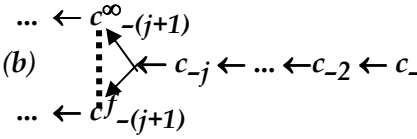
The following definition summarizes the aforesaid.

<u>Definition 12</u>. Dynamics of a d–CA $\equiv \langle Z^d, A, \tau^{(n)}, X \rangle$ ***is called reversible if and only if for each its configuration*** $c \in C(A,d)$ ***there is sole predecessor*** c^{-1} ***from the set*** $C(A,d)$ ***that*** $c^{-1}\tau^{(n)} = c$***; if not, the dynamics of such model is called irreversible.***

Graphically the given definition can be illustrated as follows**:**

(a) $\ldots \leftarrow c_{-j} \leftarrow \ldots \leftarrow c_{-3} \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_0 \in C(A,d)$ – *reversible dynamics*

(b)



– *irreversible dynamics*

(c) $c_{-j} \leftarrow ... \leftarrow c_{-3} \leftarrow c_{-2} \leftarrow c_{-1} \leftarrow c_0 \in C(A,d)$ – *irreversible dynamics*

Thus, in the state graph *(a)* a configuration c_0 has an infinite prehistory, for example, in the case of its periodicity whereas for the state graph *(b)* number of predecessors on a step $-(j+1)$ necessarily not less than **2** and they can belong to the set $C(A, d, \phi)$ and/or the set $C(A, d, \infty)$. At last, the state graph *(c)* is finite, ending by a certain ***NCF*** configuration c_{-j}.

Thus, irreversibility of dynamics of any ***d–CA*** $(d \geq 1)$ model is naturally defined or by absence of predecessors at all for a certain configuration $c^* \in C(A,d)$ or by existence of more than one predecessor c^{*-1} from the set $C(A,d)$ for a configuration c^*. Meanwhile, for case of classical ***d–CA*** $(d \geq 1)$ models the irreversibility can be ***absolute*** and ***relative.***

Above all, in view of definition of the ***NCF-1*** nonconstructability, that is conditioned by existence for classical ***CA*** models of the determinative condition $\sigma^{(n)}(x,x,...,x) = x$, where $x \in A$ – an quiescent state, specifically, $x \equiv 0$. This condition allows to differentiate quite naturally the set of all possible configurations in **2** nonoverlapping sets $C(d,A,\phi)$ and $C(d,A,\infty)$ of finite and infinite configurations accordingly; so, if the set $C(d,A,\phi)$ is closed concerning mapping $\tau^{(n)}$, i.e. $(\forall c^* \in C(d,A,\phi))(c^*\tau^{(n)} \in C(d,A,\phi))$, the set $C(d, A, \infty)$ can be nonclosed; i.e. $(\exists c^{\infty} \in C(d, A, \infty))(c^{\infty}\tau^{(n)} \in C(d, A, \phi))$, that urgently causes the necessity of interpretation specification of the *reversibility* concept for classical ***CA*** models, i.e. those ***CA*** that satisfy the above determinative condition. Existence in a ***CA*** model of the ***NCF–1*** nonconstructability in the absence even of ***NCF*** leads to *irreversibility* of such model, more precisely of its dynamics, as a whole.

This definition can be considered as an especially formal on the ground of the uniqueness of an infinite predecessor that has a rather disputable interpretation, in our opinion. Therefore, the concept of formal and real dynamic reversibility are entered below. Sure enough, from the formal standpoint we can consider any admissible possibilities whereas from applied standpoints the instant transition from an infinite configuration into finite one and vice versa, in our opinion do not admit transparent enough interpretations. However, various interpretations of transition from an infinite state to a finite, and vice versa are here possible. So, in mathematics it is possible to discover many of similar interpretations, for example, $\Sigma_k 1/p^k = 1$, $k = 1..\infty$, etc. Whereas in backward dynamics of some finite configuration it is possible to associate its instant transition into an infinite configuration with some kind of singularity. So, in case

of classical models the instant transition of an infinite configuration to a finite configuration is possible, whereas the instant transition of a finite configuration to an infinite configuration is impossible. Thus, in models of this type a singularity can arise for backward dynamics only of finite configurations. On the basis of our researches of the nonconstructability problem in classical ***CA*** models can be shown [24-28,40-43,82-87,102]**:**

There are no classical d–CA (d ≥ 1) models for which in the absence of the NCF nonconstructability and existence of the nonconstructability such as NCF–1 any finite NCF–1 configuration has 1 predecessor from the set C(A, d, ∞), whereas other finite configurations not possess such predecessors, i.e. a classical CA model without NCF in the presence of NCF–1 possesses finite configurations having at least two predecessors from the set C(A, d, ϕ)∪C(A, d, ∞), i.e. so–called absolutely constructive configurations (ACCF).

So, even on the assumption of absence of the ***NCF*** nonconstructability, however at existence of the ***NCF–1*** nonconstructability the dynamics of classical models can be considered *irreversible* as a whole that according to proposal **2** conforms to our comprehension of dynamics *irreversibility* in classical ***CA*** models.

In addition, if to consider presence for a finite configuration of a single predecessor from the set ***C(A, d, ∞)*** as a singular point of its backward dynamics that is not breaking its reversibility, then and in this case the dynamics of a classical model not possessing ***NCF,*** will be irreversible as a whole. Therefore, the dynamics of a classical model possessing the ***NCF*** and/or ***NCF–1*** nonconstructability is as a whole irreversible as we see this even under the assumption of possibility of reversible dynamics of certain finite configurations.

Absolute irreversibility for a classical ***d–CA (d ≥ 1)*** takes place if for the model at least one configuration ***c`∈C(A,d)*** exists**,** that possesses several predecessors $c^{`-1}$ from the set ***C(A,d)*** or at all has no predecessors. It is obvious if a ***d–CA (d ≥ 1)*** model possesses the ***NCF*** nonconstructability, then it is *absolutely* irreversible. On the other hand, completely different situation takes place in case of absence of the ***NCF*** nonconstructability for classical structures ***d-CA (d≥1)*** models. For this type of ***d–CA*** models the ***NCF–1*** nonconstructability that is characterized by existence of the finite configurations having predecessors only from the set ***C(A, d, ∞)*** is determined concerning finite configurations. Consequently, at research of nonconstructability of such type, an interesting class of ***d–CA (d ≥ 1)***

models has been discovered for which the following rather interesting offer takes place, namely [24-28,40-43,82-87,102,106].

Theorem 33**. There are classical d–CA ($d \geq 1$) models not possessing the NCF nonconstructability for which a finite configuration having single predecessor from the set $C(A,d,\phi)$ will has predecessors also from the set $C(A, d, \infty)$; at that, each finite configuration such as NCF–1 has at least 2 predecessors from the set $C(A,d,\infty)$, i.e. any finite configuration has at least one predecessor from the set $C(A,d,\infty)$. There are classical models 1-CA with states alphabet $A=\{0,1,...,a-1\}$ that not possess the NCF in the presence of the NCF-1 nonconstructability whose NCF-1 configurations have $N=a^{n-1}$ predecessors whereas other finite configurations have N–1 predecessors from the set $C(A, d, \infty)$, where n is a size of neighbourhood template. If a classical d-CA ($d \geq 1$) model with states alphabet $A=\{0,1,2, ..., a-1\}$ does not possess the NCF nonconstructability, and for it there are at least 2 such different configurations $c^{\infty}, b^{\infty} \in C(A,d,\infty)$ that $c^{\infty}\tau^{(n)} = b^{\infty}\tau^{(n)} = \square$, then for such model finite configurations such as NCF–1 can possess more than 2 predecessors from the set $C(A, d, \infty)$ which pairwise differ between themselves by the infinite number of states.

As a simple example it is possible to represent a classical ***1–CA*** model

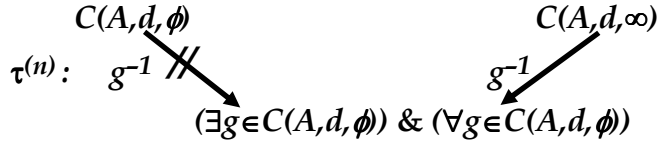
$$\sigma^{(2)}(x,y) = \begin{cases} x, & if \quad y = 0 \\ x + y \pmod 2, & if \quad < xy > \in \{11, 22\}; \\ y, & otherwise \end{cases} \qquad x, y \in A = \{0,1,2\}$$

with alphabet $A = \{0,1,2\}$, neighbourhood index $X = \{0,1\}$ and *GTF* $\sigma^{(2)}$, determined by means of the above mentioned simple formula. The next rather transparent schema confirms correctness of the above mentioned assertion for the case of dimensionality $d = 1$, namely:

$$\begin{array}{r|l|l} c^{-1} \quad =...00 & & \\ \{c^{-1} \mid c^{-2}\}\tau^{(2)} =...01 & \begin{array}{l} 1 \;\; x_1 ... x_n \square =...00 \\ x'_o \; x'_1 ... x'_n \square =...02 \\ 1 \;\; x_1 ... x_n \square =...11 \end{array} & \begin{array}{l} 2 \;\; x_1 ... x_n \square \\ x''_o \; x''_1 ... x''_n \square \\ 2 \;\; x_1 ... x_n \square \end{array} \\ c^{-2} \quad =...22 & & \end{array}$$

$$x_j, x'_j, x'_o, x''_j, x''_o \in \{0,1,2\};\ j = 1..n;\ c^{-1}, c^{-2} - \textit{predecessors}$$

Schematically such type of dynamics irreversibility of a classical ***d–CA*** $(d \geq 1)$ model concerning a configuration $g \in C(A,d,\phi)$ can be represented as follows, namely:



As an example, we shall consider the binary ***1–CA*** with neighbourhood index $X = \{0,1,2\}$ and local transition function defined by the formula**:**

$$\sigma^{(3)}(x,y,z) = y + z \pmod 2, \qquad x,y,z \in B = \{0,1\}$$

As shown, this classical binary ***1–CA*** model does not possess ***NCF.*** On the other hand, may be shown that for the model already ***NCF–1*** of the simplest kind $c^* = \square 1 \square$ exist where «□» **–** a chain of quiescent states «***0***»**,** infinite to the left *(right)*. In addition, during definition of all allowable predecessors for this configuration it is possible to show that it not only is ***NCF–1,*** i.e. has no predecessors from the set $C(B,1,\phi)$**;** furthermore**,** in the set $C(d,A,\infty)$, this configuration has **2** different predecessors, namely $c_1\tau^{(3)}=c_2\tau^{(3)}=c^*$ $\{c_1=...111\square,\ c_2 = \square 111...;\ c_1 \neq c_2\}$**,** not allowing to uniquely determine a predecessor for the configuration c^***,** i.e. unambiguously to determine its prehistory. Consequently, dynamics of the above classical ***CA*** model is ***irreversible.*** So, we can formulate important enough result relative to reversibility of classical ***CA*** models [24-28,82-87]**,** namely.

<u>Theorem 34</u>. The absence for an arbitrary classical d-CA (d≥1) model of the NCF nonconstructability is necessary but not sufficient condition for dynamics reversibility of its finite configurations.

This result demands the more precise interpretation of the reversibility concept for case of classical ***CA*** models. So, in our understanding under the *reversibility* of dynamics of a classical ***CA*** model is understood as an possibility of one-valued restoration of sole predecessor for an arbitrary finite ***c*** configuration**,** in particular, for the case of a ***1–CA*** model of a c^* configuration of the kind $c^* = \square x_1 x_2 \ldots x_p \square$; $x_1, x_p \in A \setminus \{0\}$ on the basis of analysis of its local transition function $\sigma^{(n)}$**.** Such reversibility concept is rather naturally at consideration of dynamics of finite configurations in classical ***CA*** models. Thus, owing to the above concept, the absence in a classical ***CA*** model of the ***NCF*** nonconstructability does not provide the reversibility of its dynamics that with all evidence follows of the above example of classical binary ***1–CA*** model.

Thus, among classical ***d–CA*** that do not possess the nonconstructability such as ***NCF*** but possess the ***NCF–1*** nonconstructability**,** the *irreversible* models exist according to quite natural *reversibility* definition presented above *(**d≥1**)*. In addition, the analysis confirms that all classical binary ***1–CA*** models with neighbourhood index $X = \{0, 1, 2\}$ not possessing the ***NCF*** nonconstructability but possessing ***NCF–1*** *(**i.e.** models with numbers **30, 60, 75, 86, 89, 90, 102, 105, 106** and **120;** section **2.2)*** are irreversible in the context of the above definition ***12.*** So, models with discriminating

numbers ***30, 60, 75*** and ***120*** have finite configurations possessing a sole predecessor from the set $C(B,1,\phi)$ and a predecessor from the $C(B,1,\infty)$, while models with discriminating numbers ***86, 89, 90, 102, 105, 106*** have finite configurations possessing two predecessors from the set $C(B,1,\infty)$.

Thus, existence in classical ***CA*** models of the ***NCF-1*** nonconstructability, not looking even on absence of the ***NCF*** nonconstructability can provoke the irreversibility of dynamics of models of this class as a whole. In this connection a rather interesting question arises**:** ***Whether can guarantee the global irreversibility of a classical d–CA ($d \geq 1$) model the existence for such model of the NCF–1 nonconstructability in the absence of the NCF nonconstructability?*** Taking into account our comprehension of dynamics irreversibility of finite configurations in classical ***CA*** models along with our results concerning the allowable predecessors for finite configurations the *positive* answer to this question arises all by itself**;** see some considerations below.

Meantime, owing to a number of reasons below we shall introduce two more reversibility concepts of dynamics of classical models **–** concepts of *formal* and *real* reversibility. At that, if concept of formal reversibility fully coincides with reversibility concept determined by the definition ***12,*** the concept of real reversibility is appreciably distinctive. So, under the real reversibility is understood the reversibility relative to the finite configurations only**;** i.e. existence for an arbitrary finite configuration c of such sole configuration $c`$ from the set $C(A,d,\phi)$ only that the relation $c`\tau^{(n)} = c$ takes place. As distinct from the definition ***12,*** the definition ***13*** below gives another approach to the reversibility concept of dynamics, allowing to look at important concept from different standpoints.

In addition, below this question will be considered in more detail in the network of this comprehension of the *reversibility* concept. In case of the classical ***CA*** models the condition $\square\tau^{(n)}=\square$ in the course of this book ***will not considered as openness condition of the set $C(A,d,\infty)$ relative to the GTF $\tau^{(n)}$ of the CA models because above we have agreed to ascribe the fully null configuration «$\square$» to the set $C(A, d, \phi)$.*** Obviously, if a certain ***d–CA*** *($d \geq 1$)* models possesses the ***NCF–1*** nonconstructability**,** then the following relation $(\exists c' \in C(A, d, \infty))(c'\tau^{(n)} = \square)$ will take place. Moreover, absence for classical models of the ***NCF*** and ***NCF–1*** nonconstructability at all does not guarantee the dynamics reversibility of configurations. In this connexion, it is possible to show [8,12,24,82-87]**,** that the interesting enough following offer takes place, namely.

Theorem 35. ***There are classical d–CA (d≥1) models that do not possess the NCF and NCF–1 nonconstructability, but for which will be infinite configurations having at least two different predecessors from the set C(A,d,∞) of all infinite configurations.***

Thus, the aforesaid naturally stipulates expediency of introduction for classical ***CA*** models of the concept of *relative reversibility (irreversibility)* of their dynamics. Taking into account importance of the reversibility concept for research of dynamics of classical ***CA*** models, we once again will address to three basic nonconstructability types**:** ***NCF, NCF–1,*** and ***NCF–2.*** For these nonconstructability types the following rather evident graphical interpretations can be represented**:**

$$NCF \quad c_b: \quad c \in C(A,d,\phi) \cup C(A,d,\infty) \rightarrow c_b \notin c\tau^{(n)}$$

$$NCF\text{–}1 \quad c: \quad c \Rightarrow \begin{cases} c^{\infty}\tau^{(n)}; & c^{\infty} \in C(A,d,\infty) \\ c^{*}\tau^{(n)}; & c^{*} \in C(A,d,\phi) \end{cases}$$

$$NCF\text{–}2 \quad c: \quad c \Rightarrow \begin{cases} c^{\infty}\tau^{(n)}; & c^{\infty} \in C(A,d,\infty) \\ c^{*}\tau^{(n)}; & c^{*} \in C(A,d,\phi) \end{cases}$$

where c_b is a block configuration, i.e. configuration of states of a finite block of elementary automata of an arbitrary ***CA*** model**;** $C(A, d, \phi)$ and $C(A,d,\infty)$ are sets of all finite and infinite d–dimensional configurations in alphabet $A=\{0,1, ..., a-1\}$ of the ***CA*** model accordingly**;** $\tau^{(n)}$ is a global transition function of this ***CA*** model**;** at last, $\{c, c^*\}$ and c^{∞} are finite and infinite configurations of this ***CA*** model, which have finite and infinite number of states distinct from a quiescent state «***0***» accordingly. In this case, among the presented nonconstructability types, two types of the ***NCF*** and ***NCF-1*** nonconstructability we shall consider as the *basic* types. The following result represents one rather important relation between basic types of the nonconstructability [40-43,85]. At that, it is necessary to have in mind, that completely null configuration $c = \Lambda$ by a lot of the important reasons is included into the set $C(A,d,\phi)$**.** For this reason, for an arbitrary classical ***CA*** model the set $C(A,d,\phi)$ can't consist completely of finite configurations such as ***NCF.***

Theorem 36. ***If for an arbitrary classical d–CA model the set C(A,d,∞) is nonclosed relative to the global mapping induced by a global transition function $\tau^{(n)}$ of the model then the classical CA model will possess the***

NCF/NCF–1 nonconstructability or by both nonconstructability types concurrently. A classical d–CA (d ≥ 1) model not possessing NCF–1 and NCF possesses the NCF–2 nonconstructability. Meanwhile, a classical d–CA (d ≥ 1) model which does not possess the NCF–2 will possess the NCF or/and NCF–1 nonconstructability.

Existence of the quiescent state «***0***», differentiating the set ***C(A, d)*** into **2** nonoverlapping subsets ***C(A,d,ϕ)*** and ***C(A,d,∞)*** along with **2** basic types of the ***NCF*** and ***NCF–1*** nonconstructability, allows to determine enough naturally three types of dynamics *irreversibility* for classical ***d–CA (d ≥ 1)*** models, what can be formulated as follows.

Definition 13. We shall name a configuration j∈C(A,d,ϕ) for a classical d–CA (d ≥ 1) model by configuration with irreversible dynamics of the first type, of the second type and the third type if the j configuration has 0 or at least a pair of predecessors only from the set C(A,d,ϕ), at least 1 predecessor only from the set C(A,d,∞), at last predecessors from the set C(A,d,ϕ) and the set C(A,d,∞), accordingly.

Above all, on the basis of the given definition and results concerning the nonconstructability, it is simple to make sure, for a classical ***d–CA (d≥1)*** model the set ***C(A,d,ϕ)*** cannot consists of the irreversible configurations only of the *second* or only *third* irreversibility type, but the set can quite consists of *irreversible* configurations only of the *first* irreversibility type. In the latter case, the existence of classical ***d–CA (d≥1)*** models for which any finite configuration of the set ***C(A,d,ϕ)\{□}*** is or ***NCF***, or has not less than two predecessors from the set ***C(A,d,ϕ)*** *(i.e. its predecessors are pairs of the **MEC**)* has been proved. Indeed, such ***CA*** models are characterized by simple enough dynamics of finite configurations. Meantime, similar classical ***CA*** model admits various combinations of the above **3** types of irreversibility in wide enough limits. In addition, in view of the above definition **13** the following interesting enough result takes place [82-87].

Theorem 37. Among classical d-CA (d≥1) models with an arbitrary states alphabet A = {0,1,2, ..., a–1} there are CA models such as:

1. CA models possessing the NCF nonconstructability without NCF–1 can have irreversible finite configurations of the first type only;

2. CA models without the NCF nonconstructability for which mappings $\tau^{(n)}$:C(A,d,ϕ)→C(A,d,ϕ) are bijective whereas mappings $\tau^{(n)}$: C(A,d,∞) → C(A,d,∞) are not bijective; in addition, CA models that possess the NCF nonconstructability can`t have bijective mappings such as $\tau^{(n)}$: C(A,d,ϕ) → C(A,d,ϕ), and $\tau^{(n)}$: C(A,d,∞) → C(A,d,∞);

3. CA models possessing the NCF nonconstructability can have all finite configurations excluding NCF as irreversible of the third type only;

4. CA models that do not possess the NCF nonconstructability, but at existence of the NCF–1 nonconstructability can have arbitrary finite configurations as the irreversible configurations of the second or third type; at that a finite configuration different from NCF–1 is irreversible configuration of the third type, i.e. the finite configuration is absolutely constructible, whereas each NCF–1 is irreversible configuration of the second type;

5. CA models, whose each finite configuration from the set C(A, d, ϕ) is irreversible, having the type 2 or 3; at that, each configuration such as NCF–1 has more than one predecessor from the C(A, d, ∞), whereas the others have sole predecessor from the C(A, d, ϕ) along with single from C(A,d,∞). There are such classical CA models that: (1) set C(A, d, ϕ) can be generated only from finite configurations whereas the set C(A,d,∞) is generated only from infinite configurations, and (2) the set C(A, d, ϕ) is generated from infinite configurations; in addition, its infinite subset NCF–1 is generated only from the infinite configurations, whereas the set C(A,d,ϕ)\NCF–1 is generated both from the finite, and the infinite configurations. For a classical d-CA (d≥1) model the set C(A,d,ϕ) can`t be generated only from the infinite configurations;

6. For an alphabet A={0,1, ..., a–1} (a ≥ 3) there are classical d–CA (d ≥ 1) models at the absence of the NCF nonconstructability and for which an arbitrary configuration from the set C(A, d, ϕ) possesses (a–1) different predecessors from the set C(A,d,∞) if the configuration is different from a configuration such as NCF–1; otherwise, the configuration will has a few different predecessors from the set C(A,d,∞).

If for a classical d–CA (d≥1) model with global transition function $\tau^{(n)}$ the set C(A,d,∞) is nonclosed relative to a mapping defined by GTF $\tau^{(n)}$, the mapping $\tau^{(n)}$: C(A,d,ϕ) → C(A,d,ϕ) will not be bijective. A mapping $\tau^{(n)}$: C(A,d,ϕ) → C(A,d,ϕ) for an arbitrary classical d–CA (d ≥ 1) model with GTF $\tau^{(n)}$ will be bijective if and only if the model not possess the NCF nonconstructability in the presence of the closed set C(A,d,∞) with respect the mapping defined by the GTF $\tau^{(n)}$.

So, the classical binary ***1–CA*** with connected neighbourhood template of the size three and with discriminating number ***113*** does not possess the ***NCF–1*** nonconstructability at presence in it of ***NCF.*** For this model any *irreversible* finite configuration possesses pairs of predecessors only

from the set $C(A,d,\phi)$, i.e. the irreversibility of the first type takes place. It is simple to make sure in validity and of other assertions **2–5** of this theorem; in addition, the last part of this theorem follows of the above results on the nonconstructability problematics [24,40-43,102,106].

Thus, if the first type determines the dynamics reversibility concerning the set of block configurations, and in wider understanding of the finite configurations concerning the set $C(A,d)$, then the second type relative to the set $C(A,d,\phi)$. At that, if a certain classical ***CA*** model possesses the reversibility of the *first* type then for it the reversibility of the *second* type can be absent. So, if for a classical ***CA*** model any finite configuration is *periodic*, the reversibility of the first type takes place along with absence of the second type, because a configuration from the set $C(A,d,\phi)$ does not possess predecessors from the set $C(A,d,\infty)$. It takes place only if a ***CA*** model does not possess the ***NCF*** and ***NCF–1*** nonconstructability, i.e. its dynamics is a rather simple. In the case of existence of irreversibility of the second type, configurations from the set $C(A,d,\phi)$ can have more one predecessor from the set $C(A,d,\infty)$.

So, for the above model with number ***60*** in the absence of ***NCF*** in it, i.e. in the presence of reversible dynamics of the *first* type, a configuration of the kind $\{1^{2k+1} \mid k=0,1,...\}$ is ***NCF–1*** in the model; in addition, a pair of various infinite configurations are *predecessors* for these configurations. Furthermore, from point of view of the general sense the second type of irreversibility deserves the essentially smaller attention, because both transition of an infinite configuration into finite configuration and vice versa is enough problematically interpreted. In this context there is a rather natural desire to associate dynamics of the classical models with trajectories of dynamic systems plays essential part too.

In principle, in compliance with the definition ***12*** as a reversible finite configuration a configuration having only single predecessor from the set $C(A,d,\phi)$ and, perhaps, predecessors from the set $C(A,d,\infty)$ should be regarded. The assumption is caused by the aforesaid. In this context the following interesting enough result takes place, namely:

The set $C(A,d,\phi)$ for an arbitrary classical d-CA ($d\geq 1$) model will consist only of reversible finite configurations if and only if the model does not possess the NCF and NCF–1 nonconstructability; in addition, all finite configurations in such model will generate only periodical sequences of configurations.

Moreover, if to consider the ***CA*** models with the above reversibility *(i.e.*

to deal with full reversibility) then from standpoint of modelling they not represent any especial interest due to their dynamics and insignificant quota, how it was repeatedly noted above. Consequently from this point of view only ***CA*** models possessing the ***NCF*** nonconstructability, above all, and/or ***NCF–1*** can represent the greatest interest. So, the ***CA*** models with *irreversibility* only of the *second* type possess dynamics of a limited enough complexity. While the binary classical ***1–CA*** models considered above, possessing the *full* reversibility have simple enough dynamics of finite configurations whose applied interpretation of any special interest does not represent.

It is necessary to mark, that it is enough expedient to consider also the dynamics reversibility concerning the set $C(A,d,\phi)$; i.e. when existence of single predecessor from the set $C(A,d,\infty)$ for a finite configuration does not provide the reversibility of classical ***d–CA*** $(d \geq 1)$ models. The given assumption is well founded inasmuch as infinite configurations from standpoint of a number of applied aspects not have enough satisfactory interpretation, and from the theoretical standpoint the case of potential infinity is more interesting. But in some cases infinite configurations can appear useful enough at research of certain theoretical aspects [40-43].

Roughly speaking, a reversible classical ***d-CA*** $(d \geq 1)$ model never forgets the history of a finite configuration concerning the set $C(A,d,\phi)$. If from abstract standpoint the transition in one step from *infinite* configuration into *finite* configuration is quite allowable whereas at use of classical ***CA*** models as a modelling medium the interpretation of this opportunity is problematic enough. In this case such possibility should be considered only as accompanying the base modelling process based on dynamics of finite configurations. In addition, the classical ***CA*** models of this type present especial interest for modelling of a lot of processes investigated by modern physics [7]. In connexion with that, the reversibility concept of dynamics of finite configurations for classical ***CA*** models can be quite defined as follows.

Definition 14*. An arbitrary classical d-CA* $(d \geq 1)$ *model which does not possess the NCF-1 and NCF nonconstructability is called real reversible classical model concerning the all finite configurations if not, then it is called the irreversible in the presence of possible formal reversibility.*

That allows to formulate the following result about interrelation of the nonconstructability and dynamics reversibility of the classical models concerning the finite configurations, i.e. from standpoint of trajectories of finite configurations in the classical ***CA*** models, namely [24,40–43]**:**

A classical d–CA (d ≥ 1) model for which the set C(A, d, ∞) is nonclosed concerning the global mapping defined by its global transition function $\tau^{(n)}$ is irreversible relative to dynamics of finite configurations. There is not a classical d–CA (d ≥ 1) model for which each finite configuration could has a sole predecessor from the set C(A, d, ϕ) in case of nonclosure of the set C(A,d,∞) concerning the mapping defined by global transition function $\tau^{(n)}$ of such classical CA model.

In addition, on the basis of theorem **37** the following result follows**:**

There are classical d–CA (d≥1) models for which any configuration c*∈ C(A,d,ϕ) has at least 1 predecessor from the set C(A, d, ∞). There are not the classical d–CA (d≥1) models not possessing the nonconstructability of NCF type under the condition of the presence of nonconstructability of NCF–1 type, and for which a configuration c∈C(A, d, ϕ) has only sole predecessor from the set C(A,d,ϕ) or from the set C(A,d,∞) of all infinite configurations.

However, it is enough easy to make sure, that for an arbitrary classical ***d-CA (d≥1)*** model all *finite* configurations can't have predecessors solely from the set ***C(A,d,∞).*** So, the second assertion directly follows from the above axiom that the entirely null configuration ***c=□*** is ascribed by us to the set ***C(A,d,ϕ)*** of all finite configurations**;** this assertion can be enough easily received on the basis of results on the ***NCF–1*** nonconstructability.

Having defined dynamical reversibility of classical ***CA*** models relative to the finite configurations *(Definition* ***12)*** which is quite natural in view of its research for the purpose of presence for each finite configuration of its quite determined unique trajectory reversible in time we generally speaking should have in view such essential enough circumstance.

Thus, an arbitrary configuration ***c∈C(A,d,ϕ)*** has reversible dynamics if for it the following relation takes place, namely**:**

$$(\forall t \geq 0 \mid t, t-1, \ldots, 1)(E!c^{-t} \in C(A,d,\phi))(c^{-t}\tau^{(n)} = c^{-t+1}) \qquad c^{o} \equiv c$$

Consequently**,** taking into account the aforesaid there are no classical ***CA*** models that possess the ***NCF*** nonconstructability in the presence of the ***NCF-1*** nonconstructability along with concurrent possessing as a whole reversible dynamics of finite configurations**;** in addition, certain infinite subsets of the set ***C(A,d,ϕ)*** quite can contain dynamically reversible ***CF****s*.

Hence, only classical ***d–CA (d≥1)*** models which do not possess the ***NCF*** and ***NCF–1*** nonconstructability will possess the property of dynamical reversibility *(in the sense of definition* ***12)*** as a whole but these ***CA*** models

represent a relatively simple periodical dynamics of infinite and finite configurations that excludes, in particular, presence in such ***CA*** models of such important properties as *universal computability* and *reproducibility* in the *Moore* sense of the finite configurations.

In view of differentiating of the set ***C(A, d)*** of classical ***CA*** models into **2** *nonoverlapping* subsets, the reversibility question of finite configurations concerning each of these subsets separately is quite natural. Therefore, the following result can be formulated [40-43]. We shall say that a finite configuration is *reversible* relative to the set ***C(A,d,ϕ)*** or ***C(A,d,∞)*** if it has definitely one predecessor from the appropriate set**;** otherwise, it will be *irreversible* concerning the appropriate set. The reason of it is caused by fundamental differences of the sets ***C(A,d,ϕ)*** and ***C(A,d,∞).***

Theorem 38*. A finite configuration in a classical d–CA (d ≥ 1) model is reversible relative to the set C(A, d, ϕ) if and only if the model does not have the NCF and NCF–1 nonconstructability. All finite configurations in a classical d–CA (d ≥ 1) model can`t be reversible concerning the sets C(A,d,ϕ) and C(A,d,∞) simultaneously, i.e. a finite configuration in the model can`t have only one predecessor from the set C(A,d,ϕ) and the set C(A,d,∞); at that, there are classical d–CA (d ≥ 1) models for which each finite configuration, except NCF–1, possesses sole predecessor from the set C(A, d, ϕ) along with one or two predecessors from the set C(A, d, ∞). In addition, there are classical d–CA (d ≥1) models for which any finite configuration from the set C(A, d, ϕ) possesses sole predecessor from the set C(A,d,ϕ) or will be nonconstructible configuration of the NCF-1 type with at least one predecessor from the set C(A, d, ∞). There are classical d-CA (d≥1) models in which depending on availability or absence of the NCF, NCF–1 nonconstructability the set C(A,d,ϕ) can be generated only from the set of nonconstructible configurations such as NCF–1, NCF or NCF∪NCF–1. In addition, there is not classical d–CA (d ≥ 1) model for which a finite configuration will possess the predecessors from the sets C(A,d,∞) and C(A,d,ϕ) simultaneously. If a classical d–CA (d ≥ 1) model possesses the NCF-1 nonconstructability in the absence of NCF, then for two arbitrary different configurations g, w of type NCF–1 the following relation* $\{g\tau^{(n)p}\}\cap\{w\tau^{(n)p}\} = \varnothing$, $p = 0 .. \infty$ *takes place. There are classical nontrivial d-CA (d≥1) models whose all finite configurations are NCF-2. There are classical d–CA (d≥1) models for which any constructive finite configuration possesses at least one predecessor from the set C(A,d,∞). There are classical d-CA (d≥1) models for which any finite configuration possesses at least one predecessor from the set C(A,d,∞); thus, there are*

classical d–CA (d≥1) models for which an arbitrary finite configuration will be of type ACCF or NCF–1 only.

In particular, as a confirmation of the last proposition of the theorem ***38*** certain models from the class of all binary ***d–CA (d≥1)*** models can serve. So, among ***1–CA*** models it is possible to note ***2*** binary models with the *Moore* neighbourhood index and discriminating numbers ***75*** and ***102.*** As seen from the foregoing, the reversibility problem for classical ***d–CA*** models is directly based on the predecessors concept. Meanwhile, the theoretical study of the reversibility of dynamics of classical ***CA*** models is quite difficult *(however, as well many important problems in this class of parallel dynamical systems)*, therefore the use of computer simulation for this purpose is extremely effective. Likewise as for other problems, for experimental research of ***CA*** dynamics we have used the corresponding procedures programmed in the computer mathematics systems ***Maple*** and ***Mathematica*** [24-29,40-43,48-52,102,106].

So, for experimental research of the reverse dynamics of classical ***1–CA*** models in certain cases the procedures ***Predecessors***, ***PredecessorsL*** and ***PredecessorsR*** programmed in the ***Mathematica*** system can be a rather useful. For instance, the call ***Predecessors*[*j,g,n*]** returns the predecessors set of a ***g*** configuration for a classical ***1–CA*** model with local transition function ***j*** and size ***n*** of neighbourhood template. In particular, return of the empty set speaks with certainty about availability in a model of the ***NCF*** nonconstructability and allows to determine at least one of kinds of nonconstructible configurations. And what is more, the ***Predecessors*** procedure in some cases allows to determine availability in the studied model of the ***NCF*** nonconstructability on the basis of predecessors of a block ***g*** configuration. More detailed with the source codes of the above procedures and results of their use the interested reader can familiarize in the above cited references.

It is known, that along with theoretical research of classical ***CA*** models their experimental research by means of computer modelling is enough widely used. With this purpose a rather large collection of software of various appointments and complexity was created. A lot of software of this type can be found in [7,24,48,49,106]. In large, if using the ***Maple*** or ***Mathematica*** systems for experimental research of classical ***CA*** models it should be borne in mind that considering the more large reactivity of ***Mathematica*** concerning the ***Maple***, it would seem quite natural to use ***Mathematica*** for dynamics modelling and research of a lot of properties of classical ***CA*** models however it not entirely so; with a rather detailed

consideration of this problematics the interested reader can familiarize in the above cited references, in particular, in [7,24,82,102,106,286].

Thus, full *reversibility* of finite configurations relative to the set $C(A,d,\phi) \cup C(A,d,\infty)$ demands absence of the ***NCF*** and ***NCF-1*** nonconstructability, sharply limiting the number of such ***CA*** models; in addition, concerning the dynamics, similar ***CA*** models have a rather little interest. Indeed, in a lot of examined cases the dynamics of classical ***CA*** models of such type turns out rather simple and predictable since such models for any finite initial configuration generate *periodic* sequences, not allowing to model complex enough processes, algorithms or phenomena. So, if to consider dynamics of classical ***CA*** models as abstract algebraic systems of *parallel* processing of finite words defined in finite alphabets, the full reversible classical ***CA*** models will demonstrate a rather simple dynamics.

Thus, on account of definition ***14*** of dynamics reversibility of the finite configurations in classical ***CA*** models, what seems quite natural, we are compelled to be limited to a certain subclass of ***CA*** models whose finite configurations are periodic, defining a rather simple dynamics of such models. So, the direct modelling of complex enough algorithms, objects and phenomena by means of al systems similar to ***CA*** models demands use of classical models possessing the nonconstructability such as ***NCF*** or/and ***NCF–1.*** Naturally, under certain assumptions we can simulate the irreversible classical ***CA*** models simulating those or other processes by reversible models, for example, from the standpoint of ***NCF*** absence for them, however a mediated process of modelling of the first will be far enough from the direct modelling. At that, the reversibility problem of dynamics for the classical ***CA*** models will be discussed a little below.

Having finished a discussion of the nonconstructability problem in the classical ***CA*** models we turn to algorithmic aspects of ***CA*** problematics, i.e. to questions of algorithmic solvability of those or other its problems. This theme is being presented rather important for the ***CA*** problematics from many interesting enough standpoints. First of all, the problematics has the most immediate attitude toward researches of dynamics of the classical ***CA*** models as a whole.

2.6. Algorithmical aspects of the nonconstructability problem and some connected questions of dynamics of the classical *CA* models

Algorithmic solvability of the nonconstructability problem is one of key questions of mathematical theory of the ***CA*** models and a number of its important appendices, especially by way of use of classical ***CA*** models in quality of both conceptual and practical models of *spatially-distributed* dynamical systems from which the real physical systems represent the greatest interest [7,24]. In the general formulation, the solvability of the nonconstructability problem is reduced to the question**:** ***Whether there is an algorithm for determination of, whether a classical CA model will possess the NCF, NCF–1, NCF–2 and NCF–3 nonconstructability?*** In the general formulation the problem remains open up till now but there are answers to a lot of more particular but not less important questions that represent significant independent interest [24-28,40-43,82-87,102,106].

The fullest decision of this problem is received for case of classical ***1-CA*** models. First of all, concerning an arbitrary block configuration or finite configuration the following basic result, having a number of important enough appendices takes place [24-28,40-43,82-87,102,106,286]**,** namely.

Theorem 39*. Concerning an arbitrary finite configuration as well as a block configuration the determination problem of its type (NCF-1, NCF, NCF–2, NCF–3, constructible) for an arbitrary classical 1–CA model is algorithmically solvable.*

Methods used at proof of the theorem allow not only to constructively define type of an arbitrary block configuration and finite configuration [24]**,** but also to establish structure of a set of their direct predecessors what in a lot of cases is rather important. For the general ***d-***dimensional case ***(d ≥ 1)*** the definition question of for a concrete block configuration of its type *(constructible,* ***NCF*** *or* ***NCF–3)*** is algorithmically solvable, but it nothing does speak about the solvability problem as a whole, i.e. the nonconstructability existence of the type ***NCF (NCF–3)*** for an arbitrary ***d–CA (d ≥ 2)*** model.

It is well**–**known, with transition from dimension ***d=1*** to dimension ***d=2*** the research of a lot of questions of dynamics of classical ***d–CA*** models enough appreciably becomes complicated**,** and many of the solvability problems for ***1–***dimensional case become unsolvable for ***d–***dimensional case ***(d≥2)***. In particular, in work [107] algorithmical unsolvability of the

closure problem of set of all finite configurations, distinct from the fully null configuration «□» and a number of other problems of dynamics of the classical ***d–CA*** ***(d ≥ 2)*** models has been proved. In works [7,12,40–42, 64,69,71,72,78,88,99,137,138] and others many rather interesting results on algorithmic unsolvability of problems of dynamics of the ***CA*** models is represented. Below, this theme in a certain sense will be continued.

One of known approaches to the solution of the solvability problem of existence in classical ***CA*** models of this or that nonconstructability type consists in determination of an upper limit for the minimal sizes of the internal block of ***MEC*** pairs, of sizes of ***γ–CF,*** or some nonconstructible configuration of the required type ***(NCF, NCF–1, NCF–2*** *and* ***NCF–3)***. In case of classical ***1–CA*** we acted thus and in this direction a number of results presenting a certain independent interest has been obtained [5,8, 9,12,13,24-28,40-43,82-87,102,106]. The question plays an important part for estimation of minimal size of ***γ–CF,*** study of a number of dynamical properties of classical ***CA*** models and at study of the nonconstructability problem as a whole.

In context of research of solvability of the nonconstructability problem we and a lot of other authors have studied a question of interrelation of minimal sizes of ***NCF*** and of ***IB MEC*** in the classical ***CA*** models [5,7,12, 24,106]. Meanwhile, contrary to the undertaken efforts in this direction, a satisfactory solution has not been received. However, a number of the essential results received in this direction has allowed to formulate the following interesting enough assumption.

Proposal 5. In general, for classical d–CA (d ≥ 2) models it is impossible to receive any rather satisfactory quantitative estimation for minimal size of NCF as a function from minimal size of IB MEC, and vice versa.

The proposal ***5*** has allowed to clear a number of the principal questions existing up to it [24]. However, concerning one type of classical models we have interesting enough result rather useful in a lot of applications of a theoretical nature [40-43,82-87,102,106].

Theorem 40. If for global transition function $\tau^{(n)}$ of a d–CA (d ≥ 1) model the following relation $(\forall c \in C(A,d,\phi))(|c| < |c\tau^{(n)}|)$ takes place, where |GS| is size of maximal diameter of a finite configuration GS, then the model will possess the NCF and/or NCF–1 nonconstructability. At that it is easy to see that quota of such models is asymptotically more than e^{-2}. If a classical d–CA (d ≥ 1) model of the above type will possess the NCF–1 {***NCF***} ***nonconstructability only then its set NCF–1*** {***NCF***} ***will***

generate all set C(A, d, ϕ)\{□}; in addition, detection problem of NCF–1 {NCF} of minimal size is solvable.

On basis of a number of results concerning the decomposition problem of *global transition functions (**GTF**)* in the classical ***CA*** models considered a little bit below, in certain cases it is possible to reduce the solution of the existence problem of the ***NCF*** and ***NCF–1*** nonconstructability to the solution of similar problems for essentially more simple functions ***GTF*** $\tau^{(n)}$ of the same dimension and alphabet. In a number of cases a similar approach considerably simplifies solution of the problem, but generally speaking, its direct using of any especial result does not give. However, basing on this approach, the a rather interesting result has initially been received [5,8,24-28,40-43,82-87,102,106].

Definition 15. An algorithm determining the existence of MEC pairs for classical CA models, we shall name «essentially constructive» if it for a global transition function $\tau^{(n)}$ ***not only gives answer «NO/YES» upon the question about existence of MEC but also in the case of the positive answer determines all types of MEC pairs existing for a CA model.***

Constructive algorithms present especial interest above all there, where the researcher collides with necessity of real model realization of that or other nature. But as ***CA*** models represent the most considerable interest from the standpoint of their opportunities in the constructive attitude, this definition is being presented a quite pertinent. In light of definition ***15*** along with research results of decomposition problem of ***GTF*** $\tau^{(n)}$ in classical ***d–CA*** $(d \geq 2)$ models, will be possible to throw light and on this interesting question [24-28,43,82,102,106].

Theorem 41. An essentially constructive algorithm solving the problem of existence of the MEC pairs for classical d–CA $(d \geq 2)$ ***models is absent in general case.***

The further elaboration of the proof technique of the above theorem has allowed to prove algorithmic unsolvability of the existence problem of the ***NCF-3*** nonconstructability for classical ***d–CA*** $(d \geq 2)$ models**;** namely, the following result having a number of important enough applications as an apparatus of investigation in this direction has been proved [106].

Theorem 42. The existence problem for an arbitrary classical d-CA $(d \geq 2)$ ***model of the NCF–3 nonconstructability is algorithmically unsolvable. The existence problem for an arbitrary classical d-CA*** $(d \geq 2)$ ***model of the MEC–1 pairs is algorithmically unsolvable too.***

But according to the aforesaid *(Theorem **3**)* for each ***CA*** model the set of ***NCF-3*** is a strict subset of the set ***NCF*** then a rather simple modification allows to prove the unsolvability of the existence problem of ***NCF*** in the classical ***d–CA*** *(**d**≥**2**)* models**;** that has been earlier proved by ***J. Kari*** [78] on the basis of other approach. Thus, in general case for classical ***d–CA*** *(**d** ≥ **2**)* models the existence problems of the **γ–*CF*,** the ***MEC*** and ***MEC–1*** pairs, and the ***NCF*** and ***NCF–3*** nonconstructability are algorithmically unsolvable.

Thus, the unsolvability of the general nonconstructability problem such as ***NCF*** and ***NCF–3*** for classical ***d–CA*** *(**d** ≥ **2**)* presumes development of certain partial methods of the solution of the specified problems for ***CA*** models of certain types and classes. It can have a lot of rather important theoretical and applied outcomes. The essential operational experience with classical ***CA*** models of dimensionality ***d*=2** shows that in despite of algorithmic unsolvability of the determination problem of the existence of the ***NCF*** and ***NCF–3*** nonconstructability for them in case of concrete classical ***2–CA*** models we always received solution in the form of some constructive algorithms.

Thus, here only about absence of unified decision algorithm concerning the class of all ***d–CA*** *(**d** ≥ **2**)* models makes sense to speak, while in each concrete case this problem, in our opinion, as a rule, has a constructive solution of some efficiency determined by concrete specificity of ***LTF*** of a ***2–CA.*** One of approaches to solution of a similar problem is a method *(in a series of cases**,** invariant concerning the nonconstructability property)* of modelling of one classical ***d–CA*** *(**d** ≥ **2**)* model by means of some other model of the same class and dimensionality, however with the simplest neighbourhood index *(Section **1.1**)**,** (for example**,** for case of **2–CA** the index has the kind **X** = {**(0,0),(0,1),(1,0)**})*. The increase of cardinality of alphabet ***A*** of a certain modelling model in a lot of cases is essentially balanced by significant simplification of analysis procedure of ***LTF*** $\sigma^{(n)}$ with the object of to reveal the existence in the modelled model ***d–CA*** *(**d** ≥ **2**)* of the nonconstructability phenomenon. Meanwhile, in view of a lot of the reasons such approach should be applied circumspectly enough [5,106]. Completely other picture takes place for case of the ***1–CA*** models and, above all, of the ***NCF–1*** and ***NCF–2*** nonconstructability in them. Thus, according to theorems ***10*** and ***11*** the existence problem of the ***MEC*** pairs and ***NCF*** for the classical ***d–CA*** *(**d** = **1**)* models is solvable. The following result proves the solvability problem for generalized case of ***MEC–1*** for dimension ***d* = *1*** along with unsolvability for ***d* ≥ *2*.**

Theorem 43. The existence problem of the MEC–1 pairs for an arbitrary classical d–CA is algorithmically solvable for the case d=1 whereas for the case d≥2 is algorithmically unsolvable.

The proof of theorem ***43*** can serve as a constructive test of existence of the ***MEC–1*** nonconstructability for a classical ***1–CA*** model. At the same time, the given test supposes a rather simple software implementation, making it possible to employ for the problem solution of the advanced computer facilities in a concrete case [5,8,12,24-28,40-43,82-87,106]. On the basis of theorem ***43*** and approach to its proof it is possible to obtain a lot of interesting enough results concerning the solvability of those or other aspects of the nonconstructability problem for an arbitrary ***1–CA*** model. So, from the results presented here and in a lot of other works [7, 24] concerning solvability of the basic aspects of the nonconstructability problem in the classical ***CA*** models it is easy to be convinced, that their decision essentially depends on dimensionality of the models. If in case of classical ***1–CA*** models, mainly, the algorithmic solvability takes place whereas already for two–dimensional case very much many important questions in the given direction remain unsolvable in spite of sufficient simplicity of that type of classical ***CA*** models [72,82,102,106,278,286].

The following main theorem establishes full solvability of the existence problem of the nonconstructability of every possible types for the case of classical ***1–***dimensional ***CA*** models [5,8,12,24-28,40-43,82-87,102,106].

Theorem 44. The existence problem of the NCF, NCF-1, NCF-2 and NCF-3 nonconstructability for each classical 1–CA model is algorithmically solvable. The existence problem of admissible combinations of types of nonconstructability according to the table 2 for an arbitrary classical 1–CA model is algorithmically solvable, while for the case of classical d–CA (d≥2) models the full solution of the given question remains open until now.

In the same direction the following result presents a certain interest, in the first place, from the theoretical standpoint [24-28,40-43,82-87,106].

Theorem 45. The existence problem of the NCF–1 nonconstructability in the classical d–CA models not possessing the NCF nonconstructability is solvable for case d=1 and unsolvable for case d≥2.

So, the proof of the theorem for case ***d≥2*** is based on the algorithmical unsolvability of the «***domino***» problem [209] that has been considered enough in detail in [24,106] and on the basis of definition ***4*** establishing a close connection between existence of the ***NCF–1*** nonconstructability

and closure of the set $C(A,d,\infty)$. Here, concerning the solvability of the determination problem of non–closure of the set $C(A,d,\infty)$ of all infinite configurations for an arbitrary classical d–CA $(d\geq 1)$ model with a states alphabet A the following rather important result takes place [41,82-87].

Theorem 46. ***The determination problem of closure of the set $C(A,d,\infty)$ of all infinite configurations concerning the mapping induced by a global transition function $\tau^{(n)}$ of an arbitrary classical d–CA $(d \geq 1)$ model is algorithmically solvable for case $d = 1$ and unsolvable for case $d \geq 2$.***

A proof of the theorem is based on unsolvability of the above «***domino***» problem whose brief sketch is presented in section **2.8** [26]. In addition, in the more general posing the presented arguments allow to formulate the following interesting enough result [24-28,40-43,82,102], namely.

Theorem 47. ***For a classical d–CA $(d \geq 2)$ model with an A alphabet and global function $\tau^{(n)}$, the existence problem of such configurations c that the relation $c\tau^{(n)} = c^{\infty}_{r}$ takes place is unsolvable (where c^{∞}_{r} is an infinite configuration consisting only from r states; $r \in A$).***

Essentially, under ***nonclosure (closure)*** of the set $C(A,d,\infty)$ of all infinite configurations concerning the global parallel mapping induced by ***GTF*** $\tau^{(n)}$ of a classical d–CA $(d\geq 1)$ model we understand *existence (absence)* in the set $C(A,d,\infty)$ of such configurations $c^{\infty} \in C(A,d,\infty)$ that relation $c^{\infty}\tau^{(n)} = \Lambda$ takes place, where Λ – completely null configuration which owing to a lot of essential enough reasons is attributed by us to the set $C(A, d, \phi)$ of all finite configurations. Of our results in this direction an interesting enough offer follows [24–28,40-43,82,102,106].

Theorem 48. ***The existence problem of such infinite configurations g that the relation $g\tau^{(n)} \in C(A, d, \phi)$ takes place concerning the global parallel mapping induced by the GTF $\tau^{(n)}$ of an arbitrary classical d-CA model is algorithmically solvable for case $d = 1$ and unsolvable for case $d \geq 2$.***

As one of algorithms solving the existence for classical models of such infinite g configurations that the relation $g\tau^{(n)} = \Lambda$ ($\tau^{(n)}$ – *global transition function)* takes place, was programmed in the **FullNull** procedure in the ***Mathematica*** system. The call **FullNull[***Ltf***]** returns *True* if for a model there are such infinite configurations, and *False* otherwise. Whereas, the call **FullNull[***Ltf, g***]** thru the second optional g argument – an indefinite variable – returns the list containing some basic finite subconfigurations composing required infinite configurations. ***Ltf*** defines a local transition function in the form of transition rules $x1x2 \ldots x_n \rightarrow x1$` that are coded at

the procedure call as "$x1x2...xnx1$`" where $xj \in A$ $(j=1..n)$ and $A=\{0,1,...,a-1\}$. The **FullNull** procedure rather easily extends to case when an arbitrary A alphabet and neighbourhood size n are used. The following fragment represents source code of the procedure along with examples of its use.

```
In[34]:= FullNull[x_ /; ListQ[x] && ! MemberQ[Map[StringQ, x], False],
    g___Symbol] := Module[{a, b, c = {}, t = StringLength[x[[1]]] - 1, p},
                          a = Select[x, StringTake[#, {-1, -1}] == "0" &];
                          a = Drop[Map[StringTake[#, {1, t}] &, a], 1]; b = a;
                          Do[Do[Do[AppendTo[c, If[StringTake[a[[j]], {p + 1, -1}] ==
                          StringTake[b[[k]], {1, t - 1}], a[[j]] <> StringTake[b[[k]], {-1, -1}],
                          Nothing]], {j, 1, Length[a]}], {k, 1, Length[b]}];
                          a = c; c = {}, {p, 1, t + 1}]; p = Map[StringCount[#, b] &, a];
                          If[{g} != {}, t = 1; g = Map[If[# > 1, a[[t++]], t++] &, p], Null];
                          If[a == {}, False, True]]

In[35]:= Ltf:= {"0000", "0010", "0100", "0111", "1001", "1010", "1101", "1110"}
In[36]:= Ltf1:= {"00000", "00011", "00101", "00110", "01000", "01011", "01100",
"01111", "10000", "10011", "10101", "10110", "11000", "11011", "11100", "11111"}
In[37]:= Ltf2:= {"00000", "00011", "00101", "00110", "01000", "01011", "01100",
"01111", "10000", "10011", "10101", "10110", "11000", "11011", "11101", "11110"}
In[38]:= FullNull[Ltf1]
Out[38]= False
In[39]:= FullNull[Ltf, gs]
Out[39]= True
In[40]:= gs
Out[40]= {"0101010", "0010101", "1010101", "1111111"}
In[41]:= FullNull[Ltf2, gv]
Out[41]= True
In[42]:= gv
Out[42]= {"111111111"}
```

In addition, it is shown that the following interesting result takes place.

***Theorem 49**. If for an arbitrary classical d–CA model the set C(A,d,∞) is nonclosed relative to the mapping induced by global transition function $\tau^{(n)}$, the model will possess the NCF or NCF-1 nonconstructability, or by the mentioned nonconstructability types simultaneously.*

Hence, for decision of the solvability problem of existence of the ***NCF*** or ***NCF–1*** nonconstructability for an arbitrary classical ***d–CA*** $(d \geq 1)$ model, the question of nonclosure of the set $C(A,d,\infty)$ concerning the mapping

induced by the global transition function $\tau^{(n)}$ of the model plays a rather essential part. On the assumption of rather transparent definitions of the ***NCF–1*** and ***NCF–2*** nonconstructability along with aforesaid arguments, already it is rather simple to conclude that in the absence of the ***NCF–1*** and ***NCF*** nonconstructability for a classical ***d–CA*** $(d \geq 1)$ model for it the ***NCF–2*** nonconstructability should exist**.** The above results concerning the nonconstructability problem once again confirm basic distinctions between the considered nonconstructability types in the classical ***d–CA*** models, and strong influence of dimensionality of the ***CA*** models upon the results connected to them. Obviously, since the existence problems of the ***NCF*** and ***NCF–1*** nonconstructability are unsolvable concerning the classical ***d–CA*** $(d \geq 2)$ models**,** the problem of revealing of dynamical reversibility for classical ***CA*** models is unsolvable as a whole too.

In most general posing from the point of view of researches of analogies between formal models on the basis of classical ***d–CA*** models and real physical processes and phenomena, embedded in them, then it would be extremely interesting clarify in more detail not only influence of the dimension on their global properties, but also the nonconstructability problem of directly connected to dynamical reversibility of the classical ***d–CA*** $(d \geq 1)$ models. In works [7,24–28,40-43,82-87] the influence of key parameters of classical ***CA*** models on questions of investigation of their *dynamic* properties is considered with sufficient degree of completeness. This question seems to us a rather important.

Researches of deep properties of parallel global mappings $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$**,** induced by ***GTF*** $\tau^{(n)}$ of the classical ***d–CA*** models, has the direct attitude to the nonconstructability problem and plays the fundamental part in research of dynamic properties of such ***CA*** models. Properties of the parallel mappings such as injectivity and surjectivity have *immediate* connection with the nonconstructability problem and were research by a lot of researchers while the review of their results can be found in [14, 22,24,25,28,30,32,33,40-43,63,69,70,71,79,82-84,102,106,118,137,140].

According to the theorem ***21*** the *necessary* and *sufficient* condition of the ***NCF-1*** nonconstructability existence in a classical ***d–CA*** $(d \geq 1)$ model not possessing ***NCF*** consists in *nonclosure* of the set $C(A,d,\infty)$ concerning its parallel mapping $\tau^{(n)}: C(A,d,\infty) \rightarrow C(A,d,\infty)$**.** While the existence of ***NCF*** ***(NCF-3)*** in a classical ***d–CA*** model is directly connected to ambiguity of its global mapping $\tau^{(n)}: C(A,d,\phi) \rightarrow C(A,d,\phi)$**.** In general case concerning the nonconstructability it is interesting enough to research interrelation

between existence in a classical ***CA*** model of the ***NCF–1, NCF–3*** and/or ***NCF*** nonconstructability, and mapping properties $\tau^{(n)}$:$C(A,d) \to C(A,d)$.

To this end we shall determine influence of existence of the ***NCF, NCF-3*** and/or ***NCF–1*** in a classical ***1–CA*** model upon mutual unambiguity of its parallel mapping $\tau^{(n)}$, and vice versa. Whereas for the case of classical ***d–CA*** $(d \geq 1)$ the following result takes place.

Theorem 50*. If the set* $C(A,d,\infty)$ *of infinite configurations of a classical d–CA model is nonclosed concerning the d–dimensional transformation* $\tau^{(n)}$*, then a mapping* $\tau^{(n)}: C(A) \to C(A)$ *appropriate to it will not as one–to–one, where* $C(A) = C(A,d,\infty) \cup C(A,d,\phi)$*. If a classical d–CA model does not possess the NCF nonconstructability then* $\tau^{(n)} : C(A,d,\phi) \to C(A,d,\phi)$ *is biunivocal mapping, while mapping* $\tau^{(n)}: C(A,d,\infty) \to C(A,d,\infty)$ *can be as a not biunivocal. If the set* $C(A,d,\infty)$ *of a classical d–CA* $(d \geq 1)$ *model is nonclosed relative to d–dimension transformation* $\tau^{(n)}$ *then mapping* $\tau^{(n)} : C(A,d,\phi) \to C(A,d,\phi)$ ***cannot*** *be as a bijective mapping, whereas the contrary assertion is false, generally speaking.*

On the basis of theorems ***21, 50*** it is possible to show, that at existence of the ***NCF (NCF-3)*** and/or ***NCF-1*** nonconstructability for a classical ***1–CA*** model a parallel mapping $\tau^{(n)}: C(A) \to C(A)$ is not biunique, whereas the converse proposition is generally false. Consequently, the fact of many–valuedness of a mapping $\tau^{(n)}: C(A) \to C(A)$ does not entail existence in a classical ***1-CA*** model of the ***NCF, NCF-3*** and ***NCF-1*** nonconstructability. Moreover, if ambiguity of a mapping $\tau^{(n)}: C(A,\phi) \to C(A,\phi)$ gives rise to occurrence of the ***NCF*** nonconstructability the ambiguity of a mapping $\tau^{(n)}: C(A,\infty) \to C(A,\infty)$ is not connected directly with nonconstructability in classical ***1–CA*** models. Being based on theorem ***50*** and certain other results [42,84], we can to receive a decision of the solvability problem of existence of one–to–one mapping $\tau^{(n)} : C(A) \to C(A)$ for a classical ***1–CA*** model. In this direction we have received a lot of interesting results that are expressed by the following basic theorem having a number of useful enough appendices [24,40-43,82-87,102,106].

Theorem 51*. Problems of determition of mutual one–valuedness of the parallel mappings for the case of classical 1–CA models, namely:*

$\tau^{(n)}: C(A,\phi) \to C(A,\phi)$, $\tau^{(n)}: C(A,\infty) \to C(A,\infty)$ *and* $\tau^{(n)}: C(A) \to C(A)$ *are algorithmically solvable. The existence problem of reverse parallel mapping* τ_n^{-1} *for a parallel global mapping* $\tau^{(n)}: C(A) \to C(A)$ *generated by a classical 1–CA model is algorithmically solvable too.*

In addition, the second part of this theorem represents nonconstructive proof of solvability of the existence problem of reverse global function τ_n^{-1} for ***GTF*** $\tau^{(n)}$ of an arbitrary classical ***1–CA*** model. Hence, it would be rather interesting to receive also constructive decision of the problem what will allow to receive the reverse local function $\sigma^{(n)}$ on basis of the concrete kind of ***LTF*** $\sigma^{(n)}$ of a classical ***CA*** model under the condition of its existence. On the other hand, the solvability problem of mutual one–valuedness of global mappings $\tau^{(n)}$***: C(A,d)*** $\rightarrow$ ***C(A,d)*** for general case of classical ***d–CA (d*** $\geq$ ***2)*** models the following principal theorem decides [7, 24,40-43,82-87,102,106,278,286].

Theorem 52. Problems of definition of mutual one–valuedness of global mappings $\tau^{(n)}$ ***: C(A,d,*** ϕ***)*** $\rightarrow$ ***C(A,d,*** ϕ***)*** ***and*** $\tau^{(n)}$***: C(A,d)*** $\rightarrow$ ***C(A,d)*** ***for general case of the classical d–CA (d*** $\geq$ ***2) models are unsolvable.***

Proof of the given assertion directly follows of result of the theorem ***52*** and consequences ensuing from it which determine unsolvability of the existence problem of the ***NCF (NCF-3)*** nonconstructability for a classical ***d–CA (d*** $\geq$***2)*** model. From the theorem ***52*** also follows, that ***the definition problem of existence of the reverse parallel mappings*** τ_n^{-1} ***for parallel mappings*** $\tau^{(n)}$***:C(A,d)*** $\rightarrow$ ***C(A,d) induced by classical d–CA (d*** $\geq$ ***2) models is unsolvable.*** Using results on the ***NCF*** nonconstructability along with property of compactness of the topological product, ***D. Richardson*** has proved [134] that a parallel mapping $\tau^{(n)}$ ***: C(A,d)*** $\rightarrow$ ***C(A,d)*** is biunique only if a mapping τ_n^{-1}***: C(A,d)*** $\rightarrow$ ***C(A,d)*** is being determined by global function of a certain ***d–CA (d*** $\geq$ ***1)*** model. In this respect this result plays a rather important part in theoretical research of dynamical properties of ***d–CA (d*** $\geq$ ***1)*** models**;** on the basis of it and theorem ***51,*** in particular, the above assertion easily follows.

It is known, that an arbitrary classical ***d–CA (d*** $\geq$ ***1)*** model possesses the ***NCF*** *(and, perhaps,* ***NCF-3)*** nonconstructability if and only if for the model there are configurations ***c*** $\in$ ***C(A,d,*** ϕ***)*** that have not predecessors c^{-1} from the set ***C(A,d)=C(A,d,*** ϕ***)*** $\cup$ ***C(A,d,*** ∞***)*** [82]. So, this question directly adjoins to reversibility question of dynamics of ***d–CA.*** In context of result of the solution of this problem it is necessary to note the important result of ***T. Yaku*** [141,142] consisting in that that the determination problem of the predecessors c^{*-1} for a configuration ***c*** $\in$ ***C(A,d)*** for a ***d-CA (d*** $\geq$***2)*** model is algorithmically unsolvable**.** The given result is very weighty argument that the determination problem of mutual one-valuedness of the global

mappings $\tau^{(n)}: C(A,d) \rightarrow C(A,d)$ should be algorithmically unsolvable for general case of the classical ***d–CA*** *(**d≥2**)* models. While for classical ***1–CA*** models the definition problem of direct predecessors c^{-1} for each finite configuration $c \in C(A,d,\phi)$ is algorithmically solvable and a constructive proof of it is based on the following rather important result, presenting independent interest. The question has a certain sense at working-out of various reversible computational and physical ***CA*** models [7,24,82,106].

Theorem 53. The definition problem of predecessors c^{-1} and their types for a finite configuration $c \in C(A,d,\phi)$ in a classical d–CA model for the case $d = 1$ is solvable whereas for the case $d \geq 2$ is unsolvable.

Proof of this theorem can be found in [82-87,106]**;** thus, our proof of the problem is based on nonsolvability of the known «*domino*» problem. But already for the case of classical ***1–CA*** models the definition problem of «*related*» attitudes for **2** arbitrary configurations {***c, c****} is unsolvable, i.e. the following question is unsolvable, namely**: *Whether has a place the relation $(\forall c,c^* \in C(A,1,\phi))(c^* \in <c>[\tau^{(n)}])$ for a classical 1–CA model?***

It is well–known, that resolvability problems are being investigated by means of constructive and nonconstructive methods. Whereas from the standpoint of the applied aspects of ***CA*** models the greatest interest just constructive methods present. Our research concerning the criteria and resolvability of a number of aspects of the nonconstructability problem in general case of the classical ***d–CA*** *(**d≥1**)* models confirm a rather high level of complexity of this problematics. A number of separate and most special results in this direction can be found, in particular, in [7,24,106].

Of the presented results relative to *solvability* of the nonconstructability problem for the classical ***CA*** models it is simple to make sure, that its solution essentially depends on dimensionality of the models. So, if in the ***1–***dimensional case all basic aspects of the nonconstructability are solvable with existence of constructive decision algorithms whereas in the **2–**dimensional case some questions of existence of the basic aspects of the nonconstructability are unsolvable. In this connexion the problem of influence of values of *key* parameters of the ***CA*** models *(dimensionality, local transition function, neighbourhood index, states alphabet)* on research of their dynamic properties presents indubitable interest. A number of the solvability problems of more subtle properties of dynamics of classical ***CA*** models on the basis of the theory of the creative and productive sets of finite configurations has been considered in the works [7,14,22,24,25, 28,30,32,33,40-43,63,69,70,71,79,82-84,102,106,118,137,140,278,286].

Meanwhile, the acuteness of algorithmic unsolvability of the problem of the ***NCF*** and ***NCF–3*** nonconstructability for general case of ***CA*** models is reduced also by that fact, with growth of cardinality of an alphabet ***A*** and/or size of the neighbourhood template the share of the ***CA*** models possessing ***NCF*** *(*perhaps**, NCF-3)** greatly swiftly aspires to ***1*** *(theorem* ***4****);* hence, «*almost all*» complex enough ***CA*** models will be possess the ***NCF*** nonconstructability. To some respects a similar picture takes place also for the case of the ***NCF-2*** and ***NCF-1*** nonconstructability concerning the solvability problem of their existence in case of classical ***d–CA*** *(**d*** ≥ **2)** as a whole [5,8,9,24-28,40-43,82-87,102,106].

Results presented in the section solve as a whole the solvability problem of existence for classical ***CA*** models of nonconstructible configurations of various types whereas with separate questions of this problems it is possible to familiarize more thoroughly in a lot of the works containing additional primary sources [7,9,14,22,24,25,28,30,32,33,40-43,63,69,70,71, 79,82-84,102,106,118,137,140]. The latest decades, the special attention is given to the questions relate to reversibility of ***CA*** models, in quality of some base of representation of spatially**–**distributed dynamic systems which, in turn, is closely linked to the ***NCF*** nonconstructability problem [7,75,106]. In addition, a little bit in more details questions of dynamical reversibility of classical ***CA*** models we have considered in section **2.5.**

We consider the further research on the nonconstructability problem in the classical ***CA*** models are being represented rather interesting by two principal causes, namely**:**

(1) the nonconstructability – one of fundamental concepts in research of dynamics of classical and of some other types of CA models;

(2) the base results received on the nonconstructability problem allow not only to detail this fundamental concept, but also to create a rather effective apparatus of investigation of dynamics of CA models of both classic types and of some other important enough types.

On given place a discussion of the questions concerning the property of nonconstructability for classical ***CA*** models is finished**;** meantime, from this directly follows, excepting a few fundamental questions, the given problematics by present time has received full enough resolution. But its fundamentality for mathematical ***CA*** problematics along with a lot of the open questions connected to it continue to attract attention to it a lot of researchers [7]. Te results presented in chapter **2** solve as a whole the nonconstructability problem in classical ***d–CA*** models. The more special results in this direction are enough numerous and allow to successfully

research the nonconstructability problem in detail. Along with that the results in this direction allow to form rather useful methods of research of dynamics of classical ***d–CA (d ≥ 1)*** models. A lot of the similar results are discussed below**;** many interesting enough results in this direction can be found in the above references and presented in [7,24,102,106].

This problematics composes a rather essential part of the general theory of dynamic properties of classical ***d–CA (d≥1)*** models therefore we paid such close attention to it. Furthermore, this problematics play a rather essential part in formation of general apparatus of researches in ***d–CA.***

Once more it is necessary to note that the results that are presented here bear descriptive character inasmuch as their proofs are lengthy enough and too technical to be included in this book. The interested reader has opportunity to familiarize with them in the numerous references cited in appropriate places. In addition, in order to maintain a self**–**contained presentation the definitions of the main concepts are given excepting a minimum of simplest mathematical concepts. Such approach allows to get acquainted conceptually here with this material without distraction, at times, onto voluminous proofs.

Chapter 3. Extremal constructive opportunities of the classical cellular automata

It is known that any *Turing machine (**TM**)* can be modeled by means of a classical ***1–CA*** model**,** proving the ability of universal computability for ***1–CA*****,** i.e. ability to calculate each recursive function or to realize each computational algorithm, or information processing [24]. However this universality generally speaking needs the use of some additional states for an elementary automaton of such model. Namely, for calculation of a certain function defined in a finite alphabet ***A*** the ***CA*** model can claim some expansion of alphabet ***A*****,** for example, only upon one symbol. We showed that such minimal extension is quite sufficient for possibility of calculation by means of a ***CA*** model of an arbitrary recursive function, including generation of an arbitrary finite configuration from the given initial configuration.

Apparently, this opportunity is inherent any complex enough system, i.e. within internal axiomatics of the system the solution of all problems inherent in it is impossible**;** for opportunity of decision of non**–**solvable problems is demanded an extension of axiomatics of system *(specifically of states alphabet of a system)*. Formally this axiom has rigorous proofs for a lot of formal theories.

Axiomatics of classical ***CA*** models is defined by their key parameters**:** a dimensionality ***d*** of homogeneous space Z^d**;** a states alphabet ***A*** of each elementary automaton, neighbourhood index ***X*****,** and a local transition function $\sigma^{(n)}$**.** Within of this base axiomatics the question of *constructive* opportunities of classical ***CA*** models presents special interest, namely**:** ***As far as powerful opportunities of the classical CA models (within of their base axiomatics) concerning the generation by them of the finite configurations?*** On the assumption of own interests and tastes, many researchers differently define the maximal generative opportunities of ***CA*** models within their base axiomatics.

Meanwhile, for today we have no any unique concept of the maximal generative opportunities of classical ***CA*** models, and it has appreciably subjective character [24-28,82-87,106]. In particular, as contrasted to the nonconstructability the essential enough interest presents definition of the general properties reflecting the maximal constructive means of the ***CA*** models concerning the generating by them of finite configurations. We shall consider two most known approaches**:** on the basis of *universal* and *self**–**reproducing* finite configurations [7,24-28,40-43,82-87,102,106].

3.1. Universal finite configurations in the classical cellular automata

In the well–known monograph ***S. Ulam*** has formulated one interesting problem about existence of a simple universal matrix system [143,144]. Its positive solution would give an example of simple generating formal system which could be investigated quite effectively by means of well–known mathematical methods. In view of the problem we introduce the necessary concepts and definitions used further.

Definition 16*. A square matrix* $U(n, a)$ *of order* n *with elements of a set* $A=\{0, 1, ..., a-1\}$ *is named the universal matrix concerning the class of all matrixes of order* $m < n$ *if for an arbitrary matrix* $B(k, a)$ $(k \le m)$ *there is such integer* $j>0$ *that matrix* B *will be principal minor of matrix* $U^j(n,a)$.

In terms of this definition the following theorem resolves the existence problem of the universal self–reproducing matrix system [5,9,24,82-87].

Theorem 54*. There is such integer* $n_o > 0$ *that universal matrixes* $U(n, a)$ *cannot exist for arbitrary integers* $n \ge n_o$ *and* $a \ge 2$.

From result of the theorem follows that the universal generating matrix systems of enough high order not exist. While for infinite matrixes the given question still remains open, i.e. in the initial posing of ***S. Ulam*** the existence problem of the *universal* reproducing matrix system still waits for own decision. Moreover, a lot of existence questions of such matrix systems over the field A also remain open. In terms of applied interest, the work in this direction will represent significant interest also for the theory of infinite matrixes and, above all, by the used apparatus which is developed still enough faintly.

As an interesting applied aspect of this problem it is possible to specify, in particular, the use of classical ***d–CA*** for modelling of logic deductive systems in pure mathematics. In this case the configurations from the set $C(A,d,\phi)$ are associated with offers of logic calculus, while the initial configurations of ***CA*** model with axioms and ***GTF*** with derivation rules of a system. Then a sequence of global transition function applied to an initial configuration *(axiom)*, represents a proof *(conclusion)* in the given deductive model. The problems of deducibility and completeness are basic in the such models. These both problems are directly linked with the existence problems for classical ***d–CA*** $(d\ge1)$ models of ***NCF*** and ***UCF*** *(universal finite configurations)* accordingly. Using of classical ***d–CA*** $(d\ge1)$

for modelling of developing systems of cellular nature can be noted as the second applied aspect of the existence problem of the ***UCF.***

The existence ***UCF*** problem for classical ***CA*** models formulated yet by ***S. Ulam*** for the case of regular lattices is quite closely connected to the *completeness* problem of ***H. Yamada*** and ***S. Amoroso*** for the case of the polygenic ***CA*** models [7,24,66-68,106]. This problem can be formulated as follows**:** ***Whether exists a finite configuration or a finite set of such configurations for the given classical d–CA (d ≥ 1) model of which the set C(A,d,ϕ) can be generated by means of global transition function*** $\tau^{(n)}$ ***of the model?*** In other words the question is reduced to permissibility of the following determinative relation $\cup_k<c_k>[\tau^{(n)}] = C(A, d, \phi)$ $(k = 1 .. p)$. According to what has been said, the finite configurations $c_k \in C(A, d, \phi)$ that satisfy this condition are named the ***universal*** finite configurations *(they will be denoted as* ***UCF****)*.

For the case of finite ***CA*** models the existence ***UCF*** problem has positive solution, namely**:** ***There are finite d–CA (d≥1) models which have one or all configurations as the UCF.*** So, a lot of examples of such models can be found in [1,5,24,82-87,106]. But absolutely other picture takes place in the case of research of infinite classical ***CA*** models [7,13,24-28,40-43,106]. Here we need an essential definition.

<u>Definition 16</u>. A set of configurations $c_k \in C(A,d,\phi)$ ***composes for global transition function*** $\tau^{(n)}$ ***of a classical d–CA (d≥1) model a set of the UCF if the following determinative relation*** $\cup_k<c_k>[\tau^{(n)}] = C(A,d,\phi)$ $(k = 1 .. p)$ ***takes place.***

In certain works the question relative to so**-**called «*minimal*» ***CA*** models *(for instance, in the anglo-lingual literature in* ***S. Wolfram*** *suggestion in* [23]*)* defined as follows**:** ***Starting with a finite configuration j, a minimal CA is one which provides generating from j of the sequence whose elements in the aggregate will contain all finite block configurations*** was being enough actively discussed**;** i.e. in the above**-**mentioned terminology for such classical models the following relation should be carried out**:**

$$(\exists c_o \in C(A,d,\phi))(\forall c^* \in C(A,d,\phi))(\exists t \geq 1)(c^* \subseteq c_o \tau^{(n)t}) \qquad (6)$$

Firstly in our opinion the term «*minimal*» insufficiently correctly reflects essence of the question since it, above all, concerns complexity of a ***CA*** model itself that as a rule is defined by such parameters as *cardinality* of a states alphabet of an elementary automaton**,** *dimensionality* and also *size of neighbourhood template* of a $d\text{-}CA \equiv <Z^d, A, \tau^{(n)}, X>$**,** i.e. value $SL = d*n*\#(A)$

where **#(A) –** cardinality of an alphabet ***A***. At that, many considerations about possibility of existence of similar ***CA*** models has been suggested**;** concrete examples of ***CA*** models of similar type that are based only on empirical results were presented. However, as far as we know, a strict result meanwhile is absent. Below, the given question will be a little bit detailed. We shall consider the given problem in a little bit other posing**:** ***Whether a CA model can generate all set of finite configurations, from some initial finite configuration?*** Or else**,** in the above terminology for such classical ***CA*** models the following relation should take place**:**

$$(\exists c_o \in C(A,d,\phi)) \quad \left(\bigcup_{j=0}^{\infty} \left\{ c_o, c_{j+1} = c_j \tau^{(n)} \right\} = C(A,d,\phi) \right) \qquad (7)$$

A rather simple proof of impossibility of the relation *(7)***,** using results on the nonconstructability in the classical models for the case of classical ***d–CA (d≥1)*** models can be found, for example, in [7,9,82-87]. Moreover, using results concerning the ***NCF*** and ***NCF–1*** nonconstructability**,** it is possible to show [8,40-43]**,** that the given problem even in more general formulating has a negative solution for classical ***CA*** models**;** about it the following basic result having a number of rather interesting appendices along with many theoretical aspects testifies [106]. The result is enough widely used in many considerations.

<u>Theorem 55</u>. For an arbitrary classical d–CA (d ≥ 1) model a finite set of universal finite configurations is inadmissible.

A simple enough proof basing on a lot of well**–**known properties of the ***NCF*** and ***NCF–1*** nonconstructability for classical ***d–CA (d≥1)*** models of this theorem can be found, for example, in [24]. In addition, the method used at proof of the theorem and basing on the considered properties of the ***NCF*** and ***NCF–1*** nonconstructability can be useful and in a number of other cases. The short sketch of the proof of the foregoing statements is presented below. Similar approaches are quite naturally use in many other problems studying the dynamical properties of ***CA*** models.

Let $<c\tau^{(n)}>$ **–** a sequence of finite configurations generated by means of a global transition function $\tau^{(n)}$ with a ***n*** size of neighbourhood template from a finite initial configuration $c \in C(A,d,\phi)$**,** i.e.

$$<c\tau^{(n)}> \equiv c \rightarrow c\tau^{(n)1} \rightarrow c\tau^{(n)2} \rightarrow c\tau^{(n)3} \rightarrow c\tau^{(n)4} \rightarrow \ldots$$

Suppose, that the set $C(A,d,\phi)$ of all finite ***d–***dimensional configurations determined in a finite ***A*** alphabet can be generated from a configuration $c \in C(A,d,\phi)$ by means of a global transition function $\tau^{(n)}$**.** Obviously, that

such global function $\tau^{(n)}$ will not possess the ***NCF*** nonconstructability. But then, a predecessor c^{-1}, i.e. a configuration such that $c^{-1}\tau^{(n)} = c$, will only from the set $C(A,d,\infty)$ of all infinite ***d–***dimensional configurations, otherwise the c^{-1} configuration would belong to the sequence $<c\tau^{(n)}>$, making it periodical and contradicting the assumption. Thus, the initial c configuration of the sequence $<c\tau^{(n)}>$ must be ***NCF–1,*** possessing the predecessors only from the set $C(A,d,\infty)$**.** Meanwhile, it was shown [41] that a set of finite configurations of type ***NCF–1*** *(if they there are)* for an arbitrary global transition function $\tau^{(n)}$ is infinite, proving impossibility of the existence of *universal* finite configurations for classical ***CA*** models.

The following considerations enable us to generalize the above proof. Now we will assume that there is a finite set $G = \{c^1,c^2,c^3,\ldots,c^p\}$ of finite configurations c^j $(j=1..p)$, given in a certain certain A alphabet, such that on the basis of global transition $\tau^{(n)}$ function of a classical ***CA*** model the full set $C(A,d,\phi)$ can be generated from the set ***G,*** i.e.

$$<c^1\tau^{(n)}> \equiv c^1 \rightarrow c^1\tau(n)1 \rightarrow c^1\tau(n)2 \rightarrow c^1\tau(n)3 \rightarrow c^1\tau(n)4 \rightarrow \ldots$$
$$<c^2\tau^{(n)}> \equiv c^2 \rightarrow c^2\tau(n)1 \rightarrow c^2\tau(n)2 \rightarrow c^2\tau(n)3 \rightarrow c^2\tau(n)4 \rightarrow \ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (gs)$$
$$<c^p\tau^{(n)}> \equiv c^p \rightarrow c^p\tau(n)1 \rightarrow c^p\tau(n)2 \rightarrow c^p\tau(n)3 \rightarrow c^p\tau(n)4 \rightarrow \ldots$$
$$(\forall i \neq j)(c^i \neq c^j), \qquad \cup_j <c^j\tau^{(n)}> \equiv C(A,d,\phi);\ j=1..p$$

But then there has to be the minimal set G of such finite configurations *(gs)*. Therefore, $(\forall i \neq j)(c^i \notin <c^j\tau^{(n)}> \ \&\ c^j \notin <c^i\tau^{(n)}>)$ but then for such ***CA*** model a certain set of finite configurations of the ***NCF-1*** type will be exist whose number is infinite. What proves the above thesis.

Therefore, even essentially stronger result expressed by the following theorem, that represents and independent interest, exists under certain conditions [24,40-43,82-87,102,106].

<u>Theorem 56</u>. If an arbitrary classical d–CA (d ≥ 1) model possesses the NCF nonconstructability in the presence for the model of a set W of the NCF–1 configurations then for the model there is not a finite set of such configurations $c_g \in C(A,d,\phi)$ $(g = 1 .. p)$***, that the following determinative relation takes place, namely:***

$$\bigcup_g < c_g > \left[\tau^{(n)}\right] = C(A,d,\phi) \setminus W; \qquad c_g \in C(A,d,\phi) \quad (g = 1..p)$$

The theorem proof basing on the ***NCF*** and ***NCF–1*** nonconstructability along with some other rather simple prerequisites is rather transparent and can be found in our works [24,82-87,102,106].

Moreover, from the given result directly follows, what in certain cases a narrowing of the set $C(A,d,\phi)$ of all finite configurations up to the set of only constructive configurations, which it is necessary to generate, does not lead to the positive decision of the existence problem of the ***UCF*** for classical ***CA*** models. In addition, on the basis of one interesting enough algebraic method with application of results on the nonconstructability it is possible to prove essentially more general and strong result [40-43], which gives answers to certain questions, raised in our previous works, and also forms a rather essential part of the apparatus of investigation of dynamics of the classical ***CA*** models. In addition, an analogue of this result exists also and for the ***CAoS*** models [24,82-87,102,106].

Theorem 57. If a classical d–CA (d≥1) model with states alphabet A={0, 1, ..., t–1}, where t is a prime number, and global transition function $\tau^{(n)}$ possesses a set M of NCF and/or NCF-1 then there are not finite sets of global transition functions $\tau^{(n_j)}$ and configurations $c_j \in C(A,d,\phi)$ given in the same states alphabet A that the following 2 relations will take place, namely:

$$1)\bigcup_j < c_j > \left[\tau^{(n_j)}\right] = C(A,d,\phi) \setminus M; \quad 2)\bigcup_j < c_j > \left[\tau^{(n_j)}\right] = M \quad (d \le 1; j = 1..p)$$

At that, for alphabet A (t – a composite number) the formulation of the result only with the relation (2) takes place; this statement takes place in the case of prime number t and the NCF–2 nonconstructability also; i.e. transition function $\tau^{(n)}$ will possess a set M of the nonconstructible configurations such as NCF–2.

Along with others [42] from this result a rather interesting consequence follows: ***Two sets of configurations $C(A,d,\phi)\setminus G$*** and ***G*** *(**G –** a set of **NCF**, perhaps, and **NCF–3;** and also **NCF–1** or **NCF–2** in case of an alphabet **A** for a prime **t**)* ***can`t be generated by means of finite sets $c_j \in C(A,d,\phi)$ and GTF $\tau^{(n_j)}$, that are defined in the same alphabet, regardless to an initial GTF $\tau^{(n)}$ concerning which the nonconstructability of the specified types is considered*** [24,43,82-87,102,106].

Moreover, from result of theorem ***56*** follows, that classical ***d–CA*** *(**d ≥ 1**)* models are not finite–axiomatized parallel formal systems even under condition of elimination from the set $C(A, d, \phi)$ of the nonconstructible finite configurations. Thus, each set of nonconstructible configurations *(**NCF, NCF–1, NCF–2, NCF–3**)* concerning the completeness problem in classical structures ***d–CA*** *(**d ≥ 1**)* possesses the same immunity as the set

C(A, d, ϕ). This result allows to understand essentially more deeply the essence of the nonconstructability problem in classical ***d–CA*** models. In addition, it turned out that exception of each of allowable four types of the nonconstructability does not exert serious influence on the maximal constructive opportunities, for example, in a context of existence for the classical ***CA*** models of a set of ***UCF.*** Meanwhile, use of an expanded ***A*** alphabet of the classical ***d–CA*** *(**d ≥ 1**)* models allows to successfully and simply enough generate only from a single initial configuration the set ***C(A,d,ϕ)*** of all finite configurations [24-28,40-43,102,106].

Therefore, even exception out of the set ***C(A,d,ϕ)*** of the nonconstructible configurations of any types concerning the ***GTF*** preserves in it complex enough finite configurations for generating of them even by means of a finite set of ***GTF*** $\tau^{(n_j)}$. This is direct step on a way of determination of the *complexity* concept of finite configurations in the classical ***CA*** models. In addition, existence of a finite set of ***UCF*** and for certain separate global transition function $\tau^{(n)}$, and for their finite set $\tau^{(n_j)}$ is impossible, i.e. in the expanded understanding the existence problem of ***UCF*** for classical ***CA*** models also has the negative decision.

In the extended understanding the given problem directly precedes the complexity problem of finite configurations, whereas in initial posing speaks about impossibility of generation by means of global transition function of a classical ***CA*** model of all set of finite configurations from a finite set of initial finite configurations, establishing a top unattainable limit of its generative opportunities. In this connexion a number of more weakened approaches to definition of *maximal* generative opportunities of classical ***CA*** models is considered.

Among them an approach on the basis of universal *reproducibility* in the *Moore's* sense of finite configurations occupies especial place**;** it presents along with academic interest also a certain applied interest in particular in biology of development and in a lot of research problems concerning the questions of reliability and restoration in the various ***CA*** models of technical character, including questions of self**–**reproduction of robots. This question will be considered in the following section**;** here we shall consider the existence problem for the classical ***CA*** models of universal finite configurations at reduction in requirement to their definition**:**

Whether exist classical d–CA (d ≥ 1) models together such initial finite configurations for them that the determinative relation (7) takes place?

In such posing we require not generating from an initial ***c*** configuration

of a sequence containing all finite configurations, but only of entrances in the generated finite configurations of all finite block configurations, what is essentially weaker condition in view of the concept of classical ***CA*** models. So, it is simple to make sure that at existence of a ***d-HS (d≥1)*** with the specified property all configurations, generated from a certain initial finite configuration and that contain all finite block configurations will be in the *Moore* sense as self–reproducing configurations. While for such model the ***NCF*** *(possibly,* ***NCF-3****)* nonconstructability will be absent at presence of the ***NCF–1*** nonconstructability.

At the same time in this model any finite configuration w cannot have predecessors from sets $C(A,d,\phi)$ and $C(A,d,\infty)$ simultaneously, i.e. each finite configuration w will ***NCF–1*** or absolutely constructive [5,42]. The general solution of this problem is not known to us, however in [40-43] a number of arguments in favour of its positive decision for the case of a binary classical ***1–CA*** with neighbourhood index of Neumann-Moore is given. The analysis of such class of ***1–CA*** models for detection of the specified property of generating of all block configurations has shown [7,24,40-43], that among the investigated structures it is possible to allot only ***10*** models with discriminating numbers ***30,60,75,86,89,90,102,105, 106*** and ***120*** that possess the ***NCF–1*** nonconstructability in the absence of the ***NCF*** nonconstructability and that are quite suitable as candidates for the required property. However the subsequent detailed theoretical analysis and computer simulation have shown that binary ***1-CA*** models with discriminating numbers ***30,60,75,86,89,90,102*** and ***105*** cannot serve as models with the mentioned property of *universality*. Thus, the binary ***1–CA*** models with discriminating numbers ***106*** and ***120*** remain whose more detailed analysis have been executed.

For computer research of dynamic properties of classical ***1–CA*** models by us a lot of means in various programming systems has been created; many of them programmed in the ***Maple*** system is presented in [24,48]. So, as show numerous computer experiments carried out by means of a procedure ***HS_GS*** programmed in ***Maple*** these models not only possess the universal reproducibility in the *Moore* sense of finite configurations but also a finite configuration distinct from zero that will be generate a sequence of configurations which in the aggregate will contain all block configurations in the binary states alphabet.

The computer experiments on the basis of the above ***HS_GS*** procedure with binary classical ***1-CA*** models with discriminating numbers ***106*** and ***120*** together with certain theoretical considerations based on dynamical

properties of such models which are caused by presence in them of the ***NCF-1*** nonconstructability in the absence of ***NCF*** enable us to formulate interesting enough following assumption, namely [24,42,82,102,106]**:**

For binary classical 1–CA models with discriminating numbers 106 and 120, any finite configuration which is different from zero configuration and is self-reproducing configuration in the Moore sense will generate a certain sequence of configurations which in the aggregate will contain all finite block configurations given in the binary states alphabet.

In any case along with problem of self**-**reproducing finite configurations in the *Moore* sense which belong to a so**–**called problem of the maximal *generative* possibilities of the classical ***CA*** models**,** the various facilitated generative opportunities of the models represent a rather considerable interest. In this sense the detection of ***CA*** models supporting not only of self**-**reproduction of finite configurations but also simply of the set of all finite subconfigurations from a fixed finite initial configuration presents a quite certain interest. This question we researched as theoretically and experimentally [40-43]. In particular, with the help of experiments with a rather simple procedure **SubConf** [49] that had been programmed in the ***Mathematica*** system a lot of interesting results have been obtained.

The procedure call **SubConf[***Lt, g, p, v***]** on the basis of a local transition function that is defined by the list ***Lt*** of parallel substitutions along with a finite configuration ***g,*** number of the demanded copies ***p,*** and a finite configuration ***v*** returns two**-**element list whose the first element defines number of steps used by the ***1–CA(a,n)*** with local transition function ***Lt*** for generation ***p*** copies of configuration ***g*** from an initial configuration ***v*** whereas the second element defines number of really obtained copies of the ***g*** configuration. On this basis was revealed an interesting enough class of ***1–CA(a, n)*** models a few of which from an arbitrary finite initial binary configuration ***v*** generate the set of all binary finite configurations as subconfigurations. Moreover, a binary finite configuration ***g*** in such ***1–CA(a, n)*** model is self**-**reproducing in the *Moore* sense. Let ***W*** is a set of classical binary ***1–CA(2, n) (n ≥ 3)*** models, whose global transition $\tau^{(n)}$ functions are defined by local transition functions as follows**:**

$$\begin{cases} \sigma^{(n)}(0,0,\ldots,0)=0\,; & \sigma^{(n)}(1,0,\ldots,0)=1 \\ (\forall x_n \neq x_n^*)(\sigma^{(n)}(x_1,x_2,\ldots,x_{n-1},x_n) \neq \sigma^{(n)}(x_1,x_2,\ldots,x_{n-1},x_n^*))\,, \text{otherwise} \\ x_n^*, x_k \in \{0,1\}; \quad k=1..n \end{cases}$$

It is obviously, that number of such binary ***1–CA*** models equals $2^{2^{n-1}-2}$**.** Numerous experiments with the above procedure allow us to formulate the following interesting enough assumption, namely**:**

Among all linear CA models of the set W there are models which by the generative possibilities can be characterized as follows, namely:

1. The models for which an arbitrary finite binary configuration is self–reproducing in the Moore sense;

2. The models which generate from an arbitrary finite configuration the sequence of configurations containing any beforehand given number of copies of an arbitrary binary finite block configuration.

If the first component of the assumption has been proved a rather long ago whereas the correctness of the second has substantially presumable character basing on positive results of numerous experiments with the above procedure ***SubConf.*** Thus, in the case if the second component of this assumption is valid then we have models possessing the generative property that is more strong then self–reproduction in the *Moore* sense. At that, term «*assumption*» for the above assertion is caused by its final part because according to the following section its first part is true. So, in the case of detection of classical ***CA*** models of such type we receive a rather interesting class of the ***CA*** models possessing simultaneously the property of universal reproducibility in the *Moore* sense together with property of universal generative opportunities of block configurations from any finite initial configuration. In our opinion, investigation in this direction are being represented interesting enough. In addition, on this way we would receive possibility to better make clear the essence of the complexity of finite configurations in the ***CA*** axiomatics. This question is discussed in chapter ***4*** of the present book in detail.

3.2. Self–reproduction of finite configurations in the classical *CA* models

If the existence problem of ***UCF*** characterizes generative opportunities of the classical ***CA*** models concerning the set of finite configurations as a whole, then the universal *reproducibility* combines this possibility with a structural–dynamic aspect of generating of configurations sequences by means of the ***CA*** models. The essence of universal reproducibility is that any finite configuration in a classical ***CA*** model is self–reproducing configuration in the *Moore* sense. The above class of the ***CA*** models that possessing the property of universal reproducibility is interesting from many standpoints, and a lot of attention is devoted to this question of global dynamics of ***CA*** models. In this direction a lot of such researchers as ***A. Waksman, T. Vinograd, A.R. Smith, T. Yaku, S. Amorozo, S. Ulam, T. Ostrand, A. Fredkin, G. Cooper, P. Anderson, V.Z. Aladjev,*** etc. have received rather interesting results [7,24]. In addition, our results in this direction allow to establish a lot of rather interesting interrelationships between the nonconstructability and the universal reproducibility in the classical ***CA*** models, and also to solve a lot of especially mathematical problems. For the further we shall define the concept of «*reproducibility of finite block configurations in the Moore sense*»**.**

Definition 17*. We shall speak that a configuration* $c \in C(A,d,\phi)$ *contains p copies (accurate to shift and rotation of space* Z^d*) of a certain block configuration* c_b *if exists p nonoverlapping areas of the space* Z^d *each of which contains at least one copy of the configuration* c_b*. We shall speak that a configuration* $c \in C(A,d,\phi)$ *is self–reproducing configuration in the Moore sense in a classical d–CA (d≥1) model if for any previously given integer p>0 such integer t>0 exists that configuration* $c\tau^{(n)t}$ *will contain not less p copies of the initial configuration c.*

Along with attempts of formalization of self-reproducing process at the most abstract level the self-reproducing finite configurations in a certain measure can characterize *constructive* possibilities of classical ***CA*** models and in this attitude they are to a certain extent are reverse to the ***NCF–1, NCF, NCF-2*** and ***NCF-3*** nonconstructible configurations. It is shown, the classical ***d–CA*** models can possess the sets of rather complex finite self–reproducing configurations as at absence, and at presence in them of the nonconstructability of the above types. The discovered class ***L*** of linear classical models possessing the property of universal reproducibility of

the finite configurations is the most interesting in this respect [9,82-87].

A classical ***d–CA (d ≥ 1)*** model is called the linear model if its ***LTF*** $\sigma^{(n)}$ is determined by the formula of the following kind, namely**:**

$$\sigma^{(n)}(x_1,\ldots,x_n)=\sum_{1}^{n} b_k x_k \pmod a;\ \ a, b_k - primes;\ \ x_k, b_k \in A=\{0,1,\ldots,a-1\};\ \ b_k \neq 0\ \ (k=1..n)$$

At the certain assumptions thanks to works of the above researchers the following interesting enough result takes place [5,24-28,40-43,102,106].

Theorem 58. For a classical linear d–CA (d ≥ 1) model an arbitrary finite configuration c∈C(A,d,ϕ) is self-reproducing configuration in the Moore sense, i.e. such model possesses property of universal reproducibility of finite configurations.

Using the results from [24-28,82-87], we receive the possibility not only for essential simplification of proof of this result, but in a certain degree for characterization of the whole class of such ***CA*** models further named as a class of ***linear classical d–CA (d ≥ 1) models.*** Below, we believe that for models of this linear class whose alphabet ***A={0,1,...,a-1}*** satisfies the condition $a=p^k$ where ***p, k –*** prime numbers the following result there is.

Theorem 59. A classical d-CA (d≥1) model with local transition function $\sigma^{(n)}$***, determined by the following general formula***

$$\sigma^{(n)}(x_1,x_2,\ldots,x_n)=\Big(\sum_{k=1}^{n} b_k x_k\Big)^{a^m} \pmod a$$

there is at least a pair of different integers j, p ∈ 1..n such that $b_j, b_p \neq 0$*; m - a prime or m = 1*

possesses the property of universal reproducibility in the Moore sense of the finite configurations, where $a = p^t$ ***(p – a prime; t,*** b_j ***and m – primes or m = 1),*** $b_j, x_j \in A = \{0,1,\ldots,a-1\}$***; j = 1 .. n.***

Thus, the class ***L*** of linear ***CA*** models from standpoint of their dynamics can be characterized by the property of universal reproducibility of the finite configurations. So, in this connexion a rather interesting question arises**:** ***Whether there are other classes of the d–CA (d ≥ 1) models which possess the property of universal reproducibility and whereby they can be characterized formally?***

In general terms, to this question in particular can be approached in the following way by the example of ***1–***dimension classical ***CA*** models. We note that the consideration represented below is also transferable to the case of classical ***d-CA*** models *(**d**≥2)*. Let there be a classical ***1–***dimension ***CA*** model given in an alphabet ***A={0,1,2,…,a–1}*** with neighbourhood of ***n*** size and whose local transition function $\sigma^{(n)}$ is determined by parallel

substitutions as follows:

$$\begin{array}{cccccccc}
0...00\to 0 & ... & 0...10\to x_0^2 & ... & 1...00\to 1 & ... & (a-1)...(a-1)0\to x_0^t \\
0...01\to x_1^1 & ... & 0...11\to x_1^2 & ... & 1...10\to x_1^p & ... & (a-1)...(a-1)1\to x_1^t \\
0...02\to x_2^1 & ... & 0...12\to x_2^2 & ... & 1...10\to x_2^p & ... & (a-1)...(a-1)2\to x_2^t \\
0...03\to x_3^1 & ... & 0...13\to x_3^2 & ... & 1...10\to x_3^p & ... & (a-1)...(a-1)3\to x_3^t \\
..... & ... & ..... & ... & ..... & ... & ..... \\
0...0(a-1)\to x_{a-1}^1 & ... & 0...1(a-1)\to x_{a-1}^2 & ... & 1...1(a-1)\to x_{a-1}^p & ... & (a-1)...(a-1)(a-1)\to x_{a-1}^t
\end{array}$$

$$(\forall j)(m\neq n\to x_m^j\neq x_n^j)$$

$$\text{where } x_k^j\in A=\{0,1,...,a-1\},\ j=1..a^{n-1},\ k=0..(a-1),\ (\forall d\in A\setminus\{0\})(d0...00\to d)$$

It can be shown [24,41,82] that classical ***1–***dimensional ***CA*** models with local transition functions determined in this way don`t possess the ***NCF*** nonconstructability in the presence of the ***NCF-1*** nonconstructability for them. In addition, number such ***1–CA*** models is equal $\frac{(a-1)^{a-1}}{a^a}(a!)^{a^{n-1}}$. As it turned out, that among such large number of ***1–***dimensional classical models many models were found [24,82] which differ from linear ones, but that possessing the self**–**reproducibility in the *Moore* sense of finite configurations. Meanwhile, the general criterion for the existence in a classical ***CA*** model of the property of self**–**reproducibility in the *Moore* sense of finite configurations is not known to us today.

As it was shown, along with linear classical ***1-CA*** models that belong to the above ***1–CA*** models class, there are in the class the models different from them, also possessing the possibility of self**–**reproducibility in the *Moore* sense of finite configurations. In general, based on our numerous computer experiments and a number of theoretical results on both the nonconstructability in the classical ***d–CA (d≥1)*** models and the dynamic properties of the models, the following assumption can be formulated**:**

The presence of the NCF–1 nonconstructability in classical d–CA (d≥1) models in the absence of the NCF nonconstructibility in these models is necessary, but not sufficient condition for existence of self–reproducing finite configurations in the Moore sense for such d–CA models.

In a certain sense, this assumption can serve as a filter when checking ***CA*** models for their self**–**reproducibility property in the *Moore* sense of finite configurations.

Our theoretical investigations based on the certain dynamic properties of classical ***1-CA*** models related to the ***NCF–1*** nonconstructability along with numerous computer study allow to make an assumption relative to the validity of the following proposal, namely**:**

In a classical 1-dimension CA model with the alphabet A={0,1,...,a-1}, a neighborhood template and local transition function defined as follows

$$\sigma^{(n)}(x_1,\ldots,x_n)=\sum_{1}^{n}x_k \pmod a \quad x_k\in A=\{0,1,\ldots,a-1\};\ (k=1..n),\ \text{where}$$

$$a \text{ can be represented in the form } a=p_1^{t_1}p_2^{t_2}\ldots p_g^{t_g};\quad p_j \text{ and } t_j - \text{primes};\ (j=1..g)$$

a finite block configuration is self-reproducing in the Moore sense.

Note, the generation rate of the required number of block configuration copies in such linear ***CA*** models is substantially lower than if ***a -*** primes. So, on the basis of this proposal the following conclusion follows quite naturally**:** for any strictly linear classical ***1–***dimensional ***CA*** model with alphabet ***A= {0,1, ..., a–1}*** *(**a**∈{**2,3, ..., 10**})* and an arbitrary neighborhood template, any finite block configuration given in the ***A*** alphabet, is self**–**reproducing in the *Moore* sense. Particularly, for computer study of this problem the ***Srepr*** procedure programmed in ***Mathematica*** can be used. The procedure call **Srepr[*f, z, n, m*]** returns the ***2–***element list whose first element defines the actual number of copies of a block configuration ***z,*** while the second element **–** the number of steps of a ***1–CA*** model that is required for that. The first argument ***f –*** a pure function that defines the local transition function of the ***CA*** model**;** the ***n*** argument specifies the number of block configuration ***z,*** while ***m*** argument specifies a critical number of steps of the ***1–CA*** model, after which the **Srepr** procedure is terminated with printing of the corresponding message. The fragment presents the source code of the procedure with examples of its use [49].

```
In[4683]:= Srepr[f_ /; PureFuncQ[f], z_String, n_Integer, m_Integer] :=
             Module[{a = StringTrim[z, "0"...], b, c = "", d, p, g, q = {0},
           r = 1, t = Length[ArgsPureFunc[f]], j}, b = StringRepeat["0", t];
                 Label[svg]; d = b <> a <> b; p = StringLength[d];
                                  For[j = 1, j <= p – t, j++, c = c <>
                 ToString[f @@ Map[ToExpression, StringPart[d, j ;; j + t – 1]]]];
                                  If[Set[g, StringCount[c, z]] >= n, Return[{g, r – 1}],
AppendTo[q, If[g == 0, Nothing, g]]; a = StringTrim[c, "0"...]; c = ""; r++;
                 If[r > m, Print["Configuration is not selfreproducing for " <>
                                  ToString[r – 1] <> " steps; was obtained " <>
               ToString[Sort[q][[–1]]] <> " copies only"]; Return[r – 1], Goto[svg]]]]
In[4683]:= L = Mod[#1 + #2 + #3, 10] &; Srepr[L, RString[10, 8], 8, 5000]
Out[4684]= {8, 4199}
```

Based on the above procedure, the **SelfRepr** procedure is programmed in ***Mathematica*** that provides more opportunities for computer study of the self**–**reproduction of block configurations in classic ***1–CA*** models. A successful procedure call **SelfRepr[*f, A, q, n, m*]** returns the empty list {}, i.e. all block configurations in a classical ***1-CA*** model with an alphabet ***A*** = {***0,1,...,a-1***} *(**2**≤**a**≤**10**)*, a continuous neighborhood template of ***q*** size,

a local transition function specified by a pure function ***f*** are generated in number of ***n*** copies for no more than ***m*** steps of the ***CA*** model, otherwise the procedure call returns the list of all block configurations that do not satisfy this condition. Whereas the call **SelfRepr[*f, A, q, n, m, y*]** with the sixth optional ***y*** argument **–** *an arbitrary expression* **–** in case of detection of the 1^{st} configuration not satisfying the above conditions is terminated with printing of the corresponding message. The procedure allows a lot of modifications, extending the capabilities for research of the dynamics of classical ***1–CA*** models [24,82,49,102]. The fragment below represents source code of the procedure with examples of its application.

```
In[4711]:= SelfRepr[f_/; PureFuncQ[f], A_List, q_Integer, n_Integer,
 m_Integer, y___] := Module[{u, v, t=Length[ArgsPureFunc[f]], Sr, s={}},
                Sr[f, z_String, n, m] := Module[{a = StringTrim[z, "0" ...],
                    b, c = "", d, p, g, j, k, r = 1}, b = StringRepeat["0", t];
                  Label[svg]; d = b <> a <> b; p = StringLength[d];
                                  For[j = 1, j <= p – t, j++, c = c <>
                  ToString[f @@ Map[ToExpression, StringPart[d, j ;; j + t – 1]]]];
                  If[Set[g, StringCount[c, z]] >= n, Return[{g, r – 1}],
a = StringTrim[c, "0"...]; c = ""; r++; If[r > m, Return[r – 1], Goto[svg]]]];
                                  v = Complement[Map[StringJoin,
                  Map[ToString2, Tuples[A, q]]], {StringRepeat["0", q]}];
                Do[u = Sr[f, v[[k]], n, m]; If[ListQ[u], Null, AppendTo[s, v[[k]]];
                                  If[{y} == {}, Break[], Null]], {k, Length[v]}];
 If[s == {}, Print["All tested block configurations are self–reproducing"],
 Print["Configurations " <> ToString[s] <> " are not self–reproducing"]]]

In[4712]:= Ltf = Mod[#1 + #2 + #3, 10] &;
In[4713]:= SelfRepr[Ltf, {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, 6, 10, 10000, g]
        All tested block configurations are self–reproducing
In[4714]:= t = Mod[#1 + #2 + #3, 4] &; SelfRepr[t, {0, 1, 2, 3}, 4, 16, 160]
        Block configurations {0001} are not self–reproducing
```

Meanwhile, it should be kept in mind that because of not very effective algorithms of processing of cyclic expressions ***Maple*** is more preferable then ***Mathematica*** in the problems of computer research of ***CA*** models dynamics [24,44-47,50-52,102,106].

At that, the class of linear ***CA*** models possessing property of universal reproducibility of finite configurations till now was considered relative to the connected neighbourhood index $X = \{0,1,2,3, ..., n–1\}$. Meanwhile, this property is extended to the case of a general neighbourhood index $X=\{j_1,j_2, ..., j_p\}$ $(0 = j_1 < j_2 < ... < j_p = n–1)$, including disconnected indexes

provided that in them not less than two variables in each of dimensions are the leading variables, i.e. for ***1–CA*** models their local transition $\sigma^{(n)}$ functions have the following formula representation, namely**:**

$$\sigma^{(n)}(x_1,x_2,...,x_n)=\sum_{j=j_1}^{j=j_p} a_j x_j \pmod a;\quad a_j,x_j\in A=\{0,1,...,a-1\};\quad a=p^k \qquad (8)$$
$$(0=j_1<j_2<...<j_h=n-1;\ 2\le h\le n);\quad p,\ k,\ a_j-primes$$

where $A=\{0,1,...,a-1\}$ **–** an alphabet of the ***1–CA*** under condition $a=p^k$, p and k **–** primes, while neighbourhood template has size n [82-87]. It is simple to make sure that for a neighbourhood template of n size there is $2^{n-2}-1$ different disconnected neighbourhood templates. In addition, the minimal neighbourhood index has the kind $X=\{0,n-1\}$. For example, for the case of binary classical ***1–CA, 2*** linear models with neighbourhood indices $X_3=\{0,1,2\}$ and $X_2=\{0,2\}$ possess the opportunity of universal reproducibility in the *Moore* sense of finite configurations.

Among all classical binary ***1–CA*** models with maximal neighbourhood index $X=\{0,1,...,n-1\}$ exists 2^n-n-1 linear classical models that possess the property of universal reproducibility in the *Moore* sense of the finite configurations. Hence, in this context quite pertinently to consider in a sense the *generalized* class of the linear classical models characterized by such important dynamical property as universal reproducibility in the *Moore* sense of finite configurations whose ***LTF*** are defined by the above relations of theorem ***59*** and *(8)*. In [24,48,49,106] source codes of certain procedures along with numerous examples of their use for analysis of dynamics of self**–**reproducing configurations in classical ***1–CA*** models are represented. The set of linear local transition functions of the above form *(8)* forms a semigroup with respect to the composition operation, preserving the self**–**reproducibility property in the *Moore* sense.

Meantime, there are nonlinear classical ***1–CA*** models for which a finite configuration and its reverse one will be self**–**reproducing in the *Moore* sense. Such models possess the ***NCF–1*** nonconstructability without of the ***NCF*** nonconstructability and generating of copies of both *direct* and *reverse* finite configurations are done at the same time. Some interesting examples of classical ***1–CA*** models of such type with an $A=\{0,1,...,a-1\}$ alphabet of states of elementary automaton was obtained [41-43,102].

Simple examples are binary ***1-CA*** models with local transition functions $\sigma^{(3)}(x,y,z)=$ **If**$[x=0,\ y+z\ (mod\ 2),\ x+y+z+1\ (mod\ 2)]$ and $\sigma^{(3)}(x,y,z)=$ **If**$[x=0,\ z,\ z+1\ (mod\ 2)]$. The ***Mathematica*** procedure **Reproduction** [49]

allows experimentally to verify the aforesaid.

The procedure call **Reproduction[*x, z, n*]** on actual arguments: ***x*** **–** local transition function, given by a list of format {***"x1x2…xk"*** → ***"x`1",…***}, ***z*** **–** a tested finite configuration in string format and ***n*** **–** a desired number of its copies, returns the list, whose the first element defines number of the obtained copies of *z* configuration while the second element defines number of steps of ***1–CA*** model, required for that. Below is a result of the procedure use.

In[1942]**:= x = {"000" → "0", "001" → "1", "010" → "1", "011" → "0", "100" → "0", "101" → "1", "110" → "1", "111" → "0"};**
In[1943]**:= k=34925674553; j=20;** {**Reproduction[x, IntegerString[k,2], j], Reproduction[x, StringReverse[IntegerString[k, 2]], j]**}
Out[1943]**=** {{32, 1984}, {32, 1984}}

It was succeeded to determine a group of classical ***1–CA*** models having local transition functions which satisfy the condition $\sigma^{(n)}(x1 \ldots xn) = 0$ if $x1=x2=\ldots=xn$ without the ***NCF*** that posess the self**–**reproducibility in the *Moore* sense of finite configurations and their inverse [41-43]. The ***1–CA*** models are different from *linear* models, for example with local function $\sigma^{(2)}$ = {"00" → "0", "01" → "2", "02" → "1", "10" → "1", "11" → "0", "12" → "2", "20" → "2", "21" → "1", "22" → "0"} along with other interesting models [82].

There are binary classical ***1-CA*** models that from a finite configuration ***c**** generate a sequence of configurations containing configurations of the form $c^*0^m c`$, where ***c`*** is a finite configuration which depends on ***c****, 0^m is the concatenation of ***m*** symbols ***"0"***, and ***m*** **–** an increasing integer sequence depending on ***c****.

During computer research a lot of rather interesting results concerning the self**–**reproducing finite configurations in the class of linear classical ***CA*** models has been received and with certain of them it is possible to familiarize in [24-28,40-48,106]. So, the results received in this direction allow us to speak, the property of universal or essential reproducibility in the *Moore* sense of finite configurations, apparently, is inherent also in linear models with an alphabet ***A*** **=** {***0,1, …, a–1***}, where ***a*** **–** a positive integer that can't be represented in the form of $a=p^k$, where ***p*** **–** a prime number and ***k*** **–** a positive integer. However, as against the above class of linear models the process of generating of required quantity of copies of initial finite configurations in the linear models of this type demands considerably larger quantity of steps of ***GTF*** $\tau^{(n)}$ on the assumption of

essential decrease of density of quantity of copies during generating. In addition, size of an initial finite configuration and its kind exert a rather essential influence on speed of generating. Thus, there are a lot of other interesting enough results in this direction which seem to us interesting enough for the further researches [24-28,40-48,82-87,102,106].

In particular, a number of software for experimental study of the linear classical models of dimensionalities ***1*** and ***2*** has been created**;** to a large extent such software have been focused on study of the *self-reproduction* phenomenon of finite configurations when cardinality of an alphabet of internal states of an elementary automaton of ***CA*** model is expressed by an arbitrary integer. So, the procedure ***Selfreprod,*** programmed in the ***Mathematica*** system [49], allows to obtain the number of iterations of a linear classical ***1–CA*** model that was required to generate ***p*** copies of an initial finite c configuration. In the case of a rather long run of the given procedure, it can be interrupted, by monitoring through special list the obtaining of the required number of copies of the initial ***c*** configuration.

Numerous experiments with ***SelfReprod*** procedure along with its more complex modifications allowed to research a lot of classical ***1–CA*** with different both alphabet ***A*** of internal states of elementary automata and neighbourhood template size ***m,*** allowing to formulate the following a rather interesting assumption, namely**:**

There is a rather wide class of 1–CA models with LTF $\sigma^{(n)}$ ***of the form***

$$\sigma^{(n)}(x_1,\dots,x_n)=\sum_{1}^{n}x_k \pmod p \quad x_k\in A=\{0,1,\dots,p-1\};\ (k=1..n),\ p-\text{an arbitrary integer}$$

where p is integers different from the earlier considered types of integers that possess the property of rather essential self–reproducibility in the Moore sense of the finite configurations.

Note, in many cases the above software allow to receive the structure of configurations containing the copies of self**–**reproducing configuration. Other interesting properties of the *generalized* class of linear classical ***CA*** models characterized by property of universal reproducibility of finite configurations in the *Moore* sense can be found in [82-87]. Meanwhile, it would be rather desirable to receive some determinative characteristics of the *generalized* linear classical ***CA*** models as a whole. In this direction quite definite interest represents the following basic result, used and in other purposes. In this respect we have received a general characteristic of the given class of models which is enough closely connected with the nonconstructability problem in the classical ***CA*** models [40-43,82-87].

Theorem 60. Existence of the NCF–1 nonconstructability in the absence of the NCF nonconstructability in a classical d–CA ($d \geq 1$) model is the necessary condition, but not sufficient for possession of such model of the property of universal reproducibility of finite configurations in the Moore sense.

This result represents a certain kind of testing *(a necessary condition)* for verification of classical ***CA*** models for possession by the property of the universal reproducibility and also illustrates a rather prominent aspect of interrelation between maximal constructing and nonconstructability in classical ***CA*** models. So, we obtain some kind of «***filter***» for selection of ***CA*** models for the purpose of candidates for possession of property of universal reproducibility according to the *Moore* sense. Thus, among ***CA*** models of such type it is necessary to search models with property of universal reproducibility as well as with property of high degree of reproducibility in the *Moore* sense of finite configurations.

Particularly, among the above *binary* classical ***1-CA*** models only models with discriminating numbers ***30, 45, 60, 75, 86, 89, 90, 101, 102, 105, 106, 120*** possess the ***NCF-1*** nonconstructability without ***NCF,*** yet only ***1-CA*** with numbers ***30, 45,60,75, 86, 89, 90, 101, 102, 105,106, 120*** possess the property of full *(universal)* or essential reproducibility of the finite configurations according to the following table ***3*** [24,82-87,102,106].

Table 3

	NCF	***NCF–1***	***Increase***	***Periodic***	***Self–reproduction***
30	**–**	**+**	**–**	**+**	***No***
45	**–**	**–**	**–**	**+**	***No***
60	**–**	**+**	**+**	**–**	***Yes***
75	**–**	**+**	**+**	**–**	***No***
86	**–**	**+**	**–**	**+**	***No***
89	**–**	**+**	**+**	**–**	***No***
90	**–**	**+**	**+**	**–**	***Yes***
101	**–**	**–**	**–**	**+**	***No***
102	**–**	**+**	**+**	**–**	***Yes***
105	**–**	**+**	**+**	**–**	***Yes***
106	**–**	**+**	**+**	**–**	***Yes***
120	**–**	**+**	**+**	**–**	***Yes***

Proof of the universal reproducibility in the *Moore* sense of models with numbers ***60, 90, 102*** and ***105*** is based on the fact, that they are the linear

classical ***1-CA*** models with connected and disconnected neighbourhood indexes**;** their ***LTF*** $\sigma^{(3)}$ are defined by the following formulas, namely**:**

$$\sigma^{(3)}_{60}(x,y,z)=x+y \pmod 2; \qquad \sigma^{(3)}_{90}(x,y,z)=x+z \pmod 2;\ x,y,z\in\{0,1\}$$
$$\sigma^{(3)}_{102}(x,y,z)=y+z \pmod 2; \qquad \sigma^{(3)}_{105}(x,y,z)=x+y+z \pmod 2$$

whereas the proof for models with numbers ***106*** and ***120*** is based on the kind of their ***LTF*** $\sigma^{(3)}$ determined by the following formulas**:**

$$\sigma^{(3)}_{106}(x,y,z)=\begin{cases} y+z \pmod 2, & \text{if } y=1 \\ x+y+z \pmod 2, & \text{otherwise}\end{cases}$$
$$\sigma^{(3)}_{120}(x,y,z)=\begin{cases} x+y \pmod 2, & \text{if } y=1 \\ x+y+z \pmod 2, & \text{otherwise}\end{cases}$$

or in equivalent form :

$$\sigma^{(3)}_{106}(x,y,z)=(1-y)x+y+z \pmod 2 \qquad \sigma^{(3)}_{120}(x,y,z)=x+y+(1-y)z \pmod 2$$

Thus, last two functions are different from linear functions whose ***LTF*** $\sigma^{(n)}$ are defined in the form *(8)*. Furthermore, the above ***CA*** models with discriminating numbers ***106*** and ***120*** possess not only the possibility of universal reproducibility of finite configurations in the *Moore* sense but, most probably, along with it also for them a finite configuration distinct from zero generates a sequence of configurations that in the aggregate contain all block configurations in the binary alphabet, i.e. it possess the property of universality relative to the binary block configurations. In addition, if for generation of n copies of a finite h configuration models with discriminating numbers ***106*** and ***120*** demand p and q steps, for the inverse configuration these models demand q and p steps accordingly.

It is easy to make sure that for a states alphabet $A=\{0,1,...,a-1\}$ the global transition functions $\tau^{(n)}$ $(n\geq 2)$ constitute a noncommutative subset ***T(a)*** concerning the operation of composition, i.e.

$$(\forall\tau^{(n)})(\forall\tau^{(p)})(\tau^{(n)}\tau^{(p)}\in T(a)\ \&\ (\exists\tau^{(n)},\tau^{(p)})(\tau^{(n)}\neq\tau^{(p)}\rightarrow\tau^{(n)}\tau^{(p)}\neq\tau^{(p)}\tau^{(n)}))$$

At that, the subset ***T(a)*** does not possess any finite system of generators. A rather detailed consideration of the decomposition problem of global transition functions in the classical ***CA*** models is presented below while here we use this approach for creation of *nonlinear* binary classical ***1-CA*** models possessing the property of universal reproducibility in the *Moore* sense. We illustrate the above approach on the basis of a rather simple example. Let $\tau^{(2)}$, $\tau^{(3)}_{106}$, $\tau^{(3)}_{120}$ – global transition functions whose local functions are determined as follows**:**

$$\sigma^{(2)}(x,y)=x+y \pmod 2;\quad (1)\,\tau^{(2)}\tau^{(3)}_{106},\ (2)\,\tau^{(2)}\tau^{(3)}_{120},\ (3)\,\tau^{(3)}_{106}\tau^{(2)},\ (4)\,\tau^{(3)}_{120}\tau^{(2)}$$
$$\sigma^{(3)}_{106}(x,y,z)=(1-y)x+y+z \pmod 2 \qquad \sigma^{(3)}_{120}(x,y,z)=x+y+(1-y)z \pmod 2$$

In the same place four compositions of global transition functions which were subjected to analysis are presented. It is easy to see that these four compositions present different nonlinear global transition functions $\tau^{(4)}$ according to the above numeration that are presented by the following local transition functions $\sigma^{(4)}$, namely:

$$\sigma_1^{(4)}(x,y,z,h)=\begin{cases}x+y+z+h+1 \pmod 2, & \text{if } \langle xyzh\rangle\in\{0010,0011,1100,1101\}\\ x+y+z+h \pmod 2, & \text{otherwise}\end{cases}$$

$$\sigma_2^{(4)}(x,y,z,h)=\begin{cases}x+y+z+h+1 \pmod 2, & \text{if } \langle xyzh\rangle\in\{0011,0100,1011,1100\}\\ x+y+z+h \pmod 2, & \text{otherwise}\end{cases}$$

$$\sigma_3^{(4)}(x,y,z,h)=\begin{cases}x+y+z+h \pmod 2, & \text{if } \langle xyzh\rangle\in\{0010,0011,1000,1001\}\\ z+h \pmod 2, & \text{otherwise}\end{cases}$$

$$\sigma_4^{(4)}(x,y,z,h)=\begin{cases}x+h \pmod 2, & \text{if } \langle xyzh\rangle\in\{0001,0100,1001,1100\}\\ x+y \pmod 2, & \text{otherwise}\end{cases}$$

Experimental-theoretical study of ***1***–dimension classical binary models, whose global transition functions $\tau^{(n)}$ $(n\geq 4)$ are formed by means of the above method allowed to formulate a rather interesting assertion:

For each integer n≥3 there are at least 4 nonlinear binary classical 1–CA with neighbourhood index X={0,1,...,n–1} which possess the property of universal reproducibility of finite configurations in the Moore sense; of them two binary classical models possess the universality property of finite block configurations.

In addition, if the first part of the assertion has strict theoretical basing then the second part is based on results of computer research of global transition functions $\tau_{106}^{(3)}\tau^{(2)}$, $\tau_{120}^{(3)}\tau^{(2)}$, i.e. has an especially experimental character. Meantime, the made numerous computer experiments with the above last two ***1–CA*** models allow to say about a rather high level of certainty of the second part of this assertion [24,82-87,102]. However, in this direction a theoretical confirmation would be very desirable.

In addition, for the above classical models with discriminating numbers *75* and *89* the following quite interesting regularity has been detected:

For any finite configuration c_o the 1-CA models generate configurations sequences $\langle c_o\rangle[\tau_{75}^{(3)}]$ and $\langle c_o\rangle[\tau_{89}^{(3)}]$ accordingly, each of which will contain configurations subsequences of the following kind, namely:

$$75:\ \left\{pr(c_o)\,0^{2t-|pr(c_o)|}\,c_o \,\middle|\, t=2^k;\ k=k(c_o)+j;\ j=0,1,2,\ldots\right\}$$

$$89:\ \left\{c_o\ 0^{2t-|sf(c_o)|}\,sf(c_o) \,\middle|\, t=2^k;\ k=k(c_o)+j;\ j=0,1,2,\ldots\right\}$$

where $k(c_o)$ is a positive integer depending on a finite c_o configuration, whereas $pr(c_o)$ and $sf(c_o)$ is a prefix and a suffix of the c_o configuration accordingly which depend on the configuration, and t is step number of the sequence generating.

Furthermore, for models with discriminating numbers ***60, 75, 89, 90, 102, 105, 106*** and ***120*** the relation $(\forall c \in C(A, d, \phi))(|c| < |c\tau^{(n)}|)$ takes place *(column «**Increase**» in table **3**)*, where $|S|$ is size of maximal diameter *(in **1–**dimensional case a length)* of a finite configuration ***S***, what according to theorem ***40*** provides these models with the ***NCF–1*** nonconstructability in the absence of the ***NCF*** nonconstructability and a generating of all set $C(A,d,\phi)$ from the ***NCF–1*** configurations only.

In addition for such ***CA*** models rather interesting regularities relative to quantities of copies of configurations generated by them depending on number of steps of generating have been discovered. So, for the binary ***1–CA*** models with discriminating numbers ***90, 102*** a rather interesting regularity has been found**:** ***For «almost all» finite configurations, if m copies of a configuration c* are generated by the model with number 90 during t steps, then the same number of this configuration is generated by the model with number 102 during 2*t–1 steps with the exception of configurations from the set*** $\{1^{2k-1} \mid k=1,2,3,4,...\}$.

Above we noted a lot of interesting enough properties of classical ***d–CA*** models with neighbourhood index ***X*** and states alphabet $A=\{0,1,...,a-1\}$, whose ***GTF*** $\tau^{(n)}$ satisfy the condition $(\forall c \in C(A,d,\phi))(|c\tau^{(n)}| > |c|)$, where $|c|$ is maximal diameter of a finite ***c*** configuration, i.e. models of such type produce the configurations sequences strictly increasing in the size from an arbitrary finite configuration ***w*** that is different from fully null configuration $c_o = \Lambda$. These ***CA*** models constitute a certain subclass *(let's denote as **GS-**class)* of all classical models of the same states alphabet and dimensionality. As it was noted earlier, models of this subclass possess at least the ***NCF–1*** or/and ***NCF*** nonconstructability.

On the other hand, for today namely among models of the ***GS–***class the models possessing the opportunity of universal reproducibility of finite configurations in the *Moore* sense have been found. In particular, well–known linear classical ***CA*** models also belong to the ***GS–***class. It is quite perhaps that one of tests of existence for classical ***d–CA*** $(d \geq 1)$ models of the property of universal reproducibility of finite configurations in the *Moore* sense can be formulated as follows, namely**:**

The d-CA models possessing the property of universal reproducibility in the Moore sense, it is expedient to search among classical models which in absence for them of the NCF nonconstructability satisfy the relation $(\forall c \in C(A,d,\phi))(|c|<|c\tau^{(n)}|)$ ***where*** $|j|$ ***is size of maximal diameter of a finite j configuration; thus, sought models belong to the above GS-class whose global transition function form a noncommutative subset GS in regard to the composition operation. These models possess the NCF–1 nonconstructability, and for them the set*** $C(A,d,\phi)$ ***can be generated by the NCF–1 configurations only.***

So, from the above ***8*** binary classical ***1–CA*** models satisfying the above test, i.e. ***CA*** models of the ***GS–***class, ***6*** structures possess the property of universal reproducibility of finite configurations in the *Moore* sense. So, the above test allows to reveal ***75%*** of classical binary ***1-CA*** models with neighbourhood index $X=\{0,1,2\}$. It is quite desirable to narrow the above test by means of exception out of the above ***GS–***class of ***CA*** models not having any prospects in this respect.

Having determined on the basis of experimental–theoretical research of classical ***CA*** models whose global functions $\tau^{(n)}$ satisfy the next relation $(\forall c \in C(A,d,\phi))(|c|<|c\tau^{(n)}|)$, where $|c|$ is size of maximal diameter of a finite configuration, a test for the purpose of possibility of possessing by a classical model of universal reproducibility of finite configurations in the *Moore* sense, we are hugely interested in obtaining of more concrete method for search of such models in the ***GS–***class. Inasmuch as the ***GS–***class forms a noncommutative subset of global transition functions $\tau^{(n)}$ concerning operation of composition the search of such models on the basis of this operation quite naturally arises. It was shown, composition of global transition functions $\tau^{(n)}=\tau^{(p)}\tau^{(m)}$ from ***GS–***class gives a global function $\tau^{(n)}$ that will possess the property of universal reproducibility of configurations in the *Moore* sense. But, an extension of this technique is allowable, namely: ***Composition of global transition functions*** $\tau^{(n)} = \tau^{(p)}\tau^{(h)}$ ***from GS–class when only one function*** $\{\tau^{(p)}|\tau^{(h)}\}$ ***possesses the property of universal reproducibility can give a new GTF function*** $\tau^{(n)}$ ***which also will possess the property of universal reproducibility of the finite configurations in the Moore sense.*** For example, ***6*** compositions in the form $\tau^{(3)}_{75}\tau^{(2)} \neq \tau^{(2)}\tau^{(3)}_{75}$, $\tau^{(3)}_{89}\tau^{(3)}_{105} \neq \tau^{(3)}_{105}\tau^{(3)}_{89}$ and $\tau^{(3)}_{75}\tau^{(3)}_{105} \neq \tau^{(3)}_{105}\tau^{(3)}_{75}$ can serve as an illustration where global transition functions $\tau^{(3)}_{75}$ and $\tau^{(3)}_{89}$ do not possess the property of *universal* reproducibility of finite configurations in the *Moore* sense, whereas $\tau^{(3)}_{105}$ possesses such property. Furthermore,

all compositions of such kind that have been examined have possessed by the property of universal reproducibility in the *Moore* sense.

Meanwhile, as a result of numerous computer experiments with global transition functions from the class ***GS***, we not succeeded in finding of a *pair* of global transition functions whose composition would possess the property of universal reproducibility in the *Moore* sense. So, most likely the following assertion takes place, namely**:**

Only compositions $\tau^{(n)} = \tau^{(p)}\tau^{(h)}$ ***of global transition functions from GS class, where at least one function*** $\{\tau^{(p)},\tau^{(h)}\}$ ***will possess the property of universal reproducibility in the Moore sense, can as a result give global transition functions*** $\tau^{(n)}$ ***that will possess the universal reproducibility in the Moore sense of finite configurations.***

Simple enough examples serve for illustration of the said, whereas with detailed experimental**–**theoretical aspect of the question the interested reader can familiarize in [24,82-87,102,106]**.**

Meanwhile, the result of theorem *58* allows to receive a decision of the next rather important question linked with constructive opportunities of the classical ***CA*** models, namely**:** ***Whether a classical CA model can double an arbitrary finite configuration defined in the same alphabet A?*** By dealing with complex enough questions of searching of appropriate mathematical apparatus which would be isomorphic to the developing biological organization, we have suggested in this quality the parallel τ_n**–**grammar and ***A*****–**algorithms**,** and have carried out their analysis in a context of biological interpretations [2,5,12,24-28,33,40-43,102,106]**.**

At that, the *Rozen* logical paradox connected to the phenomenon of the self**-**reproduction in formal developing systems was being investigated. The essence of this paradox consists in the condition, that the models of self**–**reproduction should include both the system of reproduction, and a certain specific environment. So, the well**–**known *Markov* algorithms defined in a certain alphabet ***A*** can`t double an arbitrary finite word in the same alphabet. It is quite reasonable to suppose that the result takes place both for τ_n**,** and ***A*****–**algorithms. Concerning the ***A*****–**algorithms we have shown [2,5] that the problem of doubling of words is being solved by introduction only of one additional symbol $b \notin A$**.** In addition, more detailed researches in this direction have allowed to formulate a rather important hypothesis representing indubitable theoretical interest from many standpoints.

Let ***P*** will be a production over a finite word ***s*** in some finite alphabet ***A*** that processes the given word into a new word ***s**** according to a certain algorithm using only alphabet ***A.*** Schema ***R*** represents some finite set of productions P_k *(**k = 1..n**)* together with algorithm of their application to any word ***s*** in the alphabet ***A.*** Then, we shall call ***F(R, A)*** a *formal system* in the alphabet ***A*** with scheme ***R.*** So, in this terminology our hypothesis assumes the following kind, namely.

Hypothesis 1. There is not a formal system F(R, A) which can double an arbitrary finite word in an arbitrary finite alphabet A.

This hypothesis remains open for today, and seems that its decision is a rather complex, meantime for the case of classical ***CA*** models the given problem has the negative solution, namely.

Theorem 61. There is not any classical d–CA (d ≥ 1) model with a states alphabet A and an arbitrary neighbourhood index which will double a d–dimensional finite configuration defined in the same alphabet A.

This result is immediate consequence of the more general result of the above theorem ***61*** that sets a certain kind of restriction on the universal reproducibility of finite configurations in classical ***CS*** models. Of results represented below it is possible to make sure in possibility of a solution of this problem for classical ***1–CA*** models with a finite alphabet ***A,*** that is expanded only onto one symbol. This proof is not constructive and the reader is recommended as an useful enough exercise to determine a classical ***1–CA*** model with an alphabet $A^* = A \cup \{\alpha\}$ *(**$\alpha \notin A$**)* which doubles an arbitrary finite configuration determined in the states alphabet ***A.***

It is well known that a number of properties intrinsic to global transition functions $\tau^{(n_j)}$ *(for some or for all)* is being inherited also by dint of some global transition function $\tau^{(n)}$ which can be represented in the form of a composition of the following general kind [24-28,40-43,82-87,102,106]**:**

$$\tau^{(n)} = \tau^{(n_1)}\tau^{(n_2)} \ldots \tau^{(n_j)} \ldots \tau^{(n_p)}; \quad n = \sum_{j=1}^{p} n_j - (p-1); \quad 2 \le n_j < n; \quad j = 1..p \qquad (9)$$

and vice versa. So, if a ***GTF*** $\tau^{(n)}$ possesses the ***NCF*** nonconstructability, then at least one from transition functions $\tau^{(n_j)}$ that compose the above decomposition *(**9**)* will possess by this property. With some other useful properties of similar type it is possible to familiarize in the book, while others can be found, for example, in [8,24,82]. Thus, we receive a rather natural mechanism of constructing the more complex global transition functions from less complex functions whose compositions will inherit

the necessary properties inherent to certain or all functions $\tau^{(n_j)}$, which form the composition *(9)*. In this context we shall consider the question of generating on the base of the above composition method of classical ***CA*** models possessing the property of universal reproducibility of the finite configurations in the *Moore* sense, that is essentially distinct from the example of classical linear ***CA*** models.

By way of illustration we shall represent a rather simple example. For a composition two binary classical ***1–CA*** models are chosen, namely: with neighbourhood index $X_2=\{0,1\}$ and ***LTF*** $\sigma^{(2)}$ determined by the formula $\sigma^{(2)}(x_o, x_1) = x_o+x_1 \ (mod\ 2)$ and with neighbourhood index $X_3=\{0,2\}$, and ***LTF*** $\sigma^{(3)}$ defined by the formula $\sigma^{(3)}(x_o,x_1,x_3)=x_o+x_2 \ (mod\ 2)$. It is simple to make sure that composition $\tau^{(4)} = \tau^{(3)}\tau^{(2)}$ whose ***GTF*** are defined by the above local σ functions as a result gives a new more complex global transition function $\tau^{(4)}$ whose local function $\sigma^{(4)}$ is defined by the next formula, namely: $\sigma^{(4)}(x_o, x_1, x_2, x_3) = x_o+x_1+x_2+x_3 \ (mod\ 2)$.

It is shown that ***1–CA*** model with this global function $\tau^{(4)}$ possesses the ***NCF–1*** nonconstructability in the absence of ***NCF.*** Wherein, this model possesses the property of universal reproducibility in the *Moore* sense. Results obtained in this direction allow to formulate a rather interesting assertion.

<u>Theorem 62</u>. Any classical d–CA (d≥1) model with global transition $\tau^{(n)}$ function given in an alphabet A = {0,1, ..., a–1} possesses the property of universal reproducibility of finite configurations in the Moore sense, if all global transition $\tau^{(n_j)}$ functions constituting a GTF composition $\tau^{(n)}$ (9) also possess the property of universal reproducibility.

In particular, at constructing ***1–***dimension ***GTF*** $\tau^{(n+m-1)}$ with property of universal reproducibility on the basis of composition of more simple ***GTF*** $\tau^{(n)}$ and $\tau^{(m)}$ which have the same property the relation $\tau^{(n+m-1)} = \tau^{(n)}\tau^{(m)} = \tau^{(m)}\tau^{(n)}$ seems a rather useful. The rather transparent relations *(without loss of generality for **1–**dimensional case)* lay in the basis of proof:

$$\sigma_1^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sum_{t=1}^{m}\left(\sum_{j=t}^{t+n-1} x_j \pmod a\right) \pmod a$$

$$\sigma_2^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sum_{t=1}^{n}\left(\sum_{j=t}^{t+m-1} x_j \pmod a\right) \pmod a$$

$$(\forall <x_1x_2...x_{n+m-1}>)(\sigma_1^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}) = \sigma_2^{(n+m-1)}(x_1, x_2, ..., x_{n+m-1}))$$

$$x_j \in A = \{0,1,...,a-1\}; \quad a - a \text{ positive integer}; \quad j = 1..n+m-1$$

In addition, global transition functions $\tau^{(n)}$ and $\tau^{(m)}$ can be either linear functions, or their compositions. At the same time it is possible to show that as a result of a composition of linear global functions $\tau^{(n)}$ and $\tau^{(m)}$ we again receive a linear ***GTF*** $\tau^{(n+m-1)}$, i.e. similar linear ***GTF*** constitute a closed subset relative to the composition operation of ***GTF*** in the form *(9)* whose elements will possess the universal *reproducibility* in the *Moore* sense of the finite configurations.

Meanwhile, among all global transition functions of the ***GS–***class there are functions that are distinct from linear functions, but which possess the universal reproducibility in the *Moore* sense. The following a rather interesting result having a lot of appendices takes place in the direction. Moreover, this result was generalized to the general case of this classical ***d–CA*** $(d \geq 1)$ models [24,40-43,82-87,102,106].

Theorem 63*. The set containing all linear classical CA models including classical models whose global transition* $\tau^{(n)}$ *functions are the result of a composition (9) of linear global transition functions will possess the universal reproducibility of finite configurations in the Moore sense; in addition, in composition (9) the global transition functions of shifts of configurations along the coordinates axes can be used additionally (in some cases results of composition can coincide with appropriate linear classical models with disconnected neighbourhood templates). Withal, there are also nonlinear classical CA models possessing the universal self–reproducibility of finite configurations in the Moore sense.*

The essence of this result **–** each composition of linear global transition functions which possess the property of universal reproducibility again gives a global transition function possessing the same property**;** so, the linear models *(8)* constitute a subset concerning composition operation *(9)* and the property of universal reproducibility**;** in addition, evidently *linearity* is not so obligatory requirement. The more special questions of dynamics of linear classical models are considered in a lot of works [7]. A lot of interesting enough properties of linear classical ***CA*** models has been studied by the Japanese mathematicians on the basis of algebraic methods using concepts of additive groups and commutative rings, and concepts and methods of dynamic systems. In particular, a class of the linear ***CA*** models was investigated by means of linear algebra[7,24,43].

By considering the universal reproducibility as a maximal constructive opportunity of the classical ***CA*** models on generating by them of finite configurations enough interestingly to discover not only new classes of

CA models with such opportunity, but also to investigate classes of ***CA*** models that possess such opportunity in a considerable extent.

So, we have determined a ***W*** class of classical ***CA*** models with connected neighbourhood template of size ***m*** along with non–binary alphabet for which each continuous finite configuration *(inside without states «**0**»)* of size ***j≥m*** is the self–reproducing in the *Moore* sense [84]. One more class ***LW*** of ***CA*** models whose local transition functions $\sigma^{(n)}$ are received on the basis of ***LTF*** of models of the classes ***L*** and ***W*** possess a rather high degree of reproducibility of finite configurations. Interesting classes of ***CA*** models possessing a rather high degree of reproducibility of finite configurations, it is possible to receive on the basis of compositions of finite number of global transition functions from the specified classes ***L*** and ***LG.*** Computer analysis has shown [82-87] that ***CA*** models of these types possess considerable enough reproducing opportunities of finite configurations of quite definite types.

One more class of the ***CA*** models possessing a rather high degree of the reproducibility of finite configurations, it is possible to determine on the basis of a special algebraical system introduced by us for a polynomial representation of ***a–***valued logic functions [82,145]. Research of a whole series of classes of *discrete parallel dynamic systems (**DPDS**),* including ***CA*** models, is very closely linked with researches of properties of their ***LTF*** $\sigma^{(n)}$, which represent ***a–***valued logic functions *(**a–VLF**)*. Among various approaches to study of similar functions the special place occupies the algebraic approach, when each ***a–VLF*** can be represented by a polynom of maximal degree ***n(a–1)*** over a field ***A*** modulo ***a*** and vice versa where ***a–VLF*** is a mapping $R^{(n)}: A^n \rightarrow A$. Meanwhile, in the case of composite number ***a*** far not each ***a-VLF*** can be presented in such polynomial form, or rather «*almost all*» functions have no such polynomial representation. Since alphabet ***A*** in a classical ***CA*** model can be arbitrary, the problem of extension of algebraic method of research of ***LTF*** $\sigma^{(n)}$ to the general case of the alphabet ***A*** arises. In this connexion arises an interesting and important from many standpoints a question**:** ***Whether it is possible to define an algebraic system that would admit polynomial presentation of each a–VLF in alphabet A for composite integer a analogously to the case of prime a?***

With this purpose we have defined an algebraical system *(**AS**)* in which «*almost all*» ***a–VLF*** have polynomial presentation for case of composite integer ***a*** [82,145]. The offered ***AS*** is defined as follows. A finite alphabet $A_a = \{0,1,2,\ldots,a-1\}$ of the system is being chosen, and on it usual binary

operation of addition modulo a is defined. At the same time, on A_a the binary operation of **#–***product* is defined according to the multiplication table of the following kind *(Table* **4***)*.

Table 4 (#–multiplication table)

#	0	1	2	3	4	5	.....	a–6	a–5	a–4	a–3	a–2	a–1
0	0	0	0	0	0	0	….	0	0	0	0	0	0
1	0	1	2	3	4	5	.....	a–4	a–3	a–2	a–1	0	a–1
2	0	2	3	4	5	6	.....	a–3	a–2	a–1	0	a–1	1
3	0	3	4	5	6	7	.....	a–2	a–1	0	a–1	1	2
4	0	4	5	6	7	8	....	a–1	0	a–1	1	2	3
5	0	5	6	7	8	9	....	0	a–1	1	2	3	4
6	0	6	7	8	9	10	....	a–1	1	2	3	4	5
.....	.....	……	……	……	.....	……	….	……	……	.....	.....	……	……
a–3	0	a–3	a–2	a–1	1	2	.....	a–9	a–8	a–7	a–6	a–5	a–4
a–2	0	a–2	a–1	1	2	3	.....	a–8	a–7	a–6	a–5	a–4	a–3
a–1	0	a–1	1	2	3	4	.....	a–7	a–6	a–5	a–4	a–3	a–2

It is easy to make sure that the operation of **#–**product on the set $A_a \setminus \{0\}$ forms the finite cyclic group $A^{\#}$ of degree *(a–1)*. Concerning the ***AS*** that is defined thus the following basic result takes place [24,82-87,102,106].

Theorem 64*. There is an algebraic system* $\langle A_a; +; \# \rangle$*, in which «almost each» a–valued logic function defined in an alphabet A (a – composite integer) can be presented in the form of polynom* $P_{\#}(n) \pmod a$ *where:*

1) (+) – traditional operation of addition modulo a (mod a);

2) (#) – an operation of product, defined according to the above table 4;

$$P_{\#} = \sum_{j=1}^{a^n-1} c_j \# X_1^{d_{j_1}} \# X_2^{d_{j_2}} \# \dots \# X_n^{d_{j_n}} \pmod a \quad - \text{ a polynomial}$$

3) which is not containing dyadic expressions of the following kind : **(10)**

$$p_d \# X_j^d + B_d \# X_j^{a-d-1} \quad (0 \le d_{i_j} \le a-1;\ \sum_{j=1}^{n} d_{i_j} \ge 1;\ X_j,\ c_j \in A_a;$$

$$p_d + B_d = a;\ p_d, B_d \ge 1;\ X_j^p = X_j \# X_j \# \cdots \# X_j;\ j = 1..n;\ j = 1..a^n - 1;$$

$$\leftarrow --- p --- \rightarrow \quad d = 1..[(a-2)/2]$$

This result has played important enough part in research of the ***DPDS*** for the case of alphabet A_a where a – *composite* number and has allowed to obtain a lot of rather interesting results concerning the ***CA*** problems, certain from which are considered below. In addition, theorem ***64*** gives

a quite satisfactory analytical representation of a lot of ***a–***valued logical functions in the case of composite ***a-***modules. Even such a rather simple logic function as**:**

$$R_1(x) = \begin{cases} 0, & if \quad x = 0; \\ 2, & if \quad x = 1; \\ 1, & otherwise \end{cases}$$

defined in alphabet A_6 cannot be represented by means of a polynomial *(**mod 6**)*, while in $AS <A_6;+;\#>$ its presentation has the following simple kind, namely**:** $R_1(y) = P_{\#}(1) = y^2 + y^3 \ (mod\ 6)$. A number of other rather interesting examples of similar type together with comparative analysis of the above ***AS*** and the classical algebraic system of the kind $<A_a;+;x>$, for which operations *(+)* and *(∗)* are usual binary operations of addition and multiplication *(**mod a**)* accordingly the reader can find in [24,82-87].

On the basis of the above *algebraic* system it is possible to determine one more interesting enough type of classical ***CA*** models possessing a rather high degree of reproducibility of finite configurations along with a lot of other interesting enough appendices. In view of the foregoing for some classical ***1–CA*** model its local transition function is defined by the next parallel substitutions, namely**:**

$$\begin{aligned} &x_1x_2x_3 \ldots x_n \to x_1^1 = 0, \quad if \quad (\forall k)(x_k = 0) \\ &x_1x_2x_3 \ldots x_n \to x_1^1 = \prod_{k=1}^{n} \#\, \delta(x_k), \quad else\ ; \quad x_1^1, x_k \in A \quad (k = 1..n) \\ &\delta(x_k) = \begin{cases} x, & if \quad x \neq 0 \\ 1, & else \end{cases} \end{aligned} \qquad (11)$$

where **#–**multiplication is being defined according to the above table ***4.***

At the made assumptions, we shall consider the set $S(a, m)$ of all finite configurations $c = \square x_1x_2x_3 \ldots x_m\square;\ x_k \in A\setminus\{0\}\ (k = 1..m)$ of size $\le m$**.** Hence**,** cardinality of the set $S(a, m)$ is $(a-1)\{(a-1)^m-1\}/(a-2)$**,** while cardinality of the set $\Sigma(a, m)$ of all finite configurations of size $\le m$ is $(a-1)a^{m-1}$**.** So, the density of the set $S(a, m)$ relative to the set $\Sigma(a, m)$ is defined as the expression $\Xi(a,m)=S(a,m)/\Sigma(a,m) \approx a(1-1/a)^m/(a-2)$ whose asymptotics is characterized by the following relations, namely**:**

$$\begin{aligned} &\lim_{a \to \infty} \Xi(a,m) = 1 \qquad\qquad \lim_{m \to \infty} \Xi(a,m) = 0 \\ &\lim_{m \to \infty} \Xi(a,m) = \lim_{a \to \infty} \Xi(a,m) = e^{-p}, \quad if \quad \lim_{m,\, a \to \infty} m/a = p = const \end{aligned} \qquad (12)$$

Of these relations it is possible to make sure, that in a lot of important enough cases the density of the set ***S(a, m)*** is quite sufficient to consider finite configurations composing it, as self–reproducing configurations, i.e. to define one more new class of the ***CA*** models substantially which possess the reproducibility property of the finite configurations in the *Moore* sense. On the basis of the detailed analysis of substitutions ***(11)*** it is possible to show, that global transition function, appropriate to them of a ***1–CA*** model possessesing the ***NCF*** and ***NCF–3*** in the absence of the ***NCF–1*** nonconstructability**;** while each configuration $c^* \in S(a,m)$ is self–reproducing configuration in the *Moore* sense for such ***CA*** models. This result does not conflict with theorem ***58*** as the considered class of ***d–CA*** models will be characterized by existence of the property of appreciable or essential reproducibility in the *Moore* sense of finite configurations, but not universal reproducibility**.**

Classical nonlinear ***1-CA*** model with the simplest neighbourhood index and states alphabet $A=\{0,1,...,a-1\}$ presents one more interesting enough example of such kind**;** local transition function of this model is defined by the following formula**:**

$$\sigma^{(2)}(x,y)=\begin{cases} x, & \text{if } y=0 \\ y, & \text{if } x=0 \\ x\cdot y \pmod a, & \text{otherwise} \end{cases} \quad ; \quad x,y \in A$$

where $a = p^k$**;** $p \geq 3$ is a prime number and k is a positive integer.

It is simple to make sure, that such model does not possess the ***NCF–1*** nonconstructability**,** possessing the ***NCF*** nonconstructability. Wherein, in such model finite configurations of the kind $c = \square x_1 \ldots x_m \square$**;** $x_k \in A \setminus \{0\}$ $(j = 1 .. m)$ are self–reproducing in the *Moore* sense. More precisely [54]**:** ***For an arbitrary configuration $c \in C(A,1,\phi)$ exists such integer $w \geq 0$, that $c\tau^{(2)w}$ is self–reproducing configuration in the Moore sense.*** In addition, full set $C(A,1,\phi)$ can be generated from a set $G \subseteq NCF$**.**

A lot of special results in this direction can be found in works [24,40-43]. The classical ***CA*** models with universal reproducibility are attractive in many respects. Results in this direction allow to discover a lot of useful correlations between nonconstructability and universal reproducibility for the classical ***CA*** models, and to solve a lot of mathematical problems [5,8,9,12,24-28,34,40-43,82-87,102,106].

From the analysis of a lot of classical ***CA*** models which in a great extent possess the reproducibility of finite configurations in the *Moore* sense it is possible to assume that necessary condition consists in that that local

transition functions $\sigma^{(n)}$ of the classical ***CA*** models with such property will define by parallel substitutions of the following kind, namely**:**

$$x_1x_2x_3 \ldots x_n \Rightarrow x^*_1 = \Phi(x_1, x_2, x_3, \ldots, x_n) \qquad x_k, x^*_1 \in A \quad (k = 1 \ldots n)$$

where function Φ is based on such operations which form finite cyclic groups of the appropriate degree on a set ***A*** or its large enough subsets. The considered models satisfy the above condition. Furthermore, in the more wide context the classical ***CA*** models known to date that possess the property of universal reproducibility in the *Moore* sense, belong to a class of the ***CA*** models possessing the ***NCF–1*** nonconstructability in the absence of the ***NCF*** nonconstructability.

Along with continuation of research of classical ***CA*** models that possess the property of universal reproducibility it is quite expedient to define other classes of ***CA*** models possessing a certain general property rather interesting from the theoretical and applied standpoints, and effectively to characterize the given classes in terms of new or earlier investigated concepts and categories. So, in view of this question, the research of the ***CSAG-***class of ***CA*** models with symmetrical functions $\tau^{(n)}$ $\{\sigma^{(n)}\}$ seems rather interesting. The analysis of a lot of classical ***d–CA (d=1,2)*** models of the ***CSAG–***class on the basis of theoretical research and by means of computer research [24,48,49,82-87,102] has allowed to formulate a rather interesting proposal, that in the ***CSAG–***subclass there is infinitely many of models possessing the property of *universal* or *essential reproducibility* in the *Moore* sense of finite configurations. Thus, the following a rather interesting proposal seems quite convincing**:**

Among classical d-CA (d≥1) models with symmetrical global transition functions not possessing the NCF nonconstructability in the presence of the NCF-1 nonconstructability there is infinitely many models that will possess the universal or essential reproducibility in the Moore sense.

It is known, the quota of classical ***d–CA (d≥1)*** models which not possess the ***NCF (NCF–3)*** tends to ***1*** with growth of neighbourhood template size and/or cardinality of an ***A–***alphabet of elementary ***CA*** automaton. Meantime, the class of models meeting conditions of the above proposal is quite representative. Thus, already for the classical binary ***1–CA*** the cardinality of this class not less then $N(n) = 2^{2^{2n-3}}$.

$$\begin{cases} (\forall < x_1, x_2, \ldots, x_{n-1} >)(\sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, x_n) \neq \sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, x^*_n)) \\ (\forall < x_1, x_2, \ldots, x_{n-1}, x_n >)(\sigma^{(n)}(x_1, x_2, \ldots, x_{n-1}, x_n) = \sigma^{(n)}(x_n, x_{n-1}, \ldots, x_2, x_1)) \\ x_k, x^*_k \in A = \{0, 1, 2, 3, \ldots, a-1\}; \quad k = 1..(n-1); \quad n \geq 2 \end{cases} \qquad (13)$$

This estimation can be obtained, considering the classical ***1–CA*** models whose ***LTF*** $\sigma^{(n)}$ is given by the above determinative relations ***(13)***. It is simple to be convinced that the classical ***CA*** models, defined thus, will possess completely by the symmetric ***LTF*** and do not possess ***NCF*** and ***NCF–3*** [82]. In the case of classical ***1–CA(a,n)***, for each whole $a > 1$ there is a ***n*** size of neighbourhood template since which the number of models with the nonlinear symmetric ***LTF*** which not possess the ***NCF (NCF–3)***, will grow quicker than the number of models with linear symmetrical ***LTF*** [82-87]. In particular, for binary ***1–CA*** models $M(n) = 2^{n-2}$ and the specified relation is valid, starting already with $n = 5$.

At first sight, the set of classical ***CA*** models determined by completely symmetric ***LTF*** which not possess the ***NCF (NCF–3)*** in the presence for them ***NCF–1*** exhausts the various ***CA*** models characterized by property of universal reproducibility of finite configurations in the *Moore* sense. But, as has been shown in [24,84], this statement most likely is incorrect and the class of ***CA*** models with the specified self-reproduction can be a few wider. The matter was investigated both by theoretical methods, and on the basis of computer simulation of dynamics of the respective classical ***CA*** models of dimensionality ***1*** and ***2***.

So, for computer researches of dynamics of classical ***1–CA*** models with symmetric ***LTF*** the procedure, that allows to research reproducibility of arbitrary finite configuration depending on the ***LTF*** has been created in the ***Mathematica*** system [49]. The ***HS*** procedure alows to do analysis on the basis of three arguments: local transition function that is given by a set of parallel substitutions, an initial finite configuration, the demanded quantity of the configuration copies. A successful conclusion of analysis prints the initial configuration researched to reproducibility along with number of its copies, and quantity of steps, required for it whereas thru global variable ***CFfin*** a configuration in which these copies were reached is returned. Along with the ***HS*** procedure many procedures for study of dynamics of classical models has been programmed also in ***Maple*** [48].

But, it is necessary to note, this procedure along with other procedures intended for experimental research of dynamics of classical ***CA*** models even in the case of dimension ***1*** and ***2*** assumes use of rather productive classes of computers. On the basis of the ***HS*** procedure substantially our assumption was confirmed, that ***the class of 1–CA models that possess the property of universal or essential reproducibility in the Moore sense is much wider than the class of linear models.*** A number of interesting enough results in this direction can be found in [24,82-87,102,106].

So, in the set of ***CSAG***, the ***VS–***subset *(isolated relative to the composition operation)* of all symmetric global transition functions which not possess by ***NCF*** nonconstructability is naturally distinguished. In our opinion, exactly the subset ***VS*** is of special interest with standpoint of question of characterization of the classical ***CA*** models possessing the universal or essential reproducibility in the *Moore* sense of finite configurations.

Therefore, it is possible to assume, that the *symmetry* of global transition functions along with absence for them of the ***NCF*** nonconstructability is one of prerequisites of the universal or essential reproducibility of finite configurations in classical ***CA*** models. In this connection the following rather interesting proposition can be formulated [24,54–56,102,640].

Among all classical d–CA models with symmetric GTF $\tau^{(n)}$ ***(d≥1; n≥d+1) which possess the NCF–1 nonconstructability in the absence of the NCF nonconstructability there is infinitely many of models (not necessarily linear models) which will possess the property of universal or essential reproducibility of finite configurations in the Moore sense; in addition, is supposed that essential reproducibility takes place if reproducibility is intrinsic to more than half all finite configurations.***

Thus, we receive well defined ***CSAG–***class of ***CA*** models possessing the specified general property**;** in addition, in the ***1–***dimensional case such class is recursive and for it there is a constructive solving algorithm. It is necessary to note, that our researces in this direction in a great extent confirm this proposal. The more detailed discussion of this question can be found in [24,40-43,82-87,102,106].

Meanwhile, the reproducibility is caused not only by symmetry of ***GTF.*** Linear global transition functions $\tau^{(n)}$, whose local transition functions are represented in the form $\sigma^{(n)}(x_1,x_2, ..., x_n)=\sum_j b_j x_j \ (mod\ p^k)$ where p is a prime, k – a positive integer**;** $b_1, b_n \in A\setminus\{0\}$, $x_j \in A$ $(j=2..n-1)$ possess the universal reproducibility property in the *Moore* sense. Quantity of such global transition functions depending on values of parameters a and n is determined by the following formula, namely**:**

$$N(a,n)=(a-1)a^{(n-2)/2}*\begin{cases}1, & \text{if } n \text{ - an even number}\\ \sqrt{a}, & \text{otherwise}\end{cases}$$

In addition, among global functions of this class a quota of symmetrical functions $\tau^{(n)}$ equals $1/[(a-1)a^{(n-k)/2}]$ where $k=2$ for an even integer n**;** otherwise $k=1$**.** So, with growth of value n and/or a their quota quickly decreases. Among all symmetric and nonsymmetric global $\tau^{(n)}$ functions

the functions possessing universal or essential reproducibility and that differ from linear global functions exist [24,40-43,82-87,102,106].

It is simple to be convinced that as an one of the simple examples of the *nonlinear* models possessing the property of universal reproducibility of finite configurations in the *Moore* sense, a ***1-CA*** model with the simplest neighbourhood index $X=\{0,1\}$, alphabet $A=\{0,1,2\}$ whose local transition function $\sigma^{(2)}$ determined by the following formula:

$$\sigma^{(2)}(x,y)=\begin{cases} x+y \pmod 3, & \text{if } x=0 \\ x+y+1 \pmod 3, & \text{if } x=1 \\ x*y+1 \pmod 3, & \text{if } x=2 \end{cases}$$

can serves. Furthermore, it was shown, that nonlinear ***CA*** models from a class of ***1–CA*** models with simplest neighbourhood index $X=\{0,1\}$, an alphabet $A=\{0,1,2,\ldots,a-1\}$ (*a is a prime number)* and with local transition functions $\sigma^{(2)}$ determined by the following formula:

$$\sigma^{(2)}(x,y)=\begin{cases} x+y \pmod a, & \text{if } x=0 \\ x*y+1 \pmod a, & \text{if } x=a-1 \\ x+y+1 \pmod a, & \text{otherwise} \end{cases}$$

will possess the property of *universal* or *essential* reproducibility of finite configurations in the *Moore* sense. There is some other rather interesting examples of the nonlinear ***CA*** models possessing universal or essential reproducibility in the *Moore* sense of finite configurations [40-43,102].

The most common standpoint up till now was, *universal* reproducibility in the *Moore* sense in linear classical models has been associated with a states alphabet whose cardinality is defined by a number $a=p^k$, where p is a prime number and k is a positive integer. Meantime, the researches carried out by us which are based on theoretical results in combination with computer modelling have allowed to prove actuality of the more general result [24,82-87,102,106], namely:

For a linear classical d–CA ($d\geq 1$) model with an alphabet $A=\{0,1,\ldots,g-1\}$ (where g can be presented in the form 2^k; k is a positive number) which possesses the property of universal reproducibility in the Moore sense there is at least one nonlinear CA model possessing the same property of universal reproducibility in the Moore sense.

As a simple example it is possible to present a subclass of classical ***1–CA*** models with the simplest neighbourhood index $X=\{0,1\}$ and an alphabet $A=\{0,1,\ldots,a-1\}$ where a number a can't be presented in the form p^k (p – *a prime number, and* k *is a positive integer)*, whose local transition functions are determined by the following formula, namely:

$$\sigma^{(2)}(x,y)=\begin{cases}0, & if\ x=y\\ a-1, & if\ x+y=a-1\\ \sigma^{(2)}(y,x), & otherwise\end{cases};\quad x,y\in A=\{0,1,2,...,a-1\}$$

$$(\forall c)(\forall b,d)(b\neq d\rightarrow\sigma^{(2)}(c,b)\neq\sigma^{(2)}(c,d));\quad b,c,d\in A$$

It was shown that ***CA*** models of this subclass will possess the property of universal or essential reproducibility of finite configurations in the *Moore* sense. As an example the ***1-CA*** model can serve with the simplest neighbourhood index, alphabet $A=\{0,1,2,3\}$, and whose local transition function $\sigma^{(2)}$ is determined by the next parallel substitutions, namely**:**

$$00\rightarrow 0\quad 10\rightarrow 1\quad 20\rightarrow 2\quad 30\rightarrow 3\quad 01\rightarrow 1\quad 11\rightarrow 0\quad 21\rightarrow 3\quad 31\rightarrow 2$$
$$02\rightarrow 2\quad 12\rightarrow 3\quad 22\rightarrow 0\quad 32\rightarrow 1\quad 03\rightarrow 3\quad 13\rightarrow 2\quad 23\rightarrow 1\quad 33\rightarrow 0$$

Such nonlinear ***CA*** model possesses the universal reproducibility in the *Moore* sense, differing from a linear model with local function $\sigma^{(2)}(x,y)=bx+cy\ (mod\ 4);\ b,c,x,y\in A$, that possesses the same property. In addition, frequently generating of the given number of copies of a certain finite configuration in the second case needs much more steps. So, generation *10* copies of configuration «*1302321200103013121303212213*» by a linear function $\sigma^{(2)}(x,y)=x+y\ (mod\ 4);\ x,y\in A=\{0,1,2,3\}$ needs *1985* steps of the model, while the above nonlinear model possessing the same property of universal reproducibility in the *Moore* sense of finite configurations demands only *481* step. The similar ratio is fair for many configurations. Indeed, it is interesting to note, in a lot of cases, the nonlinear models essentially more effectively generate the specified quantity of copies of self**–**reproducing configurations than appropriate linear ***CA*** models in the same states alphabet of an elementary automaton.

On the other hand, researches of the above ***CSAG–***class of symmetrical local transition functions has shown, that in it there are the models that possess the property of universal reproducibility in the *Moore* sense for the case of composite numbers a as a cardinality of an alphabet A while linear models under the given condition do not possess such property. So, for the case of cardinality ***6*** a linear model with the above property do not exist, whereas the nonlinear model with ***LTF*** determined by the following parallel substitutions possesses such property [24,40-43,102]**:**

$00\rightarrow 0$	$10\rightarrow 1$	$20\rightarrow 2$	$30\rightarrow 3$	$40\rightarrow 4$	$50\rightarrow 5$
$01\rightarrow 1$	$11\rightarrow 0$	$21\rightarrow 4$	$31\rightarrow 2$	$41\rightarrow 5$	$51\rightarrow 3$
$02\rightarrow 2$	$12\rightarrow 4$	$22\rightarrow 0$	$32\rightarrow 5$	$42\rightarrow 3$	$52\rightarrow 1$
$03\rightarrow 3$	$13\rightarrow 2$	$23\rightarrow 5$	$33\rightarrow 0$	$43\rightarrow 1$	$53\rightarrow 4$
$04\rightarrow 4$	$14\rightarrow 5$	$24\rightarrow 3$	$34\rightarrow 1$	$44\rightarrow 0$	$54\rightarrow 2$
$06\rightarrow 5$	$15\rightarrow 3$	$25\rightarrow 1$	$35\rightarrow 4$	$45\rightarrow 2$	$55\rightarrow 0$

There is a lot of other rather interesting examples of classical ***CA*** models possessing the property of *universal* or *essential* reproducibility of finite configurations in the *Moore* sense. Some of them have been discovered on the basis of computer modelling [9,24]. As a result of research of the problem of universal reproducibility in the *Moore* sense we are inclined to formulate the following assumption**:**

The set of all classical 1-CA models possessing the property of universal reproducibility in the Moore sense forms a subset of the models, whose local transition functions meet the following condition:

$$\left\{(\forall c \in A)(< x_2,...,x_n > \neq < y_2,...,y_n > \rightarrow \sigma^{(n)}(c,x_2,...,x_n) \neq \sigma^{(n)}(c,y_2,...,y_n))\right\} \&$$
$$\left\{(\forall < x_2,...,x_n >)(b \neq d \rightarrow \sigma^{(n)}(b,x_2,...,x_n) \neq \sigma^{(n)}(d,x_2,...,x_n))\right\}$$
$$b,c,d \in A; \quad x_j, y_j \in A; \quad j = 2..n; \quad A = \{0,1,...,a-1\}$$

Number of such structures is at least $[(a-1)!]^2$**.** The performed research allows to speak about rather high degree of veracity of the assumption for ***1-***dimensional case**.** In addition, in the case of higher dimensionality the assumption seems truthful too.

Meanwhile, use of non**–**standard methods has allowed to discover a lot of other rather interesting classes of ***CA*** models possessing the property of essential reproducibility of finite configurations. In particular, we can define a rather useful modification of the logic operation ***XOR*** over the positive integers as follows**:**

Operation "x XOR1 y" with two positive integers x and y is defined as bit–by–bit operation XOR without a carrying over into the high–order digits with binary equivalents of the given integers x, y; in addition, a length w of binary representation is defined by length of representation of the maximal integer, i.e. w = max{|x|, |y|}, for example:

$$12 \ \ XOR1 \ 19 \equiv \begin{bmatrix} 01011_{12} \\ 10011_{19} \end{bmatrix} \equiv 11000 \equiv 24$$

It is obviously, in the binary case the above operation ***XOR1*** coincides with classical *XOR* operation. Whereas for set of integers ***A*****={*0,1,2,3*}** the table of ***XOR1*** operation is determined by the following tables**:**

XOR1	*000*	*001*	*010*	*011*
000	*000*	*001*	*010*	*011*
001	*001*	*000*	*011*	*010*
010	*010*	*011*	*000*	*001*
011	*011*	*010*	*001*	*000*

XOR1	*0*	*1*	*2*	*3*
0	*0*	*1*	*2*	*3*
1	*1*	*0*	*3*	*2*
2	*2*	*3*	*0*	*1*
3	*3*	*2*	*1*	*0*

It is simple to make sure, that a set ***A*** **= {*0,1,2,...,h*}** *(**h** – an arbitrary prime)*

forms finite additive *Abel* group with a neutral element «***0***» concerning the ***XOR1*** operation**;** in addition, each element from ***A*** possesses a single inverse element conterminous with itself. A simple procedure ***&XOR1,*** programmed in the ***Maple*** system [22,48] provides execution of ***XOR1–***operation over an arbitrary finite set ***N*** of positive integers**;** ibidem the source code of ***&XOR1*** procedure along with its use is represented. So, for example, table of ***XOR1–***operation for the set ***A*****={*0,1,...,31*}** allows to establish its structural organization presenting self**–**dependent interest. The experimental and theoretical analysis of the classical ***CA*** models has shown [84] that at the made assumptions the following suggestion can be formulated as follows.

<u>Theorem 65</u>. An arbitrary classical d–CA (d≥1) model with an alphabet A={0,1,...,j} (j – an arbitrary prime) whose local transition function $\sigma^{(n)}$ is defined by formula $\sigma^{(n)}(x_1, ..., x_n)$ = &XOR1$(x_1, ..., x_n)$, will possess the universal reproducibility of finite configurations in the Moore sense.

It has been shown that this result is extrapolated on the case of classical ***d–CA (d≥1)*** models with an arbitrary neighbourhood index. So, a quite pertinently the following conclusion to formulate**:**

A class of d–CA (d≥1) models concerning the universal reproducibility in the Moore sense wider than the class of CA models defined by linear local transition functions $\sigma^{(n)}$ and their superpositions. Number of such linear 1–CA(a, n) models equals $(a–1)^2a^{(n–2)}$ (a – a prime number).

Theorem ***65*** in a great extent gives an answer to question of existence of the classical ***CA*** models, distinct from linear ***CA*** models possessing the universal reproducibility of finite configurations in the *Moore* sense. In addition, an assumption which allows to treat more widely the class of ***CA*** models possessing the *universal* or *essential* reproducibility of finite configurations in the *Moore* sense will be formulated below.

In view of complex enough dynamics of a lot of classical ***1–CA*** models even in the case of binary states alphabet the method of computer**-**based modelling with good reason can be referred to the basic components of the apparatus of research of classical ***CA*** models. Such method allows not only to empirically investigate dynamics of ***CA*** models and enough effectively to visually display it, but also gives rather good possibilities for formulation of different hypotheses, certain of which have already received stringent theoretical substantiation whereas others stimulated a lot of rather interesting researches. Along with the above procedures we programmed special software of various complexity for computer

study of various aspects of the ***CA*** problematics first of all of dynamics of ***CA*** models of dimensionality ***1*** and ***2*** [24-28,40-43,48,49,82,87,106].

So, this method has allowed to formulate and appreciably approve one rather interesting hypothesis concerning the existence of a subset of the subset of ***1–***dimensional symmetrical global transition functions which not possess the ***NCF*** nonconstructability in the presence for them of the ***NCF–1*** nonconstructability, and for which the *universal reproducibility* of finite configurations takes place. Research in this direction seems rather interesting, allowing in the case of its positive solution to obtain a lot of useful interrelations between algebraical properties of global transition functions and the nonconstructability problem.

In particular, computer simulation of ***1–CA*** with alphabet ***A={0,1, …, a}***, neighbourhood index ***X={0,1,…,n–1}*** and local transition function

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = XOR1(x_1, x_2, ..., x_n); \quad x_j \in \{0, 1, ..., a\}; \ a - an\ int\ eger; \ j = 1..n$$

allows to obtain a lot of rather interesting results [82]. So, in such models from arbitrary finite configurations ***x*** in an alphabet ***A*** are generated the sequences of configurations containing subsequences of configurations of the form $x0^{j1} \ldots x^{jk}S$ ($jk \geq 0$; $k = 1..\infty$), 0^{jk} – a string ***0…0*** of length ***jk;*** in addition, ***j1..jk*** is a palindrome. For simulating of such ***1–CA*** models a number of procedures programmed in ***Mathematica*** system had been used. These procedures along with other software allowed to obtain a lot of interesting enough results concerning constructive possibilities of classical ***d–CA (d = 1, 2)*** models, including self–reproducibility of finite configurations in the *Moore* sense. Similar results on dynamic properties of ***d–CA (d=1,2)*** can be found, in particular, in our works [24,82-87,106].

Simulation method with use of the above software has allowed to define a number of types of classical ***1–CA*** models that possess the property of essential reproducibility of finite configurations along with some other rather interesting dynamic properties of ***CA*** models of this class [24,84]. In particular, as a result of such experimental research the ***1–CA*** model has been discovered with neighbourhood index ***X={0,1}***, states alphabet ***A = {0,1,2}*** and global transition function $\sigma^{(2)}(x, y) = x\Theta y$; $x,y \in A$, where **Θ–**operation is determined by the following table, namely**:**

Θ	0	1	2
0	0	2	1
1	2	0	1
2	1	2	0

It is simple to make sure, that the ***1-CA*** model does not possess the ***NCF*** nonconstructability while a finite configuration such as $c^*=\square x_1x_2 \ldots 21\square$ $\{x_1,x_2,x_3 \in A\}$ is ***NCF–1*** for this model [24,84]. The experimental research of the above ***1–CA*** model has confirmed the existence of the property of essential reproducibility of finite configurations in the *Moore* sense for such classical ***1–CA*** model.

Meantime, a number of researches in this direction allows to formulate a lot of interesting assumptions of which we shall note the following. So, theoretical and experimental research of classical ***1–CA*** models have shown that the models possessing the *undermentioned* general property as the ***CA*** models possessing the universal reproducibility of the finite configurations in the *Moore* sense can be. As a classical ***1–CA*** model the model with neighbourhood index $X = \{0,1\}$ and alphabet $A = \{0,1, \ldots, a-1\}$ is chosen. On the set A the binary $\otimes$–operation defined by the following $\otimes$–table is given:

$\otimes$	0	1	2	.....	a–2	a–1
0	0	$x_{0,1}$	$x_{0,2}$	.....	$x_{0,a-2}$	$x_{0,a-1}$
1	$x_{1,0}$	$x_{1,1}$	$x_{1,2}$	.....	$x_{1,a-2}$	$x_{1,a-1}$
2	$x_{2,0}$	$x_{2,1}$	$x_{2,2}$	.....	$x_{2,a-2}$	$x_{2,a-1}$
.....	.....	.....	.....	.....	.....	.....
a–2	$x_{a-2,0}$	$x_{a-2,1}$	$x_{a-2,2}$	.....	$x_{a-2,a-2}$	$x_{a-2,a-1}$
a–1	$x_{a-1,0}$	$x_{a-1,1}$	$x_{a-1,2}$	.....	$x_{a-1,a-2}$	$x_{a-1,a-1}$

Elements of the above table, that determine the $\otimes$–operation satisfy the following determinative conditions, namely:

$$(\forall h,j,k)(j \neq k \rightarrow x_{h,j} \neq x_{h,k}) \& (\forall h,j,k)(j \neq k \rightarrow x_{j,h} \neq x_{k,h})$$
$$x_{h,j},\ x_{h,k},\ x_{j,h},\ x_{k,h} \in A = \{0,1,\ldots,a-1\};\quad h,k,j = 0..a-1$$

The essence of these conditions consists in that that a column and a row of the $\otimes$–table contain strictly one entrance of elements of the alphabet ***A.*** For example, we can be limited to the condition that $\otimes$–operation on the alphabet A forms the finite *Abel* group. So, from experimental and theoretical research of classical ***1-CA*** models with simple neighbourhood indexes $X1=\{0,1\}$ and $X2=\{0,1,2\}$ along with alphabet $A=\{0,1, \ldots, a-1\}$ *(*$a=2..5$*)* there are forcible enough arguments to formulate the next a rather interesting assumption:

A classical d–CA model with neighbourhood index $X=\{0,1, \ldots, n-1\}$ ***and alphabet*** $A=\{0,1, \ldots, a-1\}$ ***whose local transition function*** $\sigma^{(n)}$ ***is defined***

by formula $\sigma^{(n)}(x_0,x_1,...,x_{n-1}) = \otimes(x_0,x_1,...,x_{n-1})$ ***possesses the property of universal reproducibility of finite configurations in the Moore sense where*** x_j ***are coordinates of elementary automaton of a neighbourhood template in the homogeneous space*** Z^d $(d \geq 1; j=0..n-1)$.

For testing of this assumption the computer simulating was used as a result of which a lot of rather interesting experimental results has been received [82-87]. Furthermore, it is possible to show, that in the case of correctness of the above assumption in addition to the linear classical ***d–CA*** $(d \geq 1)$ models will exist not less $[(a-1)!]^2$ models with an alphabet $A=\{0,1,...,a-1\}$ and neighbourhood index $X=\{x_0,x_1,...,x_{n-1}\}$, whose local transition functions $\sigma^{(n)}$ are defined by the above formula that possess the property of universal reproducibility of finite configurations in the *Moore* sense. This result will allow to expand enough appreciably the class of ***CA*** models with the above interesting property of generating of finite configurations; in turn, a number of ⊗–operations can represent a certain interest in a lot of researches of classical ***d–CA*** $(d \geq 1)$ models.

The analysis of similar examples jointly with theoretical considerations allow to us to come to conclusion about the absence of some *prime cause* of linearity of classical ***CA*** models concerning the property of universal or essential reproducibility in the *Moore* sense of finite configurations. More precisely, the universal and essential reproducibility in the *Moore* sense of finite configurations has more deep hearts and their revealing presents undoubted interest.

In any case, on the assumption of the represented results, we got rather interesting examples of the classical ***CA*** models possessing the universal reproducibility of finite configurations that are fundamentally different as from class of linear classical models already becoming classical, and from wider class of ***CA*** models formed by means of composition of their ***GTF*** also possessing the property of universal reproducibility of finite configurations in the *Moore* sense. Meanwhile, research of questions of self-reproducibility in ***CA*** models in a lot of cases collides with problems of algorithmic solvability. For example, ***A. Leitsh*** has considered certain problems of self–reproduction and fertility relative to a subclass of *non–deterministic* ***CA*** models, using theory of recursive functions, and he has proved their algorithmical unsolvability [138].

According to the positive decision of the presence problem of universal reproducibility of finite configurations in the *Moore* sense, in classical models possessing this property, the ***NCF*** nonconstructability is absent

in the presence of the ***NCF–1*** nonconstructability. Hence, an interesting enough question arises, namely**:** ***Whether there are classical CA models in which any finite configuration that is distinct from NCF would as a self-reproducing in the Moore sense, or «almost all» finite constructible configurations, that are distinct from finite configurations such as NCF will possess the reproducibility property in the Moore sense?***

This question is interesting in view of possibility of self**-**reproduction in the case of narrowing of the set of all finite configurations up to the set ***C(A,d,∞)\NCF.*** Unfortunately, for today this question is open**;** it in full measure concerns the problem of essential reproducibility in the *Moore* sense, however our research of it allows to expect for this question the negative answer.

Generally speaking, the self**-**reproducibility in ***CA*** models is considered with respect to finite configurations, however this phenomenon can be also generalized and to the case of infinite configurations. One possible generalization can be defined as follows. Let ***S*** be a randomly generated string of length ***n*** of the elements from an alphabet ***A=*{*0,1,...,a–1*}** where ***a*** is a prime *(this string imitates an infinite configuration of a* ***1–CA*** *model)***,** that is divided into equal disjoint substrings of length ***m<<n.*** As a result of our numerous computer experiments**,** it became possible to formulate an assumption which defines the self**–**reproduction phenomenon in the case of infinite configurations in classical ***1–***dimension ***CA*** models**:**

For integers p and d there exists a string S (1–dimension configuration composing from an A alphabet) of such length that application to it of a global transition function which is determined by the strictly linear local transition function predetermined in the same A alphabet through not more than d steps of a classical 1–CA model with the above linear local transition function generates a string containing p disjoint copies of significant share of the above substrings of m length.

Experiments were carried out using, among other things, the ***InfiniteCF*** procedure programmed in the ***Mathematica*** software. This procedure admits seven arguments, having the following meanings**:**

n – length of alphabet ***A=*{*0,1,2,…,n–1*}** of a classical ***1–CA*** model**;**

g – neighbourhood template size of the ***1–CA*** model**;**

m – length of an initial string, composed randomly from elements of ***A*** alphabet**;** the initial string, potentially increasing on length, imitates an infinite configuration of the ***1–CA;***

p – length of disjoint substrings of the initial string**;**

t **–** the limit number of steps of the ***1–CA*** model**;**

h **–** the desired number of copies of the above substrings**;**

S **–** optional argument defining initial infinite configuration.

The procedure call ***InfiniteCF*[*n, g, m, p, t, h*]** returns the two**–**element list whose the first element determines the share of sublists that satisfy the above requirements according to the actual arguments of the procedure call relative to all sublists, while the second argument is the number of steps of the strictly linear global transition function, required for it. The following fragment presents the source code of the ***InfiniteCF*** procedure along with an example of its application.

```
In[4718]:= InfiniteCF[n_Integer, g_Integer, m_Integer, p_Integer,
             t_Integer, h_Integer, S___] := Module[{a, b, c, d = "", k, v = 0},
           a = If[{S} == {}, RString[n, m], If[StringQ[S], S, Return["The last
         argument is incorrect"]]]; b = DeleteDuplicates[StringPartition[a, p]];
                                  Do[For[k = 1, k <= StringLength[a] – g + 1, k++,
                                    d = d <> ToString[Mod[Plus @@
                                  ToExpression[Characters[StringTake[a, {k, k + g – 1}]]], n]]];
                                    a = d; d = ""; ++v; c = Map[StringCount[a, #] &, b];
                                  If[Length[Select[c, # >= h &]] > 1, Break[], 76], {j, t}];
                                  {Length[Select[b, StringCount[a, #] >= h &]]/Length[b], v}]

In[4719]:= InfiniteCF[2, 2, 40000, 8, 6000, 80]
Out[4719]= {0.996094, 14}
```

It should be noted that due to insufficiently efficient system algorithms supporting cyclic computations, the fulfillment of a lot of procedures in ***Mathematica***, which solve certain problems of simulating of dynamics of already ***1–***dimension ***CA*** models incures quite essential time costs.

Finally, the study of dynamic properties, including self-reproducibility, of ***d–CA*** $(d \geq 1)$ models, whose local transition functions are determined as $\sigma^{(n)}(x_1,x_2,...,x_n)=RandomChoice[\{x_1,x_2,...,x_n\}]$ where ***RandomChoice*** **–** *a pseudorandom choice of one of the* $\{x_1, x_2, ..., x_n\}$, seems rather interesting. A computer study of such ***d–CA*** $(d = 1,2)$ models has yielded a number of interesting properties relating to their dynamics [24,82,102,106].

```
In[4941]:= RandomLTF[S_String, n_Integer, m_Integer] :=
                                  Module[{a = "", b = StringTrim[S, ("0" | "0") ...], k, j},
                                Do[b = StringRepeat["0", n – 1] <> b <> StringRepeat["0", n – 1];
                                  For[k = 1, k <= StringLength[b] – n + 1, k++,
                                a = a <> RandomChoice[Characters[StringTake[b, {k, k + n – 1}]]]];
                                  b = StringTrim[a, ("0" | "0") ...]; a = ""; Print[b], {j, m}]]
```

```
In[4942]:= RandomLTF["10120122021", 4, 200]
          "11201200202"
          "1102200202"
           ==========
```

In particular, the procedure call **RandomLTF[*S, n, m*]** prints a sequence generated from a configuration ***S***, in ***m*** steps of the ***1–CA*** model with ***n*** size of the neighborhood template and with the above pseudorandom local transition function.

The research of dynamical properties, including self-reproducibility, of classical ***d-CA (d≥1)*** models with an alphabet ***A={0,1,…,a-1}*** whose local transition functions are defined as $\sigma^{(n)}(0,0,\ldots,0)=0$ and $\sigma^{(n)}(x_1,x_2,\ldots,x_n)$ = ***RandomChoice[{0,1,…,a–1}],*** seems also rather interesting. A computer study of such ***d–CA (d=1,2)*** models has yielded a number of interesting properties relating to their dynamics. The following **RandomLTF1** tool admits five arguments, having the following meanings**:**

A – an alphabet ***A={0,1,2,…,a–1}*** of a classical ***1–CA*** model**;**
S – an initial string of the ***1–CA*** model**;**
n – neighbourhood template size of the ***1–CA*** model**;**
m – the limit number of steps of the ***1–CA*** model**;**
p – the desired number of copies of the above initial string**;**

A successful call of the ***RandomLTF1[A, S, n, m, p]*** procedure returns the two-element list whose the first element defines the number of disjoint substrings ***S*** in a generated string, whereas the second element defines the number of steps of the ***1–CA*** model, that were required for this. The fragment below presents the source code of the above procedure along with examples of its application.

```
In[4946]:= RandomLTF1[A_List, S_String, n_Integer, m_Integer,
                                                    p_Integer] :=
             Module[{a = "", b = StringTrim[S, ("0" | "0") ...], g = 0, s, k, j, t},
           Do[b = StringRepeat["0", n – 1] <> b <> StringRepeat["0", n – 1];
                                          For[k = 1, k <= StringLength[b] – n + 1, k++,
         a = a <> RandomChoice[Set[s, Characters[StringTake[b, {k, k + n - 1}]]]];
                                 If[AllTrue[s, TrueQ[# == "0"] &], {"0"}, Map[ToString, A]]]];
                                                  b = StringTrim[a, ("0" | "0") ...]; a = ""; ++g;
        If[Set[t, StringCount[b, S]] >= p, Break[], Continue[]], {j, m}]; {t, g}]

In[4947]:= RandomLTF1[{0, 1}, "1011011101", 6, 2000, 10]
Out[4947]= {11, 1422}
```

```
In[4948]:= RandomLTF1[{0, 1, 2}, "12010112", 3, 10000, 5]
Out[4948]= {5, 4144}
```

It is necessary to note, the researches of extremal dynamic properties of finite configurations play a rather important part in the mathematical theory of classical ***CA*** models. In this context it is desirable to consider other concepts of extremality, different from universal reproducibility in the *Moore* sense or similar. In this direction a number of new concepts is considered by us. So, the first received results allow to say about their sufficient enough availability. In the future we suppose to represent the most interesting results in this direction.

Further, the study of dynamic properties, including self-reproducibility, of classical strictly linear ***d–CA (d≥1)*** models with an arbitrary alphabet ***A={0,1,2,…,a–1}*** and a neighborhood index, changing randomly within the main neighborhood index of length ***n*** seems also rather interesting. Computer study of such ***d-CA (d=1,2)*** models has yielded a number of interesting enough properties relating to their dynamics. The following **RandomLTF2** procedure admits six arguments, that have the following meanings**:**

A – an alphabet ***A={0,1,2,…,a–1}*** of a classical ***1–CA*** model**;**
S – an initial string of the ***1–CA*** model**;**
n – neighbourhood template size of the ***1–CA*** model**;**
m – the limit number of steps of the ***1–CA*** model**;**
p – the desired number of copies of the above initial string**;**
v – optional argument defining print mode of generated configurations.

A successful call of the ***RandomLTF2[A, S, n, m, p]*** procedure returns the two-element list whose the first element defines the number of disjoint substrings ***S*** in a generated string, whereas the second element defines the number of steps of the ***1–CA*** model, that were required for this. The presence of the argument ***v –*** *an arbitrary expression* **–** at a procedure call determines the printing mode. The fragment below presents the source code of the above procedure along with examples of its application.

```
In[5440]:= RandomLTF2[A_List, S_String, n_Integer, m_Integer,
    p_Integer, v___] := Module[{a = "", b = StringTrim[S, ("0" | "0") ...],
      c, d = Length[A], g = 0, gs, k, j, t}, c = Subsets[Range[1, n], {2, n}];
     Do[b = StringRepeat["0", n – 1] <> b <> StringRepeat["0", n – 1];
                                  For[k = 1, k <= StringLength[b] – n + 1, k++,
                                  gs = Characters[StringTake[b, {k, k + n – 1}]];
                                  If[AllTrue[gs, TrueQ[# == "0"] &], a = a <> "0",
```

```
                                    a = a <> ToString[Mod[Plus @@
                                  ToExpression[gs[[RandomChoice[c]]]], n]]];
      b = StringTrim[a, ("0" | "0") ...]; If[{v} == {}, 76, Print[b]]; a = ""; ++g;
       If[Set[t, StringCount[b, S]] >= p, Break[], Continue[]], {j, m}]; {t, g}]
```

```
In[5441]:= RandomLTF2[{0, 1, 2}, "22201022", 3, 10000, 8]
Out[5441]= {11, 8665}
In[5442]:= RandomLTF2[{0, 1, 2}, "2220221022", 4, 12000, 8, gs]
           "2002021223002"
            ===========
Out[5442]= {9, 10076}
```

At last, the study of dynamic properties, including self-reproducibility, of classical ***d–CA*** models with an arbitrary alphabet ***A***={***0,1,…,a–1***} and a fixed neighborhood index of length ***n;*** along with the fact that, at each step of the ***d–CA (d ≥ 1)*** the local transition function changes randomly within all admissible local functions seems also rather interesting. Our computer study of such ***d–CA (d=1,2)*** models has yielded a lot of rather interesting properties relating to their dynamics. The procedure below admits four arguments, that have the following meanings**:**

A – an alphabet ***A***={***0,1,2,…,a–1***} of a classical ***1–CA*** model**;**
S – an initial string of the ***1–CA*** model**;**
n – neighbourhood template size of the ***1–CA*** model**;**
m – the limit number of steps of the ***1–CA*** model**.**

A successful call of the **RandomLTF3[***A, S, n, m***]** procedure prints finite configurations generated during ***m*** steps from an initial ***S*** configuration, given in an alphabet ***A.*** The fragment below represents the source code of the **RandomLTF3** procedure with an example of its application.

```
In[6123]:= RandomLTF3[A_List, S_String, n_Integer, m_Integer] :=
                  Module[{a = Length[A], b = Map[ToString, A], c, d, h, j, l, g,
                u = StringRepeat["0", n – 1], s = StringTrim[S, "0" ...], w = ""},
                                  c = Map[StringJoin, Tuples[b, n]]; Print[s];
            Do[d = RandomChoice[b, a^n]; d[[1]] = "0"; h = GenRules[c, d];
                                  g = u <> s <> u; l = StringLength[g];
                                                For[j = 1, j <= l – n + 1, j++,
     w = w <> StringReplace[StringJoin[StringPart[g, j ;; j + n – 1 ;; 1]], h]];
                                  s = StringTrim[w, "0" ...]; Print[s]; w = "", m]]
```

```
In[6124]:= RandomLTF3[{0, 1, 2, 3, 4, 5, 6}, "126320140621512622", 5, 1]
           "126320140621512622"
           "3242505303420114442255"
```

This procedure allows for a number of interesting modifications which allow empirically investigating various dynamic properties of the above class of ***CA*** models [24,43,82,102,106].

A computer research of ***1–***dimension classical ***CA*** models for possession by them of the self**–**reproducibility property in the *Moore* sense of finite configurations revealed a number of classes of such models whose local transition functions are different from the above linear [24,41,82]. As an example of such models, we first consider a classical ***1–CA*** model with an alphabet $A = \{0, 1, 2, ..., a\}$ $(a+1 = p^h; p, h$ *are primes)* the neighborhood index $X = \{0, 1\}$ and with local transition function, presented by the table of parallel substitutions as follows**:**

$00 \to 0$	$10 \to a$	$20 \to a–1$	$30 \to a–2$	... ...	$a0 \to 1$
$01 \to 1$	$11 \to 0$	$21 \to a$	$31 \to a–2$	... ...	$a1 \to 2$
$02 \to 2$	$12 \to 1$	$22 \to 0$	$32 \to a$	... ...	$a2 \to 3$
$03 \to 3$	$13 \to 2$	$23 \to 1$	$33 \to 0$	... ...	$a3 \to 4$
$04 \to 4$	$14 \to 3$	$24 \to 2$	$34 \to 1$	... ...	$a4 \to 5$
$05 \to 5$	$15 \to 4$	$25 \to 3$	$35 \to 2$	... ...	$a5 \to 6$
$06 \to 6$	$16 \to 5$	$26 \to 4$	$36 \to 3$	... ...	$a6 \to 7$
$07 \to 7$	$17 \to 6$	$27 \to 5$	$37 \to 24$	... ...	$a7 \to 8$
... ...	... ...	... ...	... ...	... ...	... ...
$0a \to a$	$1a \to a–1$	$2a \to a–2$	$3a \to a–3$	... ...	$aa \to 0$

The following scheme perfectly illustrates the principle of organization of parallel substitutions that determine the local transition function of a classical ***1–CA*** model with neighborhood index $X = \{0, 1\}$ and the states alphabet $A = \{0, 1, 2, 3, 4, 5, 6\}$ of an elementary automaton of such model. Thus, from such scheme, it follows that the parallel substitutions with left parts, presented by a specially organized tuples $\{xy\}$ $(x,y \in A)$, accept values from the alphabet A as the right parts, that are symmetrically up and down relative to the line of zero values.

$00 \to 0$ $10 \to 6$ $20 \to 5$ $30 \to 4$ $40 \to 3$ $50 \to 2$ $60 \to 1$
$01 \to 1$ $11 \to 0$ $21 \to 6$ $31 \to 5$ $41 \to 4$ $51 \to 3$ $61 \to 2$
$02 \to 2$ $12 \to 1$ $22 \to 0$ $32 \to 6$ $42 \to 5$ $52 \to 4$ $62 \to 3$
$03 \to 3$ $13 \to 2$ $23 \to 1$ $33 \to 0$ $43 \to 6$ $53 \to 5$ $63 \to 4$
$04 \to 4$ $14 \to 3$ $24 \to 2$ $34 \to 1$ $44 \to 0$ $54 \to 6$ $64 \to 5$
$05 \to 5$ $15 \to 4$ $25 \to 3$ $35 \to 2$ $45 \to 1$ $55 \to 0$ $65 \to 6$
$06 \to 6$ $16 \to 5$ $26 \to 4$ $36 \to 3$ $46 \to 2$ $56 \to 1$ $66 \to 0$

The above scheme is quite transparent, applies to any finite alphabet A and does not require any special explanations.

The above scheme is easily programmed as a certain procedure, whose call **SymmetricLTF[*A*],** where ***A*** argument specifies an alphabet **A={*0,1, ..., a*}** ***(a≤9)*** of a classical ***1–CA*** model with neighborhood index ***X*={*0, 1*}**, returns the list of parallel substitutions that determine the desired local transition function of the model. With the source code of the procedure the interested reader can familiarize in [49].

In certain cases, the ***1–CA*** models thus obtained may coincide with the linear ones. The above procedure is extended on case of ***1–CA*** models with an arbitrary neighborhood index ***X*={*0,1,…,n–1*}**. It would be very interesting to extend the above technique to the case of the ***CA*** models of higher dimensionalities.

In light of the foregoing, it is advisable to clarify the effect of symmetry of local transition functions on the self–reproducibility property in the *Moore* sense of finite configurations in classical ***CA*** models. Below, is an example of a strictly linear local transition function with alphabet ***A*={*0, 1,2,3,4,5,6*}** and the neighborhood index ***X* = {*0, 1*}**, which has symmetry relative the main diagonal along with the self–reproducibility property in the *Moore* sense of the finite configurations.

00 → 0 10 → 1 20 → 2 30 → 3 40 → 4 50 → 5 60 → 6
01 → 1 11 → 2 21 → 3 31 → 4 41 → 5 51 → 6 61 → 0
02 → 2 12 → 3 22 → 4 32 → 5 42 → 6 52 → 0 62 → 1
03 → 3 13 → 4 23 → 5 33 → 6 43 → 0 53 → 1 63 → 2
04 → 4 14 → 5 24 → 6 34 → 0 44 → 1 54 → 2 64 → 3
05 → 5 15 → 6 25 → 0 35 → 1 45 → 2 55 → 3 65 → 4
06 → 6 16 → 0 26 → 1 36 → 2 46 → 3 56 → 4 66 → 5

We now define a group of classical ***1-CA*** models with symmetrical local transition functions and try to find out relations with reproducibility in the *Moore* sense of finite configurations. For clarity we consider a certain specifical local transition function of classical ***1-CA*** model with alphabet ***A*={*0,1,2,3,4*}** which belongs to this group.

00 → 0 10 → b 20 → c 30 → d 40 → e
01 → b 11 → c 21 → d 31 → e 41 → 0
02 → c 12 → d 22 → e 32 → 0 42 → b ***(aks)***
03 → d 13 → e 23 → 0 33 → b 43 → c
04 → e 14 → 0 24 → b 34 → c 44 → d

In addition, the tuples **<*b, c, d, e*>** are arbitrary permutations of elements ***b,c,d,e*∈*A*\{*0*}**. The defining principle of the symmetrical local transition

functions of ***CA*** models of the above group is easily seen from the ***(aks)*** scheme. Obviously, the number of elements of such group is ***(a–1)!.***

Theoretically and based on computer analysis shown [24,41,82] that the models of this group possess the property of universal or essential self–reproducibility in the *Moore* sense of finite configurations and have a rather interesting dynamics of generating copies of a self–reproducing finite configuration. In particular, in contrast to the linear classical ***1-CA*** models, which also belong to the above group the other models of the group differ on the whole by an essentially lower copy generation rate of self–reproducing finite configurations first of all with increasing size of an ***A*** alphabet. In addition, other things being equal, the generation speed of copies of finite configurations is maximum for classical strictly linear ***1–CA*** models.

Thus, it is completely natural to assume that the property of universal reproducibility in the *Moore* sense of finite configurations in classical ***CA*** models is in the first instance based on a certain kind of complete or substantial symmetry of local transition functions respect to the main diagonals of structurally located rules of substitutions that define local transition functions along with the lack of the ***NCF*** nonconstructability in the presence of the ***NCF-1*** nonconstructability, but not their linearity. In addition, for the alphabet ***A={0,1,2,3, ..., a}*** at ***(a+1) = p^h,*** where ***p, h –*** primes or ***h=1*** the generation speed of copies of initial configurations of the same length, as a rule, depends on the type of symmetry of local transition functions of the above described type.

So, a classical ***1–CA*** model with local transition function determined by parallel substitutions ***(aks)*** from the randomly generated configuration ***w*** = "42030334402223240011324211213123434104322424332031 30" will generates a configuration containing *62* disjoint copies of configuration ***w*** in *15500* steps, whereas the strictly linear classical ***1–CA*** model with the same alphabet ***A*** and neighbouring index generates *62* disjoint will copies of the same configuration ***w*** also in *15500* steps, while for other types of ***CA*** models that have the reproducibility property in the *Moore* sense of finite configurations, this speed is quite difficult to estimate.

So, a ***1–CA*** model from this group with alphabet ***A={0,1,2,3}*** from the initial configuration ***w***="2301202302230103110320 3" generates a certain configuration that contains *32* disjoint copies of the configuration ***w*** in *4032* steps, whereas a model from the same group with alphabet ***A={0, 1, 2, 3, 4, 5}*** from initial configuration ***w***="250102040" generates a certain configuration that contains *13* disjoint copies of the configuration ***w*** in

14328 steps. In addition, shown that the generation speed of copies of the reproducing configurations is defined by the size of neighborhood template, the length of ***A*** alphabet along with internal structure of finite configurations of the same length.

The structure of the above local transition function is generalized to the case of classical ***1–CA*** models with neighborhood index $X = \{0,1,...,n-1\}$ and the alphabet $A=\{0,1,...,a-1\}$ as follows. All possible tuples $<x1...xn>$ $(x_j \in A;\ j=1..n)$, on which the local transition function $\sigma^{(n)}(x_1,x_2,...,x_n)$ of our ***1–CA*** model is determined, are grouped as follows

$x1x2...x_{n-1}$	0	$\rightarrow x`_0$
$x1x2...x_{n-1}$	1	$\rightarrow x`_1$ *(gsv)*
========	===	$a = p^h$; p,h – *primes*; $x_j \in A$; $j=1..n-1$
$x1x2...x_{n-1}$	$a-1$	$\rightarrow x`_{a-1}$

Obviously, from all tuples $<x1x2x3x4\ ...\ xn>$ we receives a^{n-1} of such ordered groups. Then, starting from the left in the set of such ordered groups, we consistently select on ***a*** groups of format *(gsv)* on which the local transition function $\sigma^{(n)}(x_1,x_2,...,x_n)$ is defined similarly to the above ***1–CA*** with minimal neighborhood index. As an illustration of structure of local transition functions for the general case of ***1–CA*** models of the above group, we will give an example ***1–CA*** model, determined as this way, with neighborhood index $X=\{0,1,2\}$, alphabet $A=\{0,1,2\}$ along with local transition function defined by the parallel substitutions as follows:

$000 \rightarrow 0$ $010 \rightarrow 2$ $020 \rightarrow 1$ $100 \rightarrow 0$ $110 \rightarrow 2$ $120 \rightarrow 1$ $200 \rightarrow 0$ $210 \rightarrow 2$ $220 \rightarrow 1$
$001 \rightarrow 1$ $011 \rightarrow 1$ $021 \rightarrow 2$ $101 \rightarrow 1$ $111 \rightarrow 1$ $121 \rightarrow 2$ $201 \rightarrow 1$ $211 \rightarrow 1$ $221 \rightarrow 2$
$002 \rightarrow 2$ $012 \rightarrow 0$ $022 \rightarrow 0$ $102 \rightarrow 2$ $112 \rightarrow 0$ $122 \rightarrow 0$ $202 \rightarrow 2$ $212 \rightarrow 0$ $222 \rightarrow 0$

Numerous computer investigations have shown that ***1-CA*** models, thus defined, have the property of self–reproducibility in the *Moore* sense of finite configurations [24,82,102,106]. Meanwhile, it should be noted once more that the generation speed of the required number of copies in such ***1–CA*** models is incommensurably lower than in the case of linear ***1–CA*** models on condition of the identity of their alphabets and neighborhood indexes, and generation process itself is difficult to formalize as a whole.

So, the ***1–CA*** model, determined in this way, with neighborhood index $X=\{0,1,2\}$ and alphabet $A=\{0,1,2\}$ from finite configuration ***t***="*21021021*" in *17116* steps generates *12* subcopies of ***t*** in the resulting configuration, while the ***1–CA*** model with neighborhood index $X=\{0,1,2\}$ and alphabet $A = \{0,1\}$ along with local transition function determined by the parallel substitutions as follows:

$$000 \to 0 \quad 010 \to 1 \quad 100 \to 0 \quad 110 \to 1$$
$$001 \to 1 \quad 011 \to 0 \quad 101 \to 1 \quad 111 \to 0$$

from configuration w="*10111001011001001100001111010011110*" in *4032* steps generates *64* subcopies of w in the resulting configuration.

Another typical example is the ***1–CA*** model with alphabet $A=\{0,1\}$ and neighborhood index $X=\{0,1,2,3\}$ with local transition function defined by the parallel substitutions as follows**:**

$$0000 \to 0 \quad 0010 \to 1 \quad 0100 \to 0 \quad 0110 \to 1 \quad 1000 \to 0 \quad 1010 \to 1 \quad 1100 \to 0 \quad 1110 \to 1$$
$$0001 \to 1 \quad 0011 \to 0 \quad 0101 \to 1 \quad 0111 \to 0 \quad 1001 \to 1 \quad 1011 \to 0 \quad 1101 \to 1 \quad 1111 \to 0$$

By means of such ***1–CA*** model from randomly defined configuration h= "*11110110111111001101001001110011010100000000010101*" in *8128* steps are generated *128* subcopies of h in the resulting configuration, whereas *255* copies of an initial configuration of length *62* are generated in *16320* steps of the above ***CA*** model. The ***1-CA*** models, thus defined, including different from linear models have the self**–**reproduction property.

Without loss of generality, the following fragment represents a scheme of organization of symmetric local transition functions for the classical ***1–CA*** model with alphabet $A=\{0,1,2\}$ and neighborhood index $X=\{0,1,2\}$. The scheme *(j)* presents two variants of the symmetry of local transition functions — with respect to the two main diagonals of sub-blocks of the parallel substitutions that make up a common block of ordered parallel substitutions that define the local transition function of the ***1-CA*** model.

$$\begin{array}{lll} 000 \to 0 & 010 \to a & 020 \to b \\ 001 \to a & 011 \to b & 021 \to 0 \\ 002 \to b & 012 \to 0 & 022 \to a \end{array} \qquad \begin{array}{lll} 000 \to 0 & 010 \to b & 020 \to a \\ 001 \to a & 011 \to 0 & 021 \to b \\ 002 \to b & 012 \to a & 022 \to 0 \end{array}$$

(1) *(2)*

$$\begin{array}{lll} 100 \to 0 & 110 \to a & 120 \to b \\ 101 \to a & 111 \to b & 121 \to 0 \\ 102 \to b & 112 \to 0 & 122 \to a \end{array} \qquad \begin{array}{lll} 100 \to 0 & 110 \to b & 120 \to a \\ 101 \to a & 111 \to 0 & 121 \to b \\ 102 \to b & 112 \to a & 122 \to 0 \end{array}$$

(j)

$$\begin{array}{lll} 200 \to 0 & 210 \to a & 220 \to b \\ 201 \to a & 211 \to b & 221 \to 0 \\ 202 \to b & 212 \to 0 & 222 \to a \end{array} \qquad \begin{array}{lll} 200 \to 0 & 210 \to b & 220 \to a \\ 201 \to a & 211 \to 0 & 221 \to b \\ 202 \to b & 212 \to a & 222 \to 0 \end{array}$$

In the above model the states satisfy the relations $a,b \in A\setminus\{0\}$ $(a \neq b)$. The ***1–CA*** model thus defined possesses the ***NCF–1*** nonconstructability in the absence of the ***NCF*** nonconstructability. Obviously, the number of various classical ***1-CA*** models with local transition functions defined by

means of the above schemes *(j)* is ***2(a–1)!.*** Theoretically and based on a rather extensive computer analysis was shown [7,82], that the following interesting enough assumption can be formulated**:**

Classical 1-CA models whose local transition functions are determined by means of relations of the type (j) have an essential or universal self–reproducibility of finite configurations in the Moore sense. This result can be generalized for the case of d–CA models of higher dimensions.

To determine the local transition function of an arbitrary classical ***1–CA*** model with symmetry of the forms *(j)*, the following **SymmetricalLTF** procedure was programmed in ***Mathematica*** software. The procedure call **SymmetricalLTF[*f, s, m, r*]** returns the local transition function that determined by the list of symmetrical parallel substitutions in the form *(j)* in accordance with an alphabet ***f*** **=** {***0,1,2,…,a–1***} and neighbourhood index ***m*** **=** {***0,1,…,m–1***}**;** in addition, the second ***s*** argument defines a list whose the first ***0*** element is the quiescent state of a ***1-CA*** whereas others define the states for replacing of parameters ***a,b,…*** in the corresponding scheme of symmetrical local transition function of the desired ***1–CA.*** At last, the fourth ***r*** argument is optional, defining the mode of forming of symmetry type of local transition function – at its absence the type ***(j, 2)*** is used, otherwise the type ***(j, 1)*** is used. The procedure processes main espessial situations with printing of the appropriate messages. Below is the source code of the procedure with examples of its application.

```
In[4773]:= SymmetricalLTF[f_List, s_List, m_Integer, r___] :=
             Module[{a = Sort[f], h = Length[j], b, c, d = {}, n = Length[f], k},
                                    If[DeleteDuplicates[f] != f,
        Return["List " <> ToString[f] <> " contains duplicated elements"],
                                    If[DeleteDuplicates[s] != s,
        Return["List " <> ToString[s] <> " contains duplicated elements"],
                                    If[! MemberQ[f, 0] || ! MemberQ[s, 0],
        Return["Both alphabets do not contain a quiescent state 0"],
                                    If[! MemberQ3[f, s],
        Return["Invalid mismatch of elements of first two arguments"],
                                b = Map[StringJoin, Map[ToString2, Tuples[a, m]]];
                  c = Gather[b, StringTake[#1, m – 1] == StringTake[#2, m – 1] &];
                                b = Length[c]; c = If[m > 2, Partition[c, n], c];
                                    For[k = 1, k <= If[m > 2, n, 1], k++,
                                Do[AppendTo[d, GenRules[If[m > 2, c[[k]], c][[j]],
                                                        Map[ToString2,
        If[{r} != {}, RotateLeft, RotateRight][s, j – 1]]]], {j, n}]]; Flatten[d]]]]]
```

```
In[4774]:= j = SymmetricalLTF[{0, 1, 2}, {0, 2, 1}, 3, 76]
Out[4774]= {"000" → "0", "001" → "2", "002" → "1", "010" → "2",
      "011" → "1", "012" → "0", "020" → "1", "021" → "0", "022" → "2",
      "100" → "0", "101" → "2", "102" → "1", "110" → "2", "111" → "1",
      "112" → "0", "120" → "1", "121" → "0", "122" → "2", "200" → "0",
      "201" → "2", "202" → "1", "210" → "2", "211" → "1", "212" → "0",
      "220" → "1", "221" → "0", "222" → "2"}
In[4775]:= Reproduction3[j, "02121201220110212121020221101021", 122]
Out[4775]= {122, 20000}
```

Sharing of procedures **SymmetricalLTF** and **Reproduction3** for computer analysis of self-reproducibility in the *Moore* sense in the classical ***1-CA*** models proved to be rather effective. Of the above example, it can be seen that the generation speed of copies of an initial finite configuration in the above ***1-CA*** models can be rather low, depending essentially on the sizes of the alphabet of the model, its neighborhood index and the initial configuration itself first of all.

Till now, we considered the classical ***CA*** models from the standpoint of their *maximal* generative possibilities relative to the set $C(A,d,\phi)$ of finite configurations regardless of the order of their generating. However, an question about the possibility of generating by means of some classical ***CA*** model of the *preset* history of finite configurations, i.e. a sequence of configurations $<c_o>[\tau^{(n)}]$ in its dynamics directly adjoins to the problem. Thus, generally speaking the question can be formulated as follows**:**

Whether exists for a preset history of finite configurations $\Omega=\{c_o \to c_1 \to c_2 \to c_3 \to c_4 \to ... \to c_k \to ...\}$ $c_o \in C(A,d,\phi)$***, that are given in a finite states alphabet A, a global transition function*** $\tau^{(n)}$ ***that is defined in the same alphabet A and generates the above configurations history, i.e. whether the relation*** $<c_o>[\tau^{(n)}] = \Omega$ ***can take place?***

It is simple to make sure that the answer to this question is negative in general [5,8,12,82-87]. Furthermore, in algorithmic context the problem of definition of a possibility of generating of a certain **Ω-**history of finite configurations by means of a classical ***CA*** model is unsolvable [8,42,84]. With important questions of more practical approach to the problem of implementation of self**–**reproducing industrial automata the reader can familiarize in a rather interesting popular scientific book [146]**,** while in a lot of works it is possible to familiarize with other interesting enough discussions of self**–**reproducibility in ***CA*** models [7]. In light of rapidly

developing of nanotechnologies this problematics seems to us a rather actual**;** the steady growth of intense interest to the given problematics is undoubted evidence of it [7]. In addition from standpoint of research of fundamental properties of ***CA*** models it would be utterly desirable to define and research other certain fruitful concepts of the universality of ***CA*** models that are distinct from the computability and reproducibility of finite configurations.

Investigation of properties of special types of configurations *(periodical*, *passive*, *vanishing*, *etc.)* in the classical ***CA*** models represent interest both highly specific and of this or that level of community for ***CA*** problems. So, the passive configurations play an important part in the case when the classic ***CA*** models are considered as an algorithm of parallel words processing in finite alphabets and at embedding them in various types of processes and models. A number of questions relating to passive and vanishing configurations is discussed in [84]. From standpoint of study of stable trajectories of dynamics of classical ***CA*** models a quite defined interest present the periodical finite configurations. From the following follows, that the problem of determination, whether an arbitrary finite configuration as a periodic in a classical ***CA*** model will be unsolvable.

Meanwhile, in the applied attitude the classical concept of ***d–CA*** ***(d ≥ 1)*** appears inconvenient in a lot of cases at modelling of the complicated enough discrete processes, objects, and also at study of many aspects of the dynamics of the classical ***CA*** models themselves. Such modelling *(as a matter of fact undermost level of parallel symbolic programming)* becomes complex, not sufficiently visual and inefficient. In addition, the essence of some modelled processes urgently demands definite modification of classical ***CA*** concept. With this purpose we have defined a special class of models *(further designated as* ***d–CA*****)* [24-28] which are to some extent similar to neural networks or nervous tissues**;** they well enough reflect a principle of functioning of many types of electronic systems.

The ***d–CA**** models well enough meet the *base* general requirements and aspects in biology of development at the cellular level, along with base principles of functioning of parallel computing systems. All aspects of development of multicellular systems contain intercellular interactions whose mechanism in own basics is complex enough and many**–**sided. However, a number of its very important phenomena can be modelled quite well by a spreading of special control impulses in the ***CA**** models. On the basis of these ***d–CA**** models it is possible to simulate essentially more adequately also phenomena of morphogenetic fields that now are

very actively studied in various aspects. In addition, modern ideas and hypotheses in development biology [8,12,13,24-28,31,33,40-43] enough definitely specify perspective of use of ***d–CA**** models as a comfortable formal simulation environment of many phenomena from this area and applied areas adjoining to it.

In particular, ***d–CA**** ***(d≥1)*** models is a rather convenient environment of modelling of a lot of biological processes and phenomena such as *neural* networks, processes in molecular liquids and membranes, development of populations at both cellular and individual levels, etc. [24-28,106]. So, many of the listed aspects lay in the basis of neuro–computers, forming one of major components of interface between biologic and computing sciences [7,24,102,106]. Perspective use of ***CA**** models is supposed in a lot of other important fields. As a model of excitable environments the ***d–CA**** models provide their major characteristic feature – a possibility of transfer of the control impulses on distances of any length and with the necessary speeds, allowing enough simply to create wave–fronts of excitations distribution of various kinds in a modelling environment.

Nonformally the ***CA**** are defined as follows. An individual automaton in ***d–CA**** can receive information directly from the nearest neighbours and can synchronously change own state, and emit control impulses at discrete moments $t > 0$ as a function of the current state and incoming control impulses. Without loss of generality and more formally we shall define ***d–CA**** models for the most simple one-dimensional case *(fig. **9**)*. The models of this class can be considerably more easily generalized to an arbitrary dimensionality than the others $(d \geq 2)$.

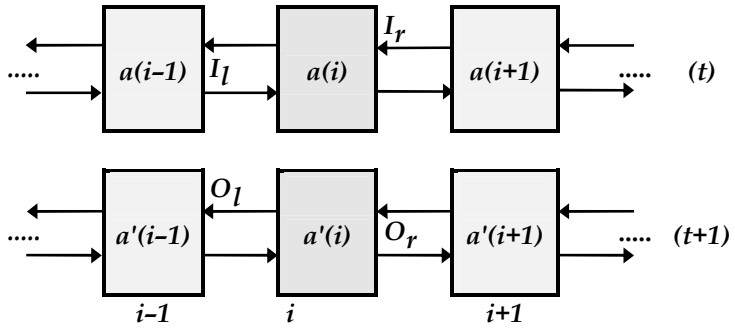


Fig. 9. The principal scheme of functioning of a 1–CA* ***model.***

A ***CA**** by definition is a four-element tuple $CA^* \equiv \langle Z^1, A, I, \varphi \rangle$, where Z^1 and A are defined just as for classical ***1-CA***, I is a set of impulses and φ is a *functional algorithm (**FA**)* of the ***CA****. We shall associate an elementary

automaton with each point i of Z^1 and shall identify an automaton with I; FA φ is defined by the following discrete equations, namely:

$$\begin{cases} a'(i)_{t+1} = S[i_r, a(i), i_l]_t \\ (j'_r)_{t+1} = R[i_r, a(i), i_l]_t \\ (j'_l)_{t+1} = L[i_r, a(i), i_l]_t \end{cases} \quad a'(i), a(i) \in A; \quad j'_r, j'_l, i_r, i_l \in I; \; t = 0,1,2,... \qquad (14)$$

where $a'(i)$, $a(i)$ – states of i–automaton; i_r, i_l – a right and left incoming impulses of an i-automaton accordingly; whereas j'_r, j'_l are right and left output impulses of an i-automaton accordingly; at last, S, R & L – choice functions defining the next state, *output* impulse to the right, and *output* impulse to the left accordingly. Thus, essence of functioning of ***1–CA****, defined thus, is rather simple and consists in following *(fig. 9)*. Being in discrete moment $t \geq 0$ in a state $a(i)$ and receiving on input the control impulses j_r *(on the right)* and j_l *(at the left)*, at the following moment $(t+1)$ the i-automaton passes into state $a'(i)$ and emits control impulses j'_r *(to the right)*, j'_l *(to the left)*, that are determined according to the equations system *(12)*. Thus, the output impulses of an arbitrary i–automaton are input impulses for all its direct neighbours.

Obviously, if input impulses of an i–automaton coincide with states of its nearest neighbours $(i-1, i+1)$, whereas output impulses coincide with its state, the ***1–CA**** models and classical ***1–CA*** models with the *Moore* neighbourhood index are identical and condition $I \equiv A$ & $I \cup A = A$ takes place. Therefore, ***d–CA**** is an equivalent modification of classical ***d–CA*** models that is considerably more adapted for study of a lot of applied aspects of ***CA*** problematics. Numerous examples of concrete use of ***CA**** models have confirmed their high enough efficiency first of all from the applied standpoint [8,12,13,24-28,31,33,40-43,102,106].

It is shown that ***d–CA**** models it is possible to exploit successfully as an an quite satisfactory intermediate stage at modelling in classical models and at researches of some questions of their dynamics [24]. This fact is put in the basis of that approach what an arbitrary ***d–CA**** $(d \geq 1)$ can be constructively embedded into a classical ***d–CA*** model. In particular, it is shown that: ***An arbitrary 1–CA**** $\equiv \langle Z^1, A, I, Fa \rangle$ ***model is equivalent to a classical 1–CA*** $\equiv \langle Z^1, A \cup I, \tau^{(7)}, X \rangle$ ***model with neighbourhood index*** $X = \{-3,-2,-1,0,1,2,3\}$ [31,33]. Using a rather simple approach [8,12,40-43] and representing the states of a modelling classical ***1–CA*** model in a special kind it is rather simple to be convinced of validity of the result.

Theorem 66. ***An arbitrary 1–CA* ≡ <Z^1,A,I=$0_l \cup 0_r$,Fa> model is modeled strictly real time by means of an appropriate classical 1-CA model with neighbourhood index X={-1,0,1} and states alphabet A*=A∪0_l∪0_r where 0_l and 0_r – sets of output impulses of elementary automata of the 1-CA* to the left and to the right accordingly.***

A number of other results concerning the equivalence, including strict equivalence, of ***CA**** models and classical ***CA*** models has been received. In any case quite pertinently to notice the following circumstance – for a theoretical study of a formal cellular model the classical ***d–CA*** models are more preferable whereas ***d–CA**** models represent in many respects more acceptable environment for modelling concrete objects, i.e. both classes of the models represent as if two different sides of the classical cellular model of parallel information processing [102]. Let's illustrate a lot of opportunities of ***CA**** models by an example of solving of a rather known *Problem of Limited Growth (**PLG**)* that is a typical representative of minimax problems in the ***CA*** theory.

In a lot of cases the research of sequences $<c_o>[\tau^{(n)}]$ includes important enough question such, as existence in the sequences of so–called *passive configurations (**PCF**)*, i.e. configurations g for which the condition $g\tau^{(n)} = g$ takes place. So, certain authors have researched a problem consisting in definition of classical ***CA*** models allowing to generate from a rather simple initial finite configurations the ***PCF*** of greatest possible size that depends on size of neighbourhood template.

As a kind of the extremal problem connected with ***PCF***, the problem of *Gaisky–Yamada* consisting in ascertainment of the greatest possible size ***PCF***, generated by a classical ***d-CA (d≥1)*** model from some simple initial configuration, but without emphasis on connection of its size with size of neighbourhood template of the models has been considered [9,24,147-149]. A number of interesting enough questions of growing of chains of automata of the given length can be found also in works [24,102,106].

Now, we shall consider so-called *Problem of Limited Growth (**PLG**)* in the classical ***CA*** models, most directly related to the above ***Gaisky–Yamada*** problem, with a class of minimax problems in the ***CA*** problematics that present doubtless gnoseological interest from the standpoint of various developing cellular systems. Due to the technical complexities, arising at embedding of rather complicated algorithms to classical ***CA*** models, we for solution of ***PLG*** chose class of the structures ***CA**** defined above. Considerations in favour of similar solution with certain accompanying

thoughts can be found in our works [24-28,42,82-87,102,106].

Without loss of generality we determine the ***PLG*** concerning a class of the simplest ***1–CA**** models. A finite w_o configuration of the following kind w_o = □*gg...gg*□ {where $|w_o|=r$} of length ***r*** of states ***g*** of elementary automata in ***1–CA**** is being set. Then ***PLG*** is reduced to definition of a functional ***Fa*** algorithm in the ***1-CA**** that allows to grow from an initial configuration w_o a passive configuration of kind ***w*=□*FFF ... FFF*□** of the greatest possible size $L = L(w_o, Fa)$. The following basic result is the best solution of ***PLG*** known for today [5,9,12,13,24-28,32,33,42,102,106].

Theorem 67. For 1–CA* ≡ <Z^1,A,I,P> model with #A=12 and #I=4m+17, {***where m – possibly minimal speed of spreading of control impulses in the model, #T – cardinality of a T set***} ***there is a functional P algorithm allowing to grow from an initial finite s_o configuration of r length a s configuration of length L of elementary automata in a «q» state where L size is determined by the following recursive formulas, namely:***

$$L = r(2m+1)^{\sum_{j=0}^{n} \varpi_j^2+2(2^r+1)}, \quad \varpi_0 = 2^{4rm(m+1)}, \quad \varpi_j = 2(L_j - r)$$
$$L_1 = r(2m+1)^{\varpi_0^2+2}, \quad L_j = L_{j-1}(2m+1)^{2^{2L_{j-1}+2}+2} \qquad (15)$$

For growing of a final configuration of chain of automata of the preset L length for functional P algorithm is required t =]1/2*m + 3/2[*L steps of the simulating 1–CA* model.

The essence of realization of one such functional algorithm solving the ***PLG*** in ***1–CA**** is represented, for example, in [84]. On the basis of such offer it is possible to research its various modifications, allowing rather substantially to improve the result presented in theorem ***67*** [32,33,102]. So, time of growing of chains of elementary automata of the specified fantastic length does not exceed their double length and with increase of ***m*** size, which very essentially influences the length of the growing chain, asymptotically approaches the limit ***t =]3/2*L[.***

Obviously, theoretical limit of growing time of chain of automata of a ***L*** length in a certain ***1–CA**** model is ***t=]L/2[,*** but the above functional ***Fa*** algorithm does not allow to achieve this limit. Meantime a modification of the ***Fa*** algorithm used for the above solution of ***PLG,*** which allows to grow a chain of elementary automata at the same initial preconditions by time, is equal asymptotically ***t =]1/2+1/2m[*L,*** and with length of the following size, namely**:**

$$L = r*(2m+1)^{4^{r+1}+3} - 2m$$

An analysis of the functional algorithms which solve the ***PLG*** allows to partition them into **2** rather large classes which essentially differ among themselves [5,9,12,13,24-28,32,33,102,106]**,** namely**:**

(1) Algorithms whose essence consists in constant backing of growth of a figure before reception of a control stopping impulse (signal);
(2) Algorithms whose essence consists in preliminary marking of shape of a growing figure with the subsequent filling of it by means of certain final F symbols (fillers).

The functional algorithm underlying the above of the first solution of the ***PLG,*** concerns to the second class, whereas algorithm optimal with respect to time **–** to the first. Meanwhile, the further complication of the given functional algorithm allows to improve a certain limit of sizes of configurations, growing in ***CA**** [8] and in this connexion an interesting enough question arises, namely**:**

Whether exist functional Fa algorithms which use any other ideas and allow to obtain the essentially better results concerning the growing of finite configurations of the maximal size other things being equal?

Detailed enough discussion of the ***PLG,*** its applied aspects along with a lot of other related problems *(firing squad synchronization problem, French Flag problem, etc.)* can be found in the works [5,7-9,24-28,31-33,40-43,106]. The above concept of ***1-CA**** models can be enough easily generalized to the general case of ***d–***dimensionality $(d \geq 2)$.

It is easy to show, that in the presence in a classic ***CA*** model of periodic configurations with minimal ***p*** period their set is infinite, and periodic configurations of infinite large size with the same ***p*** period exist. If in a ***CA*** model there are periodic configurations with minimal periods ***p*** and ***q*** $(p \neq q)$, then in it exist at least and periodic configurations with *minimal* period $g = g(p,q) = LCM(p,q)$ where ***LCM –*** the least common multiple of ***p*** and ***q.*** In this connexion the following **2** basic questions arise, namely**:** ***(1)*** obtaining of upper bounds for size of minimal periods as a function of the basic parameters of a ***CA*** model, and ***(2)*** detection of algorithmic solvability of the presence problem in a classical ***CA*** model of periodic configurations, except the trivial case of periodic zero configuration. In the case of classical ***CA*** models the low bound of size of minimal period was received which is expressed by the following result [5,12,]**,** namely**:**

There are classical d–CA models with the Moore neighbourhood index

that have periodic finite configurations with minimal period $p \geq 2|c|-2$, where $|c|$ – diameter of c configurations. Moreover, there are classical 1–CA models with the neighbourhood index $X=\{-3,-2,-1,0,1,2,3\}$ which have the periodic finite configurations with minimal period $p \geq 2L(|c|)$ where L quantity is determined from the relations (15).

The second part of the result indicates the existence in classical ***1–CA*** of periodical configurations of relatively small size with fantastically large size of a minimal period. In addition, the positive solution of the above *first* question entails algorithmic solvability of the *second* question while the algorithmic unsolvability of the *second*, in turn, entails the negative solution of the first question. Today both these questions remain open even for ***1–***dimensional case.

*The **CA** models on splitting (**CAoS**)* determined above represent a special interest for the problem of physical modelling, allowing a rather simple programming of such fundamental property as dynamics reversibility. In this connection concerning the class of ***CAoS*** models quite naturally arise questions of possibility of existence for them of *universal* and *self–reproducing* finite configurations that to a certain extent can characterize the extreme constructive possibilities of this class of models. There and then once again it is necessary to note that the used term «*models on the splitting*» is equivalent to the term «*Margolus neighbourhood index*»**.** The detailed substantiation of our term can be found in [24,40-43,102,106].

Above all, concerning the existence problem of universal configurations ***(UCF)*** in ***d–CAoS*** models the result analogous to the considered case of classical ***d–CA*** models takes place, namely**:**

An arbitrary d–CAoS ($d \geq 1$) model can't possess a finite set of the UCF.

Meanwhile, there is a very different picture concerning the question of existence in ***d–CAoS*** models of self**–**reproducing finite configurations in the *Moore* sense. It is shown that ***CAoS*** models can possess the essential or universal reproducibility of finite configurations in the *Moore* sense and their dynamics will be reversible. However, analogously to the case of classical ***CA*** models here the negative result takes place**,** namely**:**

There is not any 1–CAoS model which can double a finite configuration determined in the same states alphabet of the 1–CAoS.

With these and related questions the interested reader can familiarize in [24-28,31,33,40-43,82-87,102,106].

Chapter 4. The complexity problem of finite configurations in the classical *CA* models

Complexity in all its generality is one of the most intriguing and vague concepts of the modern natural sciences. Intuitive essence of the concept is the main reason of it to a large extent, in our opinion. Note, that the most fundamental problem of development is the understanding how a system can self–complicate itself and as far as a complexity of the initial system should be large for this purpose. One of complexities in solution of this problem grandiose in many respects is absence of a satisfactory measure of complexity. Moreover, it is quite possible that for the general concept of complexity an unified approach simply absent in spite of the fact that in this direction a lot of attempts has been done.

Thus, research in this direction are extremely desirable. Meanwhile, in view of use of the classical ***CA*** models as a formal basis of modelling in the developmental biology along with research of the parallel discrete dynamic systems the questions connected to the complexity concept in ***CA*** models seem rather actual. The special urgency for this problems is given by that circumstance, that ***CA*** models find more and more wide use as conceptual models of the spatially–distributed dynamic systems from which different physical systems seem to us the most interesting [5,8,9,24,40-43,102,106]. In this chapter our basic results of research on the complexity of finite configurations in classical ***CA*** models as well as the questions connected to them are represented.

For formal modelling of different discrete processes and phenomena in classical ***CA*** models, dynamics of initial finite configurations represents the greatest interest. Indeed, a certain modelled process is presented in the dynamics of a classical ***CA*** model by an appropriate history of initial finite configurations. In this context the question of complexity of finite configurations composing a history of process or object modelled in a classical ***CA*** model all by itself arises. Today, three basic approaches to definition of concept «*quantity of information*» associated with concept of complexity of the finite objects are known**:** *combinatory***,** *probabilistic* and *algorithmic* one basing on the theory of recursive functions and abstract automata.

So, for the first time within algorithmical approach *A.N. Kolmogorov* has determined relative complexity by the minimal length of a program of deriving of a certain finite object ***A*** from a finite object ***B*** *(complexity of an object* ***A*** *relative to an object* ***B****)*. Wherein as representatives of comparable

objects *A.N. Kolmogorov* has chosen their *binary* numbers in some formal numbering and as programs of their deriving – programs of work of the appropriate *Turing* machines [150].

The approach suggested by us to definition of the complexity of finite configurations on the basis of ***CA*** axiomatics by one's own essence also is algorithmic, however differs from the *A.N. Kolmogorov* approach. The essence of this approach to definition of the complexity concept of finite configurations consists in estimation of complexity of generating of an arbitrary finite configuration from a certain primitive configuration $c_p \in C(A,d,\phi)$ *(for example,* c_p=□1□ *for* ***1-CA****)* by means of finite number of ***GTF*** $\tau^{(n_k)}$ from a certain fixed set of functions G_f, that we shall name a basic set. In this chapter a definition of the complexity of finite configurations is introduced on the basis of the ***CA*** axiomatics along with a number of interesting enough results connected to it. Yet for strict definition of the complexity concept we need certain fundamental results relative to the dynamics of finite configurations in *classical* and *polygenic* ***CA*** models.

The nonconstructability problem takes place for monogenic *(Chapter* ***2****)*, and polygenic ***CA*** models. In the second case this problem is known as the completeness problem and is determined by the following question**:** ***Whether an arbitrary finite configuration can be generated from a preset primitive configuration by dint of a finite sequence of global transition functions of a polygenic CA model?*** The problem has attracted attention of a lot of the researchers who have received many interesting enough results in this direction, whereas the next important result of *M. Kimura* and *A. Maruoka* has finished the solution of the completeness problem [24,40-43,67,68,73,94,102,106,129].

Theorem 68*. A d–dimensional nonzero configuration* $c \in C(A,d,\phi)$ *can be generated from a primitive configuration* $c_p \in C(A,d,\phi)$ *by the agency of an appropriate finite sequence of global transition functions* $\tau^{(n_k)}$ *of a polygenic d–CA* $(d \geq 1)$ *model.*

So, the completeness problem in the definite measure characterizes the constructive opportunities of the polygenic ***CA*** models and its positive decision proves wide enough opportunities of such class of ***CA*** models concerning the generating of finite configurations. Actually, basing on result of theorem ***68,*** it is shown that from any ***d–***dimensional nonzero finite configuration $w \in C(A,d,\phi)$ by means of a finite sequence of global transition functions of a certain polygenic ***d–CA*** model it is possible to

generate any preset finite ***g*** configuration [42]. Meantime, the following result directly follows of the results of *M. Kimura* and *A. Maruoka.*

Theorem 69. An arbitrary d–dimensional configuration $c \in C(A,d,\phi)$ for a polygenic structure d–CA ($d \geq 1$) can be generated from a certain initial primitive configuration $c_p \in C(A,d,\phi)$ by means of application to it of a finite sequence of d–dimensional global transition functions $\tau^{(n}{}_k)$ of a certain fixed (base) set G_f of the global transition functions.

In addition, this result along with theoretical interest presents a rather significant applied interest, for example, in systems of processing and storage of the graphic information of various type *(for example, in picture databases)*, and also in different systems of coding and decoding of the information [12,24-28,40-43,82-87,160,161]. So, in systems of processing, storage and transfer of images of a different kind, computer graphics, cartography and a series of other important appendices the problem of compact presentation of ***d–***dimensional configurations *(discrete images)* presents significant interest. In addition, an approach to a solution of this problem along with an assumption that is associated with it is well coordinated with our presentations about the most general principles of functioning of developing systems**:** ***At the heart of developing systems a program of development lays more likely than the full description of the developed system.***

Perhaps, this problem is rather important and perspective from many points of view, demanding the further more detailed researches in this direction. With the problems of complexity of finite configurations and completeness in ***CA*** models also more applied problem of presentation and storage of the information in various picture databases in which an information is represented not by numbers and symbols but **2–** and **3–**dimensional images of the different nature is naturally being linked [42, 84]. So, this approach has been offered by us for solution of a number of problems of coding and data compression. Moreover, this approach has been used for research of certain biologically**–**motivated aspects.

Coming back to the result of theorem ***68,*** we, on the other hand, should mention, that the following fundamental result describing the dynamic properties of classical ***CA*** models and directly continuing results of the previous chapter on the general problem of the existence of universal configurations for classical ***CA*** models takes place [82**–**87]. In addition, this result can even be considered as a direct consequence of results of theorems ***72*** and ***73*** which can be enough easily received on their basis.

Theorem 70. ***There are no finite sets of d–dimensional configurations c_k from the set $C(A,d,\phi)$ and global transition functions $\tau_k^{(n_k)}$ determined in the same finite alphabet A that satisfy the following relation:***

$$\bigcup_k < c_k > [\tau_k^{(n_k)}] \equiv C(A,d,\phi) \qquad (k = 1..p)$$

We have represented several variants of the proof of theorem ***70*** with which it is possible to familiarize, in particular, in [24,84]. Theorems ***69*** and ***70*** allow to obtain a good enough grounds for a strict mathematical base of our concept of complexity of finite configurations on the basis of ***CA*** axiomatics, and also for a lot of other results in this direction. So, for example, from the theorem ***70*** the following result rather easily follows**:** *Even polygenic* ***CA*** *models are not finitely axiomatized formal systems, i.e. it is impossible to determine a finite set of configurations (axioms) from which it would be possible to deduce the all set* $C(A,d,\phi)$ *of finite configurations by dint of a certain finite set of global transition functions (derivation rules).* So, let's pass on now straightway to definition of the *complexity* concept of finite configurations, that represents doubtless theoretical and gnoseological interest. Let's assume now that G_f **–** a finite set of ***d–***dimensional global transition functions preset in a some finite states alphabet ***A*** by means of which during finite number of steps, an arbitrary finite configuration ***c**** can be generated from a certain *primitive* configuration $c_p \in C(A,d,\phi)$, i.e. there are the following derivation rules of the finite configurations from a certain primitive configuration, namely**:**

$$c^* = c_p \tau_1^{m_1} \tau_2^{m_2} \tau_3^{m_3} \ldots \tau_n^{m_n} \quad (\tau_k \in G_f;\ \tau_j \neq \tau_{j+1};\ k = 1..n;\ j = 1..n-1) \qquad (16)$$

where m_k **–** multiplicity of use of global transition functions $\tau_k \in G_f$ *(**k** = **1..n**)*. We shall speak, that a configuration $c \in C(A,d,\phi)$ is generated from some primitive configuration $c_p \in C(A,d,\phi)$ at least during $r = \Sigma_k m_k$ steps of global transition functions $\tau_k \in G_f$ *(**k** = **1..n**)*. So, for the classical ***1–CA*** models the configuration of the kind c_p = □***1***□ can be chosen as an quite clear primitive configuration.

In addition, two arbitrary finite configurations $\tau_i, \tau_j \in G_f$ are supposed as various $(\tau_i \neq \tau_j)$ only if the following relation $(\exists c \in C(A,d))(c\tau_i \neq c\tau_j)$ takes place. If in a derivation chain *(**16**)* there is *(**n–1**)* pairs of different global transition functions $<\tau_i, \tau_j>$ *(**j=1..n–1**)*, we shall speak, that in a chain of generating of configurations $c \in C(A,d,\phi)$ from a primitive configuration

$c_p \in C(A,d,\phi)$ there are $(n-1)$ levels L_k which are defined by the following signalling binary function, namely:

$$L_k = \begin{cases} 1, & if\ \tau_k \neq \tau_{k+1} \\ 0, & otherwise \end{cases} \qquad k = 1..n-1$$

The following diagram illustrates the described process of generating of an arbitrary finite configuration $c \in C(A,d,\phi)$ from a preset *primitive* finite configuration c_p *(fig. 15)*. In addition, the generating process of the finite configurations from a fixed primitive configuration c_p according to the above rules *(14)* underlies the following diagram.

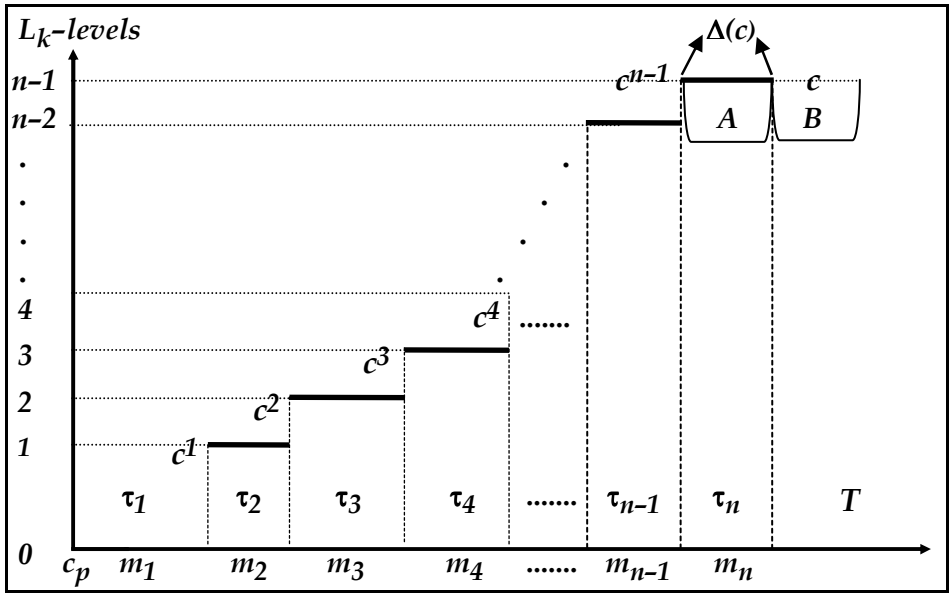


Fig. 15. The diagram explaining optimum strategy of deriving of a finite configuration $c \in C(A,d,\phi)$ [optimum graph CA(c) of derivation].

It is necessary to note that the above diagram can serve as a rather good illustration for many researches connected to the introduced concept of the complexity of finite configurations in classical *CA* models *(fig. 15)*. In view of told the *complexity* of an arbitrary finite c^{**} configuration can be determined as follows.

<u>Definition 18</u>. The complexity of a configuration $c^{**} \in C(A,d,\phi)$ $(d \geq 1)$ on the basis of CA axiomatics is being calculated according to the general formula, namely:

$$SL(c^{**}) = \min_{\tau_k \in G_k} \prod_{k=1}^{n-1} p_k^{m_k}$$

where p_k – k^{th} prime number and m_k is being determined on the basis of the generating chains of finite configurations (16) of a polygenic d–CA ($d \geq 1$) model.

The essence of this *complexity* concept is based on results of theorems ***69*** and ***70*** that assert *(on the one hand)* about the generating possibility of an arbitrary finite configuration in a polygenic ***d–CA*** from an initial finite configuration $c_p \in C(A,d,\phi)$, and *(on the other hand)* about impossibility of determination of the finite sets of initial finite configurations and global transition functions of classical ***d–CA*** models that in aggregate generate all set $C(A,d,\phi)$ $(d \geq 1)$ of finite configurations. On basis of this definition a lot of rather important properties of finite configurations in polygenic and classical ***CA*** models characterizing them concerning introduced the complexity concept has been received [24-28,42]. A number of results in this direction the following rather important theorem having a number interesting appendices in theoretical and applied aspects represents.

<u>Theorem 71</u>. For each integer $d \geq 1$ the set $C(A,d,\phi)$ of d–dimension finite configurations contains configurations of a preset complexity relative a finite base set W_f of d–dimension global transition functions, defined in some finite alphabet A, of a polygenic CA model.

Of this theorem follows, that for any finite set W_f the configurations of any preset complexity relative to it will still exist in the set $C(A,d,\phi)$. At that, other most characteristic properties of the introduced complexity concept of finite configurations in the ***CA*** models along with interesting enough consequences from them can be found in our works [24,82-87]. On the base of theorem ***71*** and some other our results in this direction it is possible to obtain the result playing a rather important part for study of dynamic properties of classical ***CA*** models and for further evolution of the complexity concept, related with basic conception of classical ***CA*** models [24,40-43,82-87,102,106].

<u>Theorem 72</u>. For a dimension $d \geq 1$ there are global transition functions $\tau \notin G_f$ generating from the preset configuration $c \in C(A,d,\phi)$ of the limited complexity, the configurations of any preset complexity in the meaning of definition 18.

This theorem asserts, if global transition functions composing the base set G_f generate finite configurations only of the limited complexity then by dint of global transition functions τ_j not belonging to the set G_f, the finite configurations of any complexity can be generated. The result of

this theorem has generated a lot of interesting enough questions, one of which is the question about number of finite configurations of the same complexity concerning the preset base set G_f. The following important enough result allows in a great extent to clear this interesting question.

Theorem 73. There is infinite number of base sets G_f of d–dimensional global transition functions, determined in an arbitrary finite alphabet A, concerning each of which the infinite sets F_j of finite configurations of the same complexity in the meaning of definition 18 exist.

Result of the theorem **73** allows to solve a number of rather interesting questions, formulated in our works [1,5]. A rather detailed research of the basic set G_f, used in definition of the complexity concept of the finite configurations in classical ***CA*** models together with properties of global transition functions, composing the set G_f, allow essentially to clear up not only new properties of the introduced complexity concept, but also will give a rather effective apparatus of research of dynamics of such ***CA*** models as classical, polygenic along with nondeterministic in a number of cases too.

So, in particular, it is enough important to investigate the minimal basic set G_f containing the least number of global transition functions $\tau_k^{(n_k)}$. Investigating the completeness problem in the polygenic ***CA*** models, ***A. Maruoka*** and ***M. Kimura*** have presented one constructive proof of the existence of a base set G_f *(Theorem* ***68);*** however, at the same time, they did not use optimizing technics. In general case the detailed research of the basic sets G_f of global transition functions till now is absent whereas concerning the narrower class of binary one–dimensional ***CA*** models a number of interesting enough results in this direction has been received [5,9,12,13,24-28,40-43,82-87,102,106].

Theorem 74. There is a minimal basic set G_f containing only four binary 1–dimensional global transition functions $\tau_k^{(n_k)}$; at least one of them possesses the NCF-1 nonconstructability. Relative to the minimal basic set G_f of 1–dimensional binary global transition functions the infinite sets of finite configurations of the same complexity there are.

In a sense this result leads to result of the previous theorem **73** in case of minimal basic sets G_f while method of its proof appears rather useful at receiving of the following rather interesting theorem which has quite a few important appendices in dynamics study of classical ***CA*** models.

***Theorem 75**. There are minimal basic sets G_f of 1–dimension transition functions $\tau_k^{(n_k)}$ in binary B alphabet in relation to each of which there are such infinite sets of functions $\tau_k^{(n_k)}$ of the same class together with configurations $c_k \in C(B,\phi)$ that the configurations sequences $<c_k>[\tau_k^{(n_k)}]$ contain finite configurations of an arbitrary predetermined complexity. However, there is not a finite basic set G_f of 1–dimension binary global transition functions $\tau_k^{(n_k)}$ relative to which each sequence $<c>[\tau_k^{(n_k)}]$ $(c \in C(B,\phi);\ \tau_k^{(n_k)} \notin G_f)$ would contain binary finite configurations only of the limited complexity.*

Theorem *75* allows to obtain the answers to a lot of questions and a little bit more deeply to reveal essence of the introduced complexity concept of finite configurations concerning the ***CA*** axiomatics. In this connexion it is necessary to note that the complexity concept of a certain algorithm depends on the concept of algorithm, and from its concrete realization. The conventional more precise definition for today is absent. Thus, the results relative to the estimation of complexity of algorithms can have essentially various character. In particular, complexity of normal *Markov* algorithm is determined by the length of recording of all its formulas of substitutions whereas under the complexity of *Turing* machine as a rule the *product* of quantity of internal states of the finite automaton and the symbols of an alphabet of external tape is understand.

Within the complexity concept a function of the algebra of logic from ***m*** variables can be realized by means of an appropriate normal algorithm with complexity of order ***2m*** whereas and by means of *Turing* machine with complexity of order $2^m/m$. Meanwhile, defining the complexity of a classical $<Z^d,A,\tau^{(m)},X>$ model as a product ***dxmxa***, we can easily make sure, that a function of algebra of logic from ***m*** variables can be realized by means of an appropriate classical ***1–CA*** model with complexity ***2m.*** Therefore, the above complexity concept of finite configurations rather essentially influences comparative characteristics of different classes of algorithms. So, the conceptual basis of the compared formal algorithms should be given more attention [5,9,12,13,24-28,40-43,82-87,102,106].

The complexity concept of description of a certain algorithm is used, as a rule, for specification of an question about minimal complexity of the algorithm generating a finite object. Similar minimal complexity very much is frequently named simply *complexity* of finite object *(at a concrete specification of the complexity concept of an algorithm description)*. As it was

already marked, definition of complexity of a finite ***g*** object for the first time has been proposed by *A.N. Kolmogorov*. At the same time, between complexity ***K(g)*** of a finite object ***g*** according to *Kolmogorov*, complexity $M_q(g)$ of the same object ***g*** expressed by the length of the normal *Markov* algorithm in alphabet with ***q*** symbols, and complexity $MT_q(g)$ which is expressed by the number of internal states of a *Turing* machine with an external alphabet of cardinality ***q*** asymptotically exact relations exist:

$$M_q(g) = K(g)/\log_2 q \qquad MT_q(g) = K(g)/(q-1)\log_2 K(g)$$

The approach suggested by us allows to estimate the complexity ***A(g)*** of such finite ***g*** objects as finite configurations in the classical ***CA*** models. At the same time, here it is necessary to take into account a number of essential aspects connected to the above conceptual circumstances [24]. So, a certain analogy takes place between complexity concepts of finite objects and complexity of finite configurations of the set $C(A,d,\phi)$.

Both approaches to the complexity concept are algorithmic but between them rather essential distinctions exist. So, one of *A. Kolmogorov* results in this direction says that MT^s_q ***with a constant program of work will print on output tape the binary words of only limited complexity.*** While for case of our complexity concept a quite other picture, whose reasons were considered in [24,82-87], takes place. These and other distinctions once again accent an question concerning the axiomatics essence of the complexity concept as a whole. Meanwhile, the existing contradiction between the *Kolmogorov* complexity concept of finite objects, on the one hand, and our complexity concept of configurations in the classical ***CA*** models can be removed, if to consider instead of finite configurations the finite block configurations, namely: there are binary ***1–***dimensional classical ***CA*** models providing generating of the set of all configurations from any finite initial configuration *(Section **3.1**)*. It is another significant difference between both types of configurations.

At proof of theorems ***73–75*** the concept of minimal basis set G_f and the certain dynamical properties of global transition functions entering in the set G_f were essentially used. At the same time, it seems that enough pertinently to present the more detailed properties of similar minimal basis sets, considering their importance from standpoint of the further study of deeper properties of dynamics of classical ***CA*** models. In this connection we confine ourselves to a case of binary polygenic ***1–CA***, the set $C(B,1,\phi)$ of finite binary configurations and binary global transition

functions $\tau_k^{(n_k)}$ determined by the appropriate local transition functions according to the following parallel substitutions:

000	⇒	*0*	*0*	*0*		*00*	⇒	*0*	
001	⇒	*1*	*1*	*1*		*01*	⇒	*1*	
010	⇒	*0*	*1*	*0*		*10*	⇒	*1*	*(17)*
011	⇒	*1*	*0*	*0*		*11*	⇒	*1*	
100	⇒	*0*	*1*	*1*				*(d)*	
101	⇒	*1*	*0*	*1*					
110	⇒	*1*	*1*	*0*					
111	⇒	*0*	*0*	*0*					
		(a)	*(b)*	*(c)*					

In view of the done assumptions the following a rather important result describing global transition functions of the minimal basis set G_f for the ***1–***dimension binary case of the polygenic ***CA*** models takes place [5,24].

<u>Theorem 76</u>. Minimal basis set G_f contains 4 1-dimension binary global transition functions $\tau_k^{(n_k)}$ whose appropriate local transition functions are determined by the above parallel substitutions (17.a–d); in addition, the global transition functions composing the basis set G_f possess the finite configurations of types according to the following table 5.

Table 5

LTF\NCF	*NCF*	*NCF–1*	*NCF–2*	*NCF–3*	*ACCF*
(15.a)	–	+	–	–	+
(15.b)	–	+	–	–	+
(15.c)	+	+	–	–	+
(15.d)	+	–	+	+	–

Minimal basis set G_f for 1-dimension nonbinary case consists of global transition functions $\tau_k^{(n_k)}$ that possess the configurations types such as NCF and/or NCF–1, NCF–2 and, perhaps, NCF–3 along with ACCF.

The choice of global transition functions $\tau_k^{(n_k)}$, determined by means of local transition functions with parallel rules *(17.a–d)* as a minimal basis set G_f has been grounded in work [41]. Moreover, it is shown that a set G_f of ***1–***dimensional binary global transition functions, defined by local transition functions with the simplest neighbourhood index $X=\{0,1\}$ that could be chosen as a minimal basis set does not exist [5,24,40-43]. In the

minimal basis set G_f, whose global transition functions are defined by ***4*** local transition functions with parallel substitutions ***(17.a-d)***, the first **2** local functions ***(17.a–b)*** have defect *one* and *three* accordingly, belonging to the isolated subset concerning the *composition* operation of the global transition functions, that possesses a lot of rather interesting properties concerning the dynamics of finite and infinite configurations [42]. This question is considered enough in detail in [24,82] along with discussion of the existence question of the nonconstructability types for the global transition functions, that compose the minimal set G_f.

So, result of theorem ***76*** has allowed to solve certain problems from the monograph [5]**;** this result can be used for researches of the complexity problem of ***1–***dimension finite configurations in a finite alphabet ***A*** [40**–**43]. In particular, on the basis of results of this theorem it is possible to give the simplest validity of the introduced complexity concept of finite configurations for ***1–***dimensional binary case. In addition, the result of such substantiation takes the form of the following theorem presenting also important independent interest as a component of the apparatus of research of the classical ***CA*** models [24-28,40-43,82-87,102,106].

Theorem 77*. An arbitrary 1–dimension binary configuration* $c \in C(B,1,\phi)$ *is monotonously generated of primitive configuration* $c_p = \square 1 \square$ *by means of global transition functions* $\tau_{jk}^{(n_k)}$ *from a certain fixed finite set G. At the same time there are no such finite system of pairs* $\{c_k, \tau_{jk}^{(n_k)}\}$ *that the following determinative relation takes place, namely:*

$$\bigcup_k < c_k > \left[\tau_{j_k}^{(n_k)} \right] = C(B,1,\phi); \quad c_k \in C(B,1,\phi) \quad (n_k \in \{2,3\};\ j_k \in \{0,1,2,3\};\ k = 1..p)$$

Set G of binary global transition functions can be chosen as base set G_f ***concerning which the complexity concept of 1–dimension binary finite configurations of classical CA models is being determined.***

It is necessary to note that result of the theorem is generalized and to the case of any finite alphabet ***A*** of internal states of elementary automata of an arbitrary ***1–CA*** model. In addition, on the basis of this result can be obtain more simple proofs, and in a lot of cases the constructive proofs of the previous theorems of this chapter can be obtain along with some other interesting results relative to complexity of finite configurations for the binary ***1–CA*** models [40-43]. At that, the first part of theorem **77** can be essentially improved, namely, the next basic result representing undoubted interest for the theory of the classical and the polygenic ***CA*** models takes place [24,82,84,102,106].

Theorem 78. ***A system of pairs in which each global transition function*** $\tau_{jk}^{(n_k)}$ ***is an arbitrary composition of finite number of functions of the set G can be chosen as a certain finite system of pairs*** $\{c_k, \tau_{jk}^{(n_k)}\}$ ***of the theorem 77. There are subsets of binary one-dimensional configurations inside of the set*** $C(B,1,\phi)$ {***of cardinality on average 3/4 of cardinality of all set*** $C(B,1,\phi)$} ***which can`t be generated by means of finite number of pairs of the kind*** $\{c_k, \tau_k^{(n_k)}\}$ ($c_k \in C(B,1,\phi)$; $k = 1 .. p$).

In a number of cases $\{c_k, \tau_k^{(n_k)}\}$ $c_k \in C(B,1,\phi)$ in formulations of theorems ***77*** and ***78*** it is possible to use a system of pairs in the equivalent form, supposing $(\forall k)(c_k \equiv c_p = \square 1 \square)$, i.e. it is possible to use a single primitive configuration c_p $(k = 1 .. m)$. Indeed, we can easily make sure that for a finite configuration $c \in C(A,1,\phi)$ it is possible to designate such transition function $\tau_c^{(h_c)}$ in the same alphabet A that $c_p \tau_c^{(h_c)} = c$. Hence, for a finite system of pairs $\{c_k, \tau_k^{(n_k)}\}$ there is a certain equivalent system of pairs of the kind $\langle c_p, \{\tau_k^{(n_k)}\} \cup \{\tau_k^{(h_k)} \mid k=1 .. m\} \rangle$, for which the global transition function $\tau_k^{(h_k)}$ satisfies the relation $c_k = c_p \tau_k^{(h_k)}$ $(k=1..m)$. This approach allows to unify the set of initial configurations c_k of a system of pairs of generatrices, displacing the basic accent onto a finite set of basis global transition functions. In view of told, formulations of theorems ***77***, ***78*** are modified in appropriate way and first of all the kind of a determinative relation of basic theorem ***70*** becomes simpler.

At the same time, these two equivalent forms of the above relation have own advantages in theoretical investigation concerning the dynamics of classical ***CA*** models. Most of all, the given moment become apparent at use of the introduced complexity concept of finite configurations in the classical ***CA*** models along with results received on its basis as a certain apparatus of research of dynamic properties of classical and polygenic ***CA*** models. It is necessary to note, that the complexity problem of finite configurations in classical ***CA*** models, in spite of the results represented above and a number of other results, has a number of open questions and perspective directions for the further researches, needing solutions from the various standpoints [5,7,9,12,24-28,40-43,82-87,102,106]. At the end of this section it is expedient to stop on two various approaches to definition of the complexity concept of finite configurations in classical ***CA*** models, namely**:** configuration approach and block approach**;** their essence was in brief marked in the above sections ***3.1–3.2.***

First of all, under the configuration complexity is being understood an opportunity of a ***CA*** model or a set of similar models of generating the set $C(A,d,\phi)$ of finite configurations from one or finite set of initial finite configurations. On the basis of definition ***18*** and theorem ***71*** follows that for an integer $d \geq 1$ the set $C(A,d,\phi)$ of ***d–***dimension finite configurations contains configurations of a predetermined complexity concerning any finite basis set G_f of ***d–***dimension global transition functions, defined in a certain finite alphabet ***A*** of a classical ***CA*** model**.** Of this result follows that at any definition of a finite basis set G_f the finite configurations of a predetermined complexity still will exist in the set $C(A,d,\phi)$ of all finite ***d–***dimensional configurations defined in an arbitrary ***A*** alphabet.

Completely other picture is quite allowable in the case of definition of the block complexity when is being taken into account essentially wider possibility of generating not of finite configurations $c = \square h x_1 x_2 x_3 \ldots x_n h \square$ [□ **–** *configuration of infinite number of symbols* $'0'; x_j \in A, j = 1..n; h \in A \setminus \{0\}$] but of block configurations, i.e. configurations of blocks $\langle x_1 x_2 x_3 \ldots x_n \rangle$ $\{x_j \in A, j = 1..n\}$ of elementary automata. In case of such approach other situation is quite real. We shall illustrate essence of similar distinction by means of a binary structure ***1–CA*** whose local transition function is determined by the following formula, namely**:**

$$\sigma^{(3)}(x,y,z) = \begin{cases} x + y \pmod 2, & \text{if } y = 1 \\ x + y + z \pmod 2, & \text{otherwise} \end{cases}$$

i.e. it is the above binary ***1-CA*** model with discriminating number ***120.*** It is shown that the model does not possess the ***NCF*** nonconstructability**,** possessing the ***NCF–1*** nonconstructible configurations of the kind, for example, $c' = \square 10 x_1 x_2 \ldots x_n 1 \square$, $c = \square 1^4 0 x_1 x_2 \ldots x_n 1 \square$ $\{x_j \in B = \{0,1\}, j = 1..n\}$**;** i.e. quota of the ***NCF–1*** nonconstructible configurations concerning all finite configurations is more than ***1/2.*** Along with that, this model does not possess the ***NCF–2*** nonconstructability**;** i.e. each finite configuration which is distinct from the ***NCF–1*** has a predecessor from the set $C(B,1,\phi)$ and the set $C(B,1,\infty)$**,** i.e. it is ***ACCF.*** This model possesses the universal reproducibility in the *Moore* sense of finite configurations too.

In view of the aforesaid along with kind of global transition function of the considered model it is possible to show that for it there is an infinite set of such configurations as ***NCF–1*** $\left\{c_1^j\right\}$ $(j = 1..\infty)$**,** that in the aggregate generate all set $C(B,1,\phi)$ of all finite configurations *(**18**)*. Hence, the set of

all configurations such as ***NCF-1*** of the above ***1-CA*** model with number ***120*** generates the set ***C(B,1,φ)*** of all finite configurations**:**

$$\tau^{(3)}:\begin{cases} c_1^1 \to c_2^1 \to c_3^1 \to c_4^1 \to \ldots \to c_k^1 \to \ldots \\ ========== \\ c_1^j \to c_2^j \to c_3^j \to c_4^j \to \ldots \to c_k^j \to \ldots \\ ========== \\ c_1^k \to c_2^k \to c_3^k \to c_4^k \to \ldots \to c_k^k \to \ldots \\ ========== \end{cases} \qquad (18)$$

$$\bigcup_{j=1}^{\infty}\left\langle c_1^j\right\rangle\left[\tau^{(3)}\right]=C(B,1,\varphi);\quad (\forall k,j)\left(k\neq j\to\left\langle c_1^k\right\rangle\left[\tau^{(3)}\right]\cap\left\langle c_1^j\right\rangle\left[\tau^{(3)}\right]=\varnothing\right)$$

In addition, as it was marked earlier, computer experiments with such ***CA*** model in combination with a lot of theoretical results basing on the dynamic properties of classical ***1–CA*** models caused by the existence in them of the ***NCF–1*** nonconstructability in the absence ***NCF, NCF–2*** and ***NCF–3*** have allowed to formulate the following interesting assumption [24,82-87,102,106]**,** namely.

<u>Proposal 6</u>. The above classical binary 1–CA model will be possess the universal reproducibility of finite configurations in the Moore sense; in addition, an arbitrary finite configuration generates in the total all set of finite binary block configurations.

In addition, in event of positive answer we shall receive an example of a rather simple binary classical ***CA*** model that possesses the property of universal reproducibility in the *Moore* sense of finite configurations with example of model for which any finite configuration generates all set of block configurations. The computer analysis we made along with other researchers persuade us about validity of the assumption, meantime, its theoretical acknowledgement up till now is absent [24,82-87,102,106].

Validity of this proposal once again will enough evidently illustrate the essence of distinctions between *universal* finite configurations and *block* configurations, generated by classical ***CA*** models along with distinction between our approach to complexity definition of finite configurations in classical ***CA*** models and approach of *A.N. Kolmogorov* to definition of complexity of finite objects. So, the possibility of generating by means of the above binary ***1–CA*** model of all block configurations from an initial configuration could be a certain analogue of generating by means of the *Turing* machine of sequences of binary words of the limited complexity.

The complexity problem of finite configurations in classical ***CA*** models has great value not only in a context of their research as certain formal deductive systems but also in case of embedding in them of developing systems of the cellular organization and their certain phenomena. Then,

this problem has the most direct attitude to a question of research of the complexity of self-organizing biological cellular systems which is actual enough for modern biology of development.

You know, till now at cybernetical research of biology of development we have no satisfactory enough approach to a question of estimation of the complexity of developing biological systems. And our mathematical approach in this direction can appear fruitful enough and perspective. So, the presented results along with other our results on the complexity problem of finite configurations in ***CA*** models not only form actually the problematics and solve a lot of its basic problems as a whole, but it also formulate a lot of open questions and rather perspective directions of the further research representing significant independent interest for theoretical and applied aspects of the ***CA*** problematics.

The results received by us concerning the complexity problem of finite configurations in a context of ***CA*** axiomatics to a certain extent allow to explain better essence of the complexity concept depending on the used axiomatics. So, in axiomatics of the classical and polygenic ***CA*** models there are binary finite configurations of any preset complexity while in other axiomatics, for example, in the *A. Kolmogorov* axiomatics all binary words printed by a *Turing* machine on output tape can have the limited complexity only. Thus, most likely, there is no any absolute complexity concept of finite objects along with the complexity concept as a whole; i.e., in a great extent the complexity concept has pronounced relatively axiomatic character.

Similarly to case of classical ***CA*** models, in the case of ***CAoS*** models it is rather naturally possible to determine the complexity concept of finite configurations that is based on an analogue of theorem **7*0*.** The majority of the presented results relative to the *complexity* of finite configurations is directly carried over to the ***CAoS*** models playing important enough part as an excellent environment of physical modelling, simulating and study of a rather wide class of the *spatially–distributed* dynamic systems. The reader can familiarize with the basic questions of the theory of this class of ***CA*** models in works [7,24,71,75,102,108,111,132,162-164], a lot of results concerning them is represented and in the references [7,24,286].

Chapter 5. Parallel formal grammars and languages determined by the classical cellular automata *(CA)*

The *theory of formal grammars (**TFG**)* takes center stage in mathematical linguistics since it gives formal resources for research of functioning of a language. At the same time, the ***TFG*** stands out against a background of other sections of mathematical linguistics by the essentially greater complexity of the used apparatus similar to apparatus of the theory of algorithms and apparatus of the general theory of automata with which it has quite a few points of contacts and intersection, and by essentially greater complexity of mathematical problems arising in it. The formal grammars of the most well studied types present the systems allowing to generate or recognize sets of chains, interpreted usually as the sets of grammatically correct sentences of some formal languages and also to associate description of their syntactic structure with chains composing these sets in terms of the systems of components or trees of subjection.

Mathematical significance of the generative grammars is defined by the circumstance, that they represent one of means of effective definition of the words sets. A class of *formal* languages that are generated by means of any grammars will coincide with class of all recursively**–**enumerable sets. From this point of view the formal grammars of classical *Chomsky* hierarchy present here a special interest [165,166]. In this connection the essential significance receives studying of classes of abstract automata, which are equivalent to one or the other classes of the formal grammars describing the same formal languages.

In particular, for instance, automaton grammars are equivalent to finite automata**,** context**–**free grammars are equivalent to automata with stack memory while context**–**dependent grammars are equivalent to linearly limited *Turing* machines. Except the *Chomsky* grammars, today, there is a number of others interesting from the various standpoints of kinds of formal grammars, destined for description of the words sets and other objects**;** among them a particular attention the *parallel* formal grammars attract which give effective enough means for the linguistic description of some important parallel processes and objects [7,24,40-43,82-87,102].

Since the ***FGT*** is a part of the automata theory, then study of dynamics of ***CA*** models from the ***FGT*** standpoint undoubtedly deserves a separate attention hence a lot of our works are devoted to these problems. At the same time, the theory of *parallel formal grammars* can be most effectively

used not only at creation of the theory of parallel programming along with architecture of computing systems of high–parallel action of new generations, but also at creation of a linguistic basis for the description of dynamics of various spatially–distributed systems of cellular nature. Therefore, for investigation of the languages generated by classical ***CA*** models by us at *1974* have been introduced formal parallel grammars, named τ_n***–grammars*** [24,32,102,106,167-169].

In addition, basically were studied the τ_n*–grammars* defined by classical and non–deterministic one–dimensional ***1–CA*** models, however similar approach can be spread to the case of ***d–CA*** $(d \geq 2)$ models, and to other certain types of models distinct from the above ones. At such approach the classical ***CA*** models can be considered as the *formal parallel grammars* ***(FPG)*** which not use non–terminal symbols and derivation of which is carried out by absolutely parallel manner. The grammars of such type are similar to known systems of *A. Lindenmayer* ***(L–systems),*** they can be quite successfully used for formal linguistic description of dynamics of various objects of cellular nature and many parallel discrete processes. Below, at a conceptual level the consideration of parallel τ_n*–grammars* is fulfilled according to traditions of the ***TFG*** as a consequence the reader receives a number of rather important characteristics of the ***FPG*** of such class interesting from many important standpoints [7,24,102,170-184].

Informally the τ_n*–grammars* are introduced as follows. By analogy to the basic concepts of the ***TFG*** the alphabet ***A*** of an elementary automaton of a classical ***1–CA*** model is named the alphabet of τ_n–grammar, its local transition function $\sigma^{(n)}$ defines a set of parallel productions or *derivation rules* of the grammar, an initial finite configuration of the model defines an *axiom* while finite configurations generated from the axiom are *words* of a language determined by such parallel τ_n–grammar. Likewise with the usual formal grammar in a certain classical ***CA*** model from an initial configuration c_o *(axiom)* due to a local transition function $\sigma^{(n)}$ *(derivation rules)* are deduced new configurations *(words of a language)*. Meanwhile, between traditional formal grammars and parallel τ_n*–grammars* **2** rather essential differences take place, namely**:**

♦ ***derivation rules in a*** τ_n***–grammar are being applied simultaneously and absolutely by parallel manner;***

♦ ***in an alphabet A of the parallel*** τ_n***–grammar are not being done any distinctions between terminal and non–terminal symbols.***

In the ***TFG*** a certain formal language is defined as a set of all terminal words that are generated from an axiom c_o through derivation rules of a grammar. $L(\tau_n)$-language is defined as a set of all finite configurations *(words)* generated from an initial configuration *(axiom)* by simultaneous application of parallel substitutions determined by the local transition function to all symbols of the current configuration *(word)*. Since a local transition function $\sigma^{(n)}$ uniquely defines global transition function, then global transition function τ_n of a ***CA*** model is frequently understood as the derivation rules in the formal parallel τ_n–grammar.

The parallel formal τ_n–grammars, defined in a similar way, are similar to the above ***L***–systems; they can be quite successfully used to linguistic description of some discrete developing systems and parallel processes. Hereinafter we need a number of definitions. In addition, it is supposed that the reader is enough well familiar with basis of the ***TFG.*** Hence, at presentation of the material below will be used standard designations and terminology of the well-established ***TFG*** [165,166]. We have studied the τ_n–grammars in conformity with traditions of the ***TFG*** with respect to a number of important enough characteristics of such class of parallel grammars. The results in this direction are presented in our works that contain the systematic exposition of the τ_n–grammars theory [8,9,13,40].

5.1. The basic properties of the parallel languages, determined by the classical cellular automata

In this section the basic concepts and properties of the formal parallel languages defined by ***1***–dimensional classical ***CA*** models will be given. In traditions of the ***TFG*** the τ_n–*grammars* and parallel formal languages determined by them have been investigated in detail enough what has found reflection in works [24,165,170-184]. According to the traditions of the ***TFG,*** we define an arbitrary $L(\tau_n)$–language as a set of all words *(finite configurations),* that are derived from an axiom $c_o \in C(A,1,\phi)$ *(initial configuration)* by sequential application to the axiom c_o of the derivation rules $\tau^{(n)}$ *(**GTF** of a certain classical **1**–dimension model)*. The more formally a parallel grammar τ_n and $L(\tau_n)$–language are defined as follows.

<u>Definition 19</u>. An arbitrary parallel τ_n–grammar is the ordered tuple in the form $\tau_n=<n,A,\tau^{(n)},c_o>$ where its components are defined as follows:

1) n – index of the grammar (size of neighbourhood template of a certain classical 1–CA model with a states alphabet A);
2) A – a finite alphabet of the τ_n-grammar (internal states of elementary automata in the classical 1–CA model);
3) $\tau^{(n)}$ – derivation rules of the grammar (global transition function of the classical 1–CA model with the states alphabet A);
4) c_o – an axiom of the grammar (an initial finite configuration in the classical 1–CA model with the states alphabet A).

A formal parallel $L(\tau_n)$–language determined by τ_n–grammar is the set of all words *(finite configurations)*, deduced from an axiom $c_o \in C(A,1,\phi)$ *(an initial configuration)* by means of sequential application to it of the derivation rules $\tau^{(n)}$ *(global transition function of a classical **1–CA** model)*, i.e. the relation $L(\tau_n) \equiv <c_o>[\tau^{(n)}]$ is here determinative. As it was already marked earlier, the especially parallel principle used by a τ_n–grammar for words processing is its essential feature appreciably distinguishing the τ_n–grammar from the traditional formal grammars.

This parallelism reflects the basic applied motivations both on the part of computational and biological sciences, and on the part of a number of highly abstract models of the real physical world that functions at space and time. Moreover, τ_n–grammars as against the traditional grammars do not use the *nonterminal* symbols performing the role of symbols that which extend alphabet of a formal grammar.

Defining the parallel τ_n–grammar and $L(\tau_n)$–language thus, we receive a possibility to research dynamics of ***1***–dimension classical ***CA*** models within of the ***TFG*** what allows to look at it from an untraditional side. The received results of investigation of classical and non–deterministic ***1–CA*** models from standpoint of the ***TFG*** are represented in a cycle of our works [7,8,9,12,13,24-28,32-33,82-87], in works of other researchers [7,62, 64,185-192] along with surveys [22,30,63,70,71,118,137,149]. If the opposite not will said then during of the present section the parallel τ_n–grammars and $L(\tau_n)$–languages defined by the classical $<Z^1, A, \tau^{(n)}, X>$ models with states alphabet $A=\{0,1,...,a-1\}$ of elementary automata and neighbourhood index $X=\{0,1,2,3, ..., n-1\}$ will be considered.

Research of the closure property of a certain class of formal languages concerning the operations, traditional in the ***TFG***, is classical approach to its mathematic characteristic. Two principal causes for consideration

of these operations concerning the parallel $L(\tau_n)$–languages exist. First of all, a possibility to more deeply elucidate the distinctions between a family of $L(\tau_n)$–languages and traditional families of formal languages that on this way exist. Second, the set of operations natural to family of $L(\tau_n)$–languages is determined till now not up to the end. In addition, the following basic result determines behaviour of the $L(\tau_n)$–languages concerning the traditional operations researched in the classical theory of formal grammars [8,9,12,13,24-28,32,33,82-87,102,106].

***Theorem 79**. Class of parallel languages $L(\tau_n)$ is non–closed relative to such operations as a finite transformation, homomorphism, iteration, union, product, addition and crossing while the class of these parallel languages is closed concerning the operation of inversion.*

J. Dassow has investigated parallel τ_n–grammars and $L(\tau_n)$–languages associated with them, concerning four new set–theoretical operations and has shown that class of $L(\tau_n)$–languages is non–closed concerning these operations, interesting from the biological standpoint [186]. The above–mentioned results have been received, basically, by constructive methods which consist in construction of the appropriate examples of $L(\tau_n)$-languages. Furthermore, a rather important fact can be ascribed to the most essential features of τ_n–grammars, namely, that the majority of approaches of standard techniques and apparatus of study in the ***TFG*** is inapplicable to the class of τ_n–grammars, presuming use of new non–standard methods. In particular, use of methods of the *recursive function* theory has made it possible to solve some questions of the theory of τ_n–grammars [8,9,12,13,24,32,33,82,87,102,106].

The given approach is based on the introduced concept of *G*–indexing relative to which biunique equivalence between some partial recursive word function $\tau^{(n)}: C(A,1,\phi) \to C(A,1,\phi)$ and a certain numerical partial recursive function $F_n(w): N \to N^* \subseteq N$ has been established. In this case the study of parallel τ_n–grammars and $L(\tau_n)$–languages associated with them is reduced to research of appropriate numerical functions $F_n(w)$ and their range of values. Within this approach a number of properties of the numerical function $F_n(w)$ has been investigated**;** for the function $F_n(w)$ the top complexity limit expressed by the following result takes place, namely**:** ***Any arbitrary numerical function $F_n(w)$ is majorized by***

a suitable primitive recursive function J(n,w). This result well conforms to the fact**:** ***A word function determined by means of a parallel mapping*** $\tau^{(n)}$***: C(A,ϕ)*** $\rightarrow$ ***(A,ϕ) is primitive recursive function*** [8,9,12,24,102,106].

Meanwhile, still a lot of questions connected to application of methods and results of the theory of recursive functions to problems of research of dynamic properties of the classical ***CA*** models remains**;** meantime, in the today's state the above approach provides essential assistance in the direction. So, for example, using the above method an extremely simple proof of a rather interesting result of the finite automata theory has been received**:** ***A function determined by a finite automaton which correctly predicts an environment is primitive recursive*** [9]. In addition, certain other rather interesting results can be received on the basis of the above approach. Whilst for the case of parallel $L(\tau_n)$**–**languages such approach allows to obtain the following rather interesting results [24,40-43,82-87].

<u>Theorem 80</u>. Generally, there is not a finite set of languages $L(\tau_n)$ ***whose union forms addition of a certain language of the same class; moreover, addition of a finite set of languages*** $L(\tau_n)$ ***can`t be again formal parallel language of the same class.***

From the represented results follows, that the family of languages $L(\tau_n)$ shows a rather strong immunity to the closing relative to the operations traditional for the ***TFG,*** and to a number of other operations presenting essential interest from the standpoint of the ***TFG*** itself and a number of interesting enough appendices. In this attitude it is rather interesting to compare among themselves ***L***-systems and τ_n-grammars. Above all, the languages of ***L***–family altogether as against languages $L(\tau_n)$ possess full immunity concerning the traditional operations of closure. In addition, a rather interesting discussion of the primary reason of distinctions of the τ_n–grammars and ***L***–systems both from the standpoint of biological appendices and as a whole can be found, for example, in [24,82-87,102].

At the same time, computing possibilities of parallel τ_n–grammars are equivalent to possibilities of the universal *Turing* machine, i.e. class of all parallel τ_n–grammars possesses the universal computability. Along with that, it is shown that each finite nonempty language is generated by a suitable τ_n–grammar whereas for each ***n***–index *(**n** is an integer* ***> 1**)* of grammar exist infinite regular languages and even finite languages which can`t be generated by means of τ_n–grammars, however quite can

be generated by means of τ_{n+1}–grammars. On the *generative* capabilities of parallel languages the τ_n–grammars form a hierarchy according to a ***n***–index of a concrete parallel τ_n–grammar.

There are non-recursive $L(\tau_n)$-languages and regular languages that are not $L(\tau_n)$ languages; the $L(\tau_n)$-languages have nonempty crossings with regular languages, context–free languages, context–sensitive languages. As consequence of the above-mentioned distinctions, the family of $L(\tau_n)$ languages is completely different from the traditional families of formal languages in the *Chomsky* hierarchy, namely – this family contains the certain non–context–free languages and even non–recursive languages in the absence in the family of rather great number of classes of regular languages [8,9,12,13,24-28,32,33,82-87,102,106].

In addition, the universally recognized method of comprehension of the generative opportunities of a certain class of generative formal systems is reduced to comparison of it with already classical *Chomsky* hierarchy. From the basic reasons in favour of that is the fact the *Chomsky* hierarchy has been investigated in the ***TFG*** most in detail. Thus, a number of our results is devoted to finding of interrelation of the $L(\tau_n)$ languages with traditional languages in the *Chomsky* hierarchy, in which the recursive and recursively enumerable languages, regular languages, context–free languages and context–sensitive languages are the basic classes. It has been shown that the set of all parallel $L(\tau_n)$ languages forms own subset of set of all *Lindenmayer* languages *(L–languages)*, while $L(\tau_n)$ languages are own subclass of a class of $L(T_n)$–languages which are determined by nondeterministic T_n–grammars, considered below.

Thus, our basic result establishes a relation between families of parallel formal languages $L(\tau_n)$ and $L(T_n)$, determined by parallel τ_n– and T_n–grammars of classical and nondeterministic ***1–CA*** models accordingly within the *Chomsky* hierarchy. In addition, with the purpose of the best understanding of place of languages $L(\tau_n)$ into the hierarchy the $<k,p>$–languages of the *Lindenmayer* have been additionally included [5,32,33]. The following theorem defines place of languages $L(\tau_n)$ and $L(T_n)$ in the *Chomsky* hierarchy concerning the basic traditional formal languages. In this case, many of interesting properties of languages $L(\tau_n)$ concerning various operations with them are represented in the above works and in the quoted references to original sources.

***Theorem 81**. The next diagram defines relations between the families of parallel formal languages $L(\tau_n)$ and $L(\tau_n)$ determined by classical and nondeterministic 1-CA models accordingly within the framework of the universally recognized Chomsky hierarchy.*

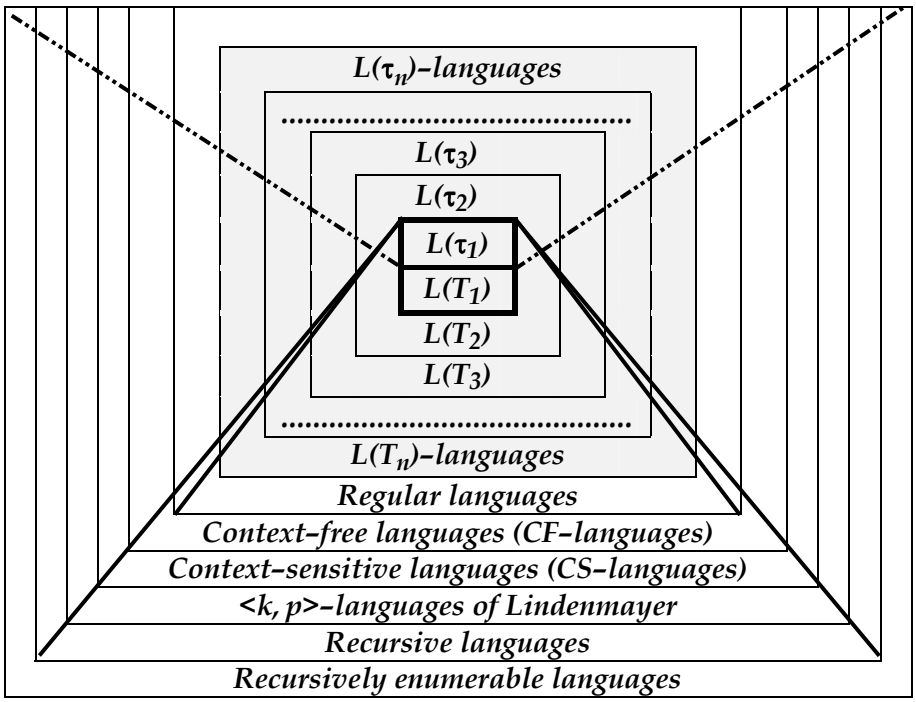


Fig. 10. Location of formal parallel languages $L(\tau_n)$ and $L(T_n)$ in the universally recognized Chomsky hierarchy.

Finding a certain class of recognizers or acceptors admitting languages generated with the help of grammars is traditional approach in the ***TFG.*** Obviously, the good automaton model of a family of formal languages gives for it a strict enough characteristic. Furthermore, concerning such model it is necessary to do one important remark, generally speaking. All reasonable models of this type *(at least in their classical sense)* have a finite automaton as a control device. Consequently, a family of formal languages admitted by similar models, should be closed concerning the operation of crossing with regular sets of words. Different classes of the $L(\tau_n)$ languages have researched from such «*programmer*» standpoint. In this direction relative to the $L(\tau_n)$ languages there is the next result [32].

***Theorem 82**. Class of all parallel $L(\tau_n)$ languages is nonclosed relative to the operation of crossing with regular sets of finite words.*

So, from this result follows, that it is not possible to find an automaton model of an acceptor in standard sense concerning the class of parallel languages $L(\tau_n)$. Concerning the languages $L(\tau_n)$ there is a lot of other interesting questions with which it is possible to familiarize in [5,32,33]. First of all, from point of view of the reversibility problem of processes embedded to classical ***CA*** models a question of existence of a language $L^{-1}(\tau_h)$ that is reversible to a language $L(\tau_n)$ represents the undoubted interest. It is simple to make sure that at existence of a biunique parallel global mapping $\tau^{(n)}: C(A) \to C(A)$ this question has the positive solution [32]**:** ***For any language $L(\tau_n)$, whose derivation rules correspond to a τ_n–grammar possessing this property, there is the reverse language $L^{-1}(\tau_h)$ of the same class.*** For example, the reversibility property can be easily programmed for the above ***CAoS–***models too. In addition, in the case of existence for global transition functions $\tau^{(n)}$ of the ***NCF, NCF–3*** and/or ***NCF–1*** nonconstructability it is possible to define the $L(\tau_n)$**–**languages having reverse languages of the same class too [24,32,41]. But in general case this question is solved negatively, namely.

Theorem 83*. There are parallel formal languages $L(\tau_n)$, for which in the general the sets $L^{-1}(\tau_h)$ of words are not languages of the same class.*

The study of a $L(\tau_n)$**-**language for the purpose of saving the property of being a $L(\tau_n)$**-**language in case of its narrowing or broadening by some finite subset of ***S*** words from the set $C(A,1,\phi)$ also is interesting enough. For lot of interesting cases the sets of words $L(\tau_n)$, $L(\tau_n) \cup S$ and $L(\tau_n) \setminus S$ are the languages of the same class, that is, the languages generated by parallel τ_n-grammars, whereas in general case the assertion is incorrect**;** lot of simple enough examples prove it. The more precisely, there is the following result.

Theorem 84*. There is a parallel $L(\tau_n)$ language and such finite subset of words $S \subset C(A,1,\phi)$, that the sets $L(\tau_n)$, $L(\tau_n) \cup S$ and $L(\tau_n) \setminus S$ can`t be as formal languages of the same class.*

Thus, results of theorems ***82–84*** present a lot of descriptive examples of nonclosure of class of parallel $L(\tau_n)$ languages relative to the operations characterizing the important properties of dynamics of classical models ***1–CA*** that determine the τ_n**–**grammars corresponding to them. Together with other results about nonclosure of class of parallel $L(\tau_n)$ languages

relative to a number of important set-theoretic operations the theorems ***82–84*** confirm strong immunity of the class of parallel formal languages in this direction. This phenomenon essentially distinguishes the class of parallel $L(\tau_n)$ languages from traditional families of formal languages considered in the classical ***TFG*** [8,9,12,13,24-28,32,33,82-87,102,106].

One of possible ways of research of structure of parallel τ_n–grammars is an approach consisting in imposing of *partial* restrictions directly on the definitions of various their component with the subsequent studying of influence of these restrictions on the languages generated by grammars. A number of the results in the given direction is presented in our book [1] and works [41-43]. The properties of parallel τ_n–grammars and $L(\tau_n)$ languages were considered earlier regardless of internal structure of the words composing such parallel languages. In this connection a rather interesting question arises.

An infinite sequence of words $S=\{c_k\}$ $(c_k \in C(A,1,\phi) \mid k=1,2,3,\ldots)$ is named ***formula sequence,*** if each its word $c_k \in S \subset C(A,1,\phi)$ can be structurally represented as one of finite number of formulas of the following kind**:**

$$C_k = C_{j_1}(k)C_{j_2}(k)C_{j_3}(k)C_{j_4}(k)\ldots C_{j_h}(k)\ldots C_{j_p}(k)$$
$$(\forall j_m)(\forall k)(C_k, C_{j_m}(k) \in C(A,1,\phi)); \quad j_m \in \{1,2,3,\ldots\}; m = 1..p$$

Formula sequences of words are examples of $L(\tau_n)$–languages in which the words composing them in certain respects contain a history of one's own development. A parallel $L(\tau_n)$–language is called *formula language* if the τ_n–grammar corresponding to it generates a formula sequence of words *(finite configurations)*. It is possible to show, that a parallel $L(\tau_n)$–language generated by an appropriate τ_n–grammar defined by a linear classical ***1–CA*** model, is a formula language [43]. It is one more kind of general characteristic of generative possibilities of such class of the ***CA*** models which is generalized to ***d***–dimension case too. There is a lot of rather complex examples of formula languages defined by classical ***CA*** models [24,41-43,82-87,102,106].

In particular, it is shown that ***1***–dimensional binary ***CA*** model different from linear model, with neighbourhood index $X = \{0,1,2\}$ and with local transition function $\sigma^{(3)}$ defined by the following formula, namely**:**

$$\sigma^{(3)}(x,y,z) = \begin{cases} x+y+z+1 \pmod 2, & \text{if } xyz \in \{001,011\} \\ x+y+z \pmod 2, & \text{otherwise} \end{cases} \quad ; \quad x,y,z \in \{0,1\}$$

produces a formula sequence of configurations from an arbitrary finite configuration c_o; in addition, any finite configuration c_o, excepting case when c_o is the simplest configuration $c_o = \square 1 \square$, is not a self–reproducing configuration in the *Moore* sense. It is easy to note that the above binary ***1–CA*** has the discriminating number ***57.*** Thus, from the following three initial configurations $c_o = \{11011 \mid 10111 \mid 100111\}$ the above binary ***1–CA*** generates the next formula sequences of configurations accordingly:

$$\left\{\begin{array}{l} c_{2k-1} = 100(10)^k 1 \\ c_{2k} = 1(10)^k 11 \end{array}\middle|\begin{array}{l} c_{2k-1} = 101(10)^k 1 \\ c_{2k} = 101(00)(10)^{k-1}11 \end{array}\middle|\begin{array}{l} c_{2k-1} = 1101(10)^k 1 \\ c_{2k} = 10(010)^k 11 \end{array}\right\} (k = 1, 2, 3, \ldots)$$

Whereas for initial configuration $c_o = \square 1111001011 \square$ the model generates the following formula sequence of configurations, namely:

$$c_1 = 11(10)^4 1,\ c_2 = 1100(10)^3 11,\ c_k = \begin{cases} (10)^{(k+9)/2} 1, & \text{if } k \text{ is odd number} \\ (10)^{(k+8)/2} 11, & \text{if } k \text{ is even number} \end{cases};\ k \geq 3$$

Thus, the concept of formula language well enough characterizes class of linear classical ***CA*** models along with a number of some other types of classical ***CA*** models. Furthermore, it is easy to show, that each finite parallel language $L(\tau_n)$ is formula language, and any formula language $L(\tau_n)$ is recursive, however the converse assertions as a whole are false. The more detailed consideration of the formula parallel $L(\tau_n)$-languages with rather interesting examples can be found in [24,32,33,102,106].

The introduced concept of formula grammars and languages represents indubitable interest at researches of syntactical structure of the parallel languages generated by τ_n–grammars. Moreover, this concept is rather closely connected to use of classical ***d–CA*** models as an environment of modelling of various parallel processes, objects and phenomena. In this connexion there is a rather actual problem of determination of formula representation of an arbitrary parallel $L(\tau_n)$–language; this problem, in our opinion, is algorithmically unsolvable. In context of the considered concept the reverse problem arises, namely: ***It is necessary to determine a parallel $L(\tau_n)$ language of the preset formula structurization,*** which is solved *negatively* already for the simplest types of formula presentation. In particular, the following set of finite formula words $L = \{c_o, c_1, \ldots, c_m; c_k = c_{k-2}c_{k-1} \mid k \geq m\}$ can`t be as a parallel $L(\tau_n)$ language. In spite of a lot of the received results, for today we have a rather meagre information concerning the problem of formula representation of the parallel $L(\tau_n)$

languages, hence the research in this direction present a certain interest. Having presented certain basic properties of parallel τ_n–grammars and languages determined by them further we pass to consideration of their interrelations with other well–known grammars, including the parallel grammars of some other types and classes.

5.2. Parallel grammars determined by the classical *CA* models in comparison with formal grammars of some other classes and types

Introducing the parallel τ_n–grammars, it is an quite natural to compare their generative possibilities with earlier investigated formal grammars of other types and classes. A number of results available in this direction allows not only to receive from many standpoints interesting enough comparative estimations of a new class of parallel grammars defined by the classical ***CA*** models, but also, on the other hand, to estimate parallel τ_n–grammars and the formal parallel languages generated by them. So, *E.S. Scherbakov* [193], occupying oneself with questions of development of the mathematical apparatus of modelling for biology of development at a cellular level, has defined a new class of parallel grammars named subsequently ***Sb(m)***–grammars which are defined as follows. In ***Sb(m)***–grammars the parallel productions of the next general kind are used as derivation rules, namely:

$$Sb: \begin{cases} x_1 x_2 x_3 \dots x_m \Rightarrow y_1 y_2 y_3 \dots y_p \\ 0\,0\,0\,0\,0 \dots 0 \Rightarrow 0\,0\,0\,0\,0 \dots 0 \end{cases} \qquad (19)$$
$$(x_k, y_j \in A = \{0,1,\dots,a-1\};\quad k = 1..m;\ j = 1..p;\ 1 \le p \le m)$$

which are characterized by simultaneous application to any finite word in a certain finite alphabet ***A***. At that, inasmuch as lengths of the right parts of the parallel substitutions can exceed ***1***, then at determination of the result of application of parallel productions of kind ***(19)*** to a certain word in the alphabet ***A*** occurrence of ambiguity in its some positions is quite possible. Hence, a simple function of choice of the posterior state is added to parallel substitutions ***(19)***, namely:

$$W(h_1, h_2, \dots, h_v) \in A \qquad h_k \in A; \quad (k = 1 .. v;\ 1 \le v \le r) \qquad (20)$$

which allows to unambiguously choose in points of ambiguity a certain state, single–valued for this concrete ambiguity, on basis of an arbitrary tuple $<h_1, \dots, h_v>$ of states. At the made assumptions the parallel ***Sb(m)***–

grammar is introduced as follows.

***Definition 20**. A Sb(m)-grammar is the ordered tuple of the kind Sb(m) = <A, c_o, Sb, W> whose components are defined as follows:*

1) A – a terminal finite nonempty alphabet;
2) Sb – parallel productions of derivation in the form (19);
3) W – a function of choice of states in points of ambiguity (20);
4) c_o – an axiom of the parallel grammar.

A set of finite words generated by such Sb(m)–grammar is named as the Sb(m)–language.

It is shown, that on generative language possibilities the parallel ***Sb(m)–*** grammars and τ_n**–**grammars are equivalent**,** namely the following basic result takes place [24,41-43,82,102,106].

***Theorem 85**. For each parallel Sb(m)–grammar exists τ_n–grammar that is strictly equivalent to the first one {$n=max_k(m_k)+max_k(p_k)-1$, if $m,p\geq 1$; and $n = 2m - 1$ otherwise}.*

The result of theorem ***85*** is one more essential argument confirming the high enough degree of the generality concept of classical ***1–CA*** models as grammars with parallel substitutions, determining derivation rules. Discussion of certain questions of interrelation of the τ_n-grammars with other types of parallel grammars *(let us say, isotonic structural grammars, parallel spatial grammars, parallel programmable spatial grammars, etc.)* along with some traditional grammars can be found in [24,82-87]. Particularly, within the theory of parallel τ_n-grammars the formal *Lipton* model of the asynchronous *linear* structures that play quite defined part in the theory of programming has been considered. It is shown, that the class of all grammars of such type composes own subclass of the class of all formal parallel τ_n**–**grammars.

Since the class of parallel languages $L(\tau_n)$ is own subclass of the class of languages of *A. Lindenmayer* that are generated by the ***L–***systems**,** a lot of certain principal questions concerning more detailed determination of attitudes between both classes of these formal languages arises [5,40-43,32,33,82-87]. So, *J. Buttler* has shown that ***an arbitrary classical 1–CA model is simulated real time by means of a certain Lindenmayer system PD(m,n)*** whereas ***an arbitrary system PD(m,n) is simulated by means of an appropriate classical 1–CA model not real time, generally speaking.*** By basing on the results on computability and modelling in the classical ***d–CA*** $(d\geq 1)$ models we proved even essentially more general result [5].

Theorem 86. An arbitrary L–system of Lindenmayer is modeled by dint of an appropriate classical 1–CA model not real time broadly speaking, and vice versa.

The interesting *comparative* analysis of both types of parallel systems of information processing in a context of their biological appendices was presented in our works [2,8,9,24-28,31-33,82,102,106,194-196].

To the problems of parallel τ_n–grammars the numerous works relative to use of ***CA*** models as acceptors and recognizers along with works on spatial grammars closely adjoin. First of all, that is related to questions of recognition by means of ***CA*** models of formal languages real time. In this direction a lot of interesting enough results has been received. For example, *A.R. Smith* [189] has researched classes of ***1–*** and ***2–***dimension formal languages, recognizable by ***CA*** models with restriction on the time. Many other rather interesting questions of recognition of various classes of formal languages both by classical**,** and nondeterministic ***CA*** models can be found in works of such famous researchers as *A.R. Smith***,** *H. Nishio***,** *S. Seki***,** *R. Vollmar***,** *T. Jebelean***,** *O. Ibarra***,** *R. Sommerhalder*, and others [7,24]. The good review of results and methods in this direction has been presented by *M. Mahajan* [198]**;** in the same place it is possible to find and certain additional examples of languages according to the above classification of ***CA*** models along with problems for the further researches, whereas the characteristic of the most important of them is represented in the review [197].

In addition, a number of interesting works is devoted to use of classical ***CA*** models as generators of languages of a special kind, for example, of fractal type [199]. The spatial grammars attracted a significant enough attention**;** the languages generated by them represent the sets of spatial figures *(areas)* instead of ***1–***dimension strings *(words)*. So, *W. Grosky* and *P. Wang* are one pair of the first researchers that considered *interrelation* between classical ***CA*** models**,** *parallel programmable spatial grammars* and *parallel spatial grammars* having large enough potentialities in theoretical and in applied aspects [187]. At that, it is shown, that τ_n–grammars are equivalent to *parallel programmable spatial* grammars, and that there is a certain constructive unilateral bridge from parallel spatial grammars to τ_n–grammars. By a lot of reasons the further researches in this direction seem to us rather perspective.

Meanwhile, a number of questions in this field remains open, however the questions of expansion of researches on parallel grammars, defined

by various types of ***CA*** models are being presented the most important. In the following section certain results presented on τ_n–grammars are generalized to the case of nondeterministic ***1***–dimensional ***CA*** models.

5.3. Parallel grammars defined by nondeterministic *CA* models

A *nondeterministic* T_n–grammar that in any discrete moment $t>0$ admits one or more but finite number of variants of choice of parallel *derivation* rules is an essential generalization of concept of parallel τ_n–grammar. A parallel nondeterministic grammar is defined as follows.

Definition 21*. A parallel nondeterministic T_n–grammar is the ordered tuple of the kind $T_n = <n, A, W_n, c_o>$, whose components are determined as follows, namely:*

1) n – index of the grammar (maximal size of neighbourhood template of the used global transition functions of classical 1–CA models);

2) A – a finite nonempty alphabet of the grammar;

3) W_n – an allowable finite set of derivation rules of the grammar;

4) c_o – an axiom of the grammar (an initial finite configuration).

A set of all words generated from an axiom $c_o \in C(A,1,\phi)$ by dint of the parallel derivation rules from an allowable W_n set of global transition functions is named the parallel language $L(T_n)$.

From definitions of parallel τ_n–grammars and T_n–grammars it is easy to notice, the *first* ones are the special case of the *second* ones. Analogously to the case of τ_n–grammars, we consider nondeterministic ***1–CA*** models determined by parallel T_n–grammars corresponding to them. One of the most important results in the finite automata theory is the fact, that the class of formal languages defined by nondeterministic finite automata, coincides with the class of all languages that are generated by means of completely determined finite automata. Whereas in the case of parallel T_n–grammars and τ_n–grammars the fully other picture characterized by the following basic result takes place [24,82-87,102,106].

Theorem 87*. There are $L(T_n)$–languages which are not $L(\tau_n)$–languages whereas there are regular languages that are neither $L(T_n)$–languages, and nor $L(\tau_n)$–languages.*

Thus, as against the finite automata the nondeterminism of derivation rules of parallel grammars determined by nondeterministic ***CA*** models expands their generative language possibilities. Let's consider now the generative opportunities of the parallel nondeterministic T_n grammars depending on their derivation rules W_n. At the earlier made definitions and assumptions in this direction the following basis result takes place.

Theorem 88. Each finite set of words defined in a finite alphabet A is a language $L(T_n)$ for an appropriate parallel nondeterministic grammar T_n. For an integer $n \geq 2$ there are infinite and finite regular sets of words which can`t be generated by any parallel nondeterministic T_n grammar, however they can be generated both by dint of an appropriate parallel T_{n+1}–grammar, and a parallel deterministic τ_{n+1}–grammar.

Of result of the theorem *88* follows, that the nondeterminism retains the *generative* language possibilities of parallel nondeterministic grammars T_n relative to regular languages similarly to the case of τ_n–grammars. Similarly to the parallel τ_n–grammars for nondeterministic grammars T_n a rather important result describing formal languages $L(T_n)$ from a «*programmer*» standpoint holds true [24,82-87,102,106].

Theorem 89. Class of all nondeterministic languages $L(T_n)$ is nonclosed concerning the crossing operation with regular sets of words.

Therefore, in the case of nondeterministic parallel T_n–grammars also it is impossible to determine a satisfactory formal *automaton* model of the recognizers admissive of an arbitrary $L(T_n)$–language. Basically, it was necessary to expect a similar result because this situation in our opinion is being defined, first of all, by *completely* parallel manner of application of derivation rules in this grammar whereas today's known traditional automaton models of the recognizers are based on pronounced serial principle of processing with the well defined centralized management. Whereas the ***CA*** models, being the parallel dynamic systems make use of decentralized control principle.

Let's consider now the question of closure of class of nondeterministic parallel languages $L(T_n)$ concerning a number of operations traditional for the classical ***TFG.*** In the previous section it was marked, the class of all deterministic languages $L(\tau_n)$ is nonclosed concerning practically all basic operations considered as basical ones in the ***TFG.*** Meanwhile, the study of these questions concerning the class of nondeterministic $L(T_n)$

languages represents the essential enough interest. The following basic theorem represents some results received in the given direction [82-87].

Theorem 90. Class of all nondeterministic languages $L(T_n)$ is nonclosed relative to such operations as union, addition, product, homomorphism, finite transformation, and crossing, however the class is closed relative to the inversion operation.

Therefore, and here for nondeterministic $L(T_n)$ languages the situation is completely similar to case of deterministic $L(\tau_n)$ languages, however this question needs the further research. Now let's introduce one more important enough operation named τ^*–operation concerning the $L(T_n)$ languages. Let a subset of words $L \subset C(A,1,\phi)$ is a certain language $L(T_n)$, while τ^* – an arbitrary global transition function, defined in an alphabet A. Let's define a set of the finite words as $\tau^*(L) = \{x \mid x=\tau^*(x');\ x' \in L\}$. An interesting enough question arises: ***Whether always the set $\tau^*(L)$ again will be language of the same class, where L – a nondeterministic $L(T_n)$ language?*** The answer to the given question is negative. Operations of the *left* and *right* division of a L–language by a finite ω–word given in A alphabet are defined and denoted as $\omega \backslash L=\{x \mid \omega x \in L\}$ и $L \backslash \omega=\{x \mid x\omega \in L\}$ accordingly. It is shown that the class of all nondeterministic languages $L(T_n)$ is nonclosed concerning the above two operations [8,33]; the next theorem being of interest from standpoint of grammatical properties of T_n models as generators of formal languages summarizes that.

Theorem 91. The class of nondeterministic $L(T_n)$ languages is nonclosed concerning the τ^*–operation and operation of the left (right) division by an arbitrary finite word determined in the same finite alphabet A.

Thus, the represented results confirm, that the class of nondeterministic parallel $L(T_n)$ languages possesses the same strong immunity relative to the *closure* operation relative to the major operations of the classical *TFG* analogously to the class of deterministic $L(\tau_n)$ languages. In addition, it is possible to familiarize with other interesting properties of languages $L(T_n)$, for example, in [40-43,82-87,31-33], whereas the position of these languages in the classical *Chomsky* hierarchy is represented on fig. ***10.*** A number of results obtained relative to the grammars τ_n and T_n showed that traditional method to implementation of programming languages for homogeneous computer systems will not allow to create an effective enough parallel software that might use maximal degree of paralleling

permitted by similar systems of finite automata. In our opinion, the T_n and τ_n grammars are powerful tools of mathematical semantics both for microprogramming languages of parallel computational structures and description of many kinds of cellular systems of the different nature. At present along with the above types of parallel formal languages we also study certain other languages of the same type, which have interesting applications, including questions of their modelling by ***CA*** models.

On this we finish presentation of the basic received results in the theory of parallel grammars τ_n and T_n that are determined by the *classical* and *nondeterministic* ***1-CA*** models accordingly. Thus, here do not considered a number of rather important and interesting questions concerning the research of the ***CA*** models as acceptors of the formal languages. These questions were enough intensively investigated by such researchers as *K. Culik*, *A.R. Smith*, *T. Jebelean*, *S. Kosaraju*, *M. Nordahl*, *A. Hemmerling*, and by many others; works in this direction can be found in [7].

5.4. Algorithmical problems of the theory of parallel grammars, determined by the classical *CA* models

Problems of *algorithmical solvability* play very important part in modern mathematics. Such problems are related to the so–called class of "*mass*" problems for which it is necessary to establish existence or absence of a common resolving algorithm. Note, in the theoretical and mathematical cybernetics the algorithmically unsolvable problems frequently enough arise in problems of the analysis of dynamics of transducers of discrete information, for example, various infinite automata of which linguistic aspects of the ***CA*** models are considered in this section. Indeed, within finite systems the problems of algorithmic solvability do not carry such actual character at worst the many solutions can be received by means of methods of simple exhaustive search of the corresponding variants and opportunities. However, in the theory of generative grammars the algorithmic problems occupy the large enough place, for example, the existence problems of algorithms, recognizing relative to grammars of some class or type, whether the formal language generated by a certain grammar possess the preset property.

Now, not onto all questions of algorithmical solvability in the theory of parallel grammars, determined by the classical ***CA*** models the answers exist. In the present section the results of solution of certain algorithmic

problems for the $L(\tau_n)$ and $L(T_n)$ parallel languages, which characterize, substantially, constructive and dynamical properties of the ***CA*** models, corresponding to them are represented. In addition, a lot of questions have remained beyond field of our sight. We present now mathematical formulations for the most known mass problems in this direction.

1. The emptiness problem: *Whether exists a solving algorithm determining, whether there will be a language generated by a formal grammar, empty?*

2. The completeness problem: *Whether exists a solving algorithm allowing to determine existence for an arbitrary formal grammar of capability to generate language which contains all nonempty finite words specified in its alphabet?*

3. The finiteness problem: *Whether there is a solving algorithm allowing to determine, whether will be finite a language generated by a formal grammar?*

4. The membership problem: *Whether exists a solving algorithm allowing to determine the fact of belonging of a finite word to a language generated by the specified grammar?*

5. The identity problem: *Whether there is any solving algorithm allowing to determine the identity of languages generated by two arbitrary grammars?*

6. The simplicity problem: *Whether there is a solving algorithm allowing to determine, whether will a language generated by the given formal grammar as a regular language, context–free language or context–sensitive language?*

7. The formularity problem: *Whether exists a solving algorithm allowing to determine, whether will a formal language generated by an arbitrary grammar as a formula language?*

8. The limit problem: *Let* ***g1*** *is a formal grammar. If for it there is such word* $c^* \in L(g1)$ *that for each word* $c \in L(g1)$ *of the language on the basis of derivation rules of the preset grammar* ***g1*** *a sequence containing a word* w^* *is generated, then it is quite natural to define the word* w^* *as a limit of process of derivation in this grammar. The limit for the case of parallel grammars determined by the classical* ***CA*** *models presents in a sense a stability point of derivation process. In many cases the limit can be potentially achievable, for example, in the case of unlimited growth of length of the deduced words. In view of these assumptions the problem is reduced to a question of existence of any solving algorithm for definition of existence for any language generated by an arbitrary grammar of a certain limit in the above sense.*

9. The problem of intersection emptiness: *Whether there is any algorithm determining the intersection emptiness of two languages generated by means of an arbitrary formal grammar?*

10. The existence problem: *Whether there is a solving algorithm determining whether will a set of finite words by a language of specified formal grammar?*

Undoubtedly the above problems of algorithmic *solvability* are the most important mathematical problems of the theory of parallel grammars, determined by the classical ***CA*** models, as, however, and certain other formal systems. On the basis of a number of researches it is possible to formulate the following basic theorem [24,40-43,82-87,102,106].

Theorem 92. Problems of identity, finiteness, membership, intersection emptiness, existence and existence of a limit are algorithmic unsolvable for $L(\tau_n)$–language generated by dint of a parallel τ_n–grammar whereas the problems of emptiness and completeness are algorithmic solvable.

These assertions are true and for the case of parallel languages $L(T_n)$. At the same time in full or in part the above *two* problems of simplicity and formularity still remain open. In addition, if the *formularity* problem still remains completely open then relative to the *simplicity* problem a rather interesting result presented by the following basic theorem takes place.

Theorem 93. The simplicity problem relatively to the regular languages and the context–free languages $L(\tau_n)$ is algorithmically unsolvable.

In view of result of this theorem, the algorithmical unsolvability of the simplicity problem is quite real. Moreover of the theorem ***93*** and results of study on finite automata the unsolvability of the *equivalence* problem of an arbitrary parallel τ_n-grammar in a context of language capabilities generated by it to a {*nondeterministic*} finite automaton, and to an *(one–way nondeterministic)* automaton with stack memory follows. Thus, the comparison problem of the formal parallel languages $L(\tau_n)$ with known traditional recognizers of the *Chomsky* hierarchy is unsolvable, what a rather strict result in the theory of formal τ_n-grammars presents [31-33].

In addition, analogously to the case of languages $L(\tau_n)$ the *algorithmical unsolvability* along with *solvability* of similar mass problems there is and for *nondeterministic* languages $L(T_n)$. However since the class of all $L(\tau_n)$ languages is own subclass of the class of all $L(T_n)$ languages, it allows to carry over the above results relative to algorithmic solvability and onto parallel nondeterministic languages $L(T_n)$. Moreover, it is necessary to have in view that unsolvability of a mass problem presumes absence of a solving algorithm in its modern understanding only. While for some problems of this class the certain decision algorithms are quite possible. In particular, in the case of ***CA*** models similar situation is widespread enough phenomenon.

On that a presentation of the obtained results in the theory of grammars defined by the classical and nondeterministic ***1–***dimension ***CA*** models which allow not only to receive satisfactory linguistic characteristics of dynamic properties of models of such type but also to give new tools of research of the ***CA*** models, as a whole, завершается. Meantime, in this direction many open problems and perspective directions of researches remain also. In addition, a spreading of the received results of theory of parallel τ_n-grammars and T_n-grammars to the case of higher dimensions seems to us rather perspective [24,31-33,40-43,82-87,102,106].

The more partial problem of identification of the infinite ***CA*** models on basis of some results of their dynamics directly adjoins to the existence problem. Its base essence is reduced to definition of sought ***CA*** model in terms of its ***LTF*** on the basis of known sequences $J_k = <c_k>[\tau^{(n)}]$ *(**k=1..v**)* of finite configurations generated by it**;** i.e. a constructive definition of the kind of ***LTF*** $\sigma^{(n)}$ of a model on the basis of known J_k**–**sequences that are generated by the appropriate ***GTF*** $\tau^{(n)}$**.** This problem is similar to the case of the theory of finite automata, when in experiments with a finite automaton is being discovered its internal logic organization, i.e. a table determining *output* of the automaton on the basis of its *internal* state and *input*. A similar experiment will consist in submission to input of a finite automaton of the input sequences of symbols and the posterior analysis of the output sequences which are generated by the automaton**;** i.e. as a result of this experiment the identification of automaton on the basis of results of analysis of its reaction to the input sequences is made.

In addition, the theory of finite automata deals with a lot of experiments of various type and purpose for identification of logic internal structure of finite automata or their separate components. The detailed discussion of the problematics can be found in works [7,24,200]. The identification problem in the case of an infinite ***CA*** model is reduced to definition of its global transition function on the basis of sequences $J_k = <c_k>[\tau^{(n)}]$ of finite configurations generated by dint of the ***CA*** model, i.e. on the basis of concrete history or finite number of histories of the model.

Thus, the identification problem is reduced to determination of internal logic structure of a ***CA*** model on the basis of its behaviour, namely**:** the input *(the current configuration – global internal state)* under the influence of ***GTF (LTF)*** is converted to the following internal state identified with output of this model**;** i.e. any ***CA*** model can be considered as the *Moore* infinite automaton. In addition, this problem, interesting by one's own

essence, for the infinite ***CA*** models does not carry such comprehensive character, because in general the problem is algorithmically unsolvable. However, in a lot of rather interesting cases the problem is solvable.

A. Adamatzky was the first who has started research of this problem for case of the ***CA*** models, having received a lot of rather interesting results concerning the identification problem for classical and some other types of the finite ***CA*** models [201-203]. Thus, for the case of finite ***CA*** models the problem of both existence, and identification is algorithmic solvable because the ***CA*** models generate only finite $L(\tau_n)$ languages. Generally, both the existence problem, and the identification problem already for the classical ***1–CA*** models are algorithmically unsolvable *(the proof of the second assertion is immediate consequence from the proof of the first one* [24]). Meanwhile, algorithmical unsolvability of both mass problems gives a very good opportunity to use it as effective components of the general apparatus of study of mass problems of the dynamic theory of classical ***CA*** models.

In spite of algorithmical unsolvability of the problems of existence and identification in general, the problem of experimental determination of a classical ***CA*** model which generates the certain J_k–sequences of finite configurations presents the indubitable interest. An interesting enough experimental approach to solution of the identification problem can be found in [24,32,82,87]. The identification problem that in our case can be defined as follows**:** ***In terms of dynamics of a CA model with alphabet A of internal states of its elementary automata it is necessary to define its local transition function;*** the problem can be constructively solved in terms of an algorithm presented in [9] for instance. This constructive algorithm has a rather simple programm realization. Generalization of the algorithm to high dimensions allows to formulate the assertion**:** ***For a classical d–CA (d ≥ 1) model the identification problem in our posing has constructive decision under the assumption of choice of a central automaton of the neighbourhood template of such model.***

So, unsolvability of the problems of existence and identification for the classical models, as a whole, presumes the further elaboration of partial approaches to their decision for ***CA*** models of certain types or classes. It would promote the occurrence of certain important enough theoretical and applied results as a whole. On the other hand, it is possible to show that the identification problem for finite ***d–CA*** models is constructively solvable and for this purpose the effective enough computer methods there are [24]. The methods of identification of the finite ***d–CA*** models

with help of artificial neural networks have been suggested**;** interesting enough questions of estimation and training of neural networks which are used for solution of the above identification problem for finite ***d–CA*** models have been considered [24]. As a whole such problematics seems interesting enough from the certain standpoints, hence in this direction a number of researches have been done by us and by a number of other researchers *(see* ***https://bbian.webs.com/Cellular_Automata.pdf****)*. Many quite interesting references in this direction can also be found in [102].

In the following chapter the questions presenting special interest from standpoint of use of classical ***CA*** models as a perspective environment for modelling of parallel discrete processes and phenomena in various theoretical and applied fields of the natural sciences are considered. In addition, the modelling concept is considered in various aspects which present certain interest from various theoretical or applied standpoints.

Chapter 6. The modelling problem in the classical cellular automata *(CA)* along with the related questions

The modelling problem in classical ***d–CA (d≥1)*** models presents a rather great theoretical and applied interest. The significant number of works containing many interesting enough results is devoted to this problem. One of basic directions of study in this field is related to the modelling of one ***d-CA (d≥1)*** model by another model, namely**:** modeling real time**;** modelling with suppression of certain properties of the modelled ***d-CA,*** parameters simplification of the simulating model, etc. If in the earlier considered fields of the ***CA*** problematics the problems of optimization character, practically, were not put, then here at modelling the use of a certain optimizing technics is already supposed. A lot of the researchers was occupied with questions of modelling in the classical ***d-CA;*** of them it is necessary to note such researchers as ***J. Neumann, A. Burks, S. Cole, K. Culik, E. Banks, H. Yamada, H. Nishio, S. Amoroso, A. Smith, E. Codd, T. Toffoli, P. Sarkar, J. Buttler, R. Volmar, A. Wuensche, A. Waksman, A. Adamatzky, A. Podkolzin, O.L. Bandman, V.Z. Aladjev, S. Ulam*** and a lot of others [7]. The more detailed information in this direction can be found in works [24,102] along with numerous references to other works available in them. And it in spite of the fact that these questions concern mainly internal ***CA*** problematics, besides numerous works considering the ***CA*** as a modelling environment for numerous applied problems.

Of the first interesting enough results in this direction, not considering results of modelling in the ***CA*** of founders of this problematics *John von Neumann* and of his direct followers, it is necessary to note the doctoral thesis of *A.R. Smith* [99]**,** where he considers a lot of principal questions of simulating of one classical ***d–CA*** model by other model of the same dimension ***d,*** but with reduction of the size of neighbourhood template of the modelling ***d–CA.*** A plenty of other rather interesting results that concern the modelling of this and similar type for case of classical ***d–CA (d≥1)*** models can be found in the works quoted above. With a history of this problematics it is possible to familiarize for example in [7,24,102].

6.1. Concepts of modelling in the classical *CA*

Above all, we shall make a general remark relative to two techniques of modelling in the ***CA*** class. Analogously to the ***CA*** founders problems *(J. von Neumann, S. Ulam, A. Burks, J. Holland, E. Codd, E. Banks, H. Yamada,*

etc.) a rather large number of researchers in this important direction use for the purposes of both theoretical, and especially applied modelling immediately the ***CA*** models, providing their by certain rules of acting with inserting in them of the modelled algorithms and processes. This approach has appreciably constructive character, when in a certain ***CA*** environment a single task of modeling can be reduced to a composition of subtasks composing it. Typical method of such type of modelling is a making in a ***CA*** environment of a lot of blocks of elementary automata that carry out determinate functions and cooperate among themselves by means of exchange of control impulses along the specially organized information channels in the environment that is formed of elementary automata of the environment. The above approach determines a direct embedding of a modelled problem in ***CA*** environment.

Whereas the second approach uses ***CA*** models at a level of the formal systems of parallel information processing, representing more general level of modelling of the researched algorithms and processes. In this respect both approaches to modelling on the basis of ***CA*** method, to a certain extent, it is possible to liken to well–known ways of modelling *(computability)* on the basis of *Turing* machines and *Markov* algorithms accordingly or other formal algebraic systems of words processing in finite alphabets. If the first approach is most suitable for the purposes of research of applied aspects of modelling on the basis of ***d–CA*** $(d \geq 1)$ whereas the second composes a basis of formal research of constructive and computing possibilities of the ***CA*** models as abstract systems of the parallel information processing that at the axiomatic level provide the properties of *homogeneity* and *localness*, whereas at program level – the *reversibility* property of ***CA*** dynamics. In addition, both these methods can be mutually complementary with *reasonable* degree of admissibility. The second approach forms a basis for the further representation of the various questions of modelling.

We shall begin the representation with the traditional approach to the modelling concept going back to *A.R. Smith* [99]. Let c_t denotes a certain configuration of a classical ***d–CA*** $(d \geq 1)$ model at a moment $t \geq 0$ and τ – a global transition function *(**GTF** τ)* of the model. Then, result of ***t–***fold application of τ-function *(denotation – τ^t)* is determined by the following recurrent relation, namely**:**

$$\tau^o(c_o) \equiv c_o\tau^o \equiv c_o\,, \quad \tau^t(c_{t-1}) \equiv c_{t-1}\tau^t \equiv \tau(\tau^{t-1}(c_{t-1})) \equiv c_t \qquad (t \geq 1)$$

At the made assumptions we introduce the modelling concept of a ***d-CA***

(d≥1) model by other model of the same ***d***–dimensionality.

Definition 22*. Let J_d is a set of all allowable global transition functions τ_j for classical d–CA (d≥1) models. Now, let's consider two classical g1 and g2 models of the given set J_d with sets of configurations and global transition functions c1, τ_1 and c2, τ_2 ($\tau_1, \tau_2 \in F_d$) accordingly. Let's speak, that the Z2 model simulates the Z1 model in real time k2/k1 if and only if there is such effectively computable injective mapping H: c1 → c2 and an effectively computable function H1: $J_d \to J_d$, that the relation takes place, namely:*

$$\tau_2^{k_2}(H(c_t)) \equiv H(\tau_1^{k_1}(c_t)); \quad \tau_2 = H1(\tau_1)$$

At fulfilling of the condition k2 = k1 we shall speak about simulation of classical d–CA (d ≥ 1) models in strictly real time.

In a number of cases at presentation of results concerning the modelling in ***d–CA (d≥1)*** it will be convenient to use for an arbitrary model ***d–CA*** ≡ $<Z^d,A,\tau^{(n)},X>$ designation ***(d, n, a)*** where the sense of parameters ***d, n, a*** fully corresponds to the definition of classical ***d–CA (d ≥ 1)*** models, not demanding any special explanations. By means of one a rather simple procedure *J. Buttler* [204] on the basis of one sufficient condition of the existence of algorithm of modelling in real time ***1/k*** of a classical ***d–CA (d≥1)*** model by dint of other model of the same dimension has proved the following a rather interesting result that is enough frequently used in research of dynamics of the classical models by means of modelling.

Theorem 94*. For an arbitrary classical (2,n,a) model there is such model $(2,p,a^{2kt})$ that simulates the first model in real time 1/k where t is length of side of minimal square containing the neighbourhood template of the simulated model.*

Without loss of generality the modelling of a classical ***1–CA*** model with alphabet ***A***={***0,1,...,a–1***} and neighbourhood index ***X***={***0,1,...,n–1***} by the models of the same class and dimension but with reduction of the size of neighbourhood template is presented in [82-87,102] with the purpose of illustration of one of possible approaches to modelling of the classical ***d–CA (d ≥ 1)*** models. The presented scheme is rather pellucid, defining a principle of reduction of a source neighbourhood template of size ***n*** of the simulated model and allows to receive the following result useful in a number of appendices.

Theorem 95*. For an arbitrary classical (1,n,a) model there is such (1,2,ψ) model with alphabet $\cup_p A^p \cup A$ {p=1..(n–2)} of elementary automaton of*

cardinality $(a^n - a) / (a - 1)$ ***that models the first one in real time*** $1/(n-1)$.

A generalization of this approach allows to receive the following useful enough result [24,43,82-87,102,106].

Theorem 96*. For an arbitrary d–CA (d≥1) model with alphabet A = {0,1, ..., a–1} and neighbourhood template* $n_1 x n_2 x \ldots x n_d$, *there is such model of the same dimension that simulates the first one in real time* $1/(\sum_{j=1}^{d} n_j - d)$ *with neighbourhood index X = {0, 1} and alphabet A' of the cardinality determined by the following formula, namely:*

$$\#A' = a + \sum_{j=1}^{d} \left(\frac{a^{\prod_{k=1}^{j} n_k + \varphi(j)} - a^{2\prod_{k=0}^{j-1} n_k}}{a^{\prod_{k=0}^{j-1} n_k} - 1} \right), \quad where \quad \varphi(j) = \begin{cases} 1, & if\ j < d \\ 0, & otherwise \end{cases}; \quad n_o = 1$$

The optimization problem of a modelling algorithm of similar or other type can be considered as a rather interesting problem. Using the above approach it is possible to prove the following result useful for problems of modeling in the classical ***d–CA*** $(d \geq 1)$ models.

Theorem 97*. A classical d-CA (d≥1) model with neighbourhood template in the form of hyperparallelepiped of size* $n_1 x n_2 x \ldots x n_d$ *and an alphabet A={0,1, ..., a–1} is simulated in real time* $1/n$ *by means of a model of the same dimension with alphabet A* and neighbourhood template in the form of hypercube with edge of length two under the defining conditions such as:*

$$n = \max_{k=1..d} \{n_k\} - 1 \qquad \#A^* = \sum_{k=1}^{n} a^{k^d}$$

In addition, a rather characteristic property of a simulating ***CA*** model is *inheritance* by it of a number of basic dynamic properties of a simulated model. This circumstance is essential enough, allowing above all at the theoretical level to research dynamics of classical ***d–CA*** $(d \geq 1)$ models with simple neighbourhood indexes with the subsequent spreading of the received results to more general types of classical ***CA*** models. Hence the simplification of neighbourhood template of a researched ***CA*** model is reached sometimes owing to essential increase of alphabet cardinality of the simulated model. In a lot of cases however simplicity of topology of connections of elementary automata of the studied ***CA*** models speaks in support of a similar approach. The above results are characterized by the fact, the classical models with rather large neighbourhood templates and with small states alphabets ***A*** can be simulated by the models of the

same type with the smaller neighbourhood templates and rather large states alphabets A^*, and vice versa.

Other strong enough result concerning simulating of arbitrary classical ***d–CA*** $(d \geq 1)$ models by means of binary models of the same dimension and type has been received by *A.R. Smith* [99], namely.

Theorem 98. For an arbitrary classical (d, n, a) model there is a (d, k, 2) model that simulates the first model in strictly real time and for which the following determining relation takes place, namely:

$$|k| \leq (2^{a-1} - 1)(n + 2) + [\log_2 a](n - 1) + 1$$

It is necessary to note that at the proof of the above *Buttler-Smith* results concerning the modelling in classical ***d–CA*** $(d \geq 1)$ models an optimizing technics was not used and in certain our works the attempts to receive definite optimization of parameters of the models has been undertaken. Questions of optimization of defining parameters of simulating models have been considered by a number of other researchers [7,205-207,286]; see also useful references in[24,42-43,82,102,106].

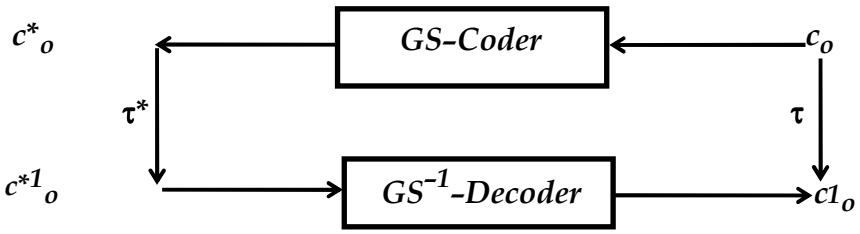


Fig. 11. The diagram illustrating the principle of 1–modelling in the classical CA models.

Below, certain of results in this direction will be represented. So, in this connection the ***1–****modelling* concept has been introduced whose essence is illustrated by the diagram presented on fig. ***11*** and is reduced to the following [5,40-43]. This diagram allows to present quite evidently the essence of ***1–***modelling in environment of classical ***d–CA*** $(d \geq 1)$ models along with illustration of the used modelling principle. In the modelled classical ***d–CA*** $(d \geq 1)$ any configuration $c_o \in C(A,d,\phi)$, defined in a finite A alphabet as a result of action of global transition function τ is converted into the following configuration $c_o\tau = c^1{}_o$. Let $GS(x)$ is a certain recursive coding method of symbols $x \in A$ whereas $GS^{-1}(y)$ is a certain method of recursive decoding of a set of symbols of an alphabet A^* of a modelling ***d-CA*** $(d \geq 1)$; let $GS(x)$ and $GS^{-1}(y)$ determine recursive methods of *coding*

and *decoding* of configurations x and y accordingly. Then for simulating classical ***d–CA*** $(d \geq 1)$ model any configuration $c^*_o = GS(c_o)$ as a result of action of global function τ^* is converted into next configuration c^{*1}_o for which the relation $GS^{-1}(c^{*1}_o)=c^1_o$ takes place. Let's speak, that a certain classical ***d–CA*** is ***1–****modeled* by means of a model of the same dimension if dynamics of the *simulated* model and the *simulating* model will submit to the conditions listed above. Let T_x is the length of edge of a minimal ***d–***dimension hypercube with neighbourhood template of the *simulated* classical ***d–CA*** $(d \geq 1)$ model. At the made assumptions the result useful in many respects takes place [8,9,24,40-43,82,102,106].

Theorem 99*. For an arbitrary classical* $<Z^d,A,\tau^{(n)},X>$ $(d \geq 1)$ *model with neighbourhood template of size* T_X *there is a classical* $<Z^d,A^*,\tau^{(p)},X^*>$ *model with neighbourhood template of size* T_{X^*} *which will 1–simulate the first classical model where* T_{X^*} *and a states alphabet* A^* *satisfy the following determinative relations, namely:*

$$\begin{cases} T_{X^*} \leq (T_X+1)([\log_2 a]+1)-1, & for \quad A^*=\{0,1,2\} \\ T_{X^*} \leq (T_X+1)(L+5)-1, & for \quad A^*=\{0,1\} \\ \quad where \quad L=[(\log_2 a-1)/(\log_2 7-1)]+2 \end{cases}$$

As far as we know, this result today is the best among results of similar type. In [1,8] a little the more special ways of simulation of one classical ***d-CA*** $(d \geq 1)$ model by another model also have been considered. So, *A.R. Smith* has shown [99] that being based on definition **22** broadly speaking it is impossible to simulate a classical ***d–CA*** $(d \geq 2)$ by a model of smaller dimension. Furthermore, we have disclosed, that at some assumptions it is possible to simulate an arbitrary classical ***d–CA*** $(d \geq 2)$ model even by means of appropriate classical ***1–CA*** model [8]. One opportunity of similar simulating of classical ***d–CA*** $(d \geq 2)$ models by means of classical ***1–CA*** model will be constructively illustrated below. This method uses a special ***1–***dimension presentation of ***2–***dimension finite configuration of classical ***2–CA*** models.

Certain works relating to the special properties of classical ***d–CA*** $(d \geq 1)$ models directly relate to the modelling problem too. In particular, some researchers dealt with certain questions of standardization of structure of neighbourhood templates of classical ***CA*** models. This problematics is presented rather important and from the theoretical standpoint, and from standpoint of numerous appendices, especially in view of use of ***d–CA*** as a rather perspective environment of modelling. So, questions

in this direction regularly were investigated in works of *H. Yamada* and *S. Amorozo* on the basis of the introduced concepts of behavioural and structural isomorphism. It has been shown, that the certain equivalent relations retentive one or both forms of isomorphism lead to structure *standardizations* of neighbourhood templates in classical ***CA*** models. At receiving of these results rather important concepts of *blocking* and the *blocked structure* of elementary automata of a ***CA*** model play the central part. At that, definition of the weak form of the *behavioural isomorphism* has led to the simplifications of standard structure of neighbourhood templates in classical models. These and other related questions can be found in [7,24,43,82,102,106,208,278,286].

In many problems of modelling in classical ***d–CA*** *(**d≥1**)* models, first of all, from the standpoint of questions of algorithmical properties of this models the ***T–modelling*** concept appears a rather effective. It is known, a classical ***CA*** model presents a certain parallel algorithm of processing of words in finite alphabets. Since research of classical ***CA*** models from this standpoint presents indubitable interest, then it is rather expedient to determine essentially important concept **– «*one algorithm (weakly) T–models other algorithm*»;** its definition can be represented as follows.

Let M_1 is some algorithm of processing of words in a finite alphabet A whereas M_2 is an algorithm of processing of words in a finite alphabet A^* $(A \subseteq A^*)$. Let's assume that $M^k{}_1 s = s^k$ $(M^o{}_1 s = s)$ is a result of k–fold processing of a word s, preset in the alphabet A by means of algorithm M_1. Then for an arbitrary finite word s in the alphabet A the algorithm M_1 generates a certain sequence of finite words of the following kind**:**

$$M^o{}_1 s,\ M^1{}_1 s,\ M^2{}_1 s,\ M^3{}_1 s,\ ...,\ M^k{}_1 s,\ ... \qquad (21)$$

Let now s^* is an arbitrary finite word in the alphabet A^* which in the A alphabet coincides with word s, and algorithm M_2 generates from the word s^* a certain sequence of words of the following kind, namely**:**

$$M^o{}_2 s^*,\ M^1{}_2 s^*,\ M^2{}_2 s^*,\ M^3{}_2 s^*,\ ...,\ M^k{}_2 s^*,\ ... \qquad (22)$$

<u>Definition 23</u>. Let's speak, that a M_2 algorithm T–models an algorithm M_1 if and only if there is a recursive procedure, permitting for a finite word s, determined in a certain alphabet A, to choose out of sequences of words (22) such subsequences of words of the kind

$$M^o{}_o s^*,\ M^{j1}{}_2 s^*,\ M^{j2}{}_2 s^*,\ M^{j3}{}_2 s^*,\ ...,\ M^{jk}{}_2 s^*,\ ...$$

that in the A alphabet the relation $(\forall k \in N)(M^{j_k}_2 s^* \equiv M^k_1 s)$ ***&*** $(j_k = T+k)$ ***takes place. In addition, if*** j_k ***value in the modelling algorithm*** M_2 ***will be depend on length of the processed words*** s $\{j_k = f(|S|)\}$***, then we shall speak – the algorithm*** M_2 ***weakly T–models the source algorithm*** M_1***, in particular, functional algorithm of classical CA models.***

The above ***T–***modelling concept is used by us rather widely, possessing a lot of positive features. Particularly, in view of this concept it is rather easy to make sure, that the concept of ***1–***modelling is a especial case of ***T–***modelling, representing, however, an independent interest first of all because of following two circumstances**:** ***(1)*** the ***1–***modelling defines the mode of strictly real time, and ***(2)*** a simple method GS^{-1} of decoding of the words generated by the modelling algorithm [24,40-43,82,102,106].

Meanwhile, along with the above concept for theoretical study of a lot of important enough questions, linked with modelling in classical ***d–CA*** models, we have determined essentially more abstract ***WM–modelling*** concept [42] that subsequently has received essential generalization in the form of the concept of ***W-modelling,*** allowing to solve subsequently a lot of rather interesting problems of the general problem of modelling in classical ***d–CA*** $(d \geq 1)$ models [24,42-43,82-87,102,106].

Meanwhile, the introduced concept of ***W–****modelling* enough essentially expands the ***WM–****modelling* concept, covering the extremely wide class of modelling methods in classical ***d–CA*** $(d \geq 1)$ models. A number of the used methods for simulation in environment of the classical ***d–CA*** $(d \geq 1)$ models corresponds to the ***W–****modelling* concept. In addition, with a lot of the related questions it is possible to familiarize in works [24,41-43].

The ***W–****modelling* concept is enough wide**,** covering not only essentially classical ***d–CA*** $(d \geq 1)$ models. As an example we shall consider the next algorithm of modelling of a classical ***1–CA*** model with a states alphabet $A=\{0,1,...,a-1\}$ and neighbourhood index $X=\{0,1\}$ by means of a model ***1–CA**** with the same alphabet A and neighbourhood index $X^*=\{-1,0,1\}$**,** whose local transition function $\sigma^{(3)}$ is determined and by configuration of neighbourhood template and by coordinates of its central automaton. More precisely, local transition function $\sigma^{(3)}$ of the ***1-CA**** model is being defined by the following formula**:**

$$\sigma^{(3)}(x_{-1},x_o,x_1)=\begin{cases}\sigma^{(2)}(x_{-1},x_1), & \text{if } [x_o] \text{ is even number}\\ x_{-1}\otimes x_o\otimes x_1, & \text{if } [x_o] \text{ is odd number}\end{cases}$$

where**:** $\sigma^{(2)}(x,y)$ **–** a local transition function of the simulated ***1–CA;*** $\otimes$ **–**

operation of addition modulo a and $[x_j]$ – a coordinate of an elementary j–automaton of the simulating ***1–CA**** model.

Depending on the definition of local function of the ***1–CA**** model, it is simple to make sure that ***1–CA**** ***1–***models a ***1–CA*** model on elementary automata with even coordinates. It is possible to show that concerning this class of the ***CA*** models the first criterion of the nonconstructability that is based on the general ***MEC*** concept is valid *(Theorem **6**)*. Analysis of existence of the ***MEC*** for models such as ***1–CA**** that are intended for modelling of an arbitrary ***1–CA*** model has allowed to formulate even a little stronger result [24,41-43,82,102,106].

Theorem 100. Class of 1-CA* models with states alphabet A={0,1,...,a-1} and neighbourhood index X={0,1,...,n-1} whose local transition functions are determined by configurations of their neighbourhood templates and coordinates of their central automata can't model an arbitrary classical 1–CA model by means of a 1–CA* model that does not possess the NCF nonconstructability.

Further in the course of the present chapter certain other approaches to modelling in classical ***d–CA*** $(d \geq 1)$ models will be used whose essence will be clear or out of the principle of modelling, or will be explained in case of need.

6.2. Modelling of the well–known formal processing algorithms of words in finite alphabets by means of *CA*

Since classical ***d–CA*** $(d \geq 1)$ models are parallel processing algorithms of ***d***-dimension words in finite alphabets it is quite interesting to compare them with known formal sequential algorithms. One of the comparative approach of such type is modeling of one type of algorithms by others, and vice versa. Above all, now we shall present results of ***T–***modelling of known formal processing algorithms of finite one–dimension words in finite alphabets by means of classical ***1–CA*** models, and vice versa. Moreover, an optimizing technics which was used at modelling allowed to receive good proportions between basic parameters of the modelled and modelling algorithms, what allows to do comparative estimations of algorithms of such type.

Further, as a parallel algorithm a classical $1\text{–}CA \equiv \langle Z,A,\tau^{(n)},X\rangle$ model is chosen, while as the first sequential algorithm – known *Turing* machine MT^s_q with alphabet S $(\#S=s)$ of symbols on output tape and alphabet Q

(#Q=q) of internal states of some finite automaton**;** MT^s_q represents the most popular formal model of sequential calculations. At that, we shall consider a machine MT^s_q with ouput tape, infinite into both sides**;** this machine is some modification of the conventional *Turing* machine and is completely equivalent to it. In this direction the following result that is rather useful to the further study of dynamics of classical ***CA*** models takes place [24,41-43,82,102,106].

Theorem 101*. For an arbitrary machine* MT^s_q *there is a classical model 1-CA with neighbourhood index X={-1,0,1} and alphabet A of cardinality (s+q+9) which 8–simulates the first. For each machine* MT^s_q *there is a classical 1-CA model with neighbourhood index X={-1,0,1} and alphabet A of cardinality (s+2q) which 2–simulates the first. For each machine* MT^s_q *there is a classical 1–CA model with neighbourhood index X={–2, –1,0,1} and an alphabet A of cardinality (s+q) that its 1–simulates. For an arbitrary machine* MT^s_q *with k (k≥1) final tapes there is a classical 1–CA model with neighbourhood index X={-1,0,1} and a states alphabet A of cardinality* $s^k(q+1)^k$ *that 1–simulates an arbitrary machine* MT^s_q**.**

For comparison the simulating problem of an arbitrary MT^s_q by means of a $CAoS\equiv<Z^1,A,2,\Psi^{(2)},\Xi>$ model was considered. This class of models is widely used for creation of physical models of a various sort, and the interrelation of this type of models with classical ***CA*** models has been considered in [24,82-87]. The result can be represented by the theorem.

Theorem 102*. An arbitrary machine* MT^s_q *is 2–simulated by means of a* $CAoS\equiv<Z^1,A,2,\Psi^{(2)},\Xi>$ *model with #A=(s+2q) and* $A=S\cup Q\cup Q^*$*, and by means of a* $CAoS \equiv <Z^1,A,3,\Psi^{(3)},\Xi>$ *model with alphabet* $A = S\cup Q$ *and #A=(s+q), where #G is cardinality of an arbitrary finite set G.*

Determining the complexity of a machine MT^s_q and a classical ***1–CA*** as $S_{MT}=sxq$ and $S_{CA}=axn$ accordingly, we receive corresponding values of three types determined by the theorem ***101,*** of the classical models that simulate an arbitrary machine MT^s_q as $S_{CA}=3(s+q+9)$, $S_{CA'}=3(s+2q)$ and $S_{CA''}=4(s+q)$ accordingly. On basis of minimal estimation of complexity for MT^s_q which for today equals $S_{MT}=24$ *(s=4, q=6; s=6, q=4)* [7,102] it is easy to make sure, that $S_{CA}=57$, $S_{CA'}=42$ and $S_{CA''}=40$**.**

In the meantime, along with the algorithmical complexity concept, the complexity concept of modelling which includes temporal and spatial

costs of a modelling algorithm is interesting enough; i.e. the modelling complexity in the case of classical ***d–CA (d≥1)*** models can be defined by the formula $SM_{CA}=dxTxaxn$, where ***d –*** dimensionality of homogeneous space of a model; ***a*** and ***n –*** cardinality of a states alphabet ***A*** and size of neighbourhood template accordingly; ***T –*** time of modelling of one step of an arbitrary modelled algorithm.

At the made assumptions the difference for three represented cases of modelling by means of ***1–CA*** becomes more striking, namely: $SM_{CA} = 1x8x3x(s+q+9) = 456$, $SM_{CA'} = 1x2x3x(s+2q)=84$, $SM_{CA''} = 1x1x4x(s+q) = 40$. So, if concerning the algorithm complexity the third way of modelling appears the most simple concerning the modelling complexity the full superiority of the third one takes place; moreover, for it values of the algorithm complexity and the modelling complexity coincide – $S_{CA''} \equiv SM_{CA''} = 40$. For estimation of the modelling complexity by means of ***CAoS*** we can use such parameter as $SM_{CAoS} = \#Axm$ which for the case of modelling of the universal MT^s_q gives value $SM_{CAoS} = 36$. However, an estimation of influence on this parameter of **Ξ–**procedure of block over–marking of Z^1 space of a modelling structure is difficult enough. As a consequence from the theorem ***101,*** a number of rather interesting results concerning algorithmic unsolvability of certain mass problems, linked with dynamics of finite configurations in the classical ***CA*** models follows [5]. Some of these problems are discussed a little bit below, but for this purpose we need to introduce a number of new concepts.

<u>Definition 24</u>. A configuration $c_o \in C(A, d, \phi)$ for a global function $\tau^{(n)}$ of a classical d–CA (d ≥ 1) model is named accordingly:

<u>limited</u>, if the relation $(\exists p)(\forall k)(c_k \in <c_o>[\tau^{(n)}] \rightarrow d(c_k) \le p)$ takes place, where d is minimal diameter of a block containing configuration c_k;

<u>(k–m)–periodic</u>, if the relation $(\exists m)(\exists k)(c_o\tau^{(n)m} = c_o\tau^{(n)k})$ $(m > 0; k–m > 1)$ takes place; <u>periodical</u> configuration, if the relation $(\exists k > 1)(c_o\tau^{(n)k} = c_o)$ takes place, and <u>passive</u> configuration for k=1; in addition, 2 last cases compose class of all fully periodical configurations; at last, a <u>vanishing configuration</u> if the relation $(\exists m)(c_o\tau^{(n)m} = «□»)$ takes place.

In view of this definition the following important enough result which determines the solution of a number of mass dynamics problems of the classical ***CA*** models can be formulated; the result is of interest also for the further research of this problematics [1,19,20,640].

***Theorem 103**. The following problems are algorithmically unsolvable for an arbitrary classical d–CA (d≥1) model, namely:*

– problem of recursiveness of a set** $<c_o>[\tau^{(n)}]$ **of finite configurations;

– problem of limitation of an arbitrary configuration** $c_o \in C(A,d,\phi)$**;

– problem of (k-m)-periodicity or periodicity of a finite configuration;

– problem of existence of passive and/or vanishing configuration in an arbitrary sequence** $<c_o>[\tau^{(n)}]$ **for a classical d–CA (d≥1) model;

– problem of existence for a classical d–CA (d≥2) model with GTF** $\tau^{(n)}$ **of such configurations c* that** $c^*\tau^{(n)} = c^{\infty}_r$**, where** $c^* \in C(A,d,\infty)$ **and** c^{∞}_r **– an arbitrary infinite configuration consisting only of states** $r \in A$**.

At proof of this theorem the ***T–***modelling opportunity of the universal *Turing* machine by classical ***1–CA*** models that directly follows of result of theorem ***101*** was essentially used. The result along with independent interest can be used as the auxiliary apparatus that bases on ***T-****modelling* at solution of some mass dynamics problems of the classical ***CA*** models representing as the theoretical, and applied interest. In particular, from this result follows, that the determination problem of type of a graph of global states of a classical ***CA*** model relative to its initial configurations is algorithmically *unsolvable*. While the unsolvability of the last problem of the theorem is based on *unsolvability* of the general «*domino*» problem considered in [24,41-43,82,102,106].

On basis of the ***T–***modelling concept certain questions of modelling by means of classical ***1–***dimensional structures of such known processing algorithms as ***TAG–***systems and ***LAG–***systems**,** *Büchi* regular systems, ***SS–***machines**,** *Marcov* normal algorithms**,** *Post* production systems and some others have been considered rather minutely [1,5,8]. At receiving of the results along with use of the ***T–****modelling* principle the optimizing technics consisting in use of especial optimizing procedures of parallel programming in classical ***1–CA*** models to some extent was applied. It has allowed to a large extent to receive optimal relations between base parameters of the modelling and the modelled algorithms.

Definitions of the processing algorithms of words in the finite alphabets used below are rather known and the reader can familiarize with them in [1,3,5,8,24,102]**,** therefore they have a rather schematic character with the purpose of explanation of certain basis parameters of the modelled sequential algorithms. Meanwhile, it is necessary to have in view, that the modelling is made at special formal level, considering the modeled

algorithm and the modelling algorithm as formal processing systems of finite words in finite alphabets without any special internal coding.

Let, a modelled formal algorithm has alphabet $C = \{c_1, ..., c_m\}$. Then an arbitrary ***TAG–***system has **ω–**number of truncation and m elementary transformations of the kind $c_k \Rightarrow b_k$ $(k = 1..m)$, where b_k – words in the alphabet ***C;*** the *Post* productions system has the same alphabet ***C,*** and p basic productions of the kind $a_jW \Rightarrow Wb_j$ $(j=1..p)$, where a_j, b_j and W are finite words in the alphabet ***C.*** The detailed description of ***SS–***machine can be found in monographs [1,5,209] whereas with concept of *Markov* normal algorithms it is possible to familiarize in the excellent book that contains rather detailed presentation of basis elements of the theory of algorithms and recursive functions [209]. The *Büchi* regular system has alphabet ***C*** and p basic transformations of the kind $a_jW \Rightarrow b_jW$ $(j = 1..p)$, where a_j, b_j, W – finite words in the alphabet ***C.*** Other natural extension of the ***TAG–***systems class are the ***LAG–***systems defined in the alphabet ***C*** by means of transformations of the following general kind, namely**:**

$$r_j = c_{j_1}c_{j_2}c_{j_3}...c_{j_q}; \quad r_jW \Rightarrow Wb_j \qquad (j = 1..p;\ p \le m^q)$$

where b_j, W – finite words in the alphabet ***C;*** in addition, if the first q symbols of a certain word s^*, processed by the system, coincide with a subword r_j, then its first symbol c_{j1} is deleted and to right end of the s^* word is added a subword b_j. Obviously, for $q = 1$ the systems ***TAG*** and ***LAG*** coincide. Thus, at the made assumptions the following basic result presenting a quite certain interest takes place, bearing in mind that the obtained relations were result of a certain optimizing procedure.

<u>Theorem 104</u>. An arbitrary TAG–system is weakly T–simulated by dint of an appropriate classical 1–CA model; in addition, the relations take place, namely:

$$a = \omega + m + \sum_{k=1}^{m} |b_k| + 3; \quad T = |s^*| + |b_k|; \quad s^* = c_k s; \quad s^* \in C \ \ (k = 1..m)$$

An arbitrary Post productions system is weakly T–simulated by dint of an appropriate classical 1–CA model; in addition, there are relations:

$$a = 3\sum_{j=1}^{p}[|a_j| + |b_j|] + 3p + m + 10; \quad T = 4|a_j| + 2|s^*| + 2|b_j|; s^* \in C \quad (j = 1..p)$$

An arbitrary SS–machine is weakly T–simulated by dint of a classical 1-CA model; in addition, there are relations $a=2n_1+n_2+4$ ***and*** $T=2|s^*|+2$ ***where*** n_1, n_2 ***– quantity of instructions*** $\{P_o, P_1\}$ ***and SD(k) of SS-machine***

accordingly. An arbitrary Büchi regular system is weakly T–simulated by means of an appropriate classical 1–CA model; in addition, there are the following relations:

$$a = 3\sum_{j=1}^{p}[|a_j|+|b_j|]-6p+m+10;\ T=|a_j|+max\{|a_j|,|b_j|\};\ s^*\in C\ \ (j=1..p)$$

In all previous cases the modelling classical 1–CA model possesses the neighbourhood index X = {–1, 0, 1}. An arbitrary LAG–system is weakly T-simulated by dint of an appropriate 1-CA model with neighbourhood index X = {0,1, ..., q+1}; in addition, there are the following relations:

$$a = m+\sum_{j=1}^{m}|b_j|+3;\quad T=|s^*|+|b_j|;\quad s^*=r_j;\quad s^*\in C\ \ (j=1..m)$$

An arbitrary Markov normal algorithm is weakly T–simulated by dint of classical 1–CA model with neighbourhood index X = {0,1} ≡ {–1,0}. In addition, it is necessary to keep in mind, that for all presented cases s* is the processed word of a modelled algorithm, |b| – length of a b word.

Particularly, a rather interesting consequence follows of modelling of an arbitrary ***SS–***machine by an appropriate ***1–CA*** model, namely [1,5,24].

<u>Proposal 7</u>. There are 1–dimensional classical CA models whose sets of finite configurations generated into null configuration are creative.

Hence, there are classical ***CA*** models whose sets of finite configurations converted directly into the null configuration are nonrecursive. In this connexion there is very interesting question about existence of classical ***CA*** models, whose analogous sets of finite configurations are simple or maximal, and what are the values of base parameters for ***1–CA*** models of such type. Thus, from theorem ***104*** follows, in spite of use in its proof of an optimizing technics of modelling for known sequential processing algorithms, it is not possible to get rid of condition of weak modelling at use of ***1–CA*** models as the modelling algorithms. A rather interesting discussion of such situation can be found in [24,40-43,82,102,106].

On basis of the *Turing* machines with ***k*** heads *(**TM***[k]*)* a lot of the formal models of parallel information processing is considered. So, one of such models allows to more simply analyze a number of situations arising in systems of parallel information processing [102]. In addition, the model receives enough interesting interpretations in terms of multiprocessing computing systems [34]. In deterministic cases this calculations model on the basis of ***TM***[***k***] is easily reduced to a traditional single–head ***TM,*** however there is an question of studying of acceleration of processing due to parallel work of ***k*** scanning heads of the machine. With principle

of functioning of ***TM***[***k***] the reader can familiarize for example in [24,34, 40-43]**;** ibidem a discussion of the related questions can be found. As a modelling algorithm a classical ***1–CA*** model with neighbourhood index ***X***={***-1,0,1***} has been chosen. At the made assumptions the following base result can be represented.

<u>Theorem 105</u>. An arbitrary Turing machine TM[k] with work program from t commands is simulated by dint of an appropriate classical 1–CA model with neighbourhood index X = {–1, 0, 1}, demanding no more than $2\{[t/2](p+1)-p\}$ ***(*** $p \geq k$***) steps of the model, where p – an initial distance between extreme scanning heads of the TM[k].***

A discussion of the results of modelling by means of classical ***1–CA*** of multihead *Turing* machines can be found, for example, in [5,24,40-43].

Simulating on basis of the classical ***d–CA (d≥1)*** modelsis linked with the problem of generality degree of either concept of a model of such type whose discussion has been started in [8]. In [24-28] and other our works the question of generality degree of the concept of classical ***CA*** models was investigated, and the basic method of this research was and there is the modelling. We shall continue discussion of this question concerning one rather interesting generalization of *parallel transformations* that have been introduced and investigated by *V.M. Glushkov* and his colleagues to the case of the homogeneous synchronous parallel processes. In this direction as such generalization *G.E. Tseitlin* has offered the so–called *heterogeneous periodically defined transformations (HPDT)*, whose essence is reduced to the following [7,24,40-43,82,102,106,210-212,286].

Let ***R*** – a bilateral infinite abstract register divided into segments each of which contains ***r*** individual elements. Then the formal object ***HPDT***

$$\Theta_s^R = \begin{bmatrix} f_1^{t_1}, & f_2^{t_2}, & \ldots\ldots, & f_r^{t_r} \\ k_1, & k_2, & \ldots\ldots, & k_r \end{bmatrix}$$

is defined by a shift function $s(q) = r^*q + k$ *(mod* ***r****)* along with a system of generating functions $\{f^t_j \mid j = 1..r\}$, where t_j – arity of f^t_j–functions which corresponds to the integral factors k_j *(*$j = 1..r$*)*. In addition, the result of application of a ***HPDT*** to an arbitrary state ***W**** *(*$W_q \mid -\infty < q < +\infty$*)* of the ***R***–register is such new state $b^*=\Theta^R_s(W^*)$ that $b^*=(b_q \mid -\infty < q < +\infty)$ where states of individual elements of q^{th} segment of ***R***–register are calculated according to the following general formulas, namely**:**

$$b_{s(q)+p-1} = f_p^{t_p}(a_{s(q)+k_p}, a_{s(q)+k_p+1}, \ldots, a_{s(q)+k_p+t_p-1}) \quad (p = 1 .. r)$$

Informally speaking, the shift function ***s(q)*** defines a periodicity of the distribution of segments on the ***R–***register, while the ordered system of generating functions with factors relating to them determines changes of states of elements of the q^{th} segment of ***R–***register. In this context the concept of the ***HPDT*** is easy generalized and to the case of the higher dimensions. This formal model seems a rather useful at consideration of a lot of such classical problems of parallel programming as pipeline translation, «*writers – readers*», the rifleman problem, etc. In particular, *G.E. Tseitlin* has shown, that on such model it is possible to sort a string of ***n*** symbols during no more than ***n*** steps**;** earlier the similar result for the case of classical ***1–CA*** models has been received by us in [8] on basis of other approach. This result has not only the theoretical character but it has appendices at practical use of parallel processors [40-43]. It turned out that such ***d–***dimension abstract ***R–***register with the ***HPDT*** that are determined on it is modeled by means of an appropriate classical ***d–CA*** ***(d≥1)*** model at quite moderate assumptions.

<u>Theorem 106</u>. Each heterogeneous periodically defined transformation determined in a states alphabet A on a d-dimension abstract R–register is 1–simulated by means of an appropriate classical d–CA (d≥1) model with the states alphabet A∪{b} (b∉A).

This result once again confirms a significant enough generality degree of the concept of classical ***CA*** models and importance of its application for problems of parallel programming, that already finds a reflection in the developed concept of parallel microprogramming [7,40-43]. Indeed, establishing equivalence of ***CA*** models concepts and ***HPDT*** on abstract ***R–***registers, it is possible to spread results, methods and approaches of a rather advanced theory of the classical models to study of theoretical questions of parallel programming studied by abstract heterogeneous periodically defined transformations. With a number of other examples of modelling of formal algorithms by means of the classical ***d–CA (d≥1)*** models it is possible to familiarize for example in references in [7,102].

It is easy to be convinced, an arbitrary classical ***1–CA*** model with states alphabet ***A={0,1,2, ..., a-1}*** and neighbourhood index ***X={0,1,2, ..., n-1}*** is ***T–***modeled by a model of the same dimension with characteristics ***(A* –*** *a states alphabet and* ***X* –*** *neighbourhood index of a simulating model)**:***

$$\#A^* = \sum_{j=0}^{T-1} a^{k^j},\ X^* = \{0,1,2,...,k-1\};\ T = \left[\frac{n-k}{k-1}\right] + Sg\left(\frac{n-k}{k-1} - \left[\frac{n-k}{k-1}\right]\right) + 1;\ \ n > k$$

where ***#A –*** the cardinality of a set ***A*** and ***Sg(x) –*** the sign function. This result allows to simply generalize the theorems ***104*** and ***105*** to the case

of the classical ***1–CA*** models as a modelling algorithm with the simplest neighbourhood index $X = \{0, 1\} \equiv \{-1, 0\}$.

In view of the presented problems it is rather interesting to consider the reverse problem of modeling of the classical ***1–CA*** models by the above processing sequential algorithms of words. Above all, we shall consider the biologically motivated algorithms to which a special attention in the theoretical biological sciences has been devoted. In view of studying of dynamic mathematical theories, isomorphical to biological developing systems the principle of biological *epimorphism* is actual enough, which reduces to the question about opportunity of mapping of one algorithm determined by a certain development onto another. In this case we can speak about *epimorphism* or *isomorphism* of two algorithms. For example because of a lot of biological reasons linked with study of applicability of dynamical mathematical theories for modelling of the development biology we defined and investigated so–called ***A–****algorithms* formulated as follows [8,9,12,13,24-28,31-33,40-43,82,102,106].

Let G is a certain finite nonempty alphabet and P_j, Q_j $(j=1 .. k)$ are finite words which can be empty in the alphabet G whereas symbols $(\Rightarrow)$ and $(*)$ do not belong to the alphabet G. Then an ***A–***algorithm is some finite set of productions of the following kind, namely**:**

$$P_j \Rightarrow Q_j; \quad P_k \Rightarrow *Q_k \qquad (j = 1 .. k–1) \qquad (23)$$

that is named *schema* of the algorithm where $P \Rightarrow (*) Q$ can be a simple production $P \Rightarrow Q$ or the final production $P \Rightarrow *Q$. Functioning of such ***A–***algorithm with the preset schema will consist in the following.

Let S_o is an arbitrary finite word defined in a finite alphabet G. Then on the first step of algorithm $S_1=A(S_o)$ the first entrance of a subword P_1 in S_o is replaced by a subword Q_1. Then the process repeats with word S_1 and so on, finishing on some word S_v which does not contain entrances of subword P_1. Next, to the word S_v the above procedure is applied but already concerning entrances of a subword P_2, and so on. Processing of the word S_o by the ***A–***algorithm finishs on a certain step f if and only if the word S_f will not contain entrances of subwords P_j $(j=1..k-1)$ or word S_{f-1} contains entrance of a subword P_k. In this case a word $S_f = A(S_o)$ is named the result of processing *(calculation)* of the word S_o by means of the ***A–***algorithm with the above schema ***(23)***. If not, it is considered that the ***A–***algorithm is inapplicable to the word S_o.

Concerning the ***A–***algorithms it is shown that they are epimorphical to classical ***1-CA*** models, having interesting biological interpretations [31-33]. Now, using the above definitions and designations, and definition of the semi–Thue system [209]**,** the basic result concerning modeling of the classical ***1–CA*** models with states alphabet ***A={0,1, ..., a–1}*** by dint of the above sequential processing algorithms of words can be formulated as follows [8,9,12,13,24-28,31-33,40-43,102,106].

Theorem 107*. Each classical 1–CA model is weakly T–simulated by an appropriate LAG–system; in addition, there is the following relations:*

$$C = A \bigcup \{\nabla\}; \quad q = n; \quad p = 2 * \sum_{j=1}^{n} a^j - a^n; \quad T = |s^*| + n - 1$$

A classical 1–CA model with the simplest neighbourhood index X={0,1} is weakly T–simulated by means of a Post productions system; at that, there are the next relations #C=3a+5 and T=[(3|s*|+10)/2]. A classical 1–CA model with the simplest neighbourhood index X={0, 1} is weakly T–simulated by the Markov normal algorithms; in addition, there are the following relations #C=2a+5 and T=3|s*|+2. Each classical 1–CA model with neighbourhood index X={0,1, ..., n-1} is weakly T–simulated by an arbitrary MT^s_q ***under the assumption T = |s*|+n–1. An arbitrary classical 1–CA model with the simplest neighbourhood index X={0,1} is weakly T–simulated by dint of an appropriate A–algorithm determined in alphabet G = A∪{b, ∇} (∇, b∉A). Each classical 1–CA model with the simplest neighbourhood index X={0,1} is weakly T–simulated by means of an appropriate Turing machine*** MT^s_q***; in addition, there are relations s = (a+1), q = 2(a+1),*** $s_x q = 2(a+1)^2$ ***and T = |s*|+1.***

At that, designations of theorem ***107*** fully correspond to designations of theorem ***104,*** meantime a classical ***1–CA*** model with alphabet ***A= {0,1,..., a–1}*** is understood as a simulated model. The optimizing technics of ***T–***modelling used at the solution of these problems has allowed to receive substantially optimum relations between a series of main parameters of the *modelled* and *modelling* algorithms**;** that presents strong reasons for a ascertaining of the fact that excepting *Turing* machines, it is not possible to narrow condition of weak ***T–****modelling* up to condition of ***T–****modelling***,** what is conditioned by the principal complexity of embedding of strictly sequential algorithms into the highly-parallel computing environment such as the ***CA*** models. Along with it, the represented results of mutual modelling to a certain extent can characterize the relative complexity of corresponding algorithms within of the used concepts of ***T-****modeling* and

classical ***CA*** as a whole. More detailed discussion of the question can be found, for example, in our works [8,9,12,13,24-28,31-33,40-43,102,106].

Moreover, on the basis of the presented results of the modelling we can receive a whole series of rather interesting consequences, first of all, of theoretical character. Particularly, results of *T. Yaku* [141] along with our result of ***T***-modelling of ***SS***-machines by means of classical ***1-CA*** models *(Theorem **104**)* have allowed to receive the following important enough result deciding one of mass dynamics problems of classical ***d–CA (d≥1)*** models that is linked with so-called *vanishing* finite configurations [1,5].

Theorem 108. For a classical d–CA (d≥1) and a configuration $j \in C(A,d,\phi)$ the definition problem whether will the j configuration as a vanishing configuration is algorithmically unsolvable.

Moreover, *T. Yaku* on basis of a special technics of embedding into the classical ***CA*** models has proved unsolvability of the existence problem of vanishing configurations for an arbitrary ***d–CA (d≥2)*** model [141]. In particular, our proof of the result is based on algorithmic unsolvability of well–known «*domino*» problem considered in [42,82].

However, for case of the classical ***1–CA*** models this problem is solvable what easily follows from theorem ***12*** that is valid as well for the passive configurations. Along with a number of other consequences this result confirms nonequivalence of the classical ***1–CA*** and ***d–CA (d≥2)*** models also concerning the solvability problems, emphasizing existence of the sharply expressed differentiation of the set of all classical ***d–CA*** models concerning their dimension [5,24,102,106].

Meanwhile, along with the above questions of modelling, the essential attention is given also to questions of modelling of one classical model by dint of other ***CA*** model of with satisfaction of the certain conditions. Similar questions are considered in the following section.

6.3. Simulating of classical *CA* models by means of *CA* models of the same class

By present time within the considered problems the greatest number of works is devoted to modelling of one classical ***d-CA (d≥1)*** model by dint of another ***CA*** model of the same type under the preset conditions. This problem presents significant theoretical and applied interest forasmuch results of researches in this direction allow to determine different types of standardization of all or separate classes of the ***CA*** models, to quite

successfully solve various optimization problems, to simulate classical ***d–CA (d≥1)*** models with suppression of those or other properties of the source models, and so on. So, from the applied standpoint the *simulating* problem of the classical ***d–CA (d ≥ 1)*** models by means of binary models of the same dimension represents a special interest, dictated on the part of computing sciences and of a number of other interesting appendices. Generally speaking, this problem can be formulated as follows**:**

For an arbitrary classical d–CA (d≥1) model with states alphabet A and neighbourhood template, contained in a minimal hyperparallelepiped of size $n_1xn_2x \ldots xn_d$ it is necessary to build a binary classical CA model of the same d dimension and with the smallest possible neighbourhood template which will simulate the source d–CA model.

As a rule, the problems of optimization character in all fields are related to the category of complex enough problems. The above problem is not exception therefore for its solution other method of research has been used [40-43]. Using certain experimental results along with a number of intuitive arguments, a certain special optimizing technics of simulating of classical ***d–CA (d ≥ 2)*** models by means of binary models with taking into account of influence of dimension of the simulated model onto the preset process has been defined. The suggested approach has allowed to receive the following basic result [5,41-43,82-87,102,106].

<u>Theorem 109</u>. An arbitrary classical d–CA (d ≥ 2) model is 1–simulated by dint of an appropriate binary model of the same dimensionality and with neighbourhood template of the following L size:

$$L = (L_1)^{d-1}(L_d+1)\prod_{k=1}^{d}(p_k+1);\ L_1=\left[V=\sqrt[d]{log_2(a-1)+2}\right];\ L_d = L_1+[2(V-L_1)]$$

where $p_1xp_2x \ldots xp_d$ – size of minimal hyperparallelepiped that contains neighbourhood template of the simulated CA model on the assumption of ensuring of the following relation, namely: $log_2\ log_2\ 4(a–1) \geq d$.

Therefore, the edge of ***d–***dimensional neighbourhood template of the simulating ***d–CA (d≥2)*** model at ensuring of the relation of theorem ***109*** is asymptotically decreased in $\sqrt[d]{log_2(a-1)+2}$ times with growth of the ***d–***dimensionality. We have considered the earlier formulated problem separately for two cases of the classical ***1–CA*** and ***d–CA (d ≥ 2)*** models because of influence of dimensionality of ***CA*** models upon technics of optimizing modelling. Earlier the nonequivalence of models ***1–CA*** and ***d–CA (d≥2)*** relative to certain phenomena was marked**;** it concerns also the modelling problem in the classical ***d–CA (d≥1)*** models. A number of

aspects of discussion of this question can be found in [24,102]. We have paid a special attention to this moment, hence the modelling technique began to proceed from the influence of dimension of a classical model ***d–CA*** ***(d ≥ 1)*** upon an optimizing factor. So, for more optimal modelling some other approaches are needed.

So, for ***1-***dimension case the optimal technics that is as much as possible taking into account specificity of functioning of classical ***1–CA*** models has been offered [5,42]. This technics is based on a principle of maximal approach of characteristics of the simulating models to the appropriate characteristics of the potentially optimal simulating models. In addition under *potentially* optimal models are understood the simulating models whose values of base parameters can be unattainable, however capable be as a good reference point for prospect of researches in this direction and for estimation of values of parameters of the earlier predetermined simulating models. The ***CA*** model with neighbourhood template of size $L_{opt} = (n+1)[\log_2 a] + 2$, that, moreover, is unattainable is supposed as a potentially optimal binary simulating model for classical ***1–CA*** models with a states alphabet ***A*** and neighbourhood template of ***n*** size. Of the standpoint of this estimation the receiving problem of optimal classical ***1–CA*** model, the closest to the potentially optimal simulating model of the same dimensionality exists. Researches in this direction allowed to define a simulating binary ***1–CA*** model with neighbourhood template of size $L = (n+1)[\log_2 a+1+\omega] + 2$, where $0 < \omega < 1$. For estimation of the affinity degree of this simulating binary model to the *potentially* optimal model it is possible to take advantage of the following obvious relation**:**

$$\frac{L}{L_{opt}} = \frac{(n+1)[\log_2 a] + (n+1)[1+\varpi] + 2}{(n+1)[\log_2 a]} = 1 + \frac{1+\varpi}{\log_2 a} + \frac{2}{(n+1)[\log_2 a]}; \quad (0 < \varpi < 1)$$

Of this relation it is easy to draw a conclusion about satisfactory affinity of the received simulating model to some kind of standard model even at a moderate cardinality of an alphabet ***A*** and a size of neighbourhood template of a simulated classical ***1–CA*** model. Furthermore, the value **ω** depends on a number of conditions and since the computer estimations reveal that in the case $a \le 2^{19}$ the **ω** value not exceed ***1,*** in calculations it is entirely possible to accept $\omega = 1$. For practical purposes it is entirely comprehensible approach because already such quantity of states of a elementary automaton of a simulated classical ***1-CA*** model is practically immense. In general case a value **ω** does not exceed two. So, in view of the aforesaid the following basic result for arbitrary classical ***d–CA*** ***(d≥1)*** models can be formulated [5,24-28,40-43,82-87,102,106].

***Theorem 110**. A classical d–CA model with alphabet A={0,1,...,a–1} and neighbourhood template that is contained in the minimal d–dimension hyperparallelepiped $p_1xp_2xp_3x...xp_d$ is 1–simulated by an appropriate binary classical d–CA (d≥1) model with neighbourhood template of size $L=(p_1+1)[\log_2 a+p_1+\lambda)]xp_2xp_3x...xp_d$ where $\lambda=4$ for $a\le 2^{19}$ and $\lambda=5$, otherwise.*

The method of proof of this theorem allows to model rather effectively classical ***1–CA*** models which have large enough alphabet ***A*** and small neighbourhood templates by means of binary classical ***1–CA*** models with quite reasonable size of neighbourhood templates. Now we shall illustrate it by one interesting example having important independent value. We shall speak, that a classical ***d–CA (d ≥1)*** model is *universal* or possesses the *universal computability* if it ***T–***models the universal *Turing* machine. Such models possess property of universal computability and play a rather important part at research of classical ***d–CA (d≥1)*** models as a formal model of parallel calculations. As a matter of principle, the *universal* computability in ***CA*** models can be defined and by means of certain other equivalent ways [24-27,40-43,82-87,102,106].

In connection with definition of the concept of universal computability on basis of the ***T–***modelling concept arises a rather important question about the minimal complexity of a classical ***CA*** model that ***T–***simulates the universal *Turing* machine, or in more general posing about the most simple classical ***1–CA*** model which possesses the property of universal computability. As a measure of complexity of an universal ***d–CA (d ≥ 1)*** model it is quite natural to use value ***d*a*n*** in which three factors define values of the base parameters of such model**:** dimension, cardinality of alphabet ***A,*** and size of neighbourhood template. Thus, in our opinion, determination of the universal ***d–CA (d ≥ 1)*** model with minimal value ***d*a*n*** is no less difficult problem than determination of universal MT^s_q with minimal value ***s*q,*** making up today the value ***s*q=26.*** Meantime, this problem seems to us unduly fundamental problem as a whole.

For classical ***1–CA*** models the best result in this direction was obtained by *A.R. Smith,* who proved availability of the universal models with the following values of ***a*n,*** namely**:** ***2*40, 3*18, 6*7, 8*5, 9*4, 12*3, 14*2*** [99]. In addition, it is necessary to note existence of universal classical ***1–CA*** models with simplest neighbourhood index. So, we have the universal classical ***1–CA*** model with value ***a*n = 28*** that almost coincides with the best for today minimal value ***s*q = 26*** for the universal machine MT^s_q**.**

The best result of similar type for the case of the universal classical ***2-CA*** models was obtained by *E. Banks* who proved existence of the universal models with value ***d*a*n=2*2*5=20*** in the case of use of an infinite initial configuration of the simulating model, and ***d*a*n=2*3*5=30,*** otherwise [100]. Meanwhile, results of *A.R. Smith* and *E.R. Banks* have allowed to make, among other things, a number of interesting enough conclusions about influence of dimension and type of the used initial configurations of the ***d–CA*** models upon the results of modelling in them. A number of similar results in this direction has been received by *A.S. Podkolzin* and by others a little bit later [7,24,102,107,112,205,206]. So, for instance, *A.S. Podkolzin* proved existence of universal ***2–CA*** models with such values as ***d*a*n=2*2*9=36*** and ***d*a*n=2*3*5=30*** [100]**;** this result repeats the *E. Banks* result. The main principle of proof is based on embedding into a model of the basic logic circuits which realize functions of disjunction, trigger, etc. Furthermore, *A.S. Podkolzin* proved the following result.

Theorem 111. The recognition problem of an universal CA model in the class of d–CA (d ≥ 2) models is algorithmically unsolvable.

On the basis of realization in the classical ***2–CA*** models of the base logic functions the universality of binary ***2–CA*** models with simple enough neighbourhood templates has been proved [7]**;** it is shown that the game ***Life*** is equivalent to the universal *Turing* machine**,** what is conditioned by an opportunity of definition for the game of processes, equivalent to the universal calculations. Depending on definition of the ***Life*** we quite naturally come to the conclusion about the existence of universal binary classical ***2–CA*** models with the value ***d*a*n = 2*2*9 = 36.***

The used optimizing method of simulating of classical ***1–CA*** models by binary ***CA*** models of the same class gives rise to rather good results and gives good *comparative* characteristics relative to the potentially optimal modelling models. However, its efficiency depends in the certain limits on values of the base parameters of the simulated classic models**:** size of neighbourhood template and cardinality of an alphabet ***A*.** Therefore, in individual cases it is possible to use some special modifications of this method that allow to receive the results of modelling the most close to optimum. This modelling aspect was considered regarding the problem of discovery of minimal universal classical ***1–CA*** models with alphabets ***A*={*0,1,2, ..., a–1*}** of cardinality ***a* ∈ {*2,3,4,8,14*},** in particular. Classical ***CA*** models with these characteristics present a certain interest from a lot of theoretical and applied standpoints. The basic result can be represented here as follows [5,24,40-43,82-87,102,106].

Theorem 112*. An arbitrary classical 1–CA model with a states alphabet of cardinality a≥6 and neighbourhood template of size n is 1–simulated by a binary classical 1–CA model with neighbourhood template of size* $n[2log_2(a+2)+1]$*-2. An arbitrary classical 1-CA model with an alphabet of cardinality 4 ÷ 21 and neighbourhood template of n size 1–simulated by a suitable classical 1–CA model with alphabet of cardinality 3 and neighbourhood template of size 6n-1. An arbitrary classical 1-CA model with an alphabet of cardinality 5 ÷ 14 and neighbourhood template of n size is 1-simulated by a suitable classical 1-CA model with an alphabet of cardinality four and the neighbourhood template of size 5n – 2.*

It is necessary to note that modelling in rigorously real time is essential enough characteristic of this result. Using method of proof of theorem ***112,*** it is easy to show, that an universal classical ***1-CA*** model with value ***a*n = 14*2*** is ***1–***simulated by a binary ***1–CA*** model with neighbourhood template of size $2[2log_2 16 + 1]–2=16$, i.e. for the simulating ***1–CA*** model the value ***a*n=2*16*** is allowable. That come to the following result [5].

Theorem 113*. There are the universal classical 1–CA models with value* ***a*n = {2*16 | 3*11 | 4*8}****; these classical models were obtained as a result of simulation in rigorously real time.*

This result proves the existence of universal classical ***1–CA*** models with relatively small neighbourhood templates along with states alphabet ***A*** of cardinality ***a;*** in addition, during a long enough time it was as one of the best results in this direction [5,7]. Meantime, *M. Cook* at *2000* shown that classical binary ***1–CA*** model with discriminating number ***118*** will possess the universal computability. In view of the above–introduced ***4*** types of nonconstructability for classical models *(**NCF**, **NCF–1**, **NCF–2**, and **NCF-3**)* can be shown that this ***CA*** model defined by local transition function $\sigma^{(3)}$ of the following kind, namely**:**

$$\sigma^{(3)}(x,y,z)=\begin{cases} x+y+z+1 \pmod 2, & \text{if } x=1 \text{ or } <yz>=<11> \\ x+y+z \pmod 2, & \text{otherwise} \end{cases}; \quad x,y,z\in\{0,1\}$$

possesses the ***NCF, NCF–1*** and ***NCF–2*** nonconstructability. As simple examples of the nonconstructible configurations of such types the finite configurations can be presented, namely: ***c1=□11001□, c2 = □1□*** and ***c3 = □01010□,*** accordingly ***NCF–2, NCF–1*** and ***NCF.*** In addition, this model possesses the ***γ–CF*** with minimal size ***1*** and pairs of ***MEC*** {***□01|01|10□, □<01|10|10□***} with ***IB*** of minimal size two along with ***NCF, NCF–1*** and ***NCF–2*** of minimal size ***5, 1*** and ***5*** accordingly.

Among all binary ***1–CA*** models with neighbourhood index ***X={0,1,2}*** up

till now only ***1*** model with the property of universal computability has been found, although other some models of such simple kind also could possess by this property. So, there are classical binary ***1-CA*** models with neighbourhood index ***X={0,1,2}*** that possess the universal *computability***.** In this sense their complexity is defined by value ***a*n=2*3=6.*** However, the *Cook* result has provoked the certain doubts**;** unfortunately, we have not any detailed enough information in this direction. Thus, for today, the above binary ***1–CA*** model can be considered as an example of the elementary ***CA*** model possessing the universal computability, allowing to formulate the following proposal.

Proposal 8. There are universal classical binary 1–CA models with the neighbourhood index X = {0,1,2} of complexity a*n = 2*3 = 6.

Meanwhile, revealing of minimal universal ***d–CA*** models, especially of binary ***1–CA*** models presents in our opinion more gnoseological, than theoretical and applied interest. Especially, if artificial technique of the proof uses occasionally as a whole disputable agreements.

Meanwhile, it is interesting to study properties of such universal ***1–CA*** models from standpoint of the *nonconstructability* problem. The analysis in this direction has allowed to formulate the interesting result [40-43].

Theorem 114. There are one–dimension universal classical models that possess the nonconstructability of all four types, namely: NCF, NCF–1, NCF–2 and NCF–3.

Discussion of results of theorems ***109–114*** together with the associated rather interesting questions can be found, for example, in [24,82-87]. In particular, the above results allow to essentially simplify the modelling technology of the complex processes demanding appreciable enough efforts for their embedding into classical ***1–CA*** models. The essence of such technology is represented both at the formal level, and on ***2*** rather interesting examples**:** problems of the limited growth in the classical ***CA*** models, and of periodic finite configurations with the maximal periods. In this direction a lot of other instructive enough examples exists [8,24].

The first example is enough closely linked with the problem of limited growth that has been considered for the case of ***1–CA*** models. Parallel algorithms of growing of chains of active automata of fantastic length**,** obtained here with use of models such as ***1–CA**** are embedded into the classical ***1–CA*** models with neighbourhood template of size ***3*** and with a states alphabet of cardinality ***4m + 29,*** where ***m –*** speed of transfer of control information in ***CA**** models. Then, on basis of theorem ***112*** it is

simple to draw the conclusion about the existence of equivalent binary classical ***1–CA*** with neighbourhood templates of size $3[\log_2(4m+31)]-2$.

The detection problem of the greatest possible minimal sizes of periods for periodical finite configurations in classical binary ***1–CA*** models can successfully serve as the second example. In [5] it is shown that there is a model ***1–CA**** whose functional algorithm admits for an integer $m \geq 3$ the existence of periodic finite configurations of length m with minimal period $p=2^m-2$ at cardinality of alphabet A and set of control impulses I, equal 3. In the same place it is shown that this ***1–CA**** is equivalent to some classical ***1–CA*** models with neighbourhood index $X=\{-1,0,1\}$ and alphabet A^* $(\#A^*=\#A+\#I=6)$. The following application to these results of the proof method of theorem ***112*** gives rise to detection of existence of binary classical ***1–CA*** models with neighbourhood template of size $n=16$, which possess the periodical finite configurations of length $5m+3$ $(m \geq 3)$ with the minimal period $p = 2^m-2$. So, 3 basic stages of the above type of simulating in classical ***d–CA*** $(d\geq 1)$ models are, generally, looked through, namely:

$$d\text{–}CA^* \Rightarrow \textit{classical d–CA models} \Rightarrow \textit{binary classical d–CA models}$$

The estimation for neighbourhood template of a simulating *binary* ***1-CA*** model in theorem ***112*** can be generalized, that in the combination with technics of simulating of classical ***1–CA*** models, offered in proof of the theorem ***112*** allows to obtain an interesting enough estimation of size of neighbourhood template of simulating binary models for case of higher dimensionalities too.

Theorem 115*. A classical d-CA* $(d\geq 1)$ *model with alphabet* $A=\{0,1,...,a-1\}$ *and neighbourhood template that is contained in minimal d–dimension parallelepiped* $p_1*p_2x*...*p_d$ *is 1–simulated by an appropriate classical binary CA model of the same dimension with neighbourhood template of size* $\{p_1[2\log_2(a+2)+1]-2\}*p_2*p_3*...*p_d$.

A number of aspects of discussion of the modelling technics used in the proof of theorems ***110–115*** along with certain related questions can be found in the works [5,8,24,40-43,82-87,102,106].

Question of simulating of the classical ***d–CA*** $(d\geq 1)$ models by means of models of the same class, but with decreasing of dimensionality of the similating models presents rather essential both theoretical and applied interest. In [213] one interesting approach to the problem of realization of simulating of classical ***3–CA*** models by means of ***2–CA*** models which

used some results of works [214,215] has been presented and analysed. A generalization of this approach allows to simulate ***d–CA (d≥3)*** models by means of ***2–CA*** models. Meanwhile this approach does not work for ***1–***dimension case, not allowing to simulate an arbitrary ***2–CA*** model by means of the corresponding ***1–CA*** model. The theorem below presents other approach which provides similation of the classical ***2–CA*** models by means of ***1–CA*** models of the same type.

Theorem 116. Finite configurations dynamics in classical 2–CA models can be simulated by means of the corresponding classical 1–CA models with neighbourhood index X = {–1,0,1}.

Essence of proof of this theorem is reduced to the following. Above all, it is known that an arbitrary classical ***d–CA (d≥1)*** model is simulated by an appropriate classical ***CA*** model of the same dimension with simplest neighbourhood index ***X*** of the kind [4,5,24,82,102,106]**:**

$$X=\{\underbrace{(0,0,0,\ldots,0,0)}_{d},\ \underbrace{(1,0,0,\ldots,0,0)}_{d},\ \ldots,\ \underbrace{(0,0,0,\ldots,0,1)}_{d}\}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{d+1}$$

i.e. its neighbourhood template contains $d+1$ elementary automata out of which one automaton is central whereas each of d automata adjoin to it along axes of the coordinates in E^d.

Hence, it is quite enough to be limited oneself to the case of a classical ***2–CA*** model with alphabet ***A = {0,1,2, ..., a–1}*** and neighbourhood index ***X={(0,0),(1,0),(0,1)}***. Then, writing down an arbitrary finite ***2–***dimension configuration on the tape of a MT^s_q machine by a special way with use of *two–*level structurization of states of cells of the tape of similar *Turing* machine, we determine an appropriate program of its work which will simulate dynamics of an arbitrary finite configuration in the simulated classical ***2–CA*** model.

The program of a modelling MT^s_q is presented in [24]. Thus, it is shown that an arbitrary classical of ***2–CA*** model with simplest neighbourhood index ***X={(0,0),(1,0),(0,1)}*** and alphabet of cardinality ***a*** is simulated by a MT^s_q machine with parameters $\#s=2a^2+3a+3$ and $\#q=a^2+2a+15$ where $\#g$ is cardinality of a set ***g.*** In addition, one step $c_o\tau$ of a simulated ***2–CA*** model the appropriate MT^s_q simulates for $(n+1)m^2+(2n^2+13n+15)m+2n(n+9)+28$ steps, where *(n×m)* is a size of minimal rectangle containing an initial configuration $c_o \in C(A,2,\phi)$.

Of that follows, that cost of such modelling is rather essential time costs determined by quantity of commands required for the modeling MT^s_q for realization of one step of the simulated ***2–CA*** model. On the other hand, according to the theorem ***101*** follows, that for a MT^s_q there is an appropriate classical ***1–CA*** model with neighbourhood index $X=\{-1,0,1\}$ and alphabet A^* of cardinality $\#A^*=s+q+9$ which ***8–***simulates the MT^s_q**.** Thus, an arbitrary classical ***2–CA*** model with the neighbourhood index $X=\{(0,0),(1,0),(0,1)\}$ and alphabet of cardinality a is simulated by means of an appropriate classical ***1–CA*** model with the *Moore* neighbourhood index and alphabet of cardinality $3a^2+5a+27$**.**

In addition, approach used in theorem ***116*** is intended for modeling of dynamics of configurations $<c_o> = \{c_o\tau^{(3)k} \mid k = 0,1,2,3, ...\}$ $[c_o \in C(A,2,\phi)]$**,** however with a rather small modification it works and in the case when configurations c_o are periodic in structural, instead of dynamical sense. In this case we shall deal not with a finite configuration c_o**,** but with its finite period. So, the above reduction of the represented results allows us to formulate the following a rather interesting assertion [24,40-43].

Theorem 117. Dynamics of finite configurations of classical d–CA (d≥1) models with an arbitrary finite states alphabet is simulated by means of an appropriate binary classical 1–CA model or nonbinary classical 1–CA model with the Moore neighbourhood index.

At the same time, it is necessary to emphasize once again, the method used in the above theorem ***116*** ensures modelling only of dynamics of finite and/or structural–periodic configurations in classical ***d–CA (d≥2)*** models and not spreads to the more general case. Discussion of certain features of such modelling can be found in [82-87]. Particularly, the way of *structurization* of states of the finite automaton of the modeling MT^s_q and symbols on its tape is a rather productive, allowing in a lot of cases it is essential to simplify programming of the MT^s_q**,** and also to embed various processes, phenomena and objects into ***CA*** models. Meanwhile, it has and own faults, first of all, of optimization character**:** on the one hand, simplifying process of programming in ***CA*** and in certain cases optimizing temporal characteristics**;** at the same time, it complicates ***CA*** models, first of all, by increase, in a number of cases enough essentially, of cardinality of a state set of the elementary automaton of ***CA*** models. For this reason similar way can be enough successfully used, above all, for the conceptual decision of problems of the ***CA*** models, instead of the optimization problems connected to the complexity of ***CA*** models. So, in

particular, use of this approach allows to obtain a rather simply result**:**

A classical 1–CA model with alphabet of cardinality s(s+1)(q+1)+1 and neighbourhood index X = {0,1} 2–simulates a MT^s_q machine. A machine MT^s_q with 2-dimension tape is 1–simulated by means of an appropriate classical 2–CA model with Neumann neighbourhood index and a states alphabet A of cardinality q(s+1)+1.

The more detailed discussion of a lot of principal questions concerning the mutual simulating of such formal computers as MT^s_q machines of sequential action along with the ***CA*** models of highly-parallel action can be found in works [7,24,49-43,82-87,102,106,278,286].

6.4. The formal parallel algorithms determined by classical one–dimensional *CA* models

Parallel algorithms of processing of the words which are determined by classical ***d–CA (d≥1)*** models are being researched especially intensively for a lot of years what is caused not only by their independent interest within of the general theory of algorithms, but also by means of use of the classical ***d–CA*** models as formal models in such fields of the modern natural sciences as mathematical modelling, mathematics, development biology, physics, discrete synergetics, computing sciences, etc.

The parallel algorithms determined by classical ***d–CA (d≥1)*** models play a rather essential part in the formal description of a number of biologic development processes and various programmable systems basing on computational homogeneous structures. Meantime owing to undoubted interest for solution of important problems of a designing of languages of multiprocessing**,** research of the formal language models functioning in especially parallel manner represent a special importance. With this purpose in chapter ***5*** were defined the formal parallel τ_n**–**grammars and formal parallel $L(\tau_n)$**–**languages appropriate to them.

While in the algorithmic attitude the parallel algorithms defined by the classical ***d–CA (d–PACA)*** models represent the further research of the classical ***d–CA (d ≥ 1)*** models as parallel processing systems of words in the finite alphabets. This problematics has the most direct attitude to a question of simulating in classical of ***d–CA (d ≥ 1)*** models both by the considered problems and by a number of base methods of research. In this section the class of parallel algorithms ***1–PACA*** is determined, and questions of their complexity relative to a number of well-known formal

processing algorithms of words are discussed.

By definition the parallel algorithms ***1–PACA(a, n)*** operate with words {*configurations of the set* $C(A,1,\phi)$} which are determined in an alphabet $A=\{0,1,2,...,a-1\}$. The manner of functioning of a ***1–PACA(a, n)*** is preset by means of neighbourhood index $X=\{0,1,...,n-1\}$ and global transition function $\tau^{(n)}$ of a classical ***1-CA*** model determined by the corresponding local transition function $\sigma^{(n)}$ with parallel rules of substitutions of the following general kind, namely**:**

$$\underbrace{00\ldots0}_{n}\Rightarrow 0 \qquad x^j{}_1x^j{}_2\ldots x^j{}_n\Rightarrow x^{*j}{}_1;\quad x^{*j}{}_1,\ x^j{}_k\in A\ (k=1..n;\ j=1..a^n-1)$$

which are simultaneously applied to each word s of the set $C(A,1,\phi)$ of all finite ***1–***dimension configurations determined in an alphabet A**,** i.e. a word s is processed by a certain parallel algorithm ***1–PACA(a,n).*** Thus, a parallel algorithm ***1–PACA(a,n)*** is fully defined by the above parallel substitutions corresponding to local transition function $\sigma^{(n)}$ of a certain classical ***1–CA*** model.

For each word $c\in C(A,1,\phi)$ a parallel algorithm ***1–PACA(a, n)*** defines a sequence of words $<c>[\tau^{(n)}]$ in which a word c_k is named the *final*, if for it the following relation $c_{k+1}=c_k\tau^{(n)}=c_k$ takes place. Let $C(A,1,\phi)$ is a set of words processed by a certain parallel algorithm ***1–PACA(a,n)*** and F **–** a partial word function in an alphabet A, if for some word $c_j\in C(A,1,\phi)$ the relation $F(c_j)=c^*{}_j\in C(A,1,\phi)$ takes place. For such word F**–**function the domain of existence and the range of values is accordingly E_F and V_F**.** In view of the made assumptions we shall speak, that the word F**–**function determined in alphabet A^***,** is ***PACA–****computable* if there is such algorithm ***1–PACA*** in the alphabet $A=\{b\}\cup A^*$ $(b\notin A^*)$ that for any word $c^*{}_o\in C(A^*,1,\phi)$, if c_o **–** representation of the word $c^*{}_o$ of the format $c^*{}_o=\square b^{p+2}0^1b^20^{p+1}c^*{}_ob^{p+2}\square$ then the following two determinative conditions are being satisfied, namely**:**

1) if a word c^* belongs to domain of existence E_F of the F**–**function**,** then the configurations sequence $<c_o>[\tau^{(n)}]$ will be contain the ***final*** word c_f of the following general kind, namely**:**

$$c_f=\square b^{p+2}0^1b^10^{p+2}F(c^*{}_o)b^{p+2}\square,\qquad F(c^*{}_o)\in V_F; \qquad (24)$$

2) if the finite word $c^*{}_o$ does not belong to domain of existence E_F of the F**–**function**,** the sequence $<c_o>[\tau^{(n)}]$ does not contain the final word c_f of

the above kind *(**24**)*.

In the light of this definition it is shown that a word ***F–***function will be ***PACA–***computable if and only if it is computable on the corresponding *Turing* machine [5]. Equivalence of strict formalization of the intuitive concept of ***PACA–***computable functions with class of *Turing* machines represents one more strong enough argument in favour of the known algorithmical thesis of *A.S. Church*. Inasmuch as in the theory of formal algorithms the great attention is given to the questions of the computing complexity, ergo with respect to the class of parallel algorithms ***1-PACA*** in this direction a rather interesting result has been received [5,24,40-43, 82-87,102,106] *(see also in https://bbian.webs.com/Cellular_Automata.pdf)*.

Theorem 118. An arbitrary partial recursive word function F determined in an arbitrary finite alphabet A is PACA–computable in the extended alphabet A* = {b}∪A (b∉A).

Of the result of theorem ***118*** follows, the parallel algorithms ***1–PACA*** in terms of *Markov–Nagorny* complexity are equivalent to normal *Markov* algorithms. We have analysed the questions of parallelism of the class of ***1–PACA*** algorithms and as a consequence of this we determined the most interesting directions for the further research**:** parallelism classes, detailing and specification of parallelism essence, choice of algorithms most suitable for effective enough realization in the computational ***CA*** models, etc. [24,40-43,82-87,102,106].

Thus, particularly, a rather interesting class of so–called *locally realizable algorithms (**LRA**)* whose the essence consists in opportunity to present a general processing algorithm as local identical algorithms over separate subwords of any processed word has been determined. The problem of *symbolic sorting* can be indicated as a simple example of such algorithm. The analysis has allowed to formulate an assumption that algorithms of the above class *LRA* can be executed in computational ***CA*** models by the most effective manner. In this connection there is an interesting enough question concerning the *existence* of other classes of algorithms the most effectively executed in the computational ***CA*** models and how they can be characterized.

Coming back to the problem of symbolic sorting, we shall estimate the efficiency of its realization by algorithms such as ***1–PACA*.** In addition, the general problem of symbolic sorting is defined as follows. Let ***G1*** is an arbitrary finite word in an alphabet ***A*,** whose symbols is allotted by a certain hierarchy *(a priority principle)*. It is necessary to define a certain

symbol-by-symbol algorithm that will quickly sort any finite word ***G1*** according to the preset priority principle of symbols of the alphabet ***A***. It is known that sequential algorithms for the solution of this problem require no more, than $M=\alpha|G1|^2$ viewings of a word ***G1***, where α is a constant and $|G1|$ is length of the word ***G1***, whereas an appropriate ***1–PACA*** can solve the preset problem for strictly linear time, namely:

There is an algorithm 1–PACA which sorts an arbitrary finite word G defined in an arbitrary finite alphabet A for no more than h steps where h is length of the word G.

The known problem of ***French flag*** is a formalization of the problem of regulation and differentiation of real biological cellular structures; and it directly adjoins to the similar problem of sorting, and in details it has been discussed in our works [5,8,24,40-43,82-87]. In the same aspect the problem of mirror inversion of an arbitrary finite symbolical string by a MT^s_q and a classical ***1–CA*** model is of interest too. Thus, it is simple to make sure that for mirror inversion of a string ***X*** of length ***m*** by means of MT^s_q is required about $2m^2$ steps, whereas an appropriate classical ***1-CA*** model can solve the same problem for linear time. In addition, use of a classical ***1–CA*** model with an alphabet of the structurized states of elementary automata allows to effectively solve this problem and a lot of other problems [24,42,102]. The used way of structurization of states alphabet of the simulating models allows to solve enough effectively in the temporal attitude a lot of rather interesting problems of modelling, by being nonoptimal from standpoint of the complexity of classical ***CA*** models; i.e. the approach is rather effective in temporal attitude, leading generally speaking to the certain complication of a simulating classical model. As a result the following assertion can be formulated [82-87]:

An appropriate classical structure 1–CA with simplest neighbourhood index X = {0, 1} decides the problem of mirror inversion of an arbitrary finite configuration of length m for time 2m–1. A classical 1–CA model decides the symmetry recognition problem of a finite configuration of length m for time about $[m/2]$. ***A classical 1–CA model with simplest neighbourhood index X={0, 1} doubles an arbitrary finite configuration determined in a certain narrowing of an alphabet A for*** $\beta|m|$ ***steps;*** β ***– a constant and*** $|m|$ ***– the length of the processed configuration.***

It is quite natural to assume that paralleling gives rather essential time advantage when temporal complexity of a certain algorithm nonlinearly depends on the input data of a problem as, in particular, in the case of

the sorting problem. However, even in the linear case the paralleling can give a rather essential temporal advantage. We shall consider a rather known problem of finding of a sample on strings as a rather interesting illustrating example. Let S and G are two strings that contain n and m symbols of an arbitrary finite alphabet A accordingly; it is necessary to check up membership of the substring S to the string G ($S \subset G$?). In this regard *D.E. Knuth, P. Pratt* and *A.J. Moris* have offered a solution of the preset problem for time no more, than $0(n+m)$. Meantime, it is possible to simply prove, that the following assertion takes place [5,24,82,102]:

A classical 1–CA model with the Moore neighbourhood index can solve the finding problem of a sample on a string for no more than $0(|n-m|)$ of steps; n and m are lengths of the string and the sample accordingly.

There are a lot of other rather interesting examples of similar character. The class ***1–PACA*** of parallel algorithms defined by the classical ***1–CA*** models presents own subclass of the class of all *local algorithms* (***LA***), i.e. algorithms establishing properties of elements of some set, in addition, by using the information in each step only about a some neighborhood of a word, processed at present. In terms of the ***LA*** the problems about existence or lack of effective algorithms for different discrete extremal problems are naturally formulated and are solved [24,102]. Hence, it is desirable to apply and to research the results and technique of the ***LA*** theory relative the class ***d–PACA*** ($d \geq 1$) of highly–parallel algorithms.

The increasing number of the works devoted to the questions of parallel algorithms as a important enough component of the general theory of algorithms is being observed at the late years. In this attitude we have researched the question of temporal complexity of parallel algorithms ***1–PACA*** concerning a number of known computing formal sequential models [5,24,40-43,82-87,102,106].

<u>Theorem 119</u>. If a partial recursive word function F is computed by an algorithm 1–PACA(a,n) for t steps, then an appropriate Turing machine calculate the same function F for no more than $\{(n+1)t^2+(n-1)t\}/2$ steps, where n – length of neighborhood template of a model corresponding to the 1–PACA(a,n).

The analysis of this result shows a sufficient degree of closeness of the received estimation to the optimal. Therefore, if some partial recursive word function is ***PACA–***computable for time t, then the corresponding *Turing* machine can calculate it for time no more than αt^2 (α – *a constant*). Thus, for formal computers on basis of classical ***1–CA*** model and *Turing*

machines the temporal difference of calculations has the *quadratic* order. Computing possibilities *(in view of their temporal complexity)* of ***1–PACA*** algorithms more evidently can be illustrated by the following example. It is known, that a *two–way stack automaton (**TWSA**)* is equivalent to an one-head *Turing* machine with time of work no more than $|x|^{\alpha|x|}$ steps where x – an input word, α – a certain constant. The detailed description of the two–way stack automaton and principle of its functioning can be found for example in [24]. Characteristic of the two-way stack automata in terms of the parallel algorithms ***1–PACA*** gives essentially best result relative to the characteristic of their temporal complexity [24,40-43,102].

***Theorem 120**. If a two–way stack automaton admits some set of finite words for t steps, then an appropriate parallel algorithm 1–PACA can admit the same set of words for no more than $2t^2$ steps.*

Thus, by parallel algorithms ***1–PACA*** for a lot of computing algorithms it is possible to receive essentially more best temporal results, than on basis of the *Turing* machines. Moreover, it is necessary to note, that the high degree of parallelism inherent in parallel algorithms ***1–PACA*** was used only at a level of the ***T–***modelling of one algorithm by another**;** at that, in the most cases a *parallelism* inherent in the modelled algorithms was essentially not used. So, the further research in this direction seems to us a rather perspective and actual.

Let's speak, a *Turing* machine admits a set g of the finite words given in an alphabet A, if the machine containing on output tape a finite ω-word determined in the alphabet A passes into a final state q' after analysis of this word if and only if $\omega \in g$. Let $s(c)$ will be a set of all finite *predecessors* of an arbitrary configuration $c \in C(A,1,\phi)$ for a classical ***1–CA*** model with an alphabet A; in addition, configuration c^* is some predecessor of the configuration c, if $c^*\tau^{(n)} = c$. The problem of finding of predecessors for finite configurations in the classical ***CA*** models plays important enough part, above all from the standpoint of research of dynamics reversibility property, fundamental at use of classical models as conceptual models of the spatially–distributed discrete dynamic systems of which physical systems present the greatest interest. Due to the above assumptions an interesting result having a lot of important appendices takes place [5].

***Theorem 121**. For any configuration $c \in C(A,1,\phi)$ an appropriate Turing machine MT^s_q admits a set s(c) for no more than $2(|c^*|+n)^2$ steps; c^* – a predecessor of the c configuration having maximal n length.*

Investigating the reversibility questions of dynamics of the classical ***CA***

models, *T. Toffoli* has shown, that in the case of classical ***d–CA (d = 1,2)*** models for a $c \in C(A,d,\phi)$ the set ***s(c)*** can be generated by means of some *nondeterministic finite automaton (**NFA**),* what allows us to do the defined conclusions about *temporal* complexity of generating and recognition by means of the ***NFA*** and MT^s_q of predecessors for a finite configuration in classical ***CA*** models [24,40-43,82-87,102,106].

Now, a lot of works concerning the research of various concepts of stack automata is known [7,24,102]. So, so**-**called ***bc-****automata* present essential significance for *structural programming****;*** they have obtained a rather wide popularity owing to works associated with presentability of the formal languages in them. On basis of the ***bc–****automaton* description it is easy to make sure, that for it exists three**–**head *Turing* machine which models it without temporal delay, and there is ***1-****head Turing* machine that models it for time $2(t+1)^2$ where ***t –*** time of processing of an input word by the ***bc–***automaton. The following result allows to estimate the *temporal* costs of a parallel algorithm ***1–PACA*** for performance of the same work, as a certain ***bc–***automaton [24,40-43,82,102,106,286].

<u>Theorem 122</u>. If a certain bc–automaton demands t steps of processing of an input finite word then an appropriate parallel PACA–algorithm can perform the same processing for no more than $2(t+1)^2$ steps.

The modelling results received in this direction have a different degree of closeness to optimum, however they allow to receive a certain extent comparative estimations of *temporal* complexities of classical ***CA*** models and other known sequential formal computing models. In combination with results of a lot of other researchers in this direction these results allow to obtain full enough picture in field of computing complexity of the classical ***CA*** models. So, for example, *A. Hemmerling* has presented a rather interesting review of study concerning the comparative analysis of *computing* complexity of classical ***CA*** models and ***d-***dimension *Turing* machines *(**d** ≥ **1**)* [24,102,216]. However, in comparison with the theory of sequential algorithms, the theory of parallel computing ***CA*** models is not so advanced.

In conclusion it is necessary to mark once again the following a rather important circumstance concerning the results represented above. We used modelling at a level of classical ***CA*** models as the abstract *algebraic* parallel systems of processing of words *(configurations)* without use of *maximum* parallel modelling in the cellular environment of such objects *(for example, by means of immersing in them of appropriate algorithms such as*

logic networks, etc.). Naturally, our approach has established results, that are a little remote from possible ones however us has interested similar approach. In the following chapter the complexity questions of classical ***CA*** models will be considered in a few other aspect characterising the properties of the models not on a level of a set of finite configurations, but on a level of global transition functions defined by them, allowing to differentiate the class of ***CA*** models from another standpoint.

6.5. Special questions of simulating in the classical *CA* models concerning their dynamics

It is known, that modelling in the environment of classical structures is the multifold problem including such important enough questions as modelling real time, optimal modelling according to the chosen criteria of optimization; methods and principles of simplification of process of modelling, receiving of estimations of complexity of mutual modelling of ***CA*** models, modelling of individual algorithms, objects, phenomena and processes, modelling in the definite classes of ***CA*** models and also at various conditions, etc. In the previous sections the questions of the simulating in classical models without any additional conditions for the simulating ***CA*** models have been considered. Below, we shall present a number of results of such modelling when the certain *restrictions* having one or another sense will be imposed on simulating classical ***CA*** models together with interpretations of such restrictions.

Studying of dynamical properties of classical ***CA*** models depending on type of their local transition functions ***(LTF)*** represents the indubitable interest. So, in the monograph [1] two large classes of ***CA*** models with *symmetrical* ***(SF)*** and *asymmetrical* ***(ASF)*** local transition functions have been differentiated. It is possible to show that the ***SF***-class of ***CA*** models composes a subset concerning the composition operation. In addition, composition of a symmetrical and asymmetricl local transition function always gives an asymmetrical function whereas there are asymmetrical local transition functions, whose compositions give a some symmetrical function. In particular, two simple local transition function, defined by formulas of the following kind:

$$\sigma_1^{(2)}(x,y) = xy^2 \ (mod\ a); \qquad \sigma_2^{(2)}(x,y) = x^2y \ (mod\ a) \qquad (a \geq 3)$$

as a result of the composition give symmetrical local transition function $\sigma^{(3)}(x,y,z)=x^2y^5z^2\ (mod\ a)$. Concerning the ***SF*** and ***ASF*** classes of the ***CA***

models it is shown that in sense of computing opportunities they are equivalent, i.e. both specified classes possess the universal classical ***CA*** models. Meanwhile, by a series of other characteristics the classes ***ASF*** and ***SF*** can essentially differ. So, for example, the essential distinctions take place concerning the constructive models possibilities and sets of the nonconstructible configurations in classical models with *symmetrical* and *asymmetrical* local transition functions what undoubtedly should be taken into account in many models appendices [3,5,8,9,24,42–43,102].

Indeed, enough many processes and algorithms have the pronounced asymmetrical character, although in their basis at the lowest levels the elements of a various degree of symmetry can take place, and they can be much simply embedded into classical ***CA*** models with asymmetrical local transition functions. Naturally, in view of the told, such processes and phenomena by classical ***CA*** models both with *symmetrical* and with *asymmetrical* local transition functions are simulated, however functions of the first type will demand as a rule of such outlay as increase of the base characteristics of a simulating ***CA*** model relative to a similar model of the second type, namely: *alphabet cardinality*, *time of modelling* and *size of neighbourhood template*.

As an example we shall represent an interesting result of simulating of a *Turing* machine by a classical ***1–CA*** model with neighbourhood index $X = \{-1,0,1\}$ and symmetrical local transition function [3]. In addition, a ***1–***dimensional local transition function $\sigma^{(n)}$, defined by parallel rules of the following kind is considered the symmetrical:

$$x_1x_2 \dots x_n \to x^*_1 \quad \textit{and} \quad (x_1x_2 \dots x_n)^R \to x^*_1 \quad (x_k, x^*_1 \in A;\ k=1 \dots n)$$

where X^R – tuple symmetrical to a tuple ***X.*** Naturally, generalization of the symmetry concept of local transition function to case of the higher dimensionality of any difficulties does not entail.

<u>Theorem 123</u>. If a Turing machine MT^s_q realizes a certain algorithm SG for time t then there is an appropriate classical 1-CA model with states alphabet of cardinality 2s+4q+2, symmetrical local transition function $\sigma^{(3)}$ and the Moore neighbourhood index that models the SG algorithm for time not more than 4t.

Method of proof of this theorem allows to spread the received result to the case of classical ***CA*** models of the higher dimension also, but then the necessity of expansion of alphabet of a simulating ***CA*** model arises. The received result not only once again corroborates the equivalence of

classical ***CA*** models with symmetrical and asymmetrial local transition functions relative to their computing possibilities but in a certain extent illustrates complexity of simulating as a whole of *asymmetric* algorithms by means of *symmetrical* ***d–CA*** ***(d ≥ 1)*** models.

In this direction interesting enough results have been received also by *H. Schwerinsky* [102] and *Y. Kobuchi* [8,217]**;** they proved an opportunity of simulating in real time of an arbitrary classical ***1-CA*** model by means of model of the same dimension with neighbourhood index $X = \{-1,0,1\}$ and symmetrical local transition function. Moreover, the simulating in their works has been considered relative to the set $C(A,1,\phi)$ of all finite configurations without any serious optimization what had an influence on certain parameters of the simulating models. In particular, we have used a certain optimization at modelling of asymmetrical algorithms by symmetrical ones.

In this connection, we would like once again to focus the attention, the both classes ***SF*** and ***ASF*** of classical ***CA*** models possess a lot of specific features but if to start with more practical reasons, then between them there are two basic differences**:** the class of ***CA*** models with symmetric local transition functions appears much easier for practical realization and is of interest from standpoint of many of biological interpretations *(for instance, the symmetry can be associated with absence of some gradient at simulating by such models of either biological phenomenon; in environment of such type of* ***CA*** *models are more naturally epresented and neuro–like systems, etc.)* while the classical ***CA*** models with asymmetric local functions are generally essentially better adapted to a modelling of various processes and algorithms, i.e. have the greater degree of constructive possibilities by a number of major factors. Indeed, as shows experience, for strongly pronounced asymmetric processes, generally speaking it is impossible to well solve in symmetric models the optimization problems [102,106].

At simulating in classical ***d–CA*** ***(d ≥ 1)*** models the problem of optimal simulating of different objects, algorithms or phenomena is important enough. Optimization is considered, as a rule, concerning such basical parameters of a simulating model as the neighbourhood template size, alphabet cardinality, model dimensionality and simulating time. In this attitude the optimization of a classical ***d–CA*** ***(d ≥ 1)*** with *symmetric* local transition function that simulates a model of the same dimension with an arbitrary local transition function represents indubitable interest. In ***1–***dimension case in this direction the following basic result takes place [5,24,40-43,82-87,102,106].

Theorem 124. ***An arbitrary classical 1–CA model with neighbourhood index X={-1,0,1} is simulated by means of an appropriate classical 1-CA model with the same neighbourhood index, an alphabet of cardinality $2(4g^2+5g+12)$ and symmetric local transition function for time no more than 4L; L – length of a processed finite configuration in the simulated classical model and g is cardinality of states alphabet of the simulated classical 1–CA model.***

Investigations on the optimization problem of basical parameters of the simulating classical ***d–CA*** $(d \geq 1)$ models represent essential interest and it is necessary to pay corresponding attention to them although similar problems are presented rather complex as, however, and the majority of the optimization problems as a whole. Together with a lot of interesting enough results on mutual simulating of classical models in work [107], the simulating of an arbitrary classical ***2–CA*** model by an appropriate classical ***2–CA*** model with the *Moore* neighbourhood index, alphabet of cardinality *7776* and symmetric local transition function was presented. It would be rather interesting to lower essentially this value that, in our opinion, is greatly exaggerated. In addition, subsequently we need an useful enough property of classical ***1–CA*** models with symmetric local transition functions $\sigma^{(2)}$ and the simplest neighbourhood index $X=\{0,1\}$.

Lemma 4. ***An arbitrary classical 1–CA model with symmetrical local transition function $\sigma^{(2)}$ and the simplest neighbourhood index $X = \{0,1\}$ possesses the NCF and/or NCF–1 nonconstructability.***

Proof of the lemma is extremely simple and can be found in [12]. On the other hand, for classical ***1–CA*** models with the simplest neighbourhood index and asymmetrical local transition functions this result, generally speaking, is incorrect.

On basis of this result can be shown, the result of theorem ***124*** already for rather general methods of modeling is unimprovable from point of view of reducing to simplest neighbourhood template of a simulating classical ***1–CA*** model with symmetrical local transition function. More precisely, in contrast to theorem ***124*** any classical ***1–CA*** model can not be simulated by means of an appropriate classical ***1–CA*** model with the simplest neighbourhood index $X=\{0,1\}$ and symmetrical local transition function at use of a rather wide class of the modelling concepts. In the heart of this affirmation the fact is put, that in conditions of a lot of the modelling concepts such properties of the simulated classical models as presence/absence of the ***NCF (NCF-3)*** & ***NCF-1*** nonconstructability along with ***MEC*** and ***γ–CF*** are kept. Furthermore, this result is used at study a

lot of aspects of the *decomposition* problem of global transition functions of ***CA*** models. Thus, a classical ***1-CA*** model is simulated by means of an appropriate ***1–CA*** model with symmetrical local transition function and the simplest neighbourhood index but only at having the ***NCF-1*** and/or ***NCF*** nonconstructability. In addition, this result with all evidence again confirms the earlier our assumption about more preferable constructive possibilities of simulating in classical ***d–CA*** models with asymmetrical local transition functions [5,24,40-43,82-87,102,106].

At studying of questions of the universal computability and simulating of physical processes in classical ***CA*** models, the dynamics reversibility problem in such structures arises. A detailed enough discussion of this problematics is represented in works [4,5,8,9,24]. In this connection the question of interrelation of the properties of reversibility and universal computability in classical ***CA*** models is a rather interesting. With this end we use the concept of «*reversibility*» of dynamics of classical models in conformity with aforesaid *(Chapter* **2***)*. In this connexion the question about interrelation of the properties of mutual erasability and universal computability in classical ***CA*** models is a rather interesting. Whereas a calculation generally speaking is an irreversible process, then is seemed quite natural that existence in the models of pairs of ***MEC–1*** and hence ***NCF*** and/or ***NCF–1*** should be closely linked with property of universal computability in classical ***CA*** models.

Meanwhile, already simple enough classical ***CA*** models possessing the pairs of ***MEC–1*** do not admit the universal computability [5,8,24]. Thus, presence for a classical ***CA*** model of ***MEC–1*** *(****NCF*** *and/or* ***NCF–1****)* is not sufficient condition for possessing the universal computability. On the other hand, on the basis of the above approach to definition of universal computability it has been shown that existence of ***MEC–1*** *(****NCF*** *and/or* ***NCF–1****)* is a necessary condition in order to some classical ***1–CA*** model possessed this property provided that the finite configurations only are used [1,5,9,24,102]. This result in one's time has provoked a rather wide discussion and stimulated the further research in this direction [7,24].

Meantime, on basis of other approaches *K. Morita* has proved existence of a reversible ***1–CA*** model which simulates any ***1–CA*** model, including irreversible ***CA*** models whereas *J. Dubacq* has proved an opportunity of simulation of the *Turing* machines by the reversible ***1–CA*** models [7]. *T. Toffoli* has proved the fact of a possibility of simulating of a ***d–CA*** model by means of a reversible *(****d+1)–CA*** *(****d*** **≥** ***1****)* model, having proved thus the computing universality of the reversible ***d–CA*** *(****d*** **≥** **2***)* models [132]. While

K. Morita and others have proved computing universality of reversible ***1–CA*** models [135,136]. Hence, in connection with the told the question of simulating of irreversible classical ***d–CA*** models by dint of reversible ones along with question of existence of the *reversible* universal classical ***d–CA*** $(d \geq 1)$ models represents a special interest. In addition, the above results have associated the reversibility mainly with absence of the ***NCF*** nonconstructability.

So, *T. Toffoli,* by working by results of *Aladjev–Smith* that are linked with the computability problem in classical ***CA*** models, has shown, in spite of the results received by them, the *reversible universal* classical ***CA*** models exist. More precisely, *T. Toffoli* has proved, that a classical ***d–CA*** model can be constructively embedded into a certain reversible classical model ***(d+1)–CA,*** for which the generation problem of finite configurations of the unlimited size is algorithmically solvable. In addition, the *T. Toffoli* result [7,218] does not contradict our results which have been received earlier and the specified situation is caused by a lot of essential enough factors defining these contradictions; first of all, it is connected to use of different concepts of *dynamics reversibility* of classical ***CA*** models whose essence is discussed enough in detail in the present book as well.

Completely other approach allows to simulate the classical ***d–CA*** $(d \geq 1)$ models, including ***CA*** models possessing the ***NCF*** nonconstructability, by means of classical ***(d+1)–CA*** models which do not possess the ***NCF*** nonconstructability, allowing to formulate the interesting enough result for a whole series of appendices.

Theorem 125. A classical d–CA $(d \geq 1)$ ***model with a states alphabet A is 1-simulated by means of an arbitrary classical (d+1)-CA model with the same states alphabet; in addition, the simulating (d+1)–CA model does not possess the NCF nonconstructability and keeps dynamics history of an arbitrary finite configuration of the simulated classical d-CA model.***

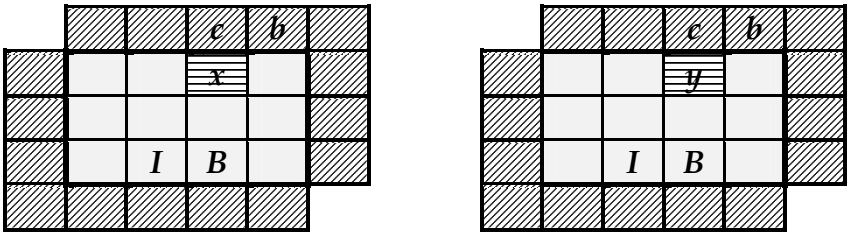
We shall consider such approach, without loss of generality, on basis of classical ***1-CA*** models. Let a certain ***1-CA*** is a classical model with states alphabet $A=\{0,1,...,a-1\}$, the simplest neighbourhood index $X=\{0,1\}$ and a local transition function $\sigma^{(2)}(x, y) = x^*;\ x,y,x^* \in A$. It is well–known, that an arbitrary classical ***1–CA*** model is simulated with time delay and the expansion of alphabet ***A*** by an appropriate model of the same class, the same dimension and with neighbourhood index $X = \{0, 1\}$. In addition, let's define a classical ***2–CA*** model which will simulate the ***1–CA*** model with the same alphabet ***A,*** the simplest neighbourhood index $X^* = \{(0,0), (0,1), (1,1)\}$ and local transition function $\sigma^{(3)}$ whose rules *(G)* are defined

by the parallel substitutions as follows.

$$S(x,y)_{t+1} = \sigma^{(2)}(S(x,y+1)_t, S(x+1,y+1)_t) \otimes S(x,y)_t$$

where: $\sigma^{(2)}(c,d)$ *– LTF of the modelled classical 1-CA;*
$c \otimes d$ *– computes* $c+d$ *(mod a);* $c,d \in A = \{0,1,2,...,a-1\}$ (25)
$S(x,y)_t$ *– a state of an elementary automaton with coordinates (x,y) at a moment* $t \geq 0$ $x,y \in \{0,\pm1,\pm2,\pm3,...\}$; $t = 0,1,2,3,...$.

First of all, we shall show, that the simulating classical ***2–CA*** model that are determined suchwise does not possess the ***NCF*** nonconstructability. Else, according to the existence criterion of the ***NCF*** nonconstructability *(Theorem **6**)* the above ***2–CA*** model should possess pairs of ***MEC*** of the following general kind**:**



In the assumption of presence for the above classical ***2–CA*** model of the ***MEC*** pairs of a specified kind, we choose a ***MEC*** pair of the rectangular form with minimal ***IB*** in which each corresponding sides *(without loss of generality we choose the upper side)* contain at least one pair of elementary automata that are being in various states $x \neq y$ *(*$x, y \in A$*)*.

Having chosen now in this pair of ***MEC*** the most right upper pair of the corresponding ***IB*** automata in different states $x \neq y$ *(*$x,y \in A$*),* it is simple to make sure that on the basis of local transition function $\sigma^{(3)}$ *(25)* of the simulating model at the following moment t we again receive for the chosen pair of elementary automata different states**;** i.e. more precisely, there is the relation**:** $(\forall c, b \in A)(x \neq y \rightarrow \sigma^{(2)}(c,b) \otimes x \neq \sigma^{(2)}(c,b) \otimes y)$**.** Thus, the simulating classical ***2–CA*** model with local transition function $\sigma^{(3)}$**,** determined by relations *(25),* does not possess the pairs of ***MEC,*** what according to the above criterion of nonconstructability says, that such model also does not possess the ***NCF*** nonconstructability.

It is simple to make sure that the above ***2–CA*** model simulates a ***1–CA*** model with states alphabet $A=\{0,1,...,a-1\}$**,** the neighbourhood index $X = \{0,1\}$ and local transition function $\sigma^{(2)}(x,y) = x^*; x,y,x^* \in A$**.** By placing, for example, in string with coordinates $\{(0, j) \mid j = 0, \pm1, \pm2, ...\}$ each finite or infinite *(without loss of generality a finite)* a ***1–***dimensional configuration

$c=x_1x_2x_3 \ldots x_n$ $(x_1, x_n \neq 0; x_j \in A; j=1..n)$, it is easy to trace dynamics of its development influenced of *GTF* $\tau^{(3)}$, determined by appropriate *LTF* $\sigma^{(3)}$ *(25)* of the simulating model. The following visual scheme illustrates the fact of simulating of an arbitrary classical *1–CA* model.

t=0	...	0	0	0	0	0	0	...	0	0	0	...
	$c_o=$	...	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
t=1	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	$x^1{}_o$	$x^1{}_1$	$x^1{}_2$	$x^1{}_3$	...	$x^1{}_{n-1}$	$x^1{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
t=2	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	$b^1{}_o$	$b^1{}_1$	$b^1{}_2$	$b^1{}_3$	...	$b^1{}_{n-1}$	$b^1{}_n$	0	...
	...	0	$x^2{}_{-1}$	$x^2{}_o$	$x^2{}_1$	$x^2{}_2$	$x^2{}_3$	...	$x^2{}_{n-1}$	$x^2{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
t=3	...	0	0	0	0	0	0	...	0	0	0	...
	...	0	0	0	x_1	x_2	x_3	...	x_{n-1}	x_n	0	...
	...	0	0	$b^1{}_o$	$b^1{}_1$	$b^1{}_2$	$b^1{}_3$	...	$b^1{}_{n-1}$	$b^1{}_n$	0	...
	...	0	$b^2{}_{-1}$	$b^2{}_o$	$b^2{}_1$	$b^2{}_2$	$b^2{}_3$	...	$b^2{}_{n-1}$	$b^2{}_n$	0	...
	...	$x^3{}_{-2}$	$x^3{}_{-1}$	$x^3{}_o$	$x^3{}_1$	$x^3{}_2$	$x^3{}_3$	...	$x^3{}_{n-1}$	$x^3{}_n$	0	...
	...	0	0	0	0	0	0	...	0	0	0	...
==========												
$x_j^1 = \sigma^{(2)}(x_j, x_{j+1}); \quad x_j^{k+1} = \sigma^{(2)}(x_j^k, x_{j+1}^k);$ $x_j, x_j^k, x_j^{k+1}, b_j^k \in A; k=1,2,\ldots; j=0,\pm1,\pm2,\ldots$												

Detailed enough description of the simulating algorithm that underlies proof of this result can be found, for example, in [24]. It is necessary to mark, the result of theorem *125* along with *T. Toffoli* result determines a high enough price of such modelling – increasing of dimensionality of a simulating classical model on one concerning the dimensionality of the simulated classical *CA* model. Of the results of theorem *117* forthwith follows, that a classical *d–CA (d ≥ 1)* model within of dynamics of finite and/or structural–periodic configurations is simulated by means of an appropriate classical *1-CA* model with neighbourhood index *X*={*-1,0,1*}. At the same time, the approach used at the proof, provides simulating of dynamics only in classical *d–CA (d≥1)* models and onto more general

case of modelling does not spread. Thus, in view of the told along with the result of theorem ***125*** it is possible to formulate the following offer.

Theorem 126. A classical d–CA (d ≥ 1) model within of dynamics of the finite and/or structural–periodic configurations is simulated by means of an appropriate classical 2–CA model that does not possess the NCF nonconstructability and it possesses the simplest neighbourhood index X = {(0,0), (0,1), (1,1)}.

At the same time, under the reversibility *T. Toffoli* along with a number of other researchers understand absence for a simulating model of the mutual erasability *(the **NCF** nonconstructability)* whereas we understand also absence for the reversible models of the ***NCF-1*** nonconstructability, the substantiation of such premise is submitted earlier. While approach used by *T. Toffoli* not only demands an increase in dimensionality of a simulating model, but also does not free it from the nonconstructability ***NCF–1***, not allowing to count dynamics of such simulating ***CA*** models reversible in the full measure. Along with it, *T. Toffoli* for creation of the reversible models used a structural approach, presenting an elementary automaton of a model by a simple logic circuit out of ***3*** elements [7,111]. Meantime, analysis of this approach shows that reversibility is reached owing to implicit increase in cardinality of states alphabet and concerns to its some subset. While the real reversibility relative to the expanded alphabet is not reached. On the other hand, we under real reversibility understand the *reversibility* of dynamics of a classical ***CA*** model relative to the set $C(A,d,\phi)$. Here again enough pertinently to discuss two levels of the *reversibility* – *real* and *formal*.

So, under the ***formal*** level is understood the reversibility of an arbitrary finite configuration c^*, namely existence for the $c^* \in C(A,d,\phi)$ of such sole configuration $c' \in C(A,d,\phi)$ irrespective of the set $C(A,d,\infty)$, that relation $c'\tau^{(n)} = c^*$ takes place. Meanwhile, under the ***real*** level is understood the reversibility relative to the finite configurations only; i.e. existence for an arbitrary finite configuration c of such sole finite configuration c^* of the set $C(A,d,\phi)$ only, that the relation $c^*\tau^{(n)} = c$ takes place. Consequently, depending on the existence criterion of the ***NCF*** nonconstructability that is based on the concept of ***MEC*** or ***γ–CF***, it is easy to make sure that the presence of formal reversibility can entail the real irreversibility whereas the converse proposition is generally speaking incorrect.

This circumstance is based on existence of two nonequivalent types of the nonconstructability in classical ***CA*** models – ***NCF*** and ***NCF–1*** which

have been considered earlier along with the real reversibility concept according to the definition ***13.*** So, if for a *formally* reversible classical ***CA*** model really reversible configurations exist, then similar ***CA*** model can possess the ***NCF-1*** nonconstructability***;*** i.e. the model is *really irreversible****;*** on the other hand, at lack of the ***NCF*** nonconstructability for a certain classical ***CA*** model, the model quite can be really irreversible. Thus, a classical model cannot simultaneously possess the properties of formal reversibility and real reversibility, i.e. these **2** properties as a whole are mutually exclusive ones. Evidently, the real reversibility in classical ***CA*** models entails the formal reversibility, while converse sentence is false, generally speaking.

One of motivations for introduction of concept of the *real* reversibility of dynamics of finite configuration in classical models is provoked as well by a quite natural requirement that a single predecessor in a prehistory $\{c\tau^{(n)k} \mid k = -1, -2, ...\}$ of a configuration $c \in C(A,d,\phi)$ should be calculated during a finite quantity of steps. In particular, under the assumption of belonging of fully zero configuration c_o = «□» to the set $C(A,d,\phi)$ of finite configurations the existence in a classical model the nonconstructability ***NCF–1*** does its really–irreversible.

From the point of view of definitions ***6*** and ***10*** of two types of erasable configurations the following a rather useful result can be presented that gives a criterion of two types of reversibility in classical models [24,84].

<u>Proposal 9</u>. A classical d-CA model is formally (really) reversible if and only if the model does not possess the pairs of MEC (MEC-1); i.e. it does not possess the NCF (NCF and NCF–1) nonconstructability.

In connection with the told an interesting question arises**:** ***Whether can an arbitrary classical d–CA (d ≥ 1) model be simulated by means of a reversible d–CA model?*** In turn, this question brings up a number of the accompanying questions, that in some extent describe the reversibility problem in classical ***CA*** models**.** Generally speaking**,** similar questions make up the *simulating* problem of arbitrary classical ***d-CA (d≥1)*** models by classical models of the same dimensionality that suppress the preset properties of the simulated models. Relative to the formal reversibility characterized by presence of the ***NCF–1*** nonconstructability**,** the basical result has been received, that plays essential enough part at research of dynamic properties of ***d–CA*** models [5,8,24,82,102,106].

<u>Theorem 127</u>. An arbitrary classical d–CA (d ≥ 1) model is 1–simulated by means of an appropriate d-CA model of the same type with minimal

expansion of a states alphabet A; wherein, the simulating d-CA model does not possess the NCF–1 nonconstructability at existence the NCF–2 nonconstructability. A classical d–CA (d ≥ 1) model that possesses the NCF–1 nonconstructability with a states alphabet of w cardinality is 1-simulated by means of an appropriate classical CA model of the same dimension and neighborhood index with a structurized states alphabet of 2w cardinality that does not possess the NCF–1 nonconstructability and VCF at existence for it of the NCF nonconstructability.

Detailed enough description of the modelling algorithm that underlies proof of this result can be found for example in [24,40-43]. Whereas the main idea of the proof of the *second* part of the theorem can be provided as follows. Without loss of generality we will consider ***1-CA*** model with the neighbourhood index ***X*={*0,1,2*}** and a states alphabet ***A*={*0,1,…,w-1*}**. The simulating ***1–CA*** model has the same neighborhood index and the ***G*** alphabet of two-level states of the following form**:**

$$G=\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}y_1\\1\end{bmatrix},\begin{bmatrix}y_2\\2\end{bmatrix},...,\begin{bmatrix}y_{w-1}\\w-1\end{bmatrix}\right\}\ where\ \begin{bmatrix}0\\0\end{bmatrix}\ is\ quiescent\ state,\ y_k\in\{0,1\},k=1..w-1$$

while a finite configuration □***x1x2 … xp***□ of the simulated model in the simulating model is represented by the *first* level, while the *second* level contains the states of the binary alphabet {***0, 1***} as follows

$$...\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}y_{-n}\\x_{-n}\end{bmatrix}...\begin{bmatrix}y_{-1}\\x_{-2}\end{bmatrix}\begin{bmatrix}y_1\\x_1\end{bmatrix}\begin{bmatrix}y_2\\x_2\end{bmatrix}...\begin{bmatrix}y_p\\xp\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}...;y_k\in\{0,1\},x_k\in A,k=-n..p$$

The local transition function of the simulating ***CA*** model, determining the substates of states of its structured state, is determined as follows**:**

$$\sigma^{(3)}(x_j^t,x_{j+1}^t,x_{j+2}^t)=x_j^{t+1};\quad y_j^{t+1}=\mu\left(\sum_{j=1}^{3}x_j^t+j_j^t\right);\quad \mu(x)=\begin{cases}0,if\ x=0\\1,otherwise\end{cases};\ t\ge 0;\ j\in\{-\infty,+\infty\}$$

where $\sigma^{(3)}$ is a local transition function of the simulated 1-CA model

On the basis of the assumptions made, it is easy to verify, that the ***1–CA*** model simulating the source model in strictly real time, has cardinality ***2w*** of states alphabet and the same neighborhood index ***X* = {*0, 1, 2*};** in addition, the simulating model possesses the ***NCF*** nonconstructability, but does not possess ***VCF*** and the ***NCF–1*** nonconstructibility.

Much more difficultly affair gets on for case of the nonconstructability such as ***NCF*** that makes up together with type ***NCF–1*** the basis of the reversibility concept of classical ***CA*** models. Within of research of this question, the concept of ***WM–***modelling covering a rather wide class of techniques of simulating of one classical model by another model of the same class and dimensionality has been defined. On this basis a result describing the opportunities for problems of modelling and useful in a

number of theoretical researches has been received [24,82,83,102,106].

Theorem 128. A classical d–CA (d ≥ 1) model cannot be WM–modeled by means of an appropriate reversible CA model (in sense of absence of the NCF nonconstructability) of the same class and dimensionality.

Of the concept of ***WM–***modelling and this result the conclusion directly follows, what for an opportunity of simulating of an arbitrary classical model by means of a reversible model of the same class and dimension it is necessary to use the coding methods of finite configurations for the simulated model that admit infinite number of equivalent members for the simulating model. That makes up a certain test of the first level for an admissibility of one or another way of simulating by ***CA*** models with the above property of reversibility. Moreover, it follows that the known traditional methods of simulating in the classical models covered with the concept of ***WM–***modelling, cannot lead to the necessary purpose, therefore new nonconventional approaches here are required.

By us in process of the further research the ***W–***modelling concept which essentially expands the concept of ***WM–***modelling and covering a wide enough class of known and potentially allowable modes of simulating in classical models has been defined. In addition, simulating of classical models by models of the same dimension is considered. But, and it has not allowed to solve positively in their midst the simulating problem by dint of an appropriate classical reversible model of the same dimension what the following basic result testifies**,** representing quite independent interest [5,8,24,40-43,82-87,102,106].

Theorem 129. A classical d–CA (d ≥ 1) model cannot be W–simulated by means of a reversible d–CA model (in sense of absence for it of the NCF nonconstructability) of the same class and dimensionality.

Thus, even within of such general enough concept as ***W–***modelling it is impossible to simulate an arbitrary classical ***d-CA (d≥1)*** model by means of a reversible model of the same dimension. Questions of distinctions of the ***NCF, NCF–1, NCF–2*** and ***NCF–3*** nonconstructability in classical models have been considered enough in detail above. Once more, if the simulating problem of ***CA*** possessing the ***NCF–1*** nonconstructability by models without the ***NCF–1*** nonconstructability is solved rather simply *(Theorem* ***127)***, then in case of the ***NCF (NCF–3)*** nonconstructability this question is a rather difficult, namely**:** at least within of two important enough concepts of ***WM–***modelling as well as ***W–***modelling that cover a broad enough spectrum of the modelling algorithms interesting both

from applied, and theoretical standpoints, the problem has the negative decision. In addition, the concepts of ***W-*** and ***WM-***modelling in the best way conform to the algorithms of simulating of classical ***CA*** models by means of classical ***CA*** models of the same dimension with suppression of existence of **γ–**configurations and ***MEC*** in the simulating models. In general, it seems quite natural that the classical ***d–CA*** $(d \geq 1)$ model can not be simulated by a suitable ***CA*** model of the same dimension for the following principal reason. Since the ***d–CA*** $(d \geq 1)$ model possessing the ***NCF*** nonconstructibility will have **γ–**configurations, i.e. to possess an unbalanced global transition function *(Definition* ***9)***, then the ***d-CA*** $(d \geq 1)$ model simulating it must have a global transition function, leveling the imbalance of the simulated model, making its global transition function unbalanced and having **γ–**configurations, i.e. it also should possess the ***NCF*** nonconstructibility. In addition, there is a proposal, having rather interesting appendices [24,40-43,82,102,106].

Proposal 10*. Within finite configurations a classical d–CA* $(d \geq 1)$ *model can`t be simulated by means of an appropriate classical CA model not possessing the NCF and NCF–1 nonconstructability, i.e. by means of an appropriate really–reversible classical CA model which not possess the property of universal computability and universal reproducibility in the Moore sense of finite configurations.*

So, concerning the simulating possibilities the classical ***CA*** models that do not possess the ***NCF*** and ***NCF–1*** nonconstructability are not of any especial interest, composing moreover a rather narrow class. Meantime, possessing substantial breadth of coverage the modelling algorithms of the above two classes are far from being exhaustive ones, therefore the further search of algorithms of simulating of *irreversible* classical models by reversible models is productive enough. For example, overstepping the limits of finiteness of states alphabet, there is a possibility to simulate an arbitrary classical model by means of models of the same dimension at absence of the ***NCF*** nonconstructability. The following result can be formulated in this direction.

Proposal 11*. A classical d–CA* $(d \geq 1)$ *model is simulated in strictly real time by means of an appropriate classical d-CA* $(d \geq 1)$ *model of the same dimensionality with an infinite* A^{∞} *alphabet of states in the absence of the NCF nonconstructability.*

Entirely other approach allows to simulate an arbitrary classical ***d–CA*** $(d \geq 1)$ model by means of a classical model of the same dimensionality, but with solvable existence problem of the nonconstructability such as

NCF–1 and ***NCF.*** The detailed description of a modelling algorithm that underlies proof of this result can be found in [24,40-43,82-87,102,106].

Theorem 130. An arbitrary classical d–CA (d ≥ 1) model is 1–simulated by means of appropriate classical d–CA* model having a rather simple neighbourhood index X and possessing the nonconstructability such as NCF and NCF–1; besides, the existence problem of nonconstructability of above types for the simulating model is algorithmically solvable.

This result turns out rather interesting for a lot of appendices including theoretical ones. Moreover this theorem absolutely does not contradict the result that, generally speaking, the existence problem of the ***NCF (NCF-3)*** nonconstructability for an arbitrary classical ***d–CA (d≥2)*** model is algorithmically unsolvable [24]. By using now the above results of *K. Morita, J. Dubacq* and of a number of others [7] together with the result of theorem ***117*** can be proved a rather interesting result [24,82,102,106].

Theorem 131. An arbitrary classical d–CA (d ≥ 1) model is simulated by means of an appropriate formally–reversible 1–CA model.

On basis of simulating of the universal *Turing* machine in the classical ***1–CA*** models it has been shown that, generally speaking, a number of mass problems for them is algorithmically unsolvable *(see, for example, Theorem* ***103****)*. Moreover, *A.R. Smith,* being based on one concept of the universal computability in classical ***CA*** models has proved that for ***CA*** models possessing the property of *universal* computability the problems of ***limitation*** and ***passivity*** of sequences $\langle w^* \rangle[\tau^{(n)}]$ are algorithmically unsolvable. Meanwhile, as a consequence of the proof of theorem ***130*** the result, interesting enough for the subsequent research on dynamics of classical ***CA*** models, easily follows.

Theorem 132. There are universal classical 1–CA models which have the Moore neighbourhood index and algorithmically solvable problem of limitation and passivity of an arbitrary sequence $\langle c \rangle[\tau^{(3)}]$ ***of the finite configurations, where*** $c \in C(A,1,\phi)$.

This contradiction in results is seeming, it is caused by distinctions of approaches to definition of the concepts of both the modelling, and the universal computability in classical ***CA*** models. So, more precisely, the computability in ***CA*** models we can define or on basis of the theory of word recursive functions directly or on basis of modelling of the known formal algorithms *(Turing machines,* ***TAG–****systems,* ***LAG–****systems, etc.)*.

Within of the general problem of simulating in the classical ***d–CA (d≥1)*** models the question of simulating of real models by means of classical

models represents a rather considerable applied interest. Under a ***real*** model such model is understood which differs from classical model if its elementary automaton at transition in the next moment into a new state determined by local transition function can pass and into a certain other state out of the same states alphabet, i.e. a new state of automaton will generally differ from a state expected according to local transition function of the classical model. Such behaviour of elementary automata in a real model can be explained by a lot of factors**:** emergency, chance failure, automaton malfunction, etc. Therefore, models of such type can be named the ***real*** with full grounds. They represent significant enough interest at research of some questions of practical realization of parallel computer systems on the basis of ***CA*** models along with a lot of other important enough motivations.

A lot of works has been devoted to investigation of the real ***CA*** models, more exactly, to the reliability problem of functioning of models of this type. The *Nishio–Kobuchi* approach is the most known technics of errors correction of functioning of real ***CA*** models, its base idea consists in the simulation of work of an arbitrary elementary automaton of some real ***CA*** model by means of ***3*** corrective neighbour automata [7,219]. In this case a classical model, simulating a real model on basis of information of ***3*** neighbours of an elementary automaton in the moment ***t>0*** *correctly* determines a new state of the automaton in the next moment ***t+1.*** But, a choice function of the states in this case is also supposed quite correct.

The block coding whose essence consists in embedding of states of an elementary automaton into some coding block organized by a special manner is represented as the most natural approach for this purpose. It corresponds to replacement of a real ***CA*** model with a neighbourhood index by means of an appropriate simulating classical ***CA*** model whose neighbourhood template includes neighbourhood template of the real model whereas its organization and way of operation of a simulating ***CA*** model allow to restore infringements occuring in the real model. A number of considerations concerning the principles of realization of the classical models correcting mistakes of functioning of real ***CA*** models can be found, for example, in [5,7,24]. The approach suggested by us to organization of reliable functioning of real ***CA*** models constitutes one of problems of the general problem of simulating of one classical ***d–CA*** ***(d ≥ 1)*** model by other model of the same dimension with suppression of a certain property of a simulated model in context of absence of this property for a simulating model.

The ***reliability*** problem of functioning of ***CA*** models of this type which consist of real elementary automata in some ways concerns the general problem of simulating in classical ***CA*** models. Till now it was supposed that the ***d–CA (d ≥ 1)*** models represent especially abstract model, while in the *real* conditions the work of models can be subject to infringements of various sort what entails the extremely undesirable consequences. In this connexion there is a rather important problem of such organization of a ***CA*** model which would allow to correct for many important cases possible failures arising in process of functioning of real ***CA*** models.

We shall name a ***CA*** model the ***self-correcting*** model if the model in the process of functioning possesses possibility to eliminate consequences of failures in operating of the elementary automata and the information channels connecting them. It is natural, that for the objects such as ***CA*** models consisting of infinite number of elementary automata, systems of switching of any elementary automaton with its direct neighbours, determined by the neighbourhood index and complex local transition functions a big enough variety of malfunctions arising at functioning of the real ***CA*** models is quite real. Here, we shall consider only two most important classes of malfunctions arising in the real ***CA*** models**:**

♦ *malfunction at definition of next state of an arbitrary elementary automaton of a real* ***CA*** *model**;** i.e. malfunction of functioning of its local function**;***

♦ *malfunction at gathering the states of elementary automata of some real* ***CA*** *model that compose its neighbourhood template**;** i.e. malfunction in means of switching of the* ***CA*** *model.*

Considering properties of ***CA*** models on behavioural *(dynamical)* level, instead of structural level, we quite can limit oneself only to two types of *malfunctions* that to a certain degree are abstraction of *real* conditions. Moreover, in view of the principle of functioning of a ***CA*** model when for obtaining of information about a configuration of neighbourhood template a time is not spent we quite can limit oneself to consideration only of the first type of malfunctions. In general case in real ***CA*** models during their functioning a lot of other malfunctions can arise, however we put into the term «***real CA model***» only the above sense. Meantime, analysis of malfunctions of a lot of other types is a rather complex and important problem demanding the detailed development at practical elaboration of ***CA*** models [102]. We shall introduce now the ***reliability*** concept of a real ***CA*** model.

<u>Definition 25</u>. A real d–CA (d≥1) model possesses the reliability (1-1/h^d) if in each d–dimension hypercube with edge of size h no more than one

elementary automaton may be subject to different malfunctions at the same moment.

It is obvious, that at boundary values $h=\infty$ or $h=1$ we deal with reliable *(classical)* model and unreliable *(real)* model accordingly. While all other intermediate values h give real models of a different level of reliability. Certain methods of organization of functioning of *real* ***CA*** models doing their by the self-correcting computational structures have been offered. So, one of similar methods allows to represent the following interesting enough result [24,40-43,82-87,102,106].

Theorem 133. For a real d–CA ($d\geq 1$) model of reliability ($1-1/h^d$) ($h\geq 3$) with a states alphabet A there is an appropriate self–correcting model of the same dimensionality with the states alphabet $A\cup\{w\}$ and global transition function $\tau^{(q)}\tau^{(p)}$ that 2–simulates the first model where $\tau^{(p)}$ is a reliable corrective function with the Moore neighbourhood index and w is a certain marker state.

The correcting problem of real models essentially becomes simpler if to assume that occurrence of failures in individual automata of a model is identified by them themselves by transition into a certain *signaling* state $g\notin A$. At that assumption it is possible to considerably simplify coding of states of some simulating model together with a corrective function $\tau^{(p)}$. Interesting possibilities for support of correction can be received at the assumption that individual automata in a ***F–***state operate without failures. In this case the states of automata of a real ***CA*** model are coded by means of a linear configuration in the form «xxF» ($x\in A$, $F\notin A$). Other things being equal the elementary automaton of the model should not be provided by the property of identification of failures. This approach allows to simplify the functioning algorithm of elementary automata of a simulating model what allows to formulate a rather useful result [24].

Theorem 134. For a real d–CA ($d\geq 1$) model of reliability ($1-1/h^d$) ($h\geq 3$) with an alphabet A there is an appropriate self–correcting model of the same dimension with alphabet $A\cup\{F, g\}$ and global transition function $\tau^{(q)}\tau^{(p)}$ that 2-simulates the first model where $\tau^{(p)}$ is a reliable corrective function with the Moore 1–dimension neighbourhood index and F, g are a marker state and signaling state accordingly. Under the condition of reliability of functioning of automata in state F for a real d-CA ($d\geq 1$) of reliability ($1-1/h^d$) ($h\geq 3$) with an alphabet A there is an arbitrary self–correcting model of the same dimensionality with alphabet $A\cup\{F\}$ and a global transition function $\tau^{(q)}\tau^{(p)}$, which 2–simulates the first model,

where a reliable function $\tau^{(p)}$ has 1–dimension neighbourhood index of format X = {–2,–1,0,1,2}.

The further research of the ***self–restoration*** problem of real ***CA*** models of different type represent a significant applied and cognitive interest, and to this direction the corresponding attention should be paid. Study on stability of the real ***CA*** models to failures of the various kind can be attributed to this direction too. In this respect the ***CA*** models out of so–called class of *threshold* models, for which the local transition functions are carried out by the principle of a threshold element represent a quite definite interest. The local transition functions of such type already in principle of own functioning contain a rather significant element of the *reliability*, i.e. stability to possible failures. May be, it – one of the reasons of high reliability of functioning of real neuro–like structures of various nature [24,102]. A discussion of a lot of practical conclusions out of the presented results and most interesting themes of research on reliability of real ***CA*** models can be found in [7,286]. Above all, the determination of the most effective self–correcting models relative to various concepts of failures is of interest. In addition, satisfactory enough solution of the reliability question of computing elements, devices and systems on the basis of ***CA*** models has the great applied significance too.

Meanwhile, the above approaches not only have formal character, but allow to consider the *reliability* problem for cellular systems of different various sort and nature from the various formal points of view. So, the suggested receptions of correction of real ***CA*** models present a certain, most likely, theoretical interest, carrying features of the general basical approach, meanwhile for the purposes of practical application they are insufficiently effective by the required resources. Therefore for practical problems it is necessary to develop more effective methods of failures correction, i.e. increasing of reliability of functioning of real ***CA*** models and concrete devices realizable on their basis. So, in the case of practical realizations of computing ***CA*** models with purpose of reliability control, the structural approach whose base essence consists in providing of the elementary automata and perhaps of switching system of *neighbourhood* template of real ***CA*** models by *special correcting logic circuits* is presented as the most natural solution.

These logic circuits on the basis of an input information and the current state of each elementary automaton should have a possibility effectively to carry out the local analysis *(testing)* of the automaton reliability and, if necessary, to carry out the corresponding *diagnostic* or some *correcting*

procedure. For these purposes the available results on self–correcting codes can be quite successfully used. Here we have rather wide field of activity; and certain useful enough ideas can be used out of the above theoretical considerations too.

Results on self–correcting ***CA*** models can appear fruitful not only from the standpoint of computing sciences, but also in a context of research of mechanisms of restoration that take place at appearance of different damages in real biological cellular structures. Other rather interesting interpretations in this direction also are possible. At present, on account of the above reasons to the problem of elaboration of the *self–recovering* ***CA*** models is paid considerable enough attention and first of all in the connexion with engineering on their base of various objects and devices with use nanotechnology [7,24,43,82,102,106,286].

In concluding this section and the chapter as a whole, we note an quite significant impact on ***CA*** models research of their computer simulation. In spite of such extremely simple concept of classical ***CA*** models, they have generally speaking complex enough dynamics. In numerous cases theoretical investigation of their dynamics collides with rather essential difficulties. Therefore, computer simulation of these models that in the empirical way allows to research their dynamics is extremely powerful tool. Thereby the question is relevant under the general problematics of ***CA*** models. The discussion of the question represented below will carry schematic enough character whereas its detailed enough discussion can be found in [7,24,43,82,102,106,278,286].

6.6. Sketch on sofrware oriented on *CA* simulating

At present, the problem of computer simulating of ***CA*** models is solved at two basic levels: *(**1**) software modelling dynamics on computing systems of traditional architecture*, and *(**2**) hardware architecture that as much as possible corresponds to the **CA** concept; so-called **CA** oriented architecture of computing systems*. Computer simulation of ***CA*** models plays a rather essential part at theoretical research of their dynamics, but it is even more important at practical realizations of ***CA*** models of different processes. By present time a number of rather interesting systems of software and hardware for the help to researchers of different types of the ***CA*** models has been developed [5,7,24-28,40-43,82-87,102,106].

J. von Neumann and *S. Ulam* perhaps were the first, who have awared of large possibilities of computer modelling. So, *S. Ulam* has considered

heuristic use of computers for a lot of interesting enough applications. Obviously, *S. Ulam* and *J. Holladay* along with *R. Schrandt* are pioneers of effective computer simulation of ***CA***–like models [7,220–222]. These authors used rather *powerful* computers for generation of huge number of configurations and have considered a number of properties of their morphology in space and time. Majority of results in this direction are empirical, however as yet there are not too many general opportunities which can be received theoretically. The similar picture takes place for ***CA*** models as a whole, thus software simulating becomes one of basic study methods in ***CA*** problematics and of especially applied directions of this type of dynamic objects [7,24,41-43,82,102,106,278,286].

The subsequent stage in development of the computing approach to research of ***CA***–like models goes back to researchers who have created the first hardware–software system for heuristical investigations of the ***CA*** models [7]. *R. Brender* was, maybe, the first, who has developed the programming system for simulation of ***CA*** models [7]. The subsequent development of the computer approach is characterized by creation of a plenty of program systems of different character for simulation and empirical research of different dynamic aspects of ***CA*** models. Thus, in our works many programs in environment of different programming languages for different computer platforms has been represented [5,24, 102]. In particular, tools of mathematical ***Mathematica*** system support algebraic substitutions rules that rather easily model the local transition functions of classical ***1–CA*** models [49]. In this context many interesting programs for simulation of ***CA*** models in the ***Mathematica*** system can be found, for example, in the books [45,46]. On the basis of the computer simulation a number of interesting theoretical results on the theory of classical ***CA*** models and their applications in such fields as computer sciences, mathematics and developmental biology has been received.

In addition, it is necessary to note four basic prerequisites of computer simulating of classical ***CA*** models**:** ***(1)*** monitoring and illustration of the dynamics of initial finite configurations during generating from them of sequences of configurations in environment of a ***CA*** model**,** ***(2)*** receipt of kind and other interesting estimations of concrete objects in a ***CA*** model *(minimal sizes of* ***MEC***, ***NCF***, ***NCF–1*** *and their form, etc.)*, ***(3)*** investigation of dynamics of simulated model for formulation of hypotheses in this or that direction**,** and ***(4)*** research of complex models basing on concept of ***CA*** models, that present direct applied interest from many points of wiev. In certain cases the computer modelling had allowed to formulate

interesting enough questions in the ***CA*** problematics, and also to draw up a plan of solving of some of them [5,24,40-47,82-87,102,106].

So, we can mark such theoretical results as types of mutually–erasable configurations in classical ***1–CA*** models, certain dynamic aspects of ***CA*** models with refractority, *H. Steinhaus* combinatory problem, etc. [12,13] while out of especially applied ones it is possible to mark such results as differentiation, regulation and regeneration of cells, the French flag problem, regulation of axial structures, etc. [2,4,17,82,223]. In addition, in these works some questions of development of effective software for research of ***CA*** models are discussed**;** in [24,102] also it is possible to find the detailed enough discussion of similar questions. Presently, a similar computer approach to study of dynamics of ***CA*** models is characterized by three basic directions, namely**:**

(1) creation of the specialized programming languages for an effective embedding of ***d–CA (d ≥ 1)*** models of various types with their practical realizations into an appropriate computing environment**;**

(2) creation of programs providing computer simulation of ***d–CA (d ≥ 1)*** models of the special types describing those or other processes**;**

(3) creation of software systems and complexes that provides computer simulation of a wide enough class of ***d–CA (d ≥ 1)*** models.

In this context on one essential enough moment it makes sense to stop separately. It is supposed the base of researchers of different aspects of ***CA*** models consists of experts in natural fields above all *mathematicians* and *physicists* that in the right degree not all possess by a programming. Meantime, the situation becomes greatly simpler in view of existence of developed systems of the computer mathematics whose the undoubted leaders **–** ***Mathematica*** and ***Maple*** [9,24,44-49]. The vast majority of the modern mathematicians and physicists use par excellence these or other systems for professional activity**;** moreover, the programming in these systems shouldn't represent especial difficulties. Hence, similar systems can serve as software environment for creation of the software oriented on the experimental research of various classes and types of ***CA*** models. Complexity of such software can fluctuate in large ranges depending on the researched objects and their dynamics.

In particular, for experimental study of dynamics and other aspects of both classical**,** and some other types of the ***CA*** models we substantially used the above ***Mathematica*** and ***Maple*** systems. On their basis a large set of the software tools solving various problems of experimental study

of classical ***1-CA*** and ***2-CA*** was created. This software bears both highly specialized, and the more all-purpose character; they are arranged or as separate programs, or grouped into the special libraries; some of them are included in libraries for ***Maple*** or packages for ***Mathematica*** [48,49]. Many of them is presented in our books, scientific reports and preprints on the mathematical theory of ***CA*** models and their appendices [24,40-47,82-87,102,106]. In addition, several examples of such tools are given and in the present book, illustrating some possibilities of experimental study of dynamics of ***1–***dimension classical ***CA*** models. While our early software for experimental study of dynamics of classical ***CA*** models has been developed in the environment of such programming languages as *Assembler*, *Basic*, *Pascal*, *PL/1* and *Reduce*, at last, in software of the *Maple* and *Mathematica* systems of computer mathematics. The latest years for the above purposes we use only these two systems.

Of means of the *first* group we can mark the following software. First of all, once more the *R. Brender's* programming system can be marked that was intended for simulating of classical ***CA*** models. Much interpreters has been created for simulating of ***CA*** models, particularly of *R. Vollmar*, *SIBICA* of *J. Pecht*, etc. [7]. Meantime, the software support of practical implementation of computing systems on the basis of the ***CA*** models in their purest kind, above all from the practical standpoint is represented more interesting. So, for software of *ML*-processors the *Hungarian group* of researchers have created language *InterCELLAS* that is well adapted to simulating of cellular automata [121]. In the works from bibliography [7] it is possible to familiarize with architecture of such *ML*–processors and their software.

In turn, software of machines *CAM* is supported by a special subset of language *Forth (CAM Forth)*, providing, above all, determination of the local transition functions. Hither it is directly possible to attribute the program *CAMEX* that covers a rather extensive collection of ***CA*** models of dimensions ***1–3.*** Whereas *CAM*–simulator created at university *ELTE (Hungary)* represents the program simulating the cellular automata on base of *CAM–6.* At the general level with *CAM* and their software it is possible to familiarize in the excellent book [162], whereas for details is recommended to address to works [71,113,120-122,124-127,163,218] and numerous references in *https://bbian.webs.com/Cellular_Automata.pdf*.

*Language of parallel substitutions (**LPS**)* can be a good enough example of a rather convenient algebraical tool for description and analysis of the parallel microprograms. The ***LPS*** is a certain linguistic formalism of a

narrow enough class of the computing ***CA*** models that is based on the concept of systems of parallel substitutions determining local transition functions for the classical ***CA*** models. We have shown, that concerning the systems of parallel substitutions the following general result takes place, namely**:** ***The consistency problem of algorithms of the LPS which are defined by means of an arbitrary system of parallel substitutions is constructively solvable*** [24,43,82-87,102,106].

This result has allowed to obtain some rather interesting consequences, including the applied ones for the further development of some control microprogrammable systems [28,29,32,34,38,114,224-226].

The Italian investigators have elaborated the high**–**level programming language *CARPET* with a lot of additional constructions for description of local transition functions of a certain cellular environment. Language *CARPET* [227] provides a rather effective support of *parallel* information processing in the computing ***CA*** models. The *Cellang* language, created by *D. Eckart* can be considered as an effective software for simulation of ***CA*** models [228]. *Cellang* **–** the specialized language for programming of a wide enough class of ***CA*** models. In our opinion, the *Cellang* **–** a rather useful programming system that is oriented above all on the computing ***d–CA*** models ***(d = 1..3)***.

Of other interesting enough programming languages of ***CA*** models it is possible to mark such languages as *CEPROL* [229]**,** *Celip* [230]**,** *Cal* [231] and some others**.** So, *CDL***–**language for cellular processing along with language *ALPACA* for descriptions of the ***CA*** models present a certain interest [7]. The specified software is intended for empirical research of a lot of the important dynamic aspects of the classical ***CA*** models. With its help many interesting enough results of both applied and theoretical character have been received [7,24,102,106]. Our experience received in process of creation and use of software of the above type have allowed to formulate a general concept of an interactive program system ***Svegal,*** that is intended for computer research of ***CA*** models of a rather broad set of types [232]. Unfortunately this system for subjective and objective reasons remains up till now only at a level of the conceptual design.

The *second* group of software for today is most numerous and allows to experimentally investigate and to conveniently visualize the required dynamic properties of the ***CA*** models for concrete applications. At the same time, the majority of tools of this group serves as for simulation of well-known game «*Life*» or its various modifications and extensions. At present, the game «*Life*» is, probably, the most popular example of the

CA models along with a rather good object for computer simulating [5, 53-61,96-98,112,117]. Of software for simulation of the game «*Life*» such useful programs as *Life32* of *J. Bontes*, *WinLife* of *J. Harper*, and *LifeLab* of *A. Trevorov* can be mentioned.

Themes of the game «*Life*» is rather extensive, the interested reader can receive rather full information on it in [7], and in the Internet by a key phrase «*Conway's Game of Life*». Many researchers for an experimental research of the ***CA*** models have used interesting enough programs out of which it is possible to mark tools presented in works [7,233-241]. As simple simulators of ***CA*** models it is possible to mark such programs as *CAPOW*, *LCAU*, *CALAB*, etc. [7]. We likewise to a lot of other authors used the special programs developed in environment of such known programming languages as *C+*, *Basic*, *PL/1*, *Pascal* and *Reduce*, and also in environments of known computer algebra systems *Mathematica* and *Maple* with the purpose of empirical study of a lot of concrete dynamic and applied aspects of classical ***d–CA*** *(**d=1 .. 3**)* models [5,24,42-43,102]. So, a lot of procedures for study of certain special aspects of dynamics of ***d–CA*** *(**d=1,2**)* models is included in *Maple* library *UserLib6789.zip* [48] and *Mathematica* package *Archive76.zip* [49].

At last, tools of the *third* group provide computer study of a rather wide class of ***CA*** models. So, here we can mark a rather interesting simulator *MCell* of *M. Wojtowicz* that is oriented on a wide enough class of types of ***CA*** models of dimensionality ***1*** and ***2*** [238]. Of other simulators of a similar type it is possible to mark such simulators as *SARCASim* [240], *CellLab* [239], *CASE* [241], etc. So, program *Dr.Cell* is intended for study of dynamics of ***d–CA*** *(**d=1,2**)* models with local transition functions and neighbourhood templates defined by the user. Program *SRCA* allowing to research dynamics of the ***CA*** models for a number of rather complex local transition functions characterized by a large cardinality of a states alphabet of a model $(a \geq 2^{32})$ presents the definite interest. Quite certain interest the program *CABuilder* of creation of ***3–CA*** models along with program *CAReader* that allows to carry out a satisfactory analysis of the models created by the *first* program is presented. Quite full data on the specified tools can be found in the extensive bibliography [7,242-246].

Simulation system *Trend* & *jTrend* is intended for simulating in classical ***2–CA*** models of self–reproduction process, i.e. the basic component of any alive system. The system has an advanced enough opportunity of tracking of reverse dynamics of a ***CA*** model; perhaps, the system – one of the most opportune simulators of ***2–CA*** models. With description of

these and a lot of other rather interesting ***CA*** simulators the reader can familiarise in the above-cited works. The appreciable interest represents also package *CAT* intended for study of some features of programming paradigm [7] on parallel computers basing on the concept of computing ***CA*** models. *CAT* can serve also as a convenient enough tool for creation of various ***CA*** models. At last, for *empirical* study of ***CA*** models a certain interest can present package *DDLab* oriented onto research of dynamics of the finite *binary networks* **–** from binary ***CA*** models upto probabilistic boolean networks. A number of both typical, and special supplements of *DDLab* for research of binary ***CA*** models and networks together with the related questions the reader can find in the above works. In addition, for computer research of the ***CA*** models by certain other researchers the program *simulators* of those or other purposes and different complexity level have been produced [7,24,43,82,102,106,242,278,286].

A new approach to computer researches of ***CA*–**like models consists in creation for their description of effective enough linguistic, i.e. highly**–** parallel programming languages. However, in this direction there are complex enough problems of organization and description of parallel information processing in similar ***CA*** models**;** they are discussed, for example, in [24]. The analysis shows that proceeding to highly**–**parallel calculations supposes a rather serious revision of many our traditional approaches to calculations. Therefore, the complexity of a paralleling procedure is defined, naturally, directly by internal specific features of some parallelizable algorithm. Multitude of very interesting examples of paralleling is represented in numerous works, for example, in [24]. A number of rather interesting problems relative the *paralleling complexity* of algorithms, including algorithms in computing ***CA*–**like models, was discussed in works [7,286]**,** and also in a lot of articles and transactions of conferences [277-250]. Similar questions have been discussed in our works [24,40-43,82-87,102,106] too.

Consequently, the theory of parallel algorithms should have a structure substantially similar to structure of the theory of sequential algorithms. Theoretical aspects of the parallel algorithms are researched a little bit better, whereas work on applied aspects is in a great extent in a stage of becoming [7]. Now, this field rapidly enough develops, and many new appendices discover crossings with a number of the important scientific disciplines [24]. Thus, a formal parallel computing model demands the specialized linguistic tools that with the maximally possible efficiency should describe a paralleling level admitted by means of a calculations

model or information processing as a whole.

Linguistic tools for the modern commercial parallel computing systems based on different formal computing models *(excepting **CA**–like models)*, and also prospect of their development are discussed enough in detail in a lot of works [7]. Now we have very great number of hardware and software systems, intended for parallel processing. These systems use different both parallel hardware architecture, and parallel software [24]. In our works [24,85,86,102] the *Parallel Control System (**PCS**)* and *Parallel System of Information Processing* for homogeneous computional systems have been presented; it is shown that on the basis of these systems the information paralleling allows to create data processing systems of the high efficiency right up to the systems with direct economic effect.

In our work [251] the description of the above ***PCS*** in terms of *systems of algorithmic algebras (**SAA**)* has been presented. The presentation is based on an automaton model of ***PCS***, allowing to use the ***SAA*** for problem of optimization of the ***PCS*** software. Furthermore, the above work at the most general level have used the ***CA*** concept as a formal parallel model of calculations. Meanwhile, a number of parallel software uses parallel algorithms inherent in the computing ***CA*** models. So, for example, the above ***PCS*** uses certain parallel algorithms inherent in a biological ***CA*** model [253], and that are based on the principle of globally–local action [102,252]. An interesting enough approach to use of highly–parallel ***CA*** algorithms can be found in [254-256]. Now, there is a number of parallel software oriented on manifold parallel architectures along with a lot of highly–parallel programming languages that are oriented, above all, on the computing ***CA*** models. These languages form both linguistic tools, and a toolkit for research of a wide enough class of parallel computing ***CA*** models [7,24,82,102,106,286].

The practical implementation of computing ***CA*** models occupies here a special place. So, group of the *Hungarian* researchers under leadership of *T. Legendi* in process of work on cellular processors has essentially simplified and modified the cellular model of *J. Neumann* and of a lot of his followers. The further researches in this direction have led them to creation of the practical implementations of the computing ***CA*** model such as «*Legendi cellular processors*», next of cellular *ML*–coprocessors for *IBM*–compatible personal computers. The long-term cooperation of research groups of *R. Vollmar* and *T. Legendi* became the main reason of creation of both functioning commercial models of cellular processors, and their satisfactory theoretical study. Within of team–work on this

specified problematics a lot of widely known international conferences *PARCELLA (**PAR**allel **CELL**ular **A**utomata)* was conducted, collections of articles and scientific reports were issued [7]. In the *PARCELLA* a lot of rather interesting results concerning the *specific* computing architecture as well as separate devices based on ***CA*** models is submitted [7]. Of the Russian practical works on realization of the computing ***CA*** models it is possible to mark rather interesting results of groups of researchers from *Novosibirsk*, *Saint-Petersburg*, *Taganrog*, *Moscow* and *Kishinev*. Perspective works executed in *Novosibirsk* under leadership of *O.L. Bandman* and in *Taganrog* under leadership of *A.V. Kaljaev* present a special interest [7].

Another interesting and perspective approach to practical realization of the computing ***CA*** model *T. Toffoli* and *N. Margolus* have offered, which have created series of so-called «*Machines of Cellular automata*» *(**C**ellular **A**utomata **M**achines – **CAM**)* [71,125-127]. The popular description of the ***CAM*** along with experiments with them the reader can find in the fine book [162]. A practical application of machines ***CAM*** appeared a rather effective at modelling of complex enough problems of hydrodynamics, ecology and study of mathematical properties of ***CA*** models, of models of images processing, physical modelling, generating of special effects and a lot of others [7]. Thus, a series of machines ***CAM*** has carried over the ***CA*** problematics onto qualitatively new unique level, having added formal ***CA*** models by their direct computing analogues.

By present time there is a number of other interesting enough practical realizations of the computing ***CA*** models from them we shall mark only several [7,257-268]. All abundance of materials on applied problematics of computing ***CA*** models does not allow to present their a satisfactory analysis. Meanwhile, the interested reader can refer to the proceedings of the corresponding scientific-practical conferences and the numerous articles in various periodicals of the corresponding themes. In view of development of technology of integrated circuits and new approaches to creation of perspective architectures of computer facilities *(quantum cellular processors, nanocomputers, etc.)* with wide use of nanotechnology the interest to the ***CA*** problematics steadily grows. However, in view of extensiveness of this theme its consideration is beyond the present book. Considering a rather essential complexity of theoretical research of ***CA*** models, in their research the computer modelling plays serious part.

Chapter 7. The decomposition problem of global transition functions in the classical *CA* models

The problem of ***decomposition*** of *global transition functions (**GTF**)* for the ***CA*** models represents the considerable enough theoretical and applied interest. The main goal of decomposition of global transition functions in ***CA*** models consists in finding of effective procedures**,** that allow on basis of the preset global transition function to define a composition of more simple functions that is equivalent to the initial global transition function. The problem is similar to the problem of partition of a complex system onto more simple subsystems, presenting a rather large interest for many directions of the ***CA*** problematics. In particular, this problem directly adjoins to the above problem of ***A(X)*** complexity. The problem has the most direct attitude to the questions of constructive complexity playing a rather important part also in concrete realization of ***CA*** models of different sort and destination.

The first posing and results concerning the decomposition problem go back to *S. Amorozo* and *J. Epstein* which have shown that in the set of all binary ***1–***dimensional global transition functions there are functions not representable in the form of *composition* of finite number of more simple functions of the same type and class [269]. Then, using a rather simple numerical procedure, *J. Buttler* has shown that in the set of all global transition functions of ***d–***dimensionality ***(d ≥ 1)*** also there are functions not representable in the form of so**–**called *minimal* composition of finite number of more simple global transition functions out of the same set of functions [270,271]. In some our works the decomposition problem has received the further development [24,40-43]**;** the results received in this direction have allowed to consider this problem from new rather interesting standpoints considered a little bit below.

First of all, the decomposition problem in a certain extent concerns the complexity problem of global transition functions, namely**:**

Whether can an arbitrary global transition function be represented by means of a certain composition of finite number of more simple global transition functions of the same class and in the same state alphabet?

In addition, below we shall speak that a global transition function $\tau^{(n)}$ is *simpler* than a global function $\tau^{(m)}$ *(both functions are defined in the same finite alphabet and have identical dimensionality)* if $n < m$; $n < m$ determines relation between quantity of the elementary automata making up the

neighbourhood templates of both models. Meantime, it turned out that such problems as *complexity* of finite configurations, *completeness* for the polygenic models along with *decomposition* of global transition functions are enough closely connected, representing a perspective and extensive field for the further research [24,43,82,102,106].

It is easy to verify that, in the general case, an arbitrary global transition function $\tau^{(n)}$ can`t be presented in the form of a composition of the finite number of more simple global transition functions of the same class and in the same state alphabet. In particular, let the following relation

$$\tau^{(n)} = \tau^{(n_1)}\,\tau^{(n_2)}\ldots\,\tau^{(n_m)} \qquad (\forall j)(n_j = 2);\ j = 1..m\ ;\ m = [n/2] \qquad (26)$$

where $[j]$ – a minimal integer, greater than j, determines a decompostion of global function $\tau^{(n)}$ of a classical ***1–CA*** model with a neighbourhood index $X=\{0,1,\ldots,n\}$ in an alphabet $A=\{0,1,\ldots,a-1\}$ onto global transition functions $\tau^{(j)}$ determined in the same alphabet and with neighbourhood index $X=\{0,1\}$. Further, on the basis of relation *(26)* it is easy to verify, the number of all possible compositions of functions functions $\tau^{(j)}$ satisfies the relation $a^{(a^2-1)m} < a^{a^n-1}$, that under the condition $m=[n/2]$ proofs our assertion. This result is generalized to the common case of ***CA*** models.

Our first results concerning the decomposition problem were based on earlier results on the nonconstructability in classical models and have allowed to solve this problem for classical ***1–CA*** models [24]. The basic result in this direction was the proof of existence of ***1–***dimension global transition function $\tau^{(n)}$ with an arbitrary neighbourhood index n and a states alphabet for which the decomposition problem has the negative decision. Meanwhile, the next discussion of the decomposition problem of global transition functions involves the introduction certain concepts and definitions introduced as required.

Definition 26*. In toto, the decomposition problem of global transition functions (d–PDF)* $\tau^{(n)}$ *is reduced to the question of an opportunity of representation of an arbitrary global transition function in the form of composition of finite number of more simple functions of the same class and in the same states alphabet* $A = \{0,1,2,\ldots, a-1\}$*, namely:*

$$\tau^{(n)} = \tau^{(n_1)}\,\tau^{(n_2)}\,\tau^{(n_3)}\ldots\,\tau^{(n_p)} \qquad (n > d+1;\ n_j < n;\ j = 1..p) \qquad (27)$$

where the global transition functions $\tau^{(n)}$ *and* $\tau^{(n_j)}$ *(j=1..p) have identical dimensionality and are defined in the same states alphabet; in addition, for global transition functions* $\tau^{(n_j)}$ *multiple occurences are admitted in*

representation (27). In case of 1–dimension global transition functions $\tau^{(n)}$ ***for decomposition (27) there is the relation*** $n = \sum_j n_j - p + 1$ $(j = 1..p)$. ***For the 1–dimension case there is the relation*** $(\forall j)(n_j \in \{2,3, \ldots, n-1\})$. ***In addition, the above representation (23) should not contain the identity*** $\tau^{(n_j)}$ ***functions for which there is the relation*** $(\forall g \in C(d,a))(g\tau^{(n_j)} = g)$.

It is shown [40] that an arbitrary global transition function with respect to representability in the form *(27)* can have one of three opportunitie, namely**:** *(1)* has no any representation in the form *(27)*, *(2)* has a single representation in the form *(27)*, and *(3)* has more than one presentation in the form *(27)*. In particular, the already simple ***1–***dimensional global transition function $\tau 1^{(3)}$, defined by the local transition function $\sigma 1^{(3)}(x, y,z)=x+y+z \pmod 2$, has no representation in the form *(27)*, whereas the global transition function $\tau 2^{(3)}$, defined by the local transition function $\sigma^{(3)}(x,y,z) = \mathbf{If}[x = 0, z, x+y+z \pmod 2]$ has a single representation in the form *(27)*, at last there exists a global transition function $\tau^{(4)}$ that admits more than one representation in the form *(27)* – $\tau^{(4)}= \tau^{(3)}\tau^{(2)}= \tau^{(2)2}$. Let representation *(27)* will be the only one for a global transition function $\tau^{(n)}$. Obviously, each global transition $\tau^{(n_j)}$ functions containing in *(27)* will have negative ***d–PDF*** decision. This is one of the simplest approach to proof of the existence of negative ***d–PDF*** decisions. Thus, for $\tau^{(n)}$ that has single representation *(27)* all its constituents aren't representable.

Consider a certain set of global transition functions the local transition functions of which are defined by the equations as follows:

$$\begin{cases} \sigma^{(n)}(x_1, x_2, \ldots, x_n) = p_1x_1 \# p_2x_2 \# \ldots \# p_nx_n; \\ x_j, p_j \in A = \{0,1, \ldots, a-1\}; \quad j = 1..n; \ \# - \text{a binary commutative} \\ \qquad \text{operation, in particular, addition modulo } a \end{cases}$$

The set defines a semigroup concerning composition operation without unity. It is easy to see that an arbitrary ***1-***dimension global transition $\tau^{(n)}$ function with an even ***n*** size of a connected neighborhood template can be presented in the form of composition of ***2*** global transition functions $\tau^{(n)} = \tau_1^{(n/2+1)}\tau_2^{(n/2)}$, whose local transition functions have neighborhood indices $X1=\{0,n/2\}$ and $X2=\{0,1,\ldots,n/2-1\}$ accordingly. Thus, the *first* one has the disjointed neighborhood template *(only two extreme variables are master while others are dummy variables)*, while the *second* has a connected one. In general, a disjointed neighborhood template for the convenience of research of ***CA*** models can be considered as a connected template, on which the corresponding local transition function has dummy variables, i.e. variables which take values from the model finite alphabet and do

not affect values of the local transition function. For example, the binary ***1–CA*** model with neighborhood index ***X1*****={*0,2*}** and ***LTF*** $\sigma^{(3)}(x,y,z)=x+z$ *(mod* **2***)* is equivalent to the binary ***CA*** model of the same dimension with neighborhood index ***X2*****={*0,1,2*}** and ***LTF*** $\sigma^{(3)}(x,y,z)=if(x=0,z,z+1$ *(mod* **2***))***.**

Taking into account a finite number of global transition functions $\tau^{(n_j)}$ in the representation *(***27***)* and a finite number of such functions in the case of a finite ***A*** alphabet**,** it is easy to show, for example, on the basis of the brute force approach *(albeit rather cumbersome)***, *the decomposition problem for an arbitrary global transition function* $\tau^{(n)}$ *into the simpler functions is solvable, including uniqueness its decomposition.***

Complete search *(or brute force approach)* — a method of the decision of mathematical problems. The complexity of complete search depends on the number of all possible decisions of the problem. If the decision space is very large, then complete search can`t yield results for a long time.

At that, it is necessary to pay attention to spurious misunderstanding that can arise from the fact of *negative* decision of the ***d–PDF*****,** on the one hand, and universality of the classical ***d–CA*** models with the simplest neighbourhood indexes, on the other hand. Really**,** any ***d–CA*** model is simulated by means of a ***CA*** model of the same dimensionality and with the simplest elementary neighbourhood index, whose neighbourhood template contains only *(**d+1**)* elementary automata. However, in case of modelling we deal, as a whole, with different sets of configurations, on which a simulated model and a simulating model operate while a global transition function and functions making up its decomposition *(***27***)* will be operate with the same set of configurations and in the same alphabet**;** thus, this distinction has very principal character.

A large enough help in investigation of the decomposition problem of global transition functions is computer modeling [5,24]. In particular, a number of procedures, programmed in software of the systems ***Maple*** and ***Mathematica*****,** are useful enough in investigating this problem for the global functions of dimensionality ***1*** and **2,** taking into account the need to use in certain cases the exhaustive enumeration method. So, the procedure call **ComposeGTF[*g*]** returns the **2-**element list, whose the 1^{st} element defines the neighbourhood template size and the 2^{nd} defines the discriminating number of ***GTF*** that is a result of composition of two or more global transition functions**,** whose the local transition functions are defined by neighbourhood template sizes {***n1,…,nk***} and discriminating numbers {***p1,…,pk***} respectively. In addition, in the call **ComposeGTF[*g*]**

the pairs {nj,pj} *(j=1..k)* are used as argument ***g***, i.e. **ComposeGTF[*n1,p1, ..., nk,pk*];** number of such pairs should be at least two. Meantime, if the procedure call contains odd number of arguments *(as the last argument can be used an expression)* the list of parallel rules determining the ***LTF*** of the resulting global transition function is returned.

This procedure is quite simply generalized to an alphabet of elementary automaton of a ***CA*** model, other than the binary alphabet. Procedure is useful at research of a lot of questions of the decomposition problem of one-dimension global transition functions. In particular, it can be useful at solution of the question of definition of possibility of decomposition of an arbitrary global transition function onto functions from the preset set of global transition functions. Then, we developed similar procedure for study of decomposition of **2-**dimension global transition functions, being based on a modification of algorithm of the above procedure. In addition, if **ComposeGTF** procedure solves the composition problem of global transition functions, the decomposition problem for each global transition function is algorithmically solvable, its constructive decision is reduced to a programm analysis of all possible variants, whose set is large enough but finite. Along with the above means we programmed a number of other software for analysis of the decomposition problem [7, 24,40,41,82,85,102,106,286].

For composition of ***1-***dimension global transition functions whose local transition functions are determined by means of lists with elements of the format {*"x1x2...xn"* → *"x`1"*}, where *xj, x`1*∈*A*={*0,1,2,...,a-1*}; *j=1..n*, the ***Mathematica*** procedure **CompGTF** was been programmed with **2** formal arguments**:** ***Alph*** **-** an alphabet common for all global transition functions and ***h*** **-** the list of local transition functions determined in the form of lists determined corresponding global transition functions with elements of the above format.

```
In[4667]:= CompGTF[Alph_List, h_List] := Module[{gtf, r, s, t},
             gtf[x_List, y_List, A_List] := Module[{a, b = {}, c, d, j, k, g},
              d = StringLength[x[[1]][[1]]]; g = StringLength[y[[1]][[1]]];
                                                  a = Tuples[A, d + g - 1];
                                    a = Map[StringJoin, Map[ToString2[#] &, a]];
                                 For[j = 1, j <= Length[a], j++, c = ""; For[k = 1, k <= g, k++,
c = c <> StringReplace[StringJoin[StringPart[a[[j]], k ;; k + d - 1 ;; 1]], x]];
                                                AppendTo[b, a[[j]] -> StringReplace[c, y]]]; b];
                                                  If[Length[h] == 1, h[[1]], s = h[[1]];
                                              For[t = 2, t <= Length[h], t++, s = gtf[s, h[[t]], Alph]]]; s]
```

The procedure is an useful enough tool in an experimental study of the composition/decomposition of global transition functions. In particular, an example of the application of the procedure well illustrates the non-commutativity of the composition of global transition functions [41,49].

```
In[4668]:= y = {"00" → "0", "01" → "1", "10" → "1", "11" → "0"};
y1 = {"00" → "0", "01" → "1", "10" → "1", "11" → "1"};
In[4669]:= CompGTF[{0, 1}, {y, y1}]
Out[4669]= {"000" → "0", "001" → "1", "010" → "1", "011" → "1",
"100" → "1", "101" → "1", "110" → "1", "111" → "0"}
In[4670]:= CompGTF[{0, 1}, {y1, y}]
Out[4670]= {"000" → "0", "001" → "1", "010" → "0", "011" → "0",
"100" → "1", "101" → "0", "110" → "0", "111" → "0"}
```

The noncommutativity of composition of global transition functions $\tau^{(n)}$ does not allow to narrow a little a framework of full search at solution of a number of problems of solvability of the common decomposition problem of global transition functions.

It is simple to be sure that the set of all linear global transition functions which have been preset in an arbitrary states alphabet $A = \{0,1,2,...,a-1\}$ *(a – a prime number)* and whose local transition functions are defined by the above function, is the closed concerning the composition operation. This assertion can be easily obtained on the basis of definition of global transition function $\tau^{(n+m-1)}$ – result of composition of 2 global transition functions $\tau_1^{(n)}$, $\tau_2^{(m)}$ determined by local transition functions $\sigma_1^{(n)}$, $\sigma_2^{(m)}$ accordingly, namely:

$$\begin{cases} \sigma_1^{(n)}(x_1,x_2,...,x_n)=\sum_{j=1}^{n} x_j \ (mod\ a); \qquad \sigma_2^{(m)}(y_1,y_2,...,y_m)=\sum_{j=1}^{m} y_j \ (mod\ a); \\ \sigma^{(n+m-1)}(z_1,z_2,...,z_{n+m-1})=\sum_{k=1}^{m}\left(\sum_{j=k}^{k+n-1} z_j \ (mod\ a)\right)(mod\ a) \\ x_n, y_m, z_2 \in A=\{0,1,...,a-1\};\ a-a\ prime;\ j=1..n;\ k=1..m;\ p=1..n+m-1 \end{cases}$$

So, the resulting global transition function $\tau^{(n+m-1)}$ of composition of 2 global functions $\tau_1^{(n)}$, $\tau_2^{(m)}$ defines by the above local function $\sigma^{(n+m-1)}$.

The question of determination of closed subsets of the above set of *linear* global transition functions presents a positive interest. In particular, the subset of binary global functions with connected neighbourhood index isn't closed relative to the composition operation.

Along with the known ***d*–PDF** it is rather interesting to investigate so-called generalized problem of decomposition of ***d***–dimensional global transition functions ***(d–GPDF)***, which consists in the question about a

possibility of presentation of an arbitrary global transition function $\tau^{(n)}$ in the form of the composition *(27)* provided that global transition $\tau^{(n_j)}$ functions of the representation are not connected with the obligatory restriction $(n_j < n)$ $(j = 1 .. k)$, permitting the sign of equality, excepting trivial cases of the representation. Thus, in case of the ***d–GPDF*** in the representation *(27)* the use of arbitrary global transition functions $\tau^{(n_j)}$ $(n_j \le n)$, that have both dimensionality and the state alphabet, identical with the initial function $\tau^{(n)}$ is permitted. It is obvious that the positive decision of the ***d–PDF*** for any global transition function $\tau^{(n)}$ entails the positive decision the ***d–GPDF*** for the function also, while the converse proposition generally speaking is incorrect. Therefore, the ***d–GPDF*** and the ***d–PDF***, generally speaking, are nonequivalent problems.

Along with the ***d–GPDF*** the question of special presentation of global transition functions in the form of composition of finite number of the more simple functions represents significant interest. Further under a special we shall understand any representation of an arbitrary global transition function $\tau^{(n)}$ in the form of *(27)* under the condition that the initial function $\tau^{(n)}$ and functions $\tau^{(n_j)}$ that make up its decomposition are chosen from a preset class of functions, but some special restrictions having a certain interpretation are imposed on them. In particular, the question of interrelation of properties of the nonconstructability of an arbitrary global transition function $\tau^{(n)}$ and global transition functions $\tau^{(n_j)}$ $(n_j < n)$ entering in its representation *(27)* will absolutely naturally interest us. In this direction the following result takes place [24,80,82].

Theorem 134*. An arbitrary global transition function $\tau^{(n)}$ determined in an arbitrary states alphabet possesses the NCF nonconstructability if and only if at least one global transition function $\tau^{(n_j)}$ $(n_j < n)$ possesses the NCF; if at least one global transition function $\tau^{(n_j)}$ $(n_j < n)$ possesses NCF–1, their composition $\tau^{(n)}=\tau^{(n_1)}\tau^{(n_2)}...\tau^{(n_k)}$ will possess the NCF or/ and NCF–1 nonconstructability. The set of all GTF without NCF will be closed relative to the composition operation whereas the set of all GTF possessing the NCF not. If a GTF $\tau^{(n)}$ in composition $\tau^{(n+m-1)} = \tau^{(n)}\tau^{(m)}$ possesses the MEC pairs of minimal size j then the GTF $\tau^{(n+m-1)}$ will be possess the MEC pairs of the same minimal j size. A composition of the linear global transition functions (GTF) again gives a GTF, i.e. the self–reproducibility property of finite configurations for linear GTF remains in force with respect to their composition.*

In addition, it is necessary to accent attention on transferability of that and a number of results presented below concerning the ***PDF*** onto the ***CA*** models distinct from classical ones. Furthermore, a lot of the results presented below is generalized onto the case of higher dimensionalities.

Along with the mentioned decomposition concept of global transition functions in some cases a certain interest another approach to definition of this concept present too. In [80-84] a number of similar approaches is considered. So, in particular, an approach to definition of this concept is interesting from the standpoint of its influence on questions connected with the nonconstructability problem as a whole. Such approach to the decomposition problem is determfined as follows. The composition of ***2*** global transition functions $\tau 1^{(n)}$, $\tau 2^{(n)}$ is designated as $\tau^{(n)}=\tau 1^{(n)} \oplus \tau 2^{(n)}$ and determined by the following relation for their local transition functions**:**

$$\left(\forall \langle x_1 x_2 ... x_n \rangle\right)\left(\sigma^{(n)}(x_1, x_2, ..., x_n) = \sigma_1^{(n)}(x_1, x_2, ..., x_n) + \sigma_2^{(n)}(x_1, x_2, ..., x_n) \pmod a\right)$$
$$x_j \in A = \{0, 1, 2, ..., a-1\}; \quad j = 1..n$$

The following simple example illustrates this concept of composition**:**

$$\begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \\ x_1x_2x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \sigma_1^{(3)} \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ \sigma_2^{(3)} \end{bmatrix} = \begin{bmatrix} 0+0 \pmod 2 = 0 \\ 1+0 \pmod 2 = 1 \\ 1+1 \pmod 2 = 0 \\ 0+1 \pmod 2 = 1 \\ 1+0 \pmod 2 = 1 \\ 0+1 \pmod 2 = 1 \\ 0+0 \pmod 2 = 0 \\ 1+1 \pmod 2 = 0 \\ \sigma^{(3)} \end{bmatrix}; \quad x_1, x_2, x_3 \in A = \{0, 1\}$$

presenting scheme of composition of local transition functions $\sigma_1^{(3)}$ and $\sigma_2^{(3)}$ with binary alphabet $A=\{0, 1\}$ and neighbourhood index $X=\{0, 1, 2\}$**,** and with discriminating numbers ***105*** and ***53*** accordingly as a result we obtain the local transition function $\sigma^{(3)}$ with discriminating number ***92.*** The global transition functions $\tau 1^{(n)}$, $\tau 2^{(n)}$ and $\tau^{(n)}$, corresponding them will possess the ***NCF*** & ***NCF–1***, ***NCF*** without ***NCF–1***, and ***NCF*** & ***NCF–1*** accordingly. It is easy to see, that an arbitrary global transition function $\tau^{(n)}$ can be represented as a finite ⊕**–**composition of global functions of the same type *(identical states alphabet and neighbourhood index)***;** the set of such ***GTF*** is closed relative to ⊕**–**operation**;** the ⊕**–**composition can be used together with standard composition for creation of more complex constructions, for example**:**

$$\tau^{(n)} = \tau_1^{(n_1)} ... \tau_p^{(n_p)} \oplus ... \oplus \tau_1^{(m_1)} ... \tau_t^{(m_t)}; \qquad n = \Sigma_{j=1}^{j=p} n_j - (p-1) = ... = \Sigma_{j=1}^{j=t} m_j - (t-1)$$

In this direction a number of interesting enough results concerning the nonconstructability problem relative to the above type of composition of global transition functions has been obtained [24,41,82,102,106,286].

7.1. Decomposition of special global transition functions in the classical *CA* models

Before consideration of questions of decomposition of global transition functions of special classes we again shall return to the case of classical ***1–***dimension models for which ***1–PDF*** generally speaking has negative decision, represented in our works [24,80-87,102,107].

Theorem 135. For arbitrary integers $a > 2$ and $n \geq 3$ there is 1–dimension global transition functions $\tau^{(n)}$ preset in an alphabet $A=\{0,1, ..., a-1\}$ for which 1–PDF has the negative decision.

How it was marked earlier, the mentioned result for the first time has been obtained by us on the basis of results on the nonconstructability in classical ***1–CA*** models and then was overproved by more constructive methods on the basis of results of *H. Yamada* and *S. Amoroso* concerning the completeness problem for *polygenic* ***CA*** models in combination with our results on the nonconstructability *(Ch. 3* [9,12–14]*)*. Having received the negative answer to solution of the ***1–PDF*** in general case, we leave aside a lot of the important questions connected to structure of the set of all global transition functions not having of the above presentation *(27)*, with influence on possibility of the solution of the decomposition problem of basis parameters and properties of classical ***1–CA*** models. For research of the marked and other questions on the decomposition problem we need a number of new approaches and methods, the part of which will be considered a little bit below.

First of all, we shall consider a class of all binary ***1-CA*** models for which ***1–PDF*** in accord with theorem ***135*** in general has the negative decision. Within of this class we have researched influence on the decision of this problem of restriction of the problem on the case of all binary global $\tau^{(n)}$ transition functions which do not possess the ***NCF*** nonconstructability, i.e. injective global functions on the set $C(A,1,\phi)$ of finite ***1–***dimensional configurations. Of results on the nonconstructability in classical models ***1–CA*** it is known that a sphere of such ***CA*** models that do not possess ***NCF*** is insignificant and rather quickly decreases with growth of size of neighbourhood template of a ***1–CA.*** In addition, the class of these ***1–CA*** models is closed concerning the composition operation what is a rather essential for the solution of ***1-PDF.*** The following result gives an answer to the earlier put question [9,24,43,82,102,106].

Theorem 136. In the class of all binary injective 1–dimensional global transition functions the 1–PDF in general case has negative decision.

So, narrowing of the class of all ***1–***dimensional binary global transition functions up to own subclass of injective functions keeps negativity of the ***1–PDF*** decision. Along with the ***d–PDF*** of the general kind a rather significant interest represents the question of special representation of global transition functions in the form of composition of finite number of more simple functions. Some of special representations of such type are considered a little bit below.

In connection with negative decision of the ***d–PDF*** in general case there are a lot of interesting concomitantproblems among which naturally to single out the following ones. Let ***M(A,d,SH)*** will be a certain subset of the set of all ***d–***dimensional global transition functions, determined in a finite alphabet ***A,*** and which possess some general property ***SH.*** Hence, the partial problem of decomposition is reduced to the question about an opportunity of presentation of any global transition function out of the set ***M(A,d,SH)*** in the form of some composition *(**27**)* of functions out of the same set ***M(A,d,SH).*** The preset partial problem ***d–PDF*** presents a certain considerable theoretical and applied interest depending on an appropriate choice of the determinative set ***M(A,d,SH)*** *(**d ≥ 1**)* of global transition functions of classical ***CA*** models.

First of all, we shall consider the ***1–PDF*** concerning the known class of all ***1–***dimension linear global transition functions which are defined in section ***3.2.*** Functions of such class possess the general property ***SH*** of the universal reproducibility in the *Moore* sense of finite configurations.

The set ***GW*** of linear ***1–***dimensional global transition functions, whose appropriate local transition functions are defined as follows

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = b_o x_1 + b_n x_n + \sum_{j=2}^{n-1} b_j x_j \pmod a; \quad x_j \in A = \{0,1,2,...,a-1\}$$

$$b_o, b_n \in A \setminus \{0\}; \quad a = p^k, \text{ where } p - a \text{ prime}, \ k - a \text{ positive integer}; \quad j = 1..n$$

composes a commutative subset concerning the composition operation and whose elements possess the property of universal reproducibility of finite configurations in the *Moore* sense. Obviously, for an arbitrary ***n*** and an alphabet ***A=*{*0,1, ..., a–1*}** the number of such models is $a^{n-2}(a-1)^2$. In addition, for any positive integers ***a*** and ***n*** there are global transition functions $\tau^{(n)} \in SL$ for which the decomposition problem has negative decision, i.e. these global transition functions can`t be presented in the form of composition of more simple global transition functions from the set ***SL*** [82]. As a simple example we shall consider linear binary global transition functions $\tau^{(4)} \in SL$ whose local transition functions are defined by the following formulas, namely**:**

$\sigma^{(4)}(x_1,...,x_4) = x_1+x_2+x_3+x_4 \ (mod\ 2)$ $\quad$ $\sigma^{(4)}(x_1,...,x_4) = x_1+x_4 \ (mod\ 2)$

$\sigma^{(4)}(x_1,...,x_4) = x_1+x_2+x_4 \ (mod\ 2)$ $\quad$ $\sigma^{(4)}(x_1,...,x_4) = x_1+x_3+x_4 \ (mod\ 2)$

Obviously, for representation of all global transition functions $\tau^{(4)} \in SL$, taking into account the commutativity of the subset SL, it is expedient to consider the following two compositions of more simple functions only: $\tau^{(2)3}$ and $\tau^{(2)}\tau^{(3)}$, where $\tau^{(2)}, \tau^{(3)} \in SL$ and for GTF $\tau^{(3)}$ two neighbourhood indices $X=\{-1,0,1\}$, $Y=\{-1,1\}$ take place. The following fragment enough visually illustrates the result of such compositions:

$$\tau^{(2)3}: \begin{vmatrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{vmatrix} \to \begin{vmatrix} x_1+x_2+x_2+x_3 \ (mod\ 2) \\ x_2+x_3+x_3+x_4 \ (mod\ 2) \end{vmatrix} \to x_1+x_2+x_3+x_4 \ (mod\ 2)$$

$$\tau^{(2)}\tau^{(3)}: \begin{vmatrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{vmatrix} \to x_1+x_4 \ (mod\ 2); \qquad x_j \in A=\{0,1\};\ j=1..4$$

$$\tau^{(2)}\tau^{(3)}: \begin{vmatrix} x_1+x_2 \ (mod\ 2) \\ x_2+x_3 \ (mod\ 2) \\ x_3+x_4 \ (mod\ 2) \end{vmatrix} \to x_1+x_2+x_3+x_4 \ (mod\ 2); \quad Y=\{-1,1\}$$

Thus, for two of 4 global transition functions $\tau^{(4)} \in SL$ the decomposition problem relative the set SL has negative decision. For these 2 functions with discriminating numbers 23205, 26265 the decomposition problem has the negative decision also as a whole. Similar picture takes place for any set of the global transition functions $\tau^{(n)} \in SL$ concerning an arbitrary integers $a \geq 2$ and $n \geq 3$ [24,41,82,84,102,106].

Class W of classical 1–CA models with a states alphabet $A=\{0,1,\ldots,a-1\}$, neighbourhood index $X=\{0,1,2,\ldots,n-1\}$ and global transition functions $\tau^{(n)}$ determined by the local transition functions as follows

$$\forall(x_1,x_2,...,x_{n-1})(h \neq t \to \sigma^{(n)}(x_1,x_2,...,x_{n-1},h) \neq \sigma^{(n)}(x_1,x_2,...,x_{n-1},t))$$
$$h,t,x_j \in A=\{0,1,2,...,a-1\}; \quad j=1..n-1$$

represents essential enough interest with standpoint of the property of self–reproducibility of finite configurations in the *Moore* sense. Shown that models, excluding trivial ones, of this class possess the property of essential or full self–reproducibility of finite configurations [24,82,102].

Class W is a rather interesting with standpoint of composition of global transition functions. Without loss of generality, a scheme that illustrates composition of two global transition functions $\tau 1^{(n)}$ and $\tau 2^{(p)}$ defined by local transition functions $\sigma 1^{(n)}$ and $\sigma 2^{(p)}$ accordingly will be represented. This scheme shows that composition of two global transition functions from the above class W gives a function of the same class.

$$\sigma 1^{(n)}: \quad x_1x_2x_3...x_nx_{n+1}x_{n+2}...x_{n+p-2}(x_{n+p-1} \equiv 0) \quad \rightarrow \quad y$$
$$\sigma 2^{(p)}: \quad x_1^1x_2^1x_3^1x_n^1...x_p^{11} \quad \rightarrow \quad x_1^2$$
$$\sigma 1^{(n)}: \quad x_1x_2x_3...x_nx_{n+1}x_{n+2}...x_{n+p-2}(x_{n+p-1} \equiv 1) \quad \rightarrow \quad z$$
$$\sigma 2^{(p)}: \quad x_1^1x_2^1x_3^1x_n^1...x_p^{12} \quad \rightarrow \quad x_1^3$$
$$y, z, x_p^{11}, x_p^{12}, x_k^1, x_1^2, x_1^3, x_j \in \{0,1\};\ j = 1..n+p-1;\ k = 1..p; \quad y \neq z \ \&\ x_1^2 \neq x_1^3$$

This scheme presents a result of application of local transition function $\sigma^{(n+p-1)}$ that corresponds to a global transition function determined by the composition of global transition functions $\tau 1^{(n)}\tau 2^{(p)}$ to two tuples of the form $x_1x_2...x_n...x_{n+p-2}0$ and $x_1x_2...x_n...x_{n+p-2}1$; $x_j \in \{0,1\}$, $j=1..n+p-2$. Without loss of generality, this scheme is based on the binary alphabet $A=\{0,1\}$. Whereas for the local transition functions $\sigma 1^{(n)}$ and $\sigma 2^{(p)}$ takes place the following relation:

$$\forall (x_1, x_2, ..., x_{n-1})(\sigma^{(n)}(x_1, x_2, ..., x_{n-1}, 0) \neq \sigma^{(n)}(x_1, x_2, ..., x_{n-1}, 1))$$
$$\forall (y_1, y_2, ..., y_{p-1})(\sigma^{(p)}(y_1, y_2, ..., y_{p-1}, 0) \neq \sigma^{(p)}(y_1, y_2, ..., y_{p-1}, 1))$$
$$x_j, y_k \in A = \{0,1\}; \quad j = 1..n-1; \quad k = 1..p-1$$

it is a rather easily be sure that the local transition function $\sigma^{(n+p-1)}$ is determined by the parallel substitutions as follows, namely:

$$\forall (x_1, x_2, ..., x_{n-1})(\sigma^{(n+p-1)}(x_1, x_2, ..., x_{n+p-1}, 0) \neq \sigma^{(n+p-1)}(x_1, x_2, ..., x_{n+p-1}, 1))$$
$$x_j \in A = \{0,1\}; \quad j = 1..n-1$$

Thus, composition of two global transition functions $\tau 1^{(n)}$ and $\tau 2^{(p)}$ as a result gives the global transition function $\tau^{(n+p-1)}$ of the same class that is interesting enough from many standpoints. And what is more, takes place the following proposal, namely:

The aforesaid set W of global transition functions is noncommutative and closed concerning the composition operation. This proposition is valid for 1-dimensional models with an alphabet different from binary and neighbourhood index of an arbitrary form. Generally, composition operation on a set of global transition functions is noncommutative.

Indeed, as a rather simple example the composition of two *1*-dimension *CA* models with binary alphabet *A* and neighbourhood index $X=\{0,1,2\}$ whose local transition functions have discriminating numbers *86* and *90* is illustrated on the base of the **ComposeGTF** procedure. In the example compositions of the above global transition functions are evaluated by means of the procedure **ComposeGTF** that is considered in [49]. Then, tuples $<x1...x5>$ $(xj \in \{0,1\};\ j=1..5)$ on which the resultant functions have different values are given. From the result follows, that the composition operation concerning the above *2* global functions is noncommutative.

So, certain sets of global functions that are rather interesting from many standpoints are closed concerning the composition operation, however are noncommutative.

The basic results concerning the decomposition problem for classes of global transition functions, including functions of a set ***M(A,d,SH)*** will be represented below. Here, we shall consider some deeper questions of the decomposition of global transition functions for one interesting subset of the set ***M(A,d,SH)*** **–** the linear ***1–***dimensional global transition functions $\tau^{(n)}$**,** whose local functions $\sigma^{(n)}$ are determined as follows**:**

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = (x_1 + x_n + \sum_{2}^{n-1} b_k x_k) \pmod a;$$

$$a = p^h;\ p - a\ prime,\ h - an\ integer; \quad b_k \in \{0,1\};\ x_1, x_n, x_k \in A \quad (k = 2..n-1)$$

This subset of the set ***M(A,d,SH)*** we shall denote as ***M*(A,1,SH).*** In this connection we shall consider the closure of the set ***M*(A,1,SH)*** relative to the composition operation**;** in addition, it is expedient to present this set in the shape of union of two noncrossing subsets ***M*$_1$(A,1,SH)*** and ***M*$_2$(A,1,SH)*** *(henceforward for brevity as* ***M*$_1$*** *and* ***M*$_2$****)* accordingly of the subsets of global transition functions $\tau^{(n)}$ *(****n*** **≥ 2***)* with the connected and disconnected neighbourhood templates of size ***n;*** i.e. ***M*$_1$*** **∩** ***M*$_2$*** **= Ø** and ***M*(A,1,SH)=M*$_1$*** **∪** ***M*$_2$*** where **Ø –** the empty set. We shall designate the global transition functions out of these subsets accordingly as $\tau_1^{(n)}$ and $\tau_2^{(n)}$**.** At the made assumptions the following base result having a lot of interesting enough appendices takes place [24,43,82,102,106].

<u>Theorem 137</u>. The subset M*$_1$ **∈** ***M*(A,1,SH) is nonclosed concerning the composition operation of global transition functions*** $\tau^{(m)}$ ***that form it. A global transition function*** $\tau_1^{(n)}$ **∈** ***M*$_1$ can't be represented in the form of a composition of two more simple functions from the set M*$_1$.***

In spite of result of this theorem, it is possible to show that expansion of a set of global transition functions that are admitted as elements of the representation *(****27****)* for an arbitrary function from the set ***M*$_1$,*** onto the set ***M*$_2$*** allows to decide the ***1–PDF*** positively for global transition $\tau^{(n)}$ functions from the set ***M*$_1$*** [42]**,** namely**:**

<u>Theorem 138</u>. An arbitrary global transition function $\tau_1^{(2k)}$ **∈** ***M*$_1$ can be represented in the following form*** $\tau_1^{(2k)} = \tau_1^{(k)} \tau_2^{(k+1)} = \tau_2^{(k+1)} \tau_1^{(k)}$ ***where*** $\tau_1^{(k)}$ **∈** ***M*$_1$ and*** $\tau_2^{(k+1)}$ **∈** ***M*$_2$ with neighbourhood index X$_2$={0,k} (k=2, ...).***

An arbitrary global transition function $\tau_1^{[p(2k+1)]} \in M^*_1$ ***can be presented in the following form*** $\tau_1^{[p(2k+1)]} = \tau_2^{[p(2k+1)-2]} \tau_1^{(p)} = \tau_1^{(p)} \tau_2^{[p(2k+1)-2]}$ ***(p=3, 5,7,9, ...; k=1,2,3,4, ...). An arbitrary global transition function*** $\tau_1^{(n)} \in M^*_1$ ***on the assumption that n=p*q can be represented in the following form*** $\tau_1^{(n)} = \tau_2^{(n-p+1)} \tau_1^{(p)} = \tau_2^{(n-q+1)} \tau_1^{(q)}$ ***where functions of the set*** M^*_2 ***which participate in representation of a global transition function*** $\tau_1^{(n)}$ ***have symmetrical neighbourhood indices.***

Of this result follows, an arbitrary global transition function $\tau_1^{(n)} \in M^*_1$ on the assumption of a composite number ***n*** can be represented in the form of a composition of two more simple global functions from the set $M^*(A,1,SH) = M^*_1 \cup M^*_2$. For the final decision of this question with the set M^*_1 quite enough to consider a case of global transition function $\tau_1^{(n)}$ on the assumption of a prime integer ***n.*** So, by not going into numerous and quite laborious details of the analysis of global transition functions $\tau_1^{(n)}$ of such type, we shall present a rather interesting result expressed by the following theorem [24,41,82,102,106].

Theorem 139*. A global transition function* $\tau_1^{(n)} \in M^*_1$ ***can be represented in the form of the composition of finite number of more simple global transition functions from the set*** $M^*(A,1,SH)$ ***if n is a composite number; for example,*** $\tau_1^{(n)} = \tau_2^{(p)} \tau_3^{(n-p+1)} = \tau_4^{(q)} \tau_5^{(n-q+1)}$, ***where*** $\tau_2^{(p)}, \tau_4^{(q)} \in M^*_1$; $\tau_3^{(n-p+1)}, \tau_5^{(n-q+1)} \in M^*_2$ $(n \geq 4)$ ***under the condition n=p*q and connected and disconnected neighbourhood template of size p,q and n–p+1, n–q+1 along with neighbourhood indices*** $X_2=\{0,1,...,p-1\}$, $X_3=\{kp \mid k=0,1,...,q-1\}$ ***and*** $X_4 = \{0,1, ..., q-1\}$, $X_5 = \{kq \mid k=0,1, ..., p-1\}$ ***accordingly.***

So, the decomposition problem of an arbitrary global transition function $\tau_1^{(n)} \in M^*_1$ concerning the set $M^*(A,1,SH)$ is *solvable* and in the case of its positive decision there are satisfactory constructive solving algorithms. Entirely other picture there is in the case of global transition functions from the set M^*_2. A great many global transition functions $\tau_2^{(n)} \in M^*_2$ can be presented in the form of *composition* of finite number of more simple functions from the set $M^*(A,1,SH)$ irrespective of nature of the number determining the size of neighborhood template of the researched global transition function. Although it is necessary to bear in mind that with a *neighborhood* template of ***n*** size there are $2^{n-2}-1$ various global transition functions $\tau_2^{(n)} \in M^*_2$ and by far not all of them have presentation *(27)*. In

particular, from **7** *binary* global transition functions $\tau_2^{(5)} \in M^*_2$ the above presentation from finite number of more simple functions from the set ***M*(B,1,SH)*** *five* functions have whereas of ***15*** binary functions $\tau_2^{(6)} \in M^*_2$ only six functions have such representation, etc. Along with that for an integer $n \geq 5$ and an alphabet A there are global functions $\tau_2^{(n)} \in M^*_2$ that have the representation *(27)* with appropriate restrictions [24,102,106].

Theorem 140. For an arbitrary integer $n \geq 5$ and an arbitrary alphabet A there are at least n–4 global transition functions $\tau_2^{(n)} \in M^*_2$ that can be represented in the form of a composition $\tau_2^{(n)} = \tau_2^{(n-1)}\tau_1^{(2)}$ of two global transition functions. A 1-dimensional binary global transition function $\tau^{(n)}$ whose local transition function is determined as $\sigma^{(n)}(x_1,...,x_n)=\sum_k x_k$ (mod 2) with the connected neighbourhood template can be represented in the form of composition $\tau^{(n)}=\tau^{(2)(n-1)}$ where $\tau^{(2)}$ is the simplest binary linear function and $n = 2^k$ ($k = 2,3,4, ...$).

However, in spite of this result, the analysis of large enough quantity of global functions of the set M^*_2 along with some theoretical results have allowed to confirm the following proposal [24,41-43,82,102,106]**:**

For an integer $n \geq 4$ and a states alphabet A there are global transition functions of the set M^*_2 which can't be represented as a composition of finite number of more simple functions from the set M*(A, 1, SH).

So, the decomposition problem of global transition functions relative to the set ***M*(A,1,SH)*** in general has the negative decision. Thus, research of the decomposition problem of global transition functions, it is rather interesting to generalize to cases of the higher dimensionalities and to observe change of the results presented above depending on increase of dimensionality of global transition functions from the considered class ***M*(A,d,SH).*** As distinct from the ***M*(A,1,SH)*** the set ***M*(B,1,SH)*** of one**–**dimension binary linear global transition functions makes up a subclass of the class of all ***1–***dimensional linear global functions which is closed concerning the composition, forming the ***subset*** concerning the preset operation. Therefore, the above results can be entirely naturally expand onto the set ***M*(B,1,SH),*** presenting in a lot of cases applied, theoretical and independent interest.

On the example of research of the ***1-PDF*** relative to the set ***M*(A,1,SH)*** of linear global transition functions a number of rather deep properties of global transition functions concerning the preset problem has been

sudied, namely**:** influence of type of neighborhood template *(connected or disconnected)***,** number defining its size *(composite or prime)***,** and type of functions composing presentation *(***27***)*. Within the further development of this problematics it is possible to extend similar study on the general case of the set ***M(A,d,SH)*** of linear functions, and also to determine the sets ***MW(A,d,G)*** of global transition functions with a certain general ***G–*** property that are interesting from the theoretical and applied points of view, in particular, *symmetry*. One more interesting subclass of the class of all classical ***CA*** models relative to the decomposition problem of their global transition functions is presented by ***CA*** models with refractority.

The above ***CA*** models have a number of biomedical interpretations, and recently they begin to be used for the problems of recognition of images, research of properties and topology of digital figures, etc. In our works a number of interesting theoretical and empirical results on dynamics of models ***d–CAR(r,p)*** {*these models have been defined for example in* [5,24,84]} can be found [24,82]. Below, these ***CA*** models are considered relative to the decomposition problem of global transition functions. A ***d–CAR(r,p)*** model is defined by such important parameters as *excitation threshold (**p**)* and *depth of refractority (**r**)*.

On the basis of these parameters and differentiation of a states alphabet ***A*** there is a rather good opportunity of researches of peculiar dynamic properties of such models consisting in excitations spread in medium of elementary automata of the models. The following result presents a solution of the decomposition problem for the case of global transition functions concerning the class of all ***d–CAR(r, p)*** models [5,24,42,82-87].

<u>Theorem 141</u>. The decomposition problem in the class of all d–CAR(r, p) (d ≥ 1) models with refractority has the negative decision.

This result speaks about the negative decision of the ***d–PDF*** in the class of all global transition functions with refractority without restrictions on excitation threshold and depth of refractority of the models**;** i.e. the class of such global transition functions is considered, whose local $\sigma^{(n)}$ functions are characterized by a switching of an elementary automaton being in an excited state into a refractority state. Now, we shall assume that certain global transition function with refractority has an arbitrary excitation threshold $1 \le p \le n-1$ and refractority depth $r \ge 1$**.** On account of the made assumption the following a rather important result follows.

<u>Theorem 152</u>. Each global transition function from the class W(r) of all 1–dimensional global transition functions with refractority depth r ≥ 1

can`t be presented in the form of composition of finite number of global functions from the same set W(r); in addition, W(r) is the set of global transition functions with refractority which are isolated concerning the composition operation.

Hence, the result of theorem ***142*** presents the exhaustive decision of the decomposition problem in the class ***W(r)*** of all one–dimensional global transition functions with refractority of ***r***–depth. Moreover, each global function from the class ***W(r)*** is located at own complexity level which is determined by its excitation threshold ***p*** and neighbourhood index. So, research of many important dynamical properties of distribution of the excited states in medium even of the ***1-CAR(r,p)*** models has much more individual character depending on the base characteristics of models of such type. The result of theorems ***141, 142*** is generalized to ***d–CAR(r, p) (d>1)*** models with neighbourhood index of a rather general kind. Many other interesting special representations of global transition functions in the form of composition of finite number of the more simple functions is considered in our works and the subsequent sections of the book [102].

7.2. Some approaches to solution of the general decomposition problem of global transition functions

In the previous section certain questions of the decomposition problem concerning the certain special classes of global transition functions have been considered, along with results of solution of the ***1–PDF*** have been represented on the basis of approach utilizing the results of *S. Amoroso* and *H. Yamada* on the *completeness* problem together with our results on the nonconstructability problem in classical ***CA*** models. In this section we shall present results of study of the general decomposition problem on the basis of new perspective approaches basing on use of results and methods of the theory of functions of algebra of logic, ***a***–valued logics, and also on use of the formal apparatus of the group theory, algebras and semigroups along with an essential generalization of a method of solution of the ***d–PDF*** on the basis of study on the nonconstructability problem in classical ***CA*** models. Along with that we will show a rather close connection of the decomposition problem of global transition $\tau^{(n)}$ functions with the complexity problem of finite configurations in the classical ***CA*** models and with the completeness problem for polygenic ***CA*** models. Approaches and methods of solution of the ***d–PDF*** that are here discussed represent the certain interest at researches of some other questions of the ***CA*** problematics and a lot of its applied aspects.

Above all, for solution of the decomposition problem the approach that is based on using of the *Shannon* function is offered**;** this approach was entered for an estimation of the complexity of realization of functions of the algebra of logic. It is known that a function of the algebra of logic is realized by the corresponding logic circuit consisting of certain basic logic elements. As the given basic elements, the elements realizing the following well**–**known logical operations are chosen, namely**:** negation**,** conjunction and disjunction.

For description of the complexity of logic circuits we act as follows. Let, to each logic circuit ***M*** that realizes a certain function of algebra of logic, a non**–**negative number ***L(M)*** is attributed **–** a complexity of the circuit, while ***L(σ)*** **–** a minimum of complexities of all circuits ***M*** which realize this logic **σ** function. In addition let $L(n)=max_{\sigma} L(\sigma)$ and the maximum is taken for all local transition functions $\sigma^{(n)}$ which depend on ***n*** logical variables. If maximum is achieved, then ***L(n)*** is such minimal number, that by means of circuits of complexity no more than ***L(n)*** we can realize an arbitrary function $\sigma^{(n)}$ of the algebra of logic, in other words define the corresponding local transition function of a certain ***d–CA*** model.

Function ***L(n)*** for the first time has been introduced by *C. Shannon* for contact networks and he has received the first significant results on it. Subsequently, such function ***L(n)*** has received the name of the author and began to be rather widely used for an estimation of complexity of function circuits. The research of behaviour of functions ***L(σ)*** and ***L(n)*** is the important enough problem**;** in addition, both aspects of research appear closely interconnected among themselves [5,24,43,82,102,106].

At present, the upper bounds which are asymptotically equal to the low bounds are received only for rather few types of functional circuits. In addition, under type of a circuit is understood, above all, a certain set of base functional elements of which is designed the circuit implementing a demanded function $\sigma^{(n)}$ of algebra of logic. Inasmuch as our problem is reduced to a *comparative* analysis of local transition functions $\sigma^{(n)}$ then it is quite possible to confine oneself to a concrete set of base functional elements, i.e. concrete type of functional circuits on the basis of which is being done the corresponding comparative analysis. In this connection the circuits from functional elements can be chosen in the basis ***G=*{*And, Or, Not*}**;** for such circuits the asymptotical expressions for the *Shannon* function under the condition of asymptotical equality of the upper and lower bounds of their complexity have been found.

Let's name the number of elements of a functional circuit M realizing a logic function $\sigma^{(n)}$, its complexity and to denote as $S = L(\sigma^{(n)})$. Let now $L(\sigma^{(n)})$ is the least complexity of the circuits realizing an arbitrary local transition function $\sigma^{(n)}$ and $L(n) = max_{\sigma} L(\sigma^{(n)})$ where the maximum is taken for all functions $\sigma^{(n)}$. In other words, a value $L(n)$ – the minimal number of elements in a functional circuit, sufficient for realization of an arbitrary boolean function from n variables. In addition, at the proof of our result the *O. Lupanov* result fundamental in the theory of boolean functions is enough substantially used, namely [13,24,82,102,106]**:**

There is asymptotic equality $L(n) = 2^n/n$***; in addition, for any value*** $\delta > 0$ ***the share of logic functions*** $\sigma^{(n)}$ ***for which the relation*** $L(\sigma^{(n)}) \le (1-\delta)*2^n/n$ ***is valid will tend to zero with growth of value n.***

In spite of all possible generalizations of this result, it is quite enough to restrict oneself to it for the further considerations. It is obviously that an arbitrary local transition function $\sigma^{(n)}$ preset in the binary alphabet can be considered as a certain function of algebra of logic. As a composition of global transition functions $\tau^{(n_j)}$ is uniquely determined by a special superposition of the corresponding local transition functions $\sigma^{(n_j)}$ in the further this question is reduced to superposition of local functions $\sigma^{(n)}$ in binary alphabet. For that for an integer n a certain local function $\sigma^{(n)}$ with greatest possible complexity $L(n)$ is chosen. Then, on the basis of a sought composition of finite number of the binary global functions $\tau^{(n_j)}$ $(n_j < n; j = 1..k)$ for an arbitrary function $\tau^{(n_j)}$ an appropriate Σ–circuit of superposition of local transition functions is defined and its complexity $L(\Sigma)$ is estimated, which is compared next with complexity $L(n)$ of the initial local transition function $\sigma^{(n)}$. The comparative analysis done after that allows to formulate the following basic result [42,43,82,102,106].

Theorem 143*. For any preset finite base set* G_f ***of 1–dimensional binary global transition functions*** $\tau^{(n)}$ ***there is such integer*** $n_o > 0$ ***that for each integer*** $n \ge n_o$ ***there is at least one binary global transition function for which 1–PDF has the negative decision.***

It is necessary to note**,** the similar result for binary case is received by us on basis of results on universal reproducibility of finite configurations in the *Moore* sense in classical ***1–CA*** models**.** Thus, result of theorem ***143*** has been received on the assumption that a local transition function $\sigma^{(n)}$ has an arbitrary kind, i.e. the obligatory determinative condition $\sigma^{(n)}(x, ..., x) = x$ is not required, allowing to spread the result of solution of the

d–PDF onto case of the nonstable ***d–CA*** models. Moreover, at proof of the theorem the concept of dimensionality did not participate, but only neighbourhood template, represented by its neighbourhood index. In addition, you need to keep in mind, that in the case of dimension $d \geq 1$ an arbitrary neighbourhood index $X=\{x_0,x_1, ..., x_{n-1}\}$ where is ***n–***tuple of ***d–***dimensional points determining coordinates of elementary automata of neighbourhood template of a ***d–CA*** $(d \geq 1)$ model. This remark play an essential enough part at generalization of results onto case of more high dimension when there is an opportunity of a special numbering of the elementary automata composing neighbourhood template of a model.

Therefore, it is rather simple to make sure that the result of theorem ***143*** is valid also for a global transition function in an arbitrary binary ***d–CA*** model. Once again it is necessary to note that to a composition of global transition functions the corresponding superposition of local transition functions corresponds that is a rather essential narrowing of the general superposition concept of functions. This fact causes the impossibility of direct carry–over of results concerning ***a–***valued logics $(a>2)$ to the case of local transition functions determined in a states alphabet ***A*** of general kind though methods of ***a–***valued logics seem promising for research of a lot of properties of dynamics of classical ***CA*** models. A rather detailed discussion of this aspect can be found, for example, in [24,42,102,106].

Inasmuch as in the general case of alphabet of a classical model it is not possible to directly generalize the above results relative to binary global transition functions, we are compelled to address above all to ***a–***valued logic $(a>2)$ relative to which is necessary to note, that at solution of a lot of its problems we collide with large enough complexities and especial circumstances [24]. So, in an ***a–***valued logic the research of an arbitrary system of functions relative to its completeness is connected to the large technical difficulties. Whilst proofs of completeness of concrete systems in a set of functions of an ***a–***valued logic are done as a rule by means of method of their reduction to deliberately complete systems of functions what in many cases meets rather essential difficulties [24,82,102,106].

Let's consider now a local transition function $\sigma^{(n)}$ defined in an alphabet $A=\{0,1,2,..., a-1\}$ $(a>2)$ as a function of ***a–***valued logic. Let's assume, that there is such finite base set ***G*** of local transition functions that each local transition function $\sigma^{(n)} \notin G$ can be presented in the form of finite number of superpositions of local transition functions from the set ***G.*** Using now terminology and designations of work [40] and designating the set of all functions of ***a-***valued logic through $L(a)$ we simply conclude that *closure*

of the set G coincides with $L(a)$, i.e. $[G]\equiv L(a)$. It is obvious that the set G is complete system in $L(a)$. But inasmuch as the set G is finite under the assumption, then we can always choose such *minimal* subset $G^*\subseteq G$ that the set G^* is complete system in $L(a)$, while any its subset will not be by a complete system in $L(a)$. So, the assumption about existence for the set $L(a)$ of a finite basic subset G^* is equivalent to existence of a finite basis in $L(a)$. Thus, the decomposition problem of global transition functions, determined in a finite states alphabet A, can be reduced to the problem of functional completeness in a–valued logic whose study is essentially connected to so–called *precomplete classes* [40]. Questions of research on a–valued logics are enough entirely presented for example in [24,102].

***Definition 27**. A class S of functions belonging to a closed class R of the set L(a), we shall name precomplete in R if S is incomplete system in R, however a supplement to it of an arbitrary function $f\in R\setminus S$ transforms S in a complete system of functions in R.*

Of the known results on a–valued logics it is easy to make sure that the number of precomplete classes in $L(a)$ is finite and does not exceed 2^{a^a}. But it is possible to show, it contradicts the condition what in case of a fixed integer a in a–valued logic always will exist a basis consisting of any but finite number of functions, including the suggested basis G^* in class of functions $L(a)$ [5,40]. Hence, in the above class of functions $L(a)$ can't exist a finite basis G^*. We need only to estimate influence of this fact upon the question of existence of a finite basis in a narrower class $L(a, 0)$ of functions of a–valued logic that satisfy the stability condition $\sigma^{(n)}(0,0, ..., 0)=0$, i.e. determinative condition of classical structures. As a result of the done estimation, absence of a finite basis in the class $L(a,0)$ of functions has been proved, that allows to formulate the result being analogue of the theorem **143** for case of functions of an a–valued logic and representing the certain theoretical interest [24,40,82,102,106].

***Theorem 144**. Class L(a,0) of local transition functions $\sigma^{(n)}$ on condition $n\geq 2$ and $a>2$ does not possess any finite basis.*

On the basis of this result similarly the *binary* case it is easy to obtain the negative decision of the decomposition problem for a global transition function $\tau^{(n)}$ as a whole [24,40-43,82,102,106].

***Theorem 145**. Among all d–CA ($d\geq 1$) models determined in an arbitrary finite alphabet A and with an arbitrary neighbourhood indixes there is an infinite set of global transition functions for which the d–PDF has the negative decision.*

Similar result can be easily received on the basis of the above results on the complexity problem of finite configurations in classical ***d–CA (d≥1)*** models [24]. The impossibility of positive decision of the ***d–PDF*** for an arbitrary global transition function allows to quite naturally introduce the complexity concept for global transition functions similarly to case of finite configurations in classical ***d–CA (d ≥ 1)*** models. Thus, from the analysis of the complexity concepts of finite configurations and global transition functions in classical ***CA*** models ensues **–** the impossibility of existence of certain finite basic sets for finite configurations and global transition functions underlies accordingly.

Of the told with all clarity it is possible to draw a conclusion about an opportunity of fruitful use in research in this direction of apparatus and methods developed in the algebra of logic and ***a–***valued logics together with results on the complexity problem of finite configurations in the classical ***d–CA (d ≥ 1)*** models. Moreover, for research of the problem of decomposition of global transition functions certain algebraic methods also can be used [24,43,82,102,106].

It is known that the global transition functions which realize a mapping of configurations out of the set ***C(A, d)*** onto themselves form a certain semigroup concerning the composition operation**;** let ***L(a,d)*** denotes the *semigroup* of all such ***d-***dimension global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$**.** It is possible to show that ***L(a,d)*** is *noncommutative* semigroup with the *group identity (for definiteness defined by* $\sigma^{(2)}(x,y) = x$*)* that leaves any ***LTF*** $\sigma^{(n)}$ by invariable within leading variables. Hence, study of properties of composition of global transition functions $\tau^{(n)}$ in many cases can be reduced to investigation of the corresponding properties of appropriate semigroup ***L(a,d).*** Therefore, for the further we shall define a number of necessary algebraical concepts.

We shall call a subgroup ***G*** of certain semigroup ***S*** with identity by the *maximal subgroup* of the semigroup ***S*** if it is strictly not contained in any other subgroup of this semigroup. Existence of the maximal subgroups for the first time has been proved by *J. Schwarz* for periodic semigroup along with *Wallace* and *Kimura* for an arbitrary semigroup. Let's present one more example of existence of maximal subgroups in semigroups, defined by global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ in classical ***d–CA (d≥1)*** models. We shall show that the semigroup ***L(a,d)*** contains one maximal group ***G,*** where under the maximal group a group ***G,*** contained in the semigroup ***L(a,d),*** is understood, which cannot be expanded by means of addition to the ***G*** of new elements out of the set $L(a,d) \setminus G$**.** With one**–**

to–one global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ the problem of research of properties of classical ***d–CA*** models by means of algebraical methods is enough closely connected.

It is known that the set of all mappings $\tau^{(n)}: C(A,d) \to C(A,d)$, generally speaking, does not meet the group axioms, rather, the axiom about an inverse element, even on the assumption of exception of global parallel mappings $\tau^{(n)}$ possessing the ***NCF, NCF–1*** & ***NCF–3*** nonconstructability. However, considering a set ***G(d)*** of all ***d***–dimensional global transition functions whose corresponding global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ are one–to–one, it is possible to show, that the set ***G(d)*** forms a group concerning the composition operation [5,40]. This result allows to apply the group methods for investigation of dynamics of classical ***d–CA*** $(d \geq 1)$ models, i.e. to reduce study of a number of properties of such classical models to research of the appropriate properties of the group ***G(d)***, and also of subgroups making up it. So, in particular, from the definition of the set ***G(d)*** $(d \geq 1)$ it is easy to receive and maximality of the group ***G(d)*** which belongs to the semigroup ***L(a,d)*** of all global parallel mappings $\tau^{(n)}: C(A,d) \to C(A,d)$.

Under the *decomposition* of a semigroup ***S*** is understood a possibility of its representation in the form of union of noncrossing sub–semigroups S_k $(k=1,2,3, \ldots)$. In order that such decomposition represents the certain significance for research of its structure and determinative properties, it is necessary in order the sub–semigroups S_k were qua the more special semigroups than ***S***, particularly, by the simple semigroups or groups. In this respect the semigroup ***L(a,d)*** of mappings decomposes into *union* of noncrossing semigroup ***L*(a,d)*** and maximal group ***G(d)***, namely: ***there are the following determinative relations, namely:***

$$L(a,d) = L^*(a,d) \cup G(d) \quad \textit{and} \quad L^*(a,d) \cap G(d) = E$$

where E – an unit group consisting only of one unit element – identity of the semigroup. The solution of very many questions concerning the opportunities of the semigroup ***L(a,d)*** can be reduced to the solution of the corresponding questions for the ***L*(a, d)*** semigroup or ***G(d)*** group, what and has been made [24,40-43,82,102,106].

Let each global transition function $\tau^{(n)} \in L(a,d)$ can be represented in the form of a composition of finite number of more simple functions from a certain finite set $G_f \subset L(a, d)$. In such case ***L(a, d)*** will be as a semigroup with the finite number of generators. Indeed, the set $G_f \subset L(a,d)$ will be

as a system of *generators* for the semigroup $L(a,d)$ if only any element of the set can be presented by not less than by one manner in the form of a composition of finite number of degrees of elements from the set G_f. But inasmuch as according to the above assumption the set G_f is finite then out of it our previous conclusion easily follows.

A system of generators for the set G_f is named as irreducible one if any true subsystem for it is not a system of generators for the semigroup $L(a,d)$. Further under the set $G_f \subset L(a,d)$ we shall understand a certain *irreducible* system of generators and name its as a *basis* of the semigroup $L(a,d)$. The above consideration, in full measure can be ascribed to the group $G(d) \subset L(a, d)$. We shall call a subgroup $G^*(d)$ of the group $G(d)$ closed, if composition g_1g_2 of its any 2 elements $g_1, g_2 \in G(d) \setminus G^*(d)$ does not belong to the subgroup $G^*(d)$. In the further, we shall need a lot of rather simple results represented in our works [24,40-43,82,102,106].

First of all, if the semigroup $L(a,d)$ have a finite basis G_f then maximal group $G(d) \subset L(a,d)$ also has such finite basis G^* that there are relations $G_f = G^* \cup G^{\#}$ and $G^* \cap G^{\#} = \varnothing$ where $G^{\#}$ – a finite basis of the semigroup $L(a,d) \setminus G(d)$. In addition, if the group $G(d)$ have not a finite basis, then the semigroup $L(a,d)$ will not have a finite basis too. If the group $G(d)$ contains n of the closed subgroups, then the number N of elements of basis G_f of the group not less, than n ($N \geq n$). Of this result it is easy to receive the important consequence: ***If the group G(d) contains infinite number of the closed subgroups, then it can`t have a finite basis.*** These results are carried over to case of arbitrary algebraic semigroups too. In further, without loss of generality we shall identify the global transition functions $\tau^{(n)}$ of classical *d–CA* models with the corresponding parallel global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$.

Let's consider now a little bit more in details one representation of the semigroup $L(a,d)$ according to our results on the nonconstructability in classical *d–CA* ($d \geq 1$) models (*Ch. 2*). It is well–known that the set of all parallel global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ can be represented in the form of union of seven noncrossing subsets, that possess the next basic determinative characteristics concerning the component parallel global mappings $\tau^{(n)}$ {*global transition functions* $\tau^{(n)}$ – *GTF*}, namely:

G_1 : GTF $\tau^{(n)}$ possess all four types of the NCF-1, NCF, NCF-2 and NCF-3 nonconstructability;

G_2 : GTF $\tau^{(n)}$ possess the NCF (NCF–3) and NCF–1 nonconstructability without the NCF–2 nonconstructability;

G_3 : GTF $\tau^{(n)}$ possess the NCF (NCF–3) and NCF–2 nonconstructability without the NCF–1 nonconstructability;

G_4 : GTF $\tau^{(n)}$ possess the NCF–2 nonconstructability only; in addition the global mappings determined by these GTF are not one–to–one;

G_5 : GTF $\tau^{(n)}$ possess the NCF–1 nonconstructability only;

G_6 : GTF $\tau^{(n)}$ possess the NCF (NCF–3) nonconstructability only;

$G_7 \subset G_6$: GTF $\tau^{(n)}$ determine one-to-one parallel global mappings of the following kind, namely: $\tau^{(n)}: C(A,d) \to C(A,d)$; $(d \geq 1)$.

It is possible to make sure, concerning the composition operation the sets G_k *(k=1..6)* make up the noncommutative semigroups while the set G_7 makes up the group [40]**.** Thus, the semigroup ***L(a, d)*** of all parallel global mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ in classical ***d–CA*** $(d \geq 1)$ models can be represented in the form of union of finite number of noncrossing semigroups and the group, i.e. $L(a, d) = \cup_k G_k$ $(k = 1..7)$. The analysis of structures of semigroups G_j $(j = 1..6)$ and the G_7 group has allowed to formulate the following rather interesting primal result concerning the decomposition operation of the semigroup ***L(a,d)*** of parallel mappings $\tau^{(n)}: C(A,d) \to C(A,d)$ for classical ***d–CA*** $(d \geq 1)$ models.

<u>Theorem 146</u>. A semigroup L(a, d) of all parallel global mappings $\tau^{(n)}$: $C(A,d) \to C(A,d)$ determined by the classical d–CA $(d \geq 1)$ models can be presented in the form of an union of six noncrossing sub–semigroups G_k (k=1..6), which have not finite systems of generators, and one maximal group G(d). Sets G_h (h = 4..6) relative to the semigroup L(a,d)\G(d) are isolated sub–semigroups.

Concerning the group ***G(d)*** one rather essential remark it is necessary to make. Analysis of a number of ***1***–dimensional binary parallel mappings $\tau^{(n)}: C(B,1) \to C(B,1)$ and, above all of the mappings composing the sub–semigroup G_4**,** has allowed to express the assumption that ***G(d)*** is the *unit group***,** i.e. will consist only of *identical* parallel global mappings. In addition, the further research have shown that the question of structure of the group ***G(d)*** remains to some degree open.

The undertaken more detailed research of binary classical ***1–CA*** models for detection of one–to–one mappings $\tau^{(n)}: C(B,1) \to C(B,1)$ which differ

from the identical ones has appeared an quite successful. The following theorem represents the best result received in this direction, presenting a certain theoretical interest [24,82-87]. Theorem ***147*** represents an quite certain interest for formal research of ***1–***dimendion classical ***CA*** models.

Theorem 147*. A semigroup L(a, 1) of all 1–dimensional parallel global mappings* $\tau^{(n)}$*: C(A,1)* $\rightarrow$ *C(A,1) (a* $\geq$ *3) determined by means of classical 1–CA models can be represented in the form of union of 6 pairwise non–equivalent noncrossing sub–semigroups* G_k *(k=1..6) that have not finite systems of generators and one maximal group G(a) that is union of the subgroup T* of all identical mappings* $\tau^{(n)}{}_o$ *(n* $\geq$ *2) with a finite system P(a,2) of generators together with the next relation* $\tau^{(n)(a-1)!} = \tau^{(2)}{}_o$ *and, perhaps, of a subgroup of biunique mappings that are distinct from the above–mentioned ones.*

Results of theorems ***146*** and ***147*** speak about necessity of continuation of research in this direction, taking into account variety of possibilities already for one–dimensional binary case. At present, our researches in this direction quite definitely say in favour of this supposition. So, the following result illustrates variety of possibilities already for the case of binary classical ***1–CA*** models [24,40-43,82,102,106].

Theorem 148*. For an arbitrary integer n*$\geq$*3 there is at least* 2^{n-1}*–n binary 1-dimension global transition functions* $\tau^{(n)}$ *each of which possesses the following properties simultaneously, namely:*

♦ *for a global transition function the nonconstructability such as NCF, NCF–1 and NCF–3 is absent in the presence of the nonconstructability of type NCF–2;*

♦ *an arbitrary configuration c*$\in$*C(B,1,*ϕ*) for a similar global transition function is periodical; there are periodical configurations with minimal p–period, whose size is defined by the relation (|c|+n–2)/(n–1) < p =* 2^k*;*

♦ *global parallel mappings* $\tau^{(n)}$*: C(B,1,*∞*)* $\rightarrow$ *C(B,1,*∞*) determined by such global transition functions (GTF) are not one–to–one;*

♦ *for these GTF* $\tau^{(n)}$ *the 1–PDF has the negative decision.*

This result is essential generalization of certain lemmas from work [40], however it does not give complete solution of the question concerning the structure even of the group ***G(1)*** that participates in the mentioned presentation of the semigroup ***L(a,1)*** of ***1–***dimension global mappings. Along with it, this result once again illustrates all variety and riches of forms of behaviour of finite configurations already for case of classical

binary ***1–CA*** models. In addition, from the results of theorems ***146–148*** follows, that the group ***G(d)*** in representation of the semigroup ***L(a, d)*** should contain nontrivial identical one–to–one global mappings while for each of six sub–semigroups G_j ***(j = 1..6)***, determined by presentation ***L(a,d)***, the ***d–PDF*** generally speaking will have the negative decision.

In closing of this section on basis of the concept of the *infinite mutually–erasable configurations* ***(∞–MEC)*** we shall define one more approach to a solution of the decomposition problem of global transition functions in the classical ***CA*** models. In this connection the sub–semigroup G_4 of ***1–***dimension global parallel mappings $\tau^{(n)}$***: C(A,1) → C(A,1)*** were studied enough in detail**;** for the G_4 a number of interesting enough dynamical properties has been received. In further, we need certain new concepts and definitions [24,40-43,82,102,106].

<u>Definition 28</u>. Two configurations $c_1, c_2 \in C(A,d,\infty)$ are called a pair of infinite mutually–erasable configurations (∞–MEC) if and only if for them there is the following relation $c_1\tau^{(n)}=c_2\tau^{(n)}=c_3\in C(A,d,\infty)\neq\square$ where $c_1\neq c_2$ and «□» is completely zero configuration of the space Z^d which in accordance with the above postulate belongs to the set C(A,d,ϕ).

We shall speak, that a binary global transition function $\tau^{(n)}$ realizes so–called «*flip–flop*» if on some pair of tuples $\langle x_1x_2...x_{n-1}0\rangle$, $\langle x_1x_2...x_{n-1}1\rangle$ the corresponding local transition function $\sigma^{(n)}$ will accept the values $\sigma^{(n)}(x_1,x_2,x_3,...,x_{n-1},x_n) = x_n+1 \ (mod\ 2)$. The number of «*flip–flops*» for an arbitrary global transition function $\tau^{(n)}$ is called its *defect* which to some extent characterizes a degree of its deviation from the identical global transition function $\tau^{(n)}{}_o$. We shall determine an useful class $E^{\#}$ of global transition functions $\tau^{(n)}$ of the sub–semigroup G_4 whose local transition functions $E^{(n)}$ are defined as follows, namely**:**

$$E^{(n)}(x_1,x_2,...,x_n)=\begin{cases}0, & if\ x_j=0,\quad x_{n-1}=x_n=1 \quad (j=1..n-2)\\ 1, & if\ x_j=0,\quad x_{n-1}=1 \qquad (j=1..n-2,\ n)\\ x_n, & otherwise\end{cases}$$

This class of global transition functions because of a number of specific dynamical properties represents an quite certain interest for the further research even regardless of the decomposition problem. Considering a subclass of global transition functions of the set $E^{\#}\subset G_4$ that have defect ***1*** it is proved, that composition of two such functions is again function

of the set G_4, but with the defect distinct from **1** [40]. On the basis of the analysis of sets of pairs of ***∞–MEC*** for global transition functions of the set $E^{\#}$ we can establish the exact kind of such pairs for each function of the class $E^{\#}$, allowing to prove the following interesting enough result**:**

For an arbitrary integer n ≥ 3 the global transition functions $E^{(n)}$ from the class $E^{\#}$ have, on the whole, the negative decision of the 1–PDF.

This result allows to receive a constructive negative decision of ***1–PDF,*** having other interesting enough appendices [24,102]. A little bit below some of them are considered in a little bit other context.

So, for decision of the existence question for the sub–semigroup G_4 of a certain finite *basis* a certain class $V \subset G_4$ of global transition functions $\tau^{(n)}$, concerning which the detailed enough research of structure of pairs of ***∞–MEC*** has been carried out is introduced. The suggested approach is constructive and allows to do persuasive enough assumptions relative to real possibility of carrying over of the results received in the preset direction onto general case of classical structures ***1–CA.*** While the base result for the further research there is the following theorem [24,40,82].

<u>Theorem 149</u>. For an arbitrary integer n≥3 there are 1–dimension binary global transition functions $\tau^{(n)} \subset G_4$ which will possess a set of pairs of ∞–MEC of the following kind only, namely:

$$C_1^{\infty} = \ldots\ldots g^1_{j+p+1}\, g^1_{j+p} \cdots g^1_{j+1}\, g^1_j\, g^1_{j-1}\, g^1_{j-2} \cdots g^1_1 X$$

$$C_2^{\infty} = \ldots\ldots g^2_{j+p+1}\, g^2_{j+p} \cdots g^2_{j+1}\, g^2_j\, g^2_{j-1}\, g^2_{j-2} \cdots g^2_1 X$$

$$\text{where } (g^1_j, g^2_j) - \text{an arbitrary pair of the } 2-\text{element set}$$

$$\left\{(x_1 x_2 \ldots x_{n-1}, x^1_1, x^1_2, \ldots, x^1_{n-1}), (x^1_1, x^1_2, \ldots, x^1_{n-1}, x_1 x_2 \ldots x_{n-1})\right\}$$

$$(x_k \in \{0,1\},\ k = 1..n-1;\ j = 1,2,3,\ldots)$$

where X^{∞} is an arbitrary infinity binary 1-dimension configuration and tuples $<x_1x_2 \ldots x_{n-1}>$, $<x^1{}_1x^1{}_2 \ldots x^1{}_{n-1}>$ are various and determined by the kind of binary tuples of length n on which the global transition $\tau^{(n)} \subset G_4$ functions will realize «flip–flop».

On the basis of a certain class of global transition functions $\tau^{(n)} \subset G_4$ that is defined by a special way and whose functions satisfy the conditions of theorem ***149,*** using the detailed analysis of structures of the ***∞–MEC*** pairs existing for them we can formulate the following result useful in a number of appendices of theoretical character [40-43]**:**

Sub–semigroup G_4 of 1–dimensional binary global transition functions does not possess any finite basis.

Of the detailed analysis of global transition functions $\tau^{(n)} \subset G_4$, which satisfy the conditions of theorem ***149*** we can deduce the conclusion that among functions of the sub-semigroup G_4 there are functions $\tau^{(n)}$ which possess the ***∞–MEC*** pairs that consist of subconfigurations having quite concrete set of components of the minimal length ***(n–1)***. In addition, for such functions there are the ***∞–MEC*** pairs which consist from periodical configurations with *minimal* period ***(n–1)***. Furthermore, a *constituent* ***(K)*** of configurations of this pair ***∞–MEC*** to a certain extent is similar to the *internal block* ***(IB)*** of classical ***MEC*** which are defined and considered in section ***2.3*** of the present book.

It is interesting to mark, that the received lower bounds for the minimal sizes of ***K*** of ***∞–MEC*** pairs for global transition functions $\tau^{(n)} \subset G_4$, ***IB*** of the classical ***MEC*** pairs and ***NCF-1*** for the case of classical ***1–CA*** models coincide and are equal ***(n–1)***, i.e. on ***1*** less of size of the neighbourhood template of the appropriate classical ***1-CA*** model. At last, on the basis of rather detailed analysis of the set of global transition functions $\tau^{(n)}$ from G_4 that satisfy conditions of theorem ***149*** and possess the ***∞–MEC*** pairs with minimal period ***(n–1)***, a new solution of the ***1–PDF*** can be received.

<u>Theorem 150</u>. Sub–semigroup L(a,1) of all 1–dimension parallel global mappings $\tau^{(j)}$: C(A,1) → C(A,1) does not possess any finite basis. For an arbitrary integer j≥3 there is 1–dimension global transition function $\tau^{(j)}$ determined in an arbitrary finite alphabet A for which the 1–PDF have the negative decision.

The analogue of our theorem ***150*** on the basis of an algebraic approach has been received by *C.A. Bodnarchuk* and *G.E. Tseitlin*. Working on the transformations that are equivalent to classical ***1–CA*** models, they on the basis of research of a class of semigroups, determined by the preset transformations, have proved, that such semigroups therefore also and ***1–***dimension global transition functions can't have finite basis [211,212]. Such algebraic approach allows to solve the decomposition problem of global transition functions for ***d–CA*** models only at the principal level **–** an impossibility or opportunity of positive decision of the ***d–PDF (d≥1)***.

On the other hand, the study method of the ***1–PDF*** on basis of theorem ***149*** allows to receive stronger result including elements of constructive determination of the kind of global transition functions $\tau^{(n)}$, for which

the ***1–PDF*** has negative decision. Besides a number of methods used by us for decision of the decomposition problem also to a certain extent has a constructive character. In this respect it would be rather interesting to spread this approach also to general case of classical ***CA*** models. It is necessary to mark, that the suggested method of solution of the ***1–PDF*** that bases on the concept of ***∞–MEC*** is an essential generalization of the decision method of the decomposition problem of global functions on the basis of results on the nonconstructability problem in the classical ***CA*** models [24,43,82,102,106].

In the next sections on basis of a polynomial representation of the local transition functions $\sigma^{(n)}$ in classical ***CA*** models the further discussion of the general problem and the generalized problem of decomposition of global transition functions along with the questions connected to them concerning the complexity of global transition functions in ***CA*** models as well as the algorithmic solvability of the decomposition problem will be considered.

7.3. Questions of solvability of the decomposition problem for global transition functions of *CA* models

In this section on the basis of one algebraic approach presenting broad enough interest for the mathematical theory of ***CA*** models along with its numerous appendices some questions of study of the *general problem* ***(d–PDF)*** and the *generalized problem* ***(d–GPDF)*** of *decomposition* of global transition functions are considered. However, first of all, we once again shall note the principal difference between the general problem and the generalized problem. As it was marked earlier, the *generalized* problem of decomposition differs from the *general* decomposition problem by the condition, what in the above presentation ***(23)*** is admitted use of global transition functions with neighbourhood templates of the same size that and an initial global transition function $\tau^{(n)}$, i.e. instead of the condition $n_j < n;\ j=1..m$ the condition $n_j \le n;\ j = 1..m$ is used.

In addition, both decomposition problems are nonequivalent – for some global transition function the ***d–GPDF*** can have a decision whereas the ***d-PDF*** no. We can adduce some rather simple examples of the decision as the compositions of binary ***1–***dimension global transition functions, namely: $\tau^{(3)}_{120} = \tau^{(2)}_{10}\tau^{(3)}_{30}$, $\tau^{(3)}_{84} = \tau^{(2)}_{10}\tau^{(3)}_{42}$ and $\tau^{(3)}_{20} = \tau^{(2)}_{9}\tau^{(3)}_{136}$; i.e. global transition functions with discriminating numbers ***120***, ***84*** and ***20***

can be represented in the form of composition of two global transition functions – with numbers ***30, 42*** and ***136*** accordingly along with global transition functions $\tau^{(2)}_{10}$ and $\tau^{(2)}_{9}$ whose local transition functions are determined as $\sigma^{(2)}_{10}(x,y)=y+1 \pmod 2$ and $\sigma^{(2)}_{9}(x,y)=x+y+1 \pmod 2$. In addition, great deal of similar examples can be adduced. This situation is due to the fact that the result of the composition is a global transition function whose local transition function has extreme elements of the neighborhood template as nonessential variables, reducing the size of real neighborhood templates. Therefore, there are global transition $\tau^{(n)}$ functions for which the ***d–PDF*** have negative decision at presence of a positive decision of the ***d–GPDF.***

Particularly, a composition $\tau^{(n)}=\tau^{(q)}\tau^{(p)}$ of ***1-***dimension global transition functions is admissible for $q<n$ and $p=n$ since the local function which is correspond to the resulting function $\tau^{(n)}$ may quite have the size of the neighborhood template, equal to n due to the presence of nonessential variables ultra, relative to the template. Meantime, the ***d–GPDF*** can be considered as a certain kind of particular case to which we will not pay a special attention, mentioning only in the more general context as an admissible case. i.e., hereinafter we will consider the representation ***(23)*** mainly under the condition $n_j < n$, bearing in mind the above case.

So, with the purpose of analysis in this attitude of binary ***1-***dimensional both classical and non–classical models with neighbourhood index $X=\{0,1,2,3\}$ the following a rather simple procedure ***PDF*** programmed in ***Maple*** had been used. Procedure call ***PDF(m)*** returns a list in the form {[a,b],[c,d,g],...,[e,f],[l,s]} whose sublists with ***2*** and/or ***3*** elements define the discriminating numbers of global transition functions making up all allowable compositions from the more simple global functions for an assumed global transition function with the given number ***m,*** if similar compositions exist**;** otherwise, the procedure call returns the empty list {}. The procedure code is presented below while for its comprehension entirely sufficiently of familiarity with ***Maple*** system in [24,44,48,106].

```
PDF := proc(m::nonnegative)
local  a, b, c, k, h, j, p, n, v, x, q, f, Res, Res1, Res2, Res3;
  if 32767 < m then error "GTF number should be in range 0..32767
     but had received %1", m end if;
  assign(c = cat("", convert(m, 'binary')), Res = {});
  c := convert(cat(seq("0", k = 1 .. 16 – length(c)), c), 'list');
  Res1 := {seq(seq([k, j], k = 1 .. 8), j = 1 .. 128)};
```

```
Res2 := {seq(seq([k, j], k = 1 .. 128), j = 1 .. 8)};
Res3 := {seq(seq(seq([k, j, q], k = 1 .. 8), j = 1 .. 8), q = 1 .. 8)};
a := table([seq(op([assign('h' = cat("", convert(k, 'binary'))),
     cat(seq("0", j = 1 .. 4 – length(h)), h)]) = c[k + 1], k = 0 .. 15)]);
c := [seq(op([assign('h' = cat("", convert(k, 'binary'))),
     cat(seq("0", j = 1 .. 4 – length(h)), h)]), k = 0 .. 15)];
b := [seq(op([assign('x' = cat("", convert(k, 'binary'))),
      assign('x' = convert(cat(seq("0", k=1 .. 8 – length(x)), x), 'list')),
      subs({1 = x[1], 2 = x[2], 3 = x[3], 4 = x[4], 5 = x[5], 6 = x[6],
      7 = x[7], 8 = x[8]}, table(["000" =1, "001" =2, "010" =3, "011" =4,
      "100" = 5, "101" = 6, "110" = 7, "111" = 8]))]), k = 0 .. 127)];
f := [seq(op([assign('x' = cat("", convert(k, 'binary'))),
     assign('x' = convert(cat(seq("0", k = 1 .. 4 – length(x)), x), 'list')),
     subs({1 = x[1], 2 = x[2], 3 = x[3], 4 = x[4]}, table(["00" = 1, "01" = 2,
     "10" = 3, "11" = 4]))]), k = 0 .. 7)];
for n to 16 do for k to 8 do for j to 8 do for q to 8 do
        if f[q][cat(seq(f[j][cat(seq(f[k][c[n][p .. p + 1]], p = 1 .. 3))
        [v..v+1]],v=1..2))] <> a[c[n]] then Res3 := Res3 minus {[k, j, q]}
        end if
      end do
    end do
  end do
end do;
Res := {op(Res), op(Res3)};
for n to 16 do for k to 128 do for j to 8 do
        if f[j][cat(seq(b[k][c[n][p .. p + 2]], p = 1 .. 2))] <> a[c[n]]
        then Res2 := Res2 minus {[k, j]}
        end if
      end do
   end do
end do;
Res := {op(Res), op(Res2)};
for n to 16 do for k to 8 do for j to 128 do
        if b[j][cat(seq(f[k][c[n][p .. p + 1]], p = 1 .. 3))] <> a[c[n]]
        then Res1 := Res1 minus {[k, j]}
        end if
      end do
   end do
end do;
Res := {op(Res), op(Res1)}  end proc
```

> PDF(27030); → {[7,7,7], [91,7], [7,91]}

> PDF(32767); → {[128,8], [8,96], [8,128], [80,8], [94,8], [96,8], [112,8], [124,8], [126,8], [8,8,8]}

> PDF(32767); → {[128,8], [8,96], [8,128], [80,8], [94,8], [96,8], [112,8], [124,8], [126,8], [8,8,8]}

> m :=[]: for k **from** 2601 to 4000 **do if** PDF(k) **<>** {} **then** m **:=** [op(m),k] **end if end do:** m**;**

[2640, 2758, 2766, 2816, 2827, 3012, 3023, 3024, 3072, 3074, 3075, 3084, 3087, 3104, 3105, 3116, 3119, 3120, 3123, 3132, 3135, 3264, 3267, 3276, 3279, 3292, 3312, 3315, 3324, 3327, 3340, 3341, 3377, 3504, 3582, 3583, 3586, 3598, 3644, 3696, 3826, 3838, 3840, 3843, 3852, 3855, 3888, 3891, 3900, 3903, 3967]

> evalf(nops(m)/(4000 – 2600)); → **0.036**

==========

[4, 5, 6, 7, 9, 10, 11, 13, 14, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 50, 52, 53, 54, 56, 57, 58, 59, 61, 62, 65, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 188, 190, 193, 194, 196, 197, 198, 199, 201, 202, 203, 205, 206, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 241, 242, 244, 245, 246, 248, 249, 250, 251]

> evalf(nops(%)/256, 2); → **0.76**

This procedure relatively simply is adapted as to more general kinds of ***1–***dimensional ***CA*** models and for more simple models**;** a number of its modifications the reader can find in our library [24,48]. Naturally, the more universal program of such purpose even for ***1–***dimensional case is appreciably more volumetric, however the essence of implemented algorithm of exhaustive search is simply perceived in the represented text of its concrete realization. Computer experiments carried out with help of the given procedure have allowed to receive diverse statistics relative to representability of binary global functions of dimensionality ***1*** and ***2*** in the form of composition ***(23)*** of more simple global transition functions given in the same state alphabet [40-43]. Only certain results of the gathered statistics are mentioned below.

In particular, in the above example of use of a procedure modification the list of numbers of all ***1–***dimension binary global transition functions with neighbourhood index $X = \{0,1,2\}$ which not possess compositions from more simple global functions is given. Very simple count shows that share of models with similar global transition functions is ***0.76,*** i.e. it exceeds ***3/4.*** In particular, among all classical ***1–***dimensional binary models with neighbourhood index $X=\{0,1,2\}$ which possess attribute of the universal reproducibility in the *Moore* sense of finite configurations only three with numbers ***60, 90*** and ***102*** can be presented in the form of compositions of two more simple functions with numbers {[7, 4], [4, 7], [13,7], [10, 13]}, {[7, 7], [10,7]} and {[10,11], [7,6], [6,7], [11,7]} accordingly. In addition, linear global transition function with number ***105*** that along with others possesses such attribute can't be represented in the form of composition of two more simple global transition functions.

On the other hand, use of the procedure has shown that linear binary global function $\tau^{(4)}_{27030}$ has ***3*** presentation in the form of composition of more simple functions, namely**:** $\tau^{(4)}_{27030}=\tau^{(2)3}_{6}=\tau^{(2)}_{6}\tau^{(3)}_{90}=\tau^{(3)}_{90}\tau^{(2)}_{6}$**;** at that, existing difference in ***1*** at indexing of both composing global transition functions is conditioned by algorithm of the procedure ***PDF.*** Thus, if the first representation follows of a result represented below, whereas the second and third have been received by experimental way.

In addition**,** using a rather simple modification of the above procedure ***PDF,*** it is possible to make sure that the given estimation satisfactorily enough conforms to analogous estimations for classical ***1–***dimensional binary ***CA*** models with an arbitrary neighbourhood index. Besides, the fulfilled numerous computing experiments with by far more complex global transition functions of dimensionality ***1*** and ***2*** have shown, that with growth of values of cardinality of a states alphabet and/or size of neighbourhood index the share of classical models for which the ***d–PDF*** has the positive decision very quickly decreases.

For example, already minimal complication of the above binary models up to ***1–***dimension binary models with neighbourhood template of size ***4*** reduces the share of ***CA*** models for which the ***d–PDF*** has the positive decision up to ***0.025,*** instead of ***0.242*** for the previous case. The call of ***Mathematica*** procedure **QdecompGTF[***ltf* **{,** *g***}]** returns *True* if ***PDF*** for ***1–CA*** with ***LTF*** given by the list ***ltf*** of parallel substitutions of the form $"x1x2...x_n" \to "x1'"$ *(xj, x1′*$\in$***A)*** has positive solution, and *False* otherwise while thru optional ***g*** argument the list of all such decisions is returned.

The above parallel substitutions are standardized for more convenient programming of the **QdecompGTF** procedure [24,49,82,102,106].

Form of representability of global transition functions also represents a rather considerable interest. More precisely, of ***62 1–***dimensional binary global transition functions with neighbourhood index $X=\{0,1,2\}$ which have positive decision of the ***1–PDF*** only two transition functions have symmetrical representations, namely: $\tau^{(3)}_{240} = \tau^{(2)}_{4}\tau^{(2)}_{13} = \tau^{(2)}_{13}\tau^{(2)}_{4}$ and $\tau^{(3)}_{170}=\tau^{(2)}_{6}\tau^{(2)}_{11}=\tau^{(2)}_{11}\tau^{(2)}_{6}$; in addition, global transition functions that allow single representations in the form *(23)* are absent. Whereas of ***823*** binary global transition functions with neighbourhood index $X=\{0,1,2,3\}$ which also have the positive decision of the ***1–PDF*** already ***185*** global transition functions have symmetrical representations, and ***382*** global functions have single representations as compositions *(23)*. At last, the number of admissible compositions *(23)* for both types of functions $\tau^{(3)}$ and $\tau^{(4)}$ are $\{2,4,6,34\}$ and $\{1..13,15,20,31,482\}$ accordingly; in addition, maximal values relate to the functions whose local transition functions have discriminating number ***0.***

A rather significant potential for the decomposition problem of global functions provides use of global functions in the same alphabet along with the unconnected neighborhood indices. And above all, it applies to the case when the decomposition problem is considered relative to the preset subclasses of global transition functions. Now, we consider the given proposition on an example of interesting class of linear ***1–CA*** whose local transition functions are defined by the following formula:

$$\sigma^{(n)}(x_1,\dots,x_n)=\sum_{1}^{n} b_j x_j \pmod a;\ x_j, b_j \in A=\{0,1,\dots,a-1\};\ (j=1..n),\ a-\text{prime} \qquad (28)$$

This interesting class of ***d–CA*** is considered in [24,43,102] with different standpoints. In particular, the simple linear ***1–CA*** with local transition function $\sigma^{(4)}(x_1, x_2, x_3, x_4) = x_1+x_2+x_3+x_4 \pmod a;\ x_j\in\{0,1,\dots,a-1\};\ j=1..4;$ a – prime can't be represented in the form of composition of the more simple ***CA*** models of the same class with the connected neighbourhood indices in a whole. Whereas this problem is easily solved by means of usage of global transition functions with unconnected neighbourhood indices, suppose that $\sigma^{(4)}(x_1, x_2, x_3, x_4) = b_1x_1+b_2x_2+b_3x_3+b_4x_4 \pmod a;$ $b_jx_j\in\{0,1,\dots,a-1\};\ j=1..4;\ a$ – prime defines a global transition function $\tau^{(4)}$ and connected neighbourhood index $X_4=\{0,1,2,3\}$. Easy to see that two global transition functions $\tau^{(3)}$ and $\tau^{(2)}$ defined by local transition

functions $\sigma^{(3)}$ and $\sigma^{(2)}$ with neighbourhood indices $X3 = \{0,3\}$, and $X2 = \{0,1\}$ accordingly, namely: $\sigma^{(3)}(x_1, x_2, x_3)=x_1+x_3 \ (mod\ a)$ and $\sigma^{(2)}(x_1, x_2) = x_1+x_2 \ (mod\ a)$; a – prime solves the *1–PDF* for *GTF* $\tau^{(4)}$ as $\tau^{(4)} = \tau^{(3)}\tau^{(2)}$.

The reader can make sure in justice of the following proposal, namely:

For an arbitrary even integer n the solution of decomposition problem for GTF $\tau^{(n)}$ ***defined by LTF*** $\sigma^{(n)}$ ***in the form (28) is defined as follows:*** $\tau^{(n)} = \tau_1^{(n/2+1)}\tau_2^{(n/2)} = \tau_3^{(2)}\tau_4^{(n-1)}$ ***where GTF*** $\tau_1^{(n/2+1)}$, $\tau_2^{(n/2)}$, $\tau_3^{(2)}$, $\tau_4^{(n-1)}$ ***are defined by local transition functions*** $\sigma_1^{(n/2+1)}$, $\sigma_2^{(n/2)}$, $\sigma_3^{(2)}$, $\sigma_4^{(n-1)}$ ***with neighbourhood indices*** $X_1 = \{0, n/2\}$, $X_2 = \{0, 1, ..., n/2-1\}$, $X_3 = \{0, 1\}$, $X_4 = \{2k \mid k = 0..n/2\}$ ***accordingly.***

Along with this, the question of preserving the fundamental dynamical properties of a global transition function that is the composition result of functions of the same class in the form *(27)*, is of definite interest. In this respect, we can confine ourselves to the class of so–called *"linear"* global transition functions whose local transition functions are defined by the formula *(28)*. Furthermore, among the set of such functions there is a subset of so–called "*strictly linear*" функций, whose local transition functions are defined by the formula *(28)* under the condition $(\forall j)(b_j \equiv 1)$. It is easy to verify that the set of all global transition functions defined in a finite alphabet $A=\{0,1,\ldots,a-1\}$ and whose local transition functions are determined by the above formula *(28)* is a *semigroup* with respect to the composition operation modulo a under the condition that a is a prime. In addition, the composition of strictly linear global transition functions always gives a linear transition function, but not a strictly linear.

Numerous computer experiments have been carried out, in particular, on the basis of rather simple ***Mathematica*** procedures ***StrictLinearGTF***, ***CompGTF*** and ***Reproduction.*** The procedure call ***StrictLinearGTF*****[*A, n*]** provides the generation of the format list $\{"x1x2\ldots x_n" \rightarrow "x1'",\ldots\}$, that defines a local transition function $\sigma^{(n)}(x_1, x_2, \ldots, x_n) = x_1+x_2+\ldots+x_n = x1' \ (mod\ a)$; $x_j \in A=\{0,1,\ldots,a-1\}$; $(xj, x1' \in A)$ which corresponds to the desired *1–*dimensional classical strictly linear ***CA*** model. Such representation of the global transition function is convenient enough for programming of many problems related to the study of the dynamics of *1-*dimension ***CA*** models. Moreover, such representation of local transition functions is in good agreement with the fact that the classical *1–CA* models are formal systems of highly parallel word processing in finite alphabets. It should

be noted that the procedure is rather easily reprogrammed for the case of a local transition function having the form $\sigma^{(n)}(x_1,\dots,x_n) = \Re[x_1,\dots,x_n]$.

Whereas the composition of ***1–***dimensional global transition functions is provided by the procedure call ***CompGTF*[*A*, *g*]** where ***A* –** an alphabet common for all global transition functions**,** included in the composition, and ***g* –** list of local transition functions defined in the form of lists with elements of the above format, defining corresponding global transition functions. At last, the procedure call ***Reproduction*[*x*,*z*,*n*]** on three actual arguments**:** ***x*** is a local transition function, given by the list of the above format which corresponds to the tested global transition function**,** ***z* –** a tested finite configuration in string format and ***n* –** a desired number of its copies, returns the list whose the first element defines number of the obtained copies of *z* configuration whereas the second element defines number of steps of ***1–CA*** model, required for that. The fragment below illustrates a principle of use of the above procedures for testing of the properties of self**–**reproduction of finite configurations in compositions of strictly linear classical ***1–CA*** models.

```
In[4939]:= StrictLinearGTF[A_List, n_Integer] :=
                                    Module[{a = Tuples[Map[ToString, A], n], b = {}},
                                  DeleteDuplicates[Flatten[Map[AppendTo[b, Rule[StringJoin[a[[#]]],
                                        ToString[Mod[Total@Map[ToExpression, a[[#]]], Length[A]]]]] &,
                                                                        Range[1, Length[A]^n]]]]]
In[4940]:= g = StrictLinearGTF[{0,1,2}, 3]; s = StrictLinearGTF[{0,1,2}, 2];
In[4941]:= h = CompGTF[{0, 1, 2}, {s, g, s}];
In[4942]:= sg = "1201020021202001212"; n = 240; Reproduction[h, sg, n]
Out[4942]= {243, 3267}
```

Unlike the ***StrictLinearGTF*** procedure the procedure call ***LinearGTF*[*A*, *f*, *n*]** provides the generation of the format list $\{"x1x2\dots x_n" \to "x1'",\dots\}$, that defines a local transition function $\sigma^{(n)}(x_1,x_2,\dots,x_n) = \Re[x_1,x_2,\dots,x_n] = x1'; x_j \in A=\{0,1,2,\dots,a-1\}; (x_j, x1' \in A)$ which corresponds to the desired ***1–***dimensional classical linear ***CA*** model. As the second actual ***f*** argument of the above procedure a pure function from ***n-1*** arguments is used. The following fragment represents the source code of this procedure with examples of generation of a linear ***1–***dimension local transition function corresponding to a pure ***f*** function, followed by its ***3–***fold composition and verification of the received local transition function for possessing the property of universal self**–**reproducibility in the *Moore* sense of the finite configurations given in an alphabet $A = \{0, 1, 2, \dots, a-1\}$.

```
In[4942]:= LinearGTF[A_List, f_/; PureFuncQ[f], n_Integer] :=
             Module[{a = Tuples[Map[ToString, A], n], b = {}},
                 If[Length[ArgsPureFunc[f]] != n–1,
  Return["The second argument does not match the third argument"],
                 DeleteDuplicates[Flatten[Map[AppendTo[b,
Rule[StringJoin[a[[#]]], ToString[f @@ Map[ToExpression, a[[#]]]]]] &,
                 Range[1, Length[A]^n]]]]]
In[4943]:= F := Mod[#1 + 2*#2 + 3*#3 + 4*#4, 5] &
In[4944]:= g0 := LinearGTF[{0, 1, 2, 3, 4}, F, 5]
In[4945]:= g = CompGTF1[g0, 3, 5];
In[4946]:= Reproduction2[g, "43210124043140003440030123 4", 25, 2000]
Out[4946]= {25, 1000}
```

Theoretical results inspired by numerous and comprehensive computer experiments using the above scheme of calculations in the ***Mathematica 11.3.0*** have allowed to formulate the following proposal [41-42,49,106].

Proposal 12*. A composition in form* ***(27)*** *of strictly linear 1–dimension global transition functions, given in the alphabet* $A = \{0,1,...,w-1\}$ *under the condition that w is a prime, gives a global transition function* $\tau^{(n)}$ *which with rare exceptions is different from strictly linear transition functions, being one of the functions of the group* ***(28)****, and has the self–reproducibility property in the Moore sense of finite configurations. The composition* $\tau^{(4)} = \tau^{(2)}\tau^{(2)}\tau^{(2)}$ *where* $\tau^{(2)}$ *is determined by local transition function* $\sigma^{(2)}(x, y) = x + y \pmod 2$; $x,y \in A=\{0,1\}$ *can serve as an exception.*

Interesting variants of the representability of global transition functions $\tau^{(n)}$ appear when using global transition functions as components in the representations ***(27, 8)*** whose local transition functions contain dummy variables, i.e. functions with unconnected neighbourhood indices.

Thus, from ***1–***dimensional strictly linear global transition functions we can generate the *linear* global transition functions whose local transition functions are determined by the above formula ***(28)*** and that posess the reproducibility in the *Moore* sense of finite configurations. Furthermore, this result essentially extends the class of *strictly linear* global transition functions, for which the property of universal self-reproducibility in the *Moore* sense of finite configurations was initially discovered.

Therefore, the problem of detection of interesting enough classes of ***CA*** models whose global transition functions admit the positive decision of ***d–PDF*** becomes enough actual owing to a significant enough oneness of this phenomenon. More precisely, as distinct from the initial posing

of the ***d–PDF*** whose decisions were reduced to proofs of existence of global transition functions which not admit representation in the form of *(27)* of more simple functions, now with good reason it is possible to speak about search of rather interesting classes of the global transition functions allowing such representations.

Along with the classical notion of composition of global functions we can consider certain other notions of composition which are of certain interest from the standpoint of the dynamics of classical ***CA*** models. A composition of two ***1***-dimension global transition functions whose local transition functions are determined by means of lists with elements of the format {***"x1x2…xn" → "x`1"***} and {***"x1x2…xn" → "y`1"***} determines a global transition function whose local transition function is defined by means of lists with elements of the format {***"x1x2x3…xn" → "x`1 + y`1 (mod a)"***}, where ***xj,x`1,y`1∈A={0,1,2,3, …, a–1}; j = 1..n.*** This type of the concatenation is called ***⊕–****concatenation*. For the purpose of the software of the given composition operation, the ***CompGTF0*** procedure was been programmed in the ***Mathematica*** system with three formal arguments**:** ***x,y –*** the lists of the above format, defining the local transition functions corresponding the global transition functions and ***p*** is cardinality of an alphabet common for both global transition functions. The procedure call returns the list determined the resultant local transition function.

```
In[4672]:= CompGTF0[x_List, y_List, p_Integer] := Module[{},
  Map[Rule[x[[#]][[1]], ToString[Mod[ToExpression[x[[#]][[2]]] +
                ToExpression[y[[#]][[2]]], p]]] &, Range[1, Length[x]]]]
```

```
In[4673]:= x = {"00" → "0", "01" → "1", "10" → "1", "11" → "0"};
y = {"00" → "0", "01" → "1", "10" → "0", "11" → "1"};
In[4674]:= CompGTF0[x, y, 2]
Out[4674]= {"00" → "0", "01" → "0", "10" → "1", "11" → "1"}
```

A lot of interesting enough properties of such composition have been discovered, among which the following should be specially noted. Concerning the composition operation of this type, a class of strictly linear ***1–***dimension global transition functions *(28)* was considered. It is shown that the result of the ***p–***fold composition of the strictly linear ***1–***dimensional global transition function $\tau^{(n)}$ $\{\tau^{(n)0}=\tau^{(n)}, \tau^{(n)p}\}$ defined in an alphabet ***A={0,1,…,a–1}***, where ***a –*** a prime and ***p={1,…,a–2}***, is the set of various ***1–***dimension global transition function $\tau^{(n)}$, each of which possesses the property self–

reproducibility in the *Moore* sense of finite configurations.

For the purpose of the software of the above composition operation, the **CompGTF1** procedure was been programmed in ***Mathematica*** system with three formal arguments**:** ***f*** **–** the list of the above format, defining a local transition function corresponding the global transition function, ***n*** is number of global transition function, participating in the composition and ***m*** is cardinality of alphabet of the global transition function.

```
In[4660]:= CompGTF1[f_List, n_Integer, m_ /; PrimeQ[m]] :=
             Module[{a = f, b = 1}, While[b <= n, a = Map[Rule[a[[#]][[1]],
                                                 ToString[Mod[ToExpression[a[[#]][[2]]] +
           ToExpression[f[[#]][[2]]], m]]] &, Range[1, Length[f]]]; b++]; a]
In[4661]:= g0 = StrictLinearGTF[{0, 1, 2}, 2]
Out[4661]= {"00" → "0", "01" → "1", "02" → "2", "10" → "1", "11" →
"2", "12" → "0", "20" → "2", "21" → "0", "22" → "1"}
In[4662]:= g1 = CompGTF1[g0, 1, 3]
Out[4662]= {"00" → "0", "01" → "2", "02" → "1", "10" → "2", "11" →
"1", "12" → "0", "20" → "1", "21" → "0", "22" → "2"}
In[4663]:= g2 = CompGTF1[g0, 2, 3]
Out[4663]= {"00" → "0", "01" → "0", "02" → "0", "10" → "0", "11" →
"0", "12" → "0", "20" → "0", "21" → "0", "22" → "0"}
```

The call **CompGTF1[*f*,*n*,*p*]** returns the list of the above format, defining a local transition function corresponding the global transition function ***f***, being ***n***–fold concatenation of the global transition function ***f***, defined in an alphabet with ***p*** cardinality, where ***p*** is prime. Based on the results of theoretical and experimental researches of ⊕–concatenation of global transition functions, the following result is formulated [41-43,102,106].

Proposal 13*. Let G be a d–dimensional classical CA model with linear global transition function F whose local transition function is defined by means of the formula (28) and is given in an alphabet A={0,1,…,a–1} (a – a prime number) then p–fold ⊕–concatenation of its global function F for p∈{1,2,3,…,a–2} generates (a–2) linear global transition functions, possessing the self–reproducibility property in the Moore sense of finite configurations defined in the A alphabet. Moreover, an arbitrary finite configuration specified in the A alphabet, and the inverse to it, are self–reproducing in the Moore sense, requiring the same number of steps of generation of the global transition function which is result from p–fold ⊕–concatenation of the global transition function F for p∈{1,2,…,a–2}.*

Generally, the generation time of a given number of copies of the tested finite configuration is maximal for the composition $\tau^{(n)0}$ *(*$\tau^{(n)}$ *is defined in an alphabet* $A = \{0, 1, ..., a-1\}$; a – *a prime),* whereas for the composition $\tau^{(n)p}$ grows identically for all *(*$p=0...a-2$*)* with increasing of strings size, i.e., the same configuration for obtaining d copies will require the same number of steps of the global transition function $\tau^{(n)p}$ for $p=0,...,a-2$.

A large enough help in investigation of the ⊕–composition problem of global transition functions was computer modelling, basing on the such Mathematica procedures as ***CompGTF0***, ***CompGTF1***, ***Reproduction*** and ***Reproduction2***. These procedures are useful enough tools in computer study of the ⊕–composition of ***1***–dimension global transition functions.

At last, it is necessary to mark, that the *general problem* of *decomposition (****d–PDF****)* of global transition functions represents the most appreciable interest from all standpoints, while the generalized problem *(****d–GPDF****)* has, rather, the more perceptual character and has been introduced by us for more exhaustive investigation of the decomposition problem of global transition functions. At the same time, the ***d–GPDF*** perhaps has rather interesting appendices too except especially theoretical interest.

Questions of algorithmical solvability play in the modern mathematics extremely important part and make up wide enough class of so–called *mass* problems. A number of problems of this class that are connected to ***CA*** models has been considered above. Below, questions of algorithmic solvability concern problems of decomposition of global functions for case of classical ***CA*** models. As noted above, the problem of algorithmic solvability of decomposition is solved positively on the basis of a brute force – full search what is as follows from the following theorem.

<u>Theorem 151</u>. For an arbitrary d–dimension global transition function determined in a states alphabet A there is constructive algorithm in the form of full search solving both the d–PDF, and the d–GPDF $(d \geq 1)$***.***

Intrinsically, it is rather simple to make sure that for an arbitrary global transition function, determined in an arbitrary finite alphabet, there is a *constructive* algorithm of ascertainment of an opportunity of decision of the ***d–PDF***, and the ***d–GPDF*** $(d \geq 1)$ for the function. The essence of this algorithm is represented enough in detail, for example in [24,41,102]. In addition, the existing theoretical possibility of a constructive decision of both the ***d–PDF***, and the ***d–GPDF*** is limited by opportunity of modern computing resources. Naturally, the suggested solving algorithm is a rather bulky, however such programmed algorithm allows to receive

(under the condition of their existence) all possible representations of any global transition function $\tau^{(n)}$ of dimensionality $d = 1,2$ that is defined in an arbitrary finite alphabet A in the form of composition of more simple global transition functions $\tau^{(nj)}$. In consideration of the specificity of the decomposition problem first of all from the applied standpoint there is the following interesting enough question:

Whether exists an effective constructive algorithm different from a full search determining the opportunity of decision of the problems d–PDF and/or d-GPDF for an arbitrary d-dimension global transition function defined in an arbitrary finite alphabet of states A ($d \geq 1$)?

In addition, labor expenditures of the constructive algorithm can be at times enough essentially reduced, if to restrict oneself to receiving of an allowable representation, or local transition function $\sigma^{(n)}$ corresponding to a tested global function $\tau^{(n)}$ has a certain special kind. In spite of the solvability of the ***d–PDF*** there is a number of the more special problems whose decision is known and is not so bulky. Consideration will begin with the most simple cases of decompositions of global functions.

Above all, we shall mark an important enough, especially from applied standpoint, a case of special presentation of a global transition function $\tau^{(n)}$ for the limited number of constituent functions in representation *(27)*, i.e. under the condition of $p \leq p^* = const$. For the given special case the decomposition problem is algorithmically resolvable, and proof of that is simply enough [41]. At the same time, the resolving algorithm is constructive and enough simply implemented already on *PC* [41-43,82].

Similarly it is possible to consider the solvability of the ***d–PDF*** and for a number of other special cases of the above representation *(27)* for global transition functions. In this connexion it is rather interesting to consider the decomposition problem not only concerning set of global transition functions, but also concerning certain its subsets.

Let $\mathfrak{R}$ is a set of all such d–dimensional global transition functions $\tau^{(n)}$ determined in an arbitrary finite alphabet A that for a function $\tau^{(n)} \in \mathfrak{R}$ there is the following relation $(\forall c \in C(A,d,\phi))(|c| \leq |c\tau^{(n)}|)$ where $|c^*|$ is the maximal diameter of an arbitrary configuration c^*. Then in class $\mathfrak{R}$ of global functions the ***d–PDF*** is algorithmically solvable, allowing a very simple and fast enough resolving constructive algorithm [41,43].

<u>Theorem 152</u>. In class $\mathfrak{R}$ of d–dimensional global transition functions the d–PDF ($d \geq 1$) is solvable, possessing a simple enough constructive solving algorithm.

The decomposition problem of a global transition function $\tau^{(n)}$, when for its composition *(27)* there is the relation $\Sigma_j n_j = n-k+1;\ n_j < n\ (j = 1..k)$, represents a rather large interest from applied and theoretical points of view. For this special case the problem also is algorithmically solvable, and the proof of its solvability is constructive**;** in addition, the solving algorithm has program realization in the ***Maple*** system, complexity of which depends on kind of a tested global transition function and/or of constraints imposed on functions making up composition *(27)* [24,102].

The so**–**called problem of the elementary representation of an arbitrary global transition function that consists in detection of functions with *minimal* sizes of neighbourhood templates making up its representation *(27)* is interesting enough, having a simple but laborious constructive resolving algorithm. Owing to practical importance of the ***d–PDF*** and the ***d–GPDF*** a receiving of essentially constructive solving algorithms allowing to establish for a global transition function the impossibility of a decision of the above problems or to give exhaustive solutions in the form of the required concrete decompositions *(27)* appears important. Thus, at level of solvability both the ***d–PDF*** and the ***d–GPDF*** $(d \geq 1)$ are equivalent while at level of a possibility to receive a solution the second problem is by far more preferable. The general problem of algorithmic solvability of the ***d–PDF*** $(d \geq 1)$ is generalization of the more particular problem of decomposition, namely**:**

Whether exists for a global transition function $\tau^{(n)}$ a positive decision of the d–PDF under the condition of belonging of functions $\tau^{(n_j)}$ which compose its representation (27) to some basic subset S of the set of all global functions defined in the same d–dimension and an alphabet A?

In this direction there is the following basic result, having a number of important enough appendices. The result is supported by rather useful software implementations in systems ***Maple*** and ***Mathematica*** [82-84].

Theorem 153. Concerning an arbitrary basic subset W of the set of all d–dimension global transition functions defined in a finite alphabet A both the d–PDF, and the d–GPDF are algorithmically solvable.

To a certain extent, the problem of uniqueness of representation of an arbitrary global transition function $\tau^{(n)}$ in the form of a composition of finite number of the more simple functions is intermediate between the general problem and particular problem of algorithmical solvability. In particular, if a global transition function $\tau^{(n)}$ has a single representation in the form composition *(27)* of simpler global transition functions $\tau^{(n_j)}$,

then these global functions are not representable in the form *(27)*.

Of the results of research of the ***d–PDF*** we know ab existence of global functions for which this problem has the negative decision along with functions permitting at least two various nontrivial positive decisions. In this connection there is an interesting enough question of existence of global transition functions permitting only one positive decision of the ***d–PDF.*** Here, it is necessary to note at once one important enough property of global transition functions making up a sole representation in the form *(27)* for an arbitrary global function, namely, the negative decision of the ***d–PDF*** for each of them. Thus, it is necessary to search similar global transition functions among global transition functions having the negative decision of the ***d–PDF.*** In particular, for that it is quite enough to consider a class W of linear global transition functions whose local transition functions $\sigma^{(n)}$ are defined by the formula *(28)*.

It is shown that in the class of similar linear global transition functions exist the functions allowing only single positive decision of the ***d–PDF.*** Furthermore, the fulfilled analysis has shown, that among already all classical ***1–***dimensional binary global functions with neighbourhood index $X = \{0, 1, 2\}$ exist functions having sole representations *(27) (with numbers* ***1,2,8,15,16,18,19,24,36,55,64,66,72,85,90,126,127****)* together with sole representations consisting of degrees of the same functions *(with discriminating numbers* ***1,15,85,90,127****)* [82]. Hence, among ***d–***dimension global transition functions defined in a finite alphabet A, it is possible to distinguish at least four *noncrossing* sets of global transition functions concerning the possible decision of the ***d–PDF*** $(d \geq 1)$, namely**:**

- ***global*** $\tau^{(n)}$ ***functions not having a positive decision of the d–PDF;***
- ***global*** $\tau^{(n)}$ ***functions having the positive decisions of the d–PDF;***
- ***global*** $\tau^{(n)}$ ***functions having a sole positive decision of the d–PDF;***
- ***global*** $\tau^{(n)}$ ***functions having a sole positive decision of the d–PDF which consists of a degree of some more simple function.***

Therefore, an interesting enough question of dependence of the above four types of representation *(27)* of global functions on a chosen set of global functions arises. So, the enlargement of classical ***1–***dimensional binary global functions with neighbourhood index $X = \{0,1,2\}$ up to all binary global functions of the same type violates the oneness of global functions representations with numbers ***1, 15, 85, 90, 127,*** ensuring for them double representations in the form *(27)*. Concerning the class of ***1–***dimensional linear global transition functions $\tau^{(n)}$ along with results

represented above there are some interesting enough which have a lot of rather useful applications in investigations of dynamics of classical ***CA*** models**.** In particular**,** the above results can be rather useful in case of researches of representations of ***GTF,*** namely [24,43,82,102,106]**:**

An arbitrary global transition function $\tau^{(n)}$ ***defined by a local function*** $\sigma^{(n)}(x_1, ..., x_n) = \sum_j x_j \pmod a$; $x_j \in A=\{0,1,...,a-1\}$ ***has a presentation in the form*** $\tau^{(n)}=\left[\tau^{(a)}\right]^{(a^k-1)/(a-1)}$ ***if and only if n is represented as*** a^k ***where k is a positive integer, a is a prime number and*** $\tau^{(a)}$ ***is linear global function with connected neighbourhood template of a size a. Each linear global transition function*** $\tau^{(n)}$ ***with local function*** $\sigma^{(n)}(x_1, ..., x_n) = \sum_j x_j \pmod a$ ***has presentation in the form*** $\tau^{(n)}=\tau_1^{(n/2+1)}\tau_2^{(n/2)}$**,** ***where a is an arbitrary integer, n is an even number and linear global transition functions*** τ_1 ***and*** τ_2 ***forming the similar representation have neighbourhood indices*** $X1=\{0, n/2\}$ ***and*** $X2=\{0,1, ..., n/2-1\}$ ***accordingly. For another values of n a linear function*** $\tau^{(n)}$ ***has the negative decision of the PDF in the form of functions composition of the same kind. For arbitrary integers*** $n \geq 3$ ***and*** $a \geq 2$ ***the class (17) of functions contains N(n, a) linear functions allowing compositions from more simple functions of the same class, where N(n, a) is a function increasing in variables n and a.***

With the representability problem of global transition functions $\tau^{(n)}$ in the form of composition *(27),* arises the question of interrelation of the nonconstructability types of global functions $\tau^{(n)}$ and global functions forming their decompositions. So, closure of the set $C(A,d,\phi)$ relative to an arbitrary global mapping, defined by a global transition function $\tau^{(n)}$ plays a rather considerable part in the question of possessing the ***NCF, NCF–1, NCF–2*** and ***NCF–3*** nonconstructability.

The question of influence on nonclosure of the set $C(A,d,\infty)$ relative to a mapping, defined by a global transition function $\tau^{(n)}$**,** of the existence of similar property for global transition functions $\tau^{(n_j)}$ $(j = 1..p)$**,** that make up the above-mentioned decomposition represents a certain interest. In this direction a number of rather interesting results was obtained [82].

Possibility of the ***NCF*** nonconstructability to do infinite configurations $c^{\infty}_j \in C(A,d,\infty)$**,** for which the relation $c^{\infty}_j \tau^{(n_j)} = \square$ $(j \geq 2)$ is possible, by the nonconstructible ones, lays at the heart of receiving an answer to this question**;** i.e. there is a possibility to inhibit arising of nonclosure of the set $C(A,d,\infty)$ concerning a global function $\tau^{(n)}$ as a whole. In view of the

told, an example of a composition *(generally speaking* **27***)*, basing on this possibility has been found, allowing to formulate the result [24,41,82].

Theorem 154*. In order that a global transition function representable in the form* **(27)** *define a global mapping concerning which the set* $C(A,d,\infty)$ *would be nonclosed is necessary and sufficient the existence of at least one global transition function* $\tau^{(n_j)}$ $(j = 1..p)$ *of the representation* **(27)** *which determines a global mapping concerning which the set* $C(A, d, \infty)$ *is nonclosed. A global transition function representable in the form* **(27)** *will posess the NCF nonconstructability if and only if at least 1 global transition function* $\tau^{(n_j)}$ $(j = 1..p)$ *of its representation* **(27)** *possesses the NCF nonconstructability.*

With the question uniqueness of solution of the ***d–PDF*** for certain types of global transition functions, the absence of the positive decision of the ***d–PDF*** $(d \geq 1)$ as a whole is closely connected. Hence, each example of global transition function having the single solution of the ***d–PDF*** $(d \geq 1)$ proves negativity of the solution of the ***d–PDF*** as a whole. A number of interesting enough examples of such type of global transition functions can be found in our works [24,43,82,102,106].

Let's consider one more algebraical approach to research of the ***d–PDF*** and the ***d–GPDF*** that represent a rather significant interest for research of dynamic properties of classical models ***d–CA*** $(d \geq 1)$ as a whole. In this context certain earlier presented results on the decomposition problem have been overproved, and also certain new results have been received thanks to this approach. The possibility of representation of some local transition function $\sigma^{(n)}$ defined in an alphabet A in the polynomial form of the following general kind underlies this approach**:**

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = \sum_{j=1}^{j=p} b_j Y_j \pmod a \qquad (29)$$

where $b_j \in A$ and Y_j is a variable of the set $\{x_1,...,x_n\}$ $(j=1..p)$ or product of degrees of these variables. One of the basic results concerning to the ***a–***valued logics $(a > 2)$ says**:** ***An arbitrary function of a–valued logic can be represented in the polynomial form* (29) *if and only if a is a prime.***

This result appears a rather useful not only for polynomial presentation of the kind *(29)* of ***1–***dimension local transition functions, defined in an alphabet ***A.*** The result can be quite successfully used for ***d–***dimensional local functions $\sigma^{(n)}$**,** representing the neighbourhood index of classical models ***d–CA*** $(d \geq 1)$ in a special way. In addition, in our opinion, the way of polynomial representation of local transition functions will find

wide enough application in researches on classical ***CA*** models and their numerous appendices. This reception was used for the further research of both the *general* problem and the *generalized* problem of composition of global transition functions in classical models ***d–CA*** *(**d≥1**)*. Moreover, as a theoretical basis of this reception the following basic result serves [24,41-43,82,102,106].

Theorem 155. An arbitrary local transition function $\sigma^{(n)}$, determined in a finite alphabet A can be represented in the form of a polynomial over S from n variables of a degree not higher than n*(a–1) if and only if an algebraical system S = <A; +; x> is the field.

If a number ***a*** is prime, the field ***M* = <A; +; x>*** also will be prime. As a prime field ***M**** is isomorphic to a ring of classes of residues of a ring of integers modulo ***a*** then we can limit oneself only to the representative polynomials modulo ***a.*** It is necessary to mark that at $a = p^k$ *(**p** – a prime number and **k>a** is an integer)* an algebraic system ***M* = <A; +; x>*** also can be reformed into a field, defining peculiar operations of multiplication and/or addition**,** but in such case the representative polynomials over the field ***M**** will be inconvenient enough for research of properties of local transition functions. Therefore, in impossibility of representation of local transition functions by polynomials modulo ***a,*** in a lot of cases it is expedient to apply a little bit other approaches.

Besides, a rather interesting example of algebraic system for polynomial presentations of ***a–***valued logic functions in case of composite ***a*** number is given by theorem ***64*** and enough in detail discussed in works [82,145]. The problem of polynomial representability of local transition functions has an quite constructive decision for cases of both *prime* number ***a,*** and *composite* number ***a*** in this algebraic system, supposing a rather effective computer realization basing on the simple algorithm whose description can be found, for example, in [24,43,82,102,106].

The question of polynomial presentability of local transition functions over a field ***A*** is closely related to the question about the reducibility of the given polynomials over the field ***A,*** i.e. a possibility of presentation in the form of a product of a finite number of simpler polynomials over the field ***A,*** namely**:**

$$\sigma^{(n)}(x_1, x_2, ..., x_n) = \prod_{1}^{k} F_j(x_{j_1}, x_{j_2}, ..., x_{j_n}); \quad j_n \in \{1, 2, ..., n\} \quad (j = 1..p; p \le n)$$

In more general posing the given question is reduced to a possibility of representation of a polynomial, adequate to a local transition function

$\sigma^{(n)}$ defined in a finite alphabet A, in the form of a certain function F of simplest polynomials or polynomials of the definite type over the field A. This question is of interest as from the standpoint of research of a lot of properties of local transition functions in classical *CA* models and in practical implementation of computational and certain other discrete systems of various purpose on their base [7,24,82,85,86,102,106,286].

The factoring problem of the representative polynomials modulo a is directly connected to questions of research of dynamics of classical *CA* models by means of algebraic methods. So, if for a certain local function the polynomial representing it admits decomposition of the next kind:

$$P(x_1, ..., x_n) = (x_j - b)^k P_1(x_1, ..., x_n) \pmod a; \; j \in \{1,...,n\}; \; b \in A \setminus \{0\}; \; k \geq 1$$

then the appropriate global transition function of a classical *1-CA* model will possess the *NCF (NCF–3)* and/or *NCF–1* nonconstructability. Thus, the *reducibility* problem of the representing polynomial modulo a seems as a rather important means of the dynamics analysis of classical *d–CA (d≥1)* models. However, practical use of this approach is rather difficult because for a numerical field A the structure of polynomials irreducible over the A is various enough, and any general algorithms establishing reducibility or irreducibility of polynomials from $n > 1$ variables do not exist. Thus, for revealing of reducibility rather frequently it is necessary to use skilful enough receptions, or to resolve problems of dynamics of classical *CA* models by means of other more acceptable methods [102].

Among set of all polynomials over a field A it is possible to single out a lot of interesting enough classes. Polynomials in normal form of which all additive monomials have the same degree concerning the totality of variables play a certain peculiar part. These polynomials are called the *homogeneous* polynomials or *forms*. Owing to the fact that forms can be enough essentially simplified with the help of replacement of variables, it is possible to carry out the research enough effectively of properties of local transition functions corresponding to them [24,41-43,82,86,106].

The so–called symmetrical polynomials constitute one more interesting enough class. A polynomial from $n>1$ variables over a field A is called *symmetric* polynomial if it does not vary at any reversible permutations of its essential variables. It is well–known that the set of all symmetrical polynomials from n variables forms a subring of ring of all polynomials from n variables over a field A. For purposes of our researches a special interest represents the case when polynomial *PP* over a field A can be presented by a complex function in the form of a polynomial from the

symmetrical polynomials. In addition, the symmetric polynomials over a field ***A*** that are called the *elementary* polynomials play a special part**:**

$$p_k(x_1, x_2, ..., x_n) = \sum_{1}^{r} R_j(k, x_1, x_2, ..., x_n); \qquad r = C_n^k / k!$$

$$p_1(x_1, x_2, ..., x_n) = \sum_{j=1}^{n} x_j; \qquad p_n(x_1, x_2, ..., x_n) = \prod_{j=1}^{n} x_j$$

where $R_j(k, x_1, x_2, x_3, ..., x_n)$ are various arrangements *(within symmetry)* on k from elements of the set $\{x_1, x_2, ..., x_n\}$. In particular, the *elementary* polynomials $\{P_1, P_n\}$ have the kind, represented above. The basic result here can be formulated as follows**:**

An arbitrary symmetric polynomial over a field A can be presented by only single way in the form of a certain polynomial from elementary symmetrical polynomials P_k ***(k=1..n).***

For a practical decision of this problem it is possible to use the known method of indefinite coefficients, applying it to elementary symmetric polynomials onto which each symmetric polynomial will decompose. The theory of symmetric polynomials basing on the above base result has numerous appendices in various fields of the polynomials theory.

The reasons determining the role of symmetric polynomials lay deeply enough and are discovered only at research of a lot of properties of the automorphisms of algebraical fields. In context of the ***CA*** problematics the *symmetric polynomials* defining symmetric local transition functions of the classical ***CA*** models also represent heightened interest since the models of such type under the certain conditions possess, for example, the property of universal or essential reproducibility in the *Moore* sense of finite configurations. Besides, the ***CA*** models with symmetrical local transition functions $\sigma^{(n)}$ represent special both theoretical and applied interest in bio–medical sciences, physics, modelling, mathematics, and computing sciences, and also in a lot of other appendices [7,24,27,31,33].

The class of elementary symmetrical polynomials from ***n*** variables over a field ***A*** is enough closely connected to the decomposition problem of global transition functions $\tau^{(n)}$ in the classical ***CA*** models. Let's denote the class of global transition functions $\tau^{(n)}$ of the classical ***1–CA*** models, whose local transition functions $\sigma^{(n)}$ are representable by elementary symmetrical polynomials, through $\Psi(n, a)$**.** It is easy to make sure, that any global transition function $\tau_j^{(n)} \in \Psi(n, a)$ excepting the first (P_1) and last (P_n) has at any rate the general presentation of the following kind**:**

$$\tau_j^{(n)} = \tau_1^{(n-j+1)}\tau_j^{(j)};\quad \tau_j^{(n)} \in \psi(j,a);\quad \tau_1^{(n-j+1)} \in \psi(n-j+1,a)$$

$$\sigma_1^{(n-j+1)}(x_1,\ldots,x_{n-j+1}) = \sum_1^{n-j+1} x_k \pmod a;\ \sigma_j^{(j)}(x_1,\ldots,x_j) = \prod_1^{j} x_k \pmod a\ \ (1 < j < n)$$

The question of representability of the first global transition function $\tau_1^{(n)} \in \Psi(n, a)$ was enough in detail discussed in section **7.1.** Inasmuch as the product is a certain kind of analogue of operation of addition, the results for function $\tau_1^{(n)} \in \Psi(n, a)$ are enough easily disseminated onto function $\tau_n^{(n)} \in \Psi(n, a)$. At that, for an integer $n \geq 2$ the global transition functions $\tau_1^{(n)}$ and $\tau_n^{(n)}$ are called *basic* functions of the set $E(n, a)$ of all symmetrical global transition functions $\tau^{(n)}$ from not more n variables defined in a certain state alphabet A. So, an arbitrary global transition function $\tau_j^{(n)} \in E(n, a)$, excepting basic functions can be presented in the form of a composition of two basic functions $\tau_1^{(n-j+1)}\tau_j^{(j)}$ $(1 < j < n)$. On the other hand, not any basic function of the set $E(n, a)$ is representable in the form of a composition of more simple *basic* functions. In addition, the presented arguments allow to formulate the following interesting enough result [24,43,82,102,106].

Theorem 156. ***Every global transition function $\tau^{(n)} \in E(n,a)$, excepting the basic functions, is representable in the form of the composition of two more simple basic functions of the following kind $\tau_1^{(n-j+1)}\tau_j^{(j)}$ $(1<j<n)$. Meantime, by far not each basic function of the set E(n, a) has similar representations in the form of composition of more simple functions.***

The suggested method of polynomial presentation of a local transition function $\sigma^{(n)}$ from n variables over a field $A=\{0,1,2,3,\ldots,a-1\}$ in case of a prime number a is naturally extended and onto the binary case $(a = 2)$. In the further we need certain concepts and definitions.

Definition 29. ***Elementary conjunction is called monotonous if it does not contain the negation of variables; in addition, the formula of the following kind:***

$$P(x_1, x_2, x_3, \ldots, x_n) = \sum_{k=1}^{s} \Theta_k \pmod 2$$

represents the Zegalkin polynomial, where Θ_k $(k=1..s)$ define pairwise–various monotonous elementary conjunctions over the set of various binary tuples $<x_1,\ldots,x_n>$. Moreover, the greatest of ranks of elementary conjunctions, entering in such polynomial will be called the degree of the Zegalkin polynomial.

Of the theory of boolean functions follows, that a boolean function can be presented in the form of the ***Zegalkin*** polynomial, i.e. a binary local transition function $\sigma^{(n)}$ can be *uniquely* presented by means of a certain ***Zegalkin*** polynomial of n variables of degree not above n. At the same time, for creation of the ***Zegalkin*** polynomial realizing any binary local transition function $\sigma^{(n)}$ a number of methods is used among which it is possible to mark the known method of indefinite coefficients which are similar to the case of ***a–***valued logics. Other method having a computer realization can serves as basis for proof of possibility of presentation of any boolean function by means of the ***Zegalkin*** polynomial [24,43,82].

In more general posing it is necessary to keep in mind, that binary ***CA*** models are the most convenient for research within ***Zegalkin`s*** algebra being a version of algebra of logic [7,24]. About that speaks the simple fact, that at the ***Zegalkin's*** algebra a function of algebra of logic from n variables can be uniquely presented by some reduced polynomial from variables of degree not higher 1, whereas its coefficients are elements of the binary field $\{0, 1\}$ $(k=1..n)$. In addition, the ***Zegalkin*** algebra admits natural generalization onto case of ***a–***valued logics, if a is a degree of a certain prime number. It allows to apply enough effectively apparatus of the polynomials theory over finite fields to research of both ***a–***valued logics, and classical ***d–CA*** $(d \geq 1)$ models for case of more general types of a finite alphabet A. Rather interesting discussions in this direction can be found, for example, in works [24,43,82,102,106].

Using the possibility of polynomial representation of a local transition function, defined in an alphabet $A=\{0,1, ..., a-1\}$ *(a – a prime number)* we receive the possibility to overprove, generalize or improve a number of earlier received results concerning the decomposition problem, but also essentially advance research in this direction. In particular, on the basis of research of class G of local transition functions that can be presented in the form of polynomials of the following kind:

$$\sigma^{(n)}(x_1, x_2, x_3, ..., x_n) = g\sum_{k=1}^{k=n} x_k \pmod a, \qquad x_j \in A;\ j = 1..n;\ g \in A \setminus \{0\}$$

over the above field A along with the class of all binary local transition functions, representable by the ***Zegalkin*** polynomials, it is possible to receive the following basic result [24,41,82,102,106].

<u>Theorem 157</u>. For prime a and n by far not each local function $\sigma^{(n)} \in G$ can be represented in the form of superposition of finite number of more simple functions in the same alphabet $A=\{0,1, ..., a-1\}$. For an arbitrary prime integer $n \geq 3$ the binary local transition functions $\sigma^{(n)} \in G$ can`t be

represented in the form of superposition of finite number of the simplest local transition functions $\sigma^{(j)} \in G$ in the same binary alphabet.

Of proof of this theorem follows, on basis of polynomial representation of local transition functions $\sigma^{(n)}$ by polynomials modulo a, except case of *composite* number a, it is possible to receive constructive decisions of the decomposition problem of global transition functions, not using the above concept of the basis. In addition, on the basis of this theorem the absence of a finite basis for the set of all global transition functions $\tau^{(n)}$ of classical ***d–CA*** $(d \geq 1)$ models is easily proved. In view of the told, the general criterion of the decision of the decomposition problem for an arbitrary global transition $\tau^{(n)}$ function, determined in a finite alphabet $A = \{0,1,...,a-1\}$ $(a$ – *a prime number)* can be received [12,43,82,102,106].

Theorem 158*. A global transition function $\tau^{(n)}$ can be represented in the form of a composition of a finite number of the simplest functions in the same alphabet A if and only if a polynomial P_n (mod a) appropriate to its local transition function $\sigma^{(n)}$ can be represented by a superposition of polynomials of the following general kind, namely:*

$$P_{n_k}(P_{n_{k-1}}(P_{n_{k-2}} \cdots (P_{n_1}) \cdots))) \pmod{a}; \qquad n_j < n,\ j = 1..k$$

Nonetheless, a method of decision of the decomposition problem on the basis of this criterion contains, at times, insuperable complexities for a rather large number of variables, though in simple cases can turn out a rather effective tool [24,43]. As the suggested method of research of ***CA*** models is based on basis of polynomial presentation of local transition functions $\sigma^{(n)}$ over a field A, later under alphabets A_p and A_c we shall understand the set $A = \{0,1,...,a-1\}$ with composite and prime number a accordingly. The aforetold, concerning this algebraic approach, refers to the alphabet A_p. Now we represent a number of interesting enough results concerning both the ***d–PDF***, and the ***d–GPDF*** which have been received on the basis of polynomial representations of local transition functions $\sigma^{(n)}$ over a field A_p [24,43,82,102,106].

Theorem 159*. For an arbitrary global transition function defined in an alphabet A_p the d–GPDF has the negative decision generally speaking, excepting the trivial cases.*

Thus, removal of restrictions on global transition functions entering in decomposition *(27)* of a global function $\tau^{(n)}$ does not change, generally speaking, the *negativity* of decision of the decomposition problem of this

type. As generally the ***d–PDF*** and the ***d–GPDF*** are nonequivalent, the following result represents particular interest and, first of all, from the standpoint of theoretical research of classical ***CS*** models [24,43,82,102].

Theorem 160. If for a d–dimensional global transition function $\tau^{(n)}$ the d–PDF and the d–GPDF are equivalent, then for such global transition function they are algorithmically solvable too.

Consideration of the decomposition problem in all its generality for the global transition functions defined in an alphabet A_p allows to receive the following important enough result [24,43,82,102,106].

Theorem 161. For each d–dimensional ($d \geq 1$) global transition function $\tau^{(n)}$, determined in an arbitrary alphabet A_p the d-PDF and the d-GPDF are equivalent and algorithmically solvable.

The result of theorem ***161*** allows to receive answers to some questions put up early. Moreover, results of theorems ***160*** and ***161*** reveal, that the structure of states alphabet ***A*** has a rather essential significance for the question of equivalence of the ***d–PDF*** and the ***d–GPDF*** ($d \geq 1$). Namely, for case of a finite alphabet A_p the equivalence and solvability of both problems takes place, while in general case of a states alphabet A_c these problems generally speaking are nonequivalent and question of their solvability has been considered on other base above. Further, a rather important result allows to substantially make clear an interrelation of solution of the ***d–PDF*** and the ***d–GPDF*** for general case of an alphabet $A=\{0,1,...,a-1\}$ (*a – a prime number)*; it gives a rather simple constructive decision of these two problems for global transition functions $\tau^{(n)}$ of the classical ***CA*** models [24,43,82,102,106].

Theorem 162. For a d–dimension global transition function $\tau^{(n)}$ defined in an alphabet A_p the d–PDF and the d–GPDF have positive solutions if and only if the global transition function $\tau^{(n)}$ can be presented in the form of composition $\tau^{(n)} = \tau^{(m)}\tau^{(q)}$ ($m, q < n$; $m + q - 1 = n$; $n > d + 1$) of the more simple two d-dimensional global transition functions determined in the same alphabet A_p.

The presented approaches to the representability problem of the global transition functions in the form of a composition of more simple global functions allows in some cases to solve the ***d-PDF*** more acceptably than by means of the procedure of exhaustive search. Notwithstanding, such approach despite of its labouriousness allows to solve both the ***d–PDF,*** and the ***d–GPDF*** in the most general case.

Questions of the general decomposition problem of abstract automata are directly connected to the optimization problem which in case of the classical ***CA*** models has two basic aspects, namely**:** ***(1) decomposition of an arbitrary global transition function into a finite number of the most simple global transition functions,*** and ***(2) decomposition of a global transition function into minimal number of more simple functions.*** It is simple to make sure in presence of a constructive solving algorithm for the ***PDF/GPDF*** for an arbitrary ***d*-**dimensionality, neighbourhood index and a states alphabet on the basis of the procedure of exhaustive search. The previous proofs of this fact illustrate only an admissibility of the used technique. Furthermore, both aspects of the above minimization problem have the constructive positive decision, generally speaking. Besides, on basis of theorem ***162*** it is possible to receive the following a rather interesting result [24,43,82,102,106].

Theorem 163. For an arbitrary integer $n > d + 1$ there are d–dimensional global transition functions determined in an alphabet A_p, for which the d–PDF and the d–GPDF are equivalent and have negative decision.

This result presents one more proof of negativity of the solution of both the ***d–PDF*** and the ***d–GPDF*** generally. In addition**,** on the basis of proof of theorem ***163*** it is possible to solve interesting question of estimation of number of global transition functions $\tau^{(n)}$ defined in a states alphabet A_p for which the ***d–PDF*** and the ***d–GPDF*** have positive decisions. The following base result, presenting independent interest too summarizes the research of this question [24,43,82,102,106].

Theorem 164. For nearly all d–dimensional global transition functions, defined in an arbitrary alphabet A_p both the d–PDF, and the d–GPDF possess the negative decision ($d \geq 1$).

Consequently, we receive a rather unexpected result, namely**:**

Share of all d–dimension global transition functions, determined in an arbitrary alphabet A_p that admit positive decisions of the d–PDF, and the d–GPDF equals zero ($d \geq 1$).

Thus, the study of the ***d–PDF*** instead of proof of existence of negativity of its decision turned into search of its rare enough positive decisions. Hence**,** relative to the composition operation the set of all ***d–***dimension global transition functions defined in an alphabet A_p**,** appears enough legibly differentiated and has good enough prerequisites for definition of an appropriate hierarchy of complexity of global transition functions.

7.4. The complexity problem for global transition functions in the classical *CA* models

Of our previous results of research of the ***d–PDF,*** and the ***d–GPDF*** it is possible to establish enough easy that among all ***d–***dimensional global transition functions $\tau^{(n)}$ $(n \geq d+1)$ which are defined in an alphabet A_p, a certain hierarchy of complexity of global transition functions $\tau^{(n)}$ can be introduced concerning the decomposition problem as follows.

Definition 30*. An arbitrary global transition function* $\tau^{(n)}$ *belongs to a s–level of complexity* [$s < n$; *designation:* $\tau^{(n)} \in L(s)$] *if and only if for this global transition function there are representations in the form:*

$$\tau^{(n)} = \tau_1^{(n_1^p)} \tau_2^{(n_2^p)} \tau_3^{(n_3^p)} \dots \tau_{k_p}^{(n_{k_p}^p)}; \quad n > d+1; \quad s = min\left\{ max\left\{ n_1^p, \dots, n_{k_p}^p \right\} | p = 1..m \right\}$$

i.e. for the global function $\tau^{(n)}$ *the PDF has the positive decision. If the PDF for a global transition function* $\tau^{(n)}$ *has the negative decision then such global function is ascribed to a complexity class L(n).*

On account of the above results, definitions and proof of theorem ***164*** it is possible to receive the following asymptotic relations having a lot of important enough applications in the ***CA*** problematics [24,82]**,** namely**:**

$$(\forall s \geq 2)(\#L(s) > 0); \qquad \lim_{s \to \infty} \#L(s) / a^{a^s} \geq 1 \qquad \textit{(a is a prime number)}$$

where #G is cardinality of a finite set G.

In addition, on the basis of theorem ***161*** and the complexity concept of ***d–***dimension global transition functions concerning the ***d–PDF*** and the ***d–GPDF*** $(d \geq 1)$**,** the following important enough result on solvability of complexity levels of global transition functions of classical ***CA*** models which represent, above all, theoretical interest at study of algorithmical properties of dynamics of classical ***d–CA*** $(d \geq 1)$ models as the conceptual models of the spatially-distributed dynamic systems takes place [24,82].

Theorem 165*. The determination problem of belonging of an arbitrary d–dimension global transition function* $\tau^{(n)}$*, determined in an alphabet A, to a s–level of complexity* $(s \leq n)$ *is algorithmically solvable.*

On basis of the introduced complexity concept of the global transition functions concerning the ***d–PDF*** *(**d–GPDF**)* we can receive interesting enough characteristics of global transition functions $\tau^{(n)}$**.** Of the above results follows, that we essentially used algebraic properties of a finite

alphabet A_p, since a local transition function can be uniquely presented by a polynomial modulo a of maximal degree $n(a-1)$ over the field A_p, and vice versa. Whereas in case of an alphabet A_c by far not each local transition function defined in an alphabet of such type can be presented in the polynomial form of the above kind. Namely, the following basic result takes place [24,43,82,102,106].

Theorem 166. ***For an arbitrary finite alphabet $A_c = \{0,1,...,a-1\}$ the share (W) of local transition functions $\sigma^{(n)}$, that are defined in an alphabet of such type and admit polynomial presentations modulo a, satisfies the following relation, namely:***

$$\frac{1}{a^{a^n-4^n}} \le W \le \frac{1}{a^{a^n-(a-2)^n}}$$

Of this result follows, for case of a composite integer a nearly all local transition functions, determined in an alphabet A_c can`t be presented in the polynomial form modulo a for large enough values of parameters n and/or a. In this connection the following question can be formulated:

Whether it is possible to define such algebraical system within which a polynomial representation for a local transition function determined in an alphabet A_c could be determined?

As a result of the fulfilled analysis, one interesting enough example of such algebraical system has been proposed; within this system nearly all local transition functions defined in an alphabet A_c, can be uniquely represented by polynomials modulo a *(Theorem* ***64****)* [5,145]. On basis of proofs of theorems ***64, 160, 161*** the following interesting enough result relative to the ***d-PDF*** and the ***d-GPDF*** in case of general states alphabet A_c of classical ***d–CA*** models has been received; this result represents a rather essential theoretical interest for the ***CA–***problematics as a whole along with the certain appendices [7,24,43,82,102,106,278,286].

Theorem 167. ***Concerning nearly all global transition functions, defined in an arbitrary alphabet A_c and whose appropriate local transition $\sigma^{(n)}$ functions admit polynomial presentations in the above form the d-PDF and d–GPDF are equivalent and algorithmically solvable.***

Thus, on the basis of theorems ***64*** and ***167*** the above results of research of the ***d-PDF*** and the ***d-GPDF*** are spread on nearly all global transition functions defined in an alphabet A_c. However, in spite of it we can`t so far spread the results obtained here on general case of a states alphabet

A which needs additional researches, except an approach basing on the above exhaustive search.

Along with the above algebraic method which is based on polynomial representations of local transition functions, for their formal researches the methods and results of the algebraic theory of many–valued logics, for example, the *Post* iterative algebras can be enough successfully used [7]. In this respect it is interesting to define and study a class of certain nonconventional algebraical systems within the framework of which satisfactory presentations of local transition functions $\sigma^{(n)}$ defined in an arbitrary states alphabet ***A*** are possible. In particular, as elements of an alphabet ***A***, it is possible to choose certain objects that been researched in the algebraic theory of numbers *(circular integers, divisors, etc.)* along with usage of results and methods of the advanced algebraic theory of numbers [7,24,82]. At last, research of different sets of global transition functions, closed concerning the composition operation will represent essential enough interest from many standpoints. In this connection a number of similar sets interesting from the applied standpoints and in context of dynamical, and extremal possibilities of classical ***CA*** models have been researched. Along with representation of global transition functions that is based on the composition operation, a number of other representations rather useful in many theoretical study of classical ***CA*** models and their applied aspects is interesting too. A lot of interesting enough examples of use of some other operations over global transition functions along with more detailed discussion of the ***PDF/GPDF*** can be found in our works [7,24,41-42,82,102,106].

On that the discussion of basis results, received on the decomposition problem of global transition functions in classical ***CA*** models that solve this problem and determine a lot of interesting questions for the further researches in this direction is being completed. At the same time, we once again shall note that negativity of decision of the decomposition problem along with its algorithmic solvability is proved simply enough whereas the used approaches for these purposes carry for problems of research of ***CA*** models more general character. Finally, the results that base on solution of the global decomposition problem compose a rather essential part of basic methods for research of ***d–CA*** $(d \geq 1)$ models and are used sufficiently widely.

Chapter 8. Some applied aspects of *CA* problematics

The sphere of appendices of ***CA*** models at all their generality now is sufficient extensive and needs a special consideration that lays beyond the present monograph. So, here we only in brief shall mark the most advanced fields of appendices**;** in addition, with the more detailing we shall touch applied aspects of the ***CA*** problematics *(with the emphasis on some results received by us)* in such fields as mathematics, biological and computational sciences. In addition, at present the ***CA*** concept along with properly independent interest with different degree of intensity is used as an important mathematical object of research in a rather broad range of appendices, namely**:** cybernetics and synergetics, applied and pure mathematics, pattern recognition and signal processing, coding theory, cryptography, theory of computing, mathematical and physical modelling, theoretical and mathematical biology, computer sciences, information processing, artificial intellect, urbanistics, geology, etc. In addition, various ***CA*** objects can successfully simulate the most general phenomenological aspects of the real world along with direct physical laws and processes at microscopic level [7,24,102]. ***CA*** models present a certain kind of the formal recursive worlds, whereas itself recursion is one of fundamental concept in computer science, mathematics, physics, biology, art and even in such field as linguistics. For this reason today the such models are enough claimed, attracting the increasing number of researchers from the very various fields.

In many cases the ***CA*** models represent an alternative approach to the analysis of dynamics of the complex systems instead of the differential equations. As the spatial differentiation onto elementary automata is an immanent property of ***CA*** models, they are simply irreplaceable there where the differential equations are ineffective or cannot be applied at all. In a lot cases there is no other way to find out dynamics of an initial configuration of a certain dynamical system, than to simply simulate its behaviour by means of the appropriate ***CA*** model [7,22,24,82,102,140].

Such fundamental properties as *locality* and *homogeneity* that a priori is provided at level of ***CA*** axiomatics along with reversibility property of dynamics at programmable level allow to consider the ***CA*** models as a perspective enough environment of physical modelling. These and a lot of other essential circumstances carry over the ***CA*** problematics to the serious enough interdisciplinary level, turning the problematics into a certain conceptual environment of modeling, description and research

of phenomena, processes, and objects from various naturally–scientific fields and some other fields.

At the same time, possibilities of use of ***CA*** as a nontrivial environment of physical modelling allow to consider them much more widely than simply independent objects for research. In this connection they can be considered as a new perspective *conceptual* approach to organization of the reversible computing processes in context of research of the general theory of computations and working–out of new perspective means of computer facilities [7,24]. Now, it is quite possible to ascertain that the ***CA*** concept presents a rather perspective environment of modelling for realization of the *conceptual* and applied aspects of spatially–distributed dynamical systems of which the physical and biological systems along with various systems of parallel information processing are the most essential prototypes. A rather full representation in the field of applied aspects of the ***CA*** problematics can be found in [7,22,63,70,71,102,106, 278,286], and in numerous original sources quoted in them.

After a rather brief discussion of certain mathematical appendices of the ***CA*** concept we shall dwell on its application for research of interesting enough problems from combinatorial analysis, theory of numbers and discrete mathematics though the mathematical appendices presented in this section are by far not exhaustive. An interesting enough survey of appendices of the ***CA*** concept in mathematics, including our results, can be found, for example, in [7,22,24,43,63,70,71,82,102,106,278,286].

8.1. Solution of the *Steinhaus* combinatory problem

Polish mathematician ***H. Steinhaus*** more *85* years ago has formulated a rather interesting combinatory problem named «*pluses–minuses*» whose essence in our terminology is reduced to the following [10,24,41,82,102]. Let $c(k) = p(1,1)p(1,2)p(1,3)\ \ldots\ p(1,k)$ will be the first string of the binary elements $p(1, j)\in\{0, 1\};\ \ j=1\ ..\ k.$ Furthermore, values k are chosen of the set $M=\{3+4t,\ 4+4t\ |\ t=0,1,2,3,4,\ \ldots\}$ only. Then elements of the j–th string of length $(k-j+1)$ are calculated in terms of elements of the $(j-1)$-th string of length $(k - j + 2)$ according to the following simple recurrent rule:

$$p(j, i) = p(j–1, i) + p(j–1, i+1) + 1 \pmod 2; \qquad (i=1\ ..\ k–j+1;\ j=2\ ..\ k)$$

It is simple to make sure, that as a result a triangular shape $T(k)$ which will consist of $N=k(k+1)/2$ symbols $\{0,1\}$ will be obtained. Inasmuch as N are even numbers for values $k\in M$, we can formulate the following a rather interesting question, namely:

Whether it is possible to determine for an arbitrary allowable value k from M the shapes T(k) that will contain the same number k(k+1)/4 of each of symbols «0» and «1»?

In case of the positive answer we shall speak, that similar string ***c(k)*** is a solution of the *Steinhaus* problem for the specified number ***k;*** further for brevity the term «***S–problem***» is being used. A number of professional mathematicians along with a lot of amateurs was involved in solution of the ***S–***problem which have received a number of interesting enough results. Meanwhile, solution of the general ***S–***problem remained open. And only on the basis of a number of results on classical ***2–CA*** models together with computer modeling it was possible to obtain a number of new rather interesting results along with the exhaustive decision of the general ***S–***problem [10,41,82,102,223]. For the further we need a number of the basic definitions, notions and denotations.

Definition 31. Solution S(k) of the S–problem for an arbitrary integer k of $M = \{3+4t, 4+4t \mid t=0,1,2, ...\}$ ***is named the derivative solution if it can be presented in the form of concatenation*** $D(k) = S(k_1)S(k_2)S(k_3) ... S(k_n)$ ***of solutions for values*** $k_j < k$ ***at*** $\sum_j k_j = k$ $(j=1..n)$ [***denotation: D(k)***]. ***Let S(k) is a set of various solutions of the S–problem for some value k. It is easy to make sure that S(3) = {000, 011, 110, 101} and S(4) = {1101, 1011, 0011, 1100, 1010, 0101}; these two sets of solutions are named the basic sets. A derivative solution D(k) is named basic solution if in its D(k)–representation the defining relationship*** $S(k_j) \in S(3) \cup S(4)$ $(j = 1..n)$ ***takes place*** {***we will use the designation: B(k)***}.

Sets of derivative and basic solutions *(together with their elements)* of the ***S–***problem for any integer ***k*** are denoted ***D(k)*** and ***B(k)*** accordingly. We can mark, the basic solutions represent an especial interest because they consist of elementary base solutions and to some extent illustrate one of interesting examples of the self–complication phenomenon. The overall research in terms of a classical ***2–CA*** model defined by a special method has allowed to receive a few rather interesting properties of solutions of the ***S–***problem that reveal their internal structure while a rather general result in this direction the following basic theorem represents [43,82].

Theorem 168. Let S(k), D(k) and B(k) – sets of all, derivative and basic solutions of the S–problem for some integer $k \in M$ ***accordingly; then for each admissible integer*** $k > 2$ ***the set S(k) is nonempty and for arbitrary admissible integer*** $k > 10$ ***there is the relation*** $\#S(k) > \#B(k)$, ***where #R is cardinality of an arbitrary finite set R.***

Thus, this theorem gives the exhaustive decision of the ***S–***problem, that has been formulated more ***85*** years ago. Use of the combined methods *(the theoretical analysis of the appropriate* ***2–CA*** *models along with computer modelling)* has allowed to obtain a number of interesting estimations for all types of solutions of the above ***S–***problem [24,41,82,102,106,223].

<u>Theorem 169</u>. For each integer $k\in\{3+4t,\ 4+4t \mid t=0,1,2,3,...\}$ the following determinative relations there are, namely:

$$\#S(k) \succ 2^{k-r(k)} \quad for\ \ r(k)\le[k/2]; \qquad \#B(k) \ge \begin{cases} 2^{3t-2}, & if\ \ k\in\{3+4t \mid t=1,2,3,...\} \\ 2^{3t}, & if\ \ k\in\{4+4t \mid t=1,2,3,...\} \end{cases}$$

Similar determinative relations are valid and for derivative solutions D(k) of the S–problem too.

It is necessary to note, the ***S–***problem can be essentially generalized as follows. Instead of two symbols {***0, 1***}, an alphabet ***A***={***0,1,2, ..., a–1***} that is typical for classical ***CA*** models, is used, whereas elements of a string ***c(k)*** are chosen of the alphabet ***A.*** In addition, integers ***k*** are chosen of the set ***M***={***3+4t, 4+4t*** | ***t=0,1,2, ...***} only. Then elements of the ***j–***th string of length ***(k-j+1)*** are calculated in terms of elements of the string ***(j-1)*** of length ***(k – j + 2)*** according to the following simple recurrent rule**:**

$$p(j,i) = p(j-1,i) + p(j-1,i+1) + 1 \ \ (mod\ a); \qquad (i=1\ ..\ k-j+1;\ j=2\ ..\ k) \qquad (30)$$

It is simple to make sure, that as a result a triangular shape ***T(k)*** which will consist of $N=k(k+1)/2a$ symbols of an alphabet ***A*** will be obtained. Inasmuch as ***N*** are integers for values $k\in M$ then we can formulate the following a rather interesting question, namely**:**

Whether it is possible to determine for an arbitrary admissible value k of M the shapes T(k) that will contain the same number m = k(k+1)/2a of each of symbols of an arbitrary alphabet A={0,1,2,...,a–1}?

The ***S–***problem in the given posing is named the generalized. It is quite reasonable to assume, the generalized ***S–***problem can receive the wider interpretation, namely**:** an neighbourhood index ***X***={***0,1, ..., n–1***} for it is supposed arbitrary. The allowable integers ***k*** in this posing are chosen of the set $M^*=\{n+t(n-1) \mid t=0,1,...\}$, and step**-**like shapes ***R(k)*** that contain $L=[(n-1)t^2+(3n-1)t+2(n+1)]/2$ symbols of the alphabet ***A*** are considered. At the made assumptions the general ***S–***problem will be reduced to an question of existence for each allowable integer ***k*** of a shape ***R(k)*** which will be contain equal number L/a of entries of each symbol from above general alphabet ***A.*** Meanwhile, one generalization of methods of the decision of the classical ***S–***problem allows to formulate the result being

basic in this direction [24,41,82,102,106,223].

***Theorem 170**. For an arbitrary alphabet $A = \{0,1,2,...,a-1\}$ and allowable integer $k \geq 2a$ the generalized S–problem has at least $2a$ solutions. The number G(k) of solutions of the generalized S-problem for $A=\{0,1,2\}$ and allowable integers $k \in \{2+3t, 3+3t \mid t=1,2,...\}$ equals $G(k) > 2^{k-1}$. For each allowable integer $k \in M^*$, neighbourhood index $X=\{0,1,2,...,n-1\}$ and the alphabet A the general S–problem has at least $2k$ solutions.*

In conclusion it is necessary to note, the results of theorems ***168–170*** can be generalized to cases of the higher dimensionality and recurrent rules ***(30)*** of the more general kind too. In addition, a number of interesting enough results of research in this direction can be found in [24,102,223].

8.2. Solution of the *Ulam* problem from number theory

Heuristic research of the growth problem already for case of two and three dimensions discloses all variety of the growing figures which it is enough difficultly to satisfactorily characterize by the formal methods. Therefore, with the purpose of simplification of research of the given problem ***S. Ulam*** has tried to introduce the corresponding definitions in one–dimensional case with hope, that certain basic properties of so–called *sequences of uniquely defined sums (**SUDS**)* will help to make clear picture in this direction [143,144,220-222]. However, the given problem has received a rather large popularity and in one's time has attracted attention of a lot of researchers not only from the standpoint of a formal problem of growth, but, first of all, in connection with number theory. For the last case the problem represents even more interest. The basic essence of the given problem is a rather simple and can be represented as follows, previously introducing requisite notions.

On the set $M = \{1,2,3,4,...\}$ of positive integers a simple binary operation is defined as follows $\varphi: x+y \Rightarrow z$, where $x,y,z \in M$. Elements z make up a subset $M^* \subset M$. The following restrictions are put on the operation φ:

(1) starting with integers a and b $(a<b)$ all subsequent elements $z = x + y$ are obtained as sum of any two previous elements $x, y \in M$ of the earlier obtained sequence, however we not include in them those sums which can be obtained by more than one way; *(2)* themselves numbers are not added, and at addition operation the extreme element of the generated numerical segment (a, b) of the ***SUDS*** should participate. A numerical sequence obtained thus will be named the ***SUDS(a, b)***. In particular, the

first twelve elements of the ***SUDS(1, 2)*** the following natural numbers make up, namely**:** ***1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28.***

Pairs of the adjacent elements in the ***SUDS(a,b)*** which differ by a value ***p = p(a, b)*** are named the *twins*. Further any sets of twins pairs we shall denote ***T(p).*** So, ***T(a + b)*** is a set of twins pairs of the kind ***p(a, b) = a + b.*** The original posing of the ***S. Ulam*** problem consists in the definition of a cardinality of the set ***T(2)*** for the ***SUDS(1, 2),*** i.e. pairs of the adjacent elements of the set ***M**** that differ by value two. Thereupon, ***S. Ulam*** has put forward the hypothesis about infinity of the set ***T(2).*** Inasmuch as we investigated this problem in more general posing we need a number of additional notions and definitions.

In addition to the sequence ***SUDS(a, b)*** we shall consider the sequence ***SUDS1(a, b)*** which differs from the ***SUDS(a, b)*** only by absence for it of obligatory participation in binary **φ-**operation of the extreme element of already generated numeric segment ***(a,b)*** for the ***SUDS.*** In addition, the both presented variants of the ***SUDS*** along with independent interest in number theory have a lot of rather interesting biological interpretations connected to the growth problem formalized for simple ***1–***dimensional case. To tell the truth, this formalization is tense concerning the natural processes of growth, consequently each interpretation of the obtained results is relative in many respects. Concerning this problem we have investigated a number of questions of behaviour of the ***SUDS;*** they can be formulated as follows [1,2,8,9,13,24,31,33,82,102,106,272]**:**

- ***determination of partial densities of a SUDS, starting with the given element;***
- ***growth degree of elements of a certain SUDS, starting with the given element;***
- ***change of partial densities of twins pairs concerning the SUDS;***
- ***change of distance between the nearest twins pairs in the SUDS;***
- ***rating of number of twins pairs in the given segment of the SUDS.***

In addition, all enumerated questions concern both sequences such as ***SUDS(a,b),*** and ***SUDS1(a,b)*** for any positive integers ***a*** and ***b.*** The basic results obtained in this direction can be characterized as follows. First of all, we have established that a sequence ***SUDS1(a, b)*** should possess the infinite set of twins pairs at least of one of three types**:** ***T(a), T(b)*** or ***T(a + b).*** It is shown that if a_k is ***k–***th element of the ***SUDS1(a, b)*** then ***k–***th element of the sequence ***SUDS1(da, db)*** will be a number da_k**.**

This property is valid also for sequences such as ***SUDS(a,b).*** Fully other

picture there is for sequences ***SUDS(a, b)***, where practically exhaustive decisions have been obtained for a number of variants of the *generalized* ***Ulam*** problem. For example, sequence ***SUDS(1, b)*** for $b \geq 5$ possesses the infinite sets ***T(b)*** and ***T(b+1)*** of twins pairs, whereas its elements a_k are calculated according to the simple recurrent formulas, namely:

$$a_k = \begin{cases} b+k-2, & if \quad k \in \{3,4,...,b+2\} \\ 4*b-2, & if \quad k = b+3 \\ (k-b+1)*b+[(k-b-3)/2]-2, & otherwise \end{cases}$$

The density of this sequence concerning the set ***N*** equals $\rho = 2/(2b+1)$. The sequence ***SUDS(a, b)*** for $a>1$ and $b/a - [b/a] > 0$ has the infinite set ***T(a)*** of twins pairs, while its density relative to the set ***N*** equals $\rho = 1/a$. Elements of this sequence, starting with number $k \geq 3$, are calculated by the simple recurrent formula $a_k = b+(k-2)a$. In our works [8,33,102] it is possible to find a lot of other interesting examples of the ***SUDS(a,b)*** for which it is possible to determine explicit functional relations of the kind $a_k = F(k,a,b)$ and to find out a lot of other rather interesting behavioural properties of numerical sequences of such type.

For description of behaviour of the ***SUDS(1, 2)***, i.e. the ***S. Ulam*** classical problem and consequently of the ***SUDS(a, 2a)*** we shall act as follows. Along with the sets ***K*** and ***A(k)*** of numbers ***k*** and values of elements a_k of the sequence accordingly, we determine the set ***P*** of differences of the kind $\Delta_k = a_{k+1} - a_k$ of changes of values of elements of the ***SUDS***. It turned out that structure of the above set ***P*** is much more convenient for study. Namely, since the number $k = 14$, in the set ***P*** quite definite regularity is being already traced.

An element $\Delta_k \in P$ is named the ***jump***, if $\Delta_k \neq \{2, 8\}$. An element $\Delta_k \in P$ is named the ***growing jump***, if it has the maximal value among all jumps Δ_k *(j<k)*. The growing jumps are not limited from above, whereas their values grow with growth of ***k***–value. An interval out of four elements $<2, 8, 2, \Delta_k>$ (Δ_k – *an jump not necessarily the growing jump)* is named the *basic* interval. It is possible to show, that since the number $k = 14$, the set ***P*** will consist only of *basic* intervals adjoining to each other, i.e. ***P*** can be represented in the form of the following sequence, namely:

$$P = \{<2,8,2,\Delta_1>, <2,8,2,\Delta_2>, <2,8,2,\Delta_3>, <2,8,2,\Delta_4>, ..., <2,8,2,\Delta_j>\}$$

Hence, research of the set ***P*** is reduced to research of behaviour of the subset $P1 = \{\Delta_k\}$ of its jumps. It is possible to show, that distribution of

the growing jumps in the set P submits to the quite definite regularity, allowing to determine the following functional relation $a_k = F(k, d)$ for the $SUDS(d, 2d)$ [1,2,8,9,13,24,31,33,102,106,272], allowing to obtain the full decision of the *Ulam* classical problem.

Theorem 171. ***The SUDS(1,2) has infinite set T(2) of twins pairs whereas its density concerning the set N of integers is defined by the relation:***

$$\lim_{k \to \infty} \frac{4*(2^{k+2} - 4) + 14}{(12 + P_o)*(2^{k+2} - 4) + 72*P_o*5^{2k-10}} = 0$$

In particular, for experimental research of the $SUDS$ of various types a special simulation program, which has allowed to receive a lot of very interesting empirical results has been developed [33,82]. For particular, it has been shown that partial densities of the $SUDS(1, 2)$ monotonically tend to a limit in compliance with the empirical formula $r(k) = 14/k^4 + m$ $(0 \le m < 0.002)$, whereas rate of convergence to the given limit is defined by the empirical formula $\Delta(k) = 1150*k^{-2.31}$. A number of certain other estimations and discussions in this direction can be found in [33,102].

The results obtained by us on the *Ulam* *generalized* problem along with pure mathematical interest are of interest as well for research of formal models of growth in elementary 1–dimensional cases, and also from the standpoint of the applied theory of *complexity* of computing algorithms and the applied aspects of the CA problematics as a whole.

Research of a lot of other types of numerical sequences, that in a certain measure have rather formal analogies with process of growth and other biological phenomena in 1–dimensional case, represents a quite certain interest. At present, in this direction we investigated a lot of interesting enough numerical sequences that with a certain degree of formalization can be associated with certain biological phenomena in 1–dimensional case. Particularly, on the basis of specifical types of classic 2–CA models we researched some interesting nontrivial arithmetical properties of the generalized *Pascal* triangle (GPT) and *Fibonacci* numbers [24,43,82,102]. Since *Pascal* triangle and *Fibonacci* numbers have close connection with binomial coefficients, factorials, a number of important mathematical formulas and tables, therefore this research represents a certain interest for parallel computations based on CA models. In addition, the *linearity* of local transition function of CA model which generates GPT allows to try to relate purely mathematical properties of such 2–CA models with universal reproducibility in the *Moore* sense of finite configurations [82].

8.3. Certain applied aspects of *CA* models in biological sciences

The latest years are characterized by utterly intensive penetration of the newest mathematical concepts and approaches in biomedical sciences. Above all, that is linked to the further becoming of both the theoretical biology and the mathematical biology, along with mass application of the modern computing means, allowing with high enough visualization degree to investigate various biomedical models. In addition, the most intensive, interesting, and perspective attempts are made for study of *evolutionary* biological systems. One of the most intriguing and complex fields of the modern biology – *developmental biology* of alive systems has been subjected to the mathematical analysis.

Considering the questions of *discrete modelling* of developmental biology on the basis of ***CA*** concept, it is necessary to have in mind, at the same time, that this approach can be rather successfully used for creation and study of a lot of other biologically motivated formal models. Moreover, it is necessary to have in mind that the ***CA*** concept has essentially more general character, involving both the fundamental medical and biologic problematics along with a lot of biophysical directions. In addition, the more detailed discussion of similar questions can be found in literature cited in [7,24,102] and also in *Internet* by the appropriate key phrases.

The detailed enough consideration of applied aspects of ***CA*** models by way of an environment of research of biologically motivated problems can be found in our works [24,102] whereas here only their brief sketch is submitted. In particular, with the basic prerequisites of such model approach to biology of development together with its historical stages the reader can familiarize oneself in [7], in numerous sources, quoted in them, along with well–known journals such as «*Mathematical Biology*», «*Biophysics*», «*Mathematical Biosciences*», «*Journal of Theoretical Biology*», etc. Our interests in this respect were in the sphere of modelling of the developmental biology from the cybernetical standpoint with a certain emphasis on discrete aspect of modelling.

Growth and ***regeneration*** of an organism are carried out via continuous process of self–reproduction of cells in an organism, mainly, while, the differentiation of cells during the growth is considerably more complex for comprehension inasmuch as in the general opinion of the modern biologists all cells contain the same set of genetic rules – new cells will be genotypically identical to the predecessors. Thus, the most important

question arises, namely**:** ***How organism cells which different from each other develop into carefully produced and stable spatial forms?***

Moreover, all development is strictly controllable from within in such a manner, that various parts of a developing organism develop in certain proportions relative to each other and organism as a whole. In addition, the organism during development and even after in significant limits is capable to eliminating the damages caused by the external reasons, i.e. the organism in the certain limits is capable to regeneration. Naturally, the development uses strict mechanisms of the control, regulation and adaptation. Meanwhile, to date we do not know the best answer to all these questions, except as solution of the similar problems for artificial systems, i.e. using of the certain modelling principle for research of the development. In addition, it is necessary to have in mind, the studying of a development phenomenon has led a lot of researchers to a rather important conclusion that an organism can`t be considered as a certain artificial machine **–** an *automaton*.

Therefore from the standpoint of biological science, cybernetics and the general systems theory it is extremely important to try to elucidate the following important gnoseological question**:** ***Whether can an artificial automaton develop similarly to alive systems in general, and how we can secure that?***

The essence of the given question enough in detail is discussed in [102]. Biological development includes two important phenomena**:** ***growth*** of an organism along with ***differentiation*** of cells making up the organism. It is known that growth is simple increase of an organism size, mainly, due to controlled self**-**reproduction of its cells whereas differentiation is much more complex phenomenon, therefore it is enough expedient to distinguish at least two its types **–** *spatial* and *phenotypic* differentiation, which *M. Apter* has named the *functional differentiation* [273,274].

So, in a growing tissue it is possible to differentiate change of the form and *configuration* of *intercellular connection (spatial differentiation)* along with increase at differentiation of separate types of its cells *(phenotypical differentiation)*. It is necessary to mark, that for spatial differentiation in developmental biology the settled term «*morphogenesis*» exists, but for the purposes of our modelling the first term as a more natural is used. Certainly, the phenotypic differentiation is presented and in the spatial differentiation, however, with the purpose of the greater transparency of modelling problems this will not be taken into account by us under the condition of the predominating role of spatial differentiation [31].

But a developing organism is characterized not only by a possibility of achievement of complex spatial and phenotypic differentiation, but also in the greater or smaller extent by presence of ability to regulation and regeneration during the development and the subsequent functioning. Under the regulation is understood as an opportunity of an organism to develop into a normal person even in case of occurrence of obstacles during a development, for example, if removal or reorganization of its cells take place, excepting critical cases, lethal for the organism. While under the regeneration we understand a possibility of the organism to restore in the certain limits any infringement which the organism has obtained during his own full development.

Regardless of importance of understanding of processes of biological development including *spatial* and *phenotypic* differentiation**,** *regulation* and *regeneration***,** along with phenomenon of self**–**reproduction the first attempts to achieve success in the given direction can be attributed to the *first* stage of the model approach that is characterized by modelling of *separate* phenomena of the general development as a whole. The base role of the *first stage* of biological modeling can be characterized by that satisfactory formalization that has been given to a lot of rather complex phenomena of the general process of biological development, and later on was being corrected on the basis of the analysis of numerous formal models. The subsequent analysis of a number of models has allowed to look in a new fashion to certain important regulator mechanisms of the development. However we had a number of models, non**–**linked by the general theoretical base. Of course, this position did not assist formation of the unified apparatus of modelling in the developmental biology.

Meanwhile, already within the framework of the first stage two formal apparatus of modeling of certain phenomena of biological development have arisen**:** *cellular automata (**CA**)* and the *Lindenmayer parallel grammars*. The ***CA*** models for the first time have been used by *John von Neumann* for research of the self**–**reproduction problem [15,16,275]**,** while parallel grammars for the first time have been introduced by *A. Lindenmayer* [7, 70,188-192] for modelling of processes of morphogenesis of plants and subsequently has received the name «***L–systems***». At present, ***CA*** and ***L–*** systems represent the most general and popular apparatus of discrete modeling in the developmental biology, while the mathematical theory of formal means of modeling is very well advanced and allows to study by formal methods at cellular level such phenomena of development as growth, self**–**reproduction, differentiation, regulation and regeneration.

Along with these problems the ***CA*** models allow to study satisfactorily a lot of the questions of development such as complexity of developing systems**;** processes controlling the growth, regulation and regeneration**;** the necessary and sufficient conditions of regeneration and regulation, stability of the development, etc. [31-33]. Meantime**,** the apparatus of ***CA*** models causes also a number of rather essential complexities at research of certain biologically**–**motivated phenomena. The basic difficulties are connected to large enough sensitivity of ***CA*** models to such important factor as dimensionality, and with serious restrictions on possibility of cell fission within an arbitrary modelled developing organism, i.e. with existence of a rather rigid system of coordinates in the ***CA*** models.

Taking into account essential complexities of modelling in ***CA*** of some biological phenomena and processes, *A. Lindenmayer* has introduced the systems known nowadays as ***L–****systems* [7,24]. Within the framework of the ***L–***systems for modelling of morphogenesis and growing structures *A. Lindenmayer* has offered the branching algorithms, whereas a lot of investigators for modelling of development and growth has introduced the graphic generative systems [191]. On the basis of ***L–***systems a lot of interesting enough growing algorithms has been realized**;** a rather good review can be found in fine work [192]. Lately on the basis of ***L–****systems* the increasing number of models of both actually growth and of growth accompanying the general *phenomenon* of development is being worked up. Meanwhile, in spite of the large preferability of the ***L–***systems as an environment of modeling in the developmental biology, the ***CA*** models represent interesting enough means of research of many processes and phenomena of development as they well meet the cellular nature of the biological systems and allow to create effective models of development which are qualitatively visualized by means of the modern computers.

It is possible to ascertain with all definiteness, that the ***CA*** models and the ***L–***systems well supplement each other, stimulating creation of the modern apparatus inheriting the best features of both specified systems of modelling of processes in the developmental biology. A definition of ***L–***systems along with more detailed discussion of their applicability for solution of problems of biological modeling can be found in [24,82,102].

Particularly, it is shown that ***L–***systems enough essentially expand one**–**dimensional ***CA*** models in sense of sets of words generated by them. In addition, from the standpoint of biologic adequacy the ***L–****systems* obtain rather satisfactory interpretations, perfectly showing themselves at the description of a number of biological processes**;** they are now the most

developed, perhaps, in the mathematical plan and represent adequate apparatus in the biological attitude for many problems of the discrete modelling in the developmental biology. Concerning the methodology, strictly speaking, the ***L*–systems** are more abstract than the ***CA*** models because they are not linked rigidly with co–ordinates and, as a matter of fact, they are one of types of parallel formal grammars that now are intensively studied [5,7,24]. In addition, it is necessary to have in mind, the ***CA*** models can be considered as a certain type of parallel formal $\boldsymbol{\tau_n}$**–** grammars *(Chapter* ***5****)*. In [24,31-33,82] the ***CA*** models and the ***L–****systems* are analyzed in detail in a context of potential possibilities for modeling in the developmental biology, disclosing a lot of defects of both systems.

So, the defects intrinsic to both systems suppose the urgent necessity of continuation of works on working out of a mathematic apparatus, most adequate to problems of biological modelling. In this direction a rather intensive study with essential use of both interdisciplinary approaches and attracting of a lot of the known experts working in adjacent fields are conducted. The detailed discussion of possible ways of making up of a new apparatus for discrete modelling along with the predictable stages of making up and the further development of the discrete model approach in the developmental biology and in certain other fields of the mathematical biology can be found in [24,31-33,82,102,106].

With other aspects of modeling of the developmental biology the reader can familiarize rather in detail in literature cited above and in numerous sources contained in it. We shall start discussion with modeling of such general phenomenon for all alive substance as the *self–reproduction*. The problem made up a corner stone of the basic discussions concerning the opportunities of automata and alive systems, and was one of the basical catalysts of stimulation of study on abstract automata as the analogues or even substitutes of the mature alive systems, including the man. The actuality of this problematics has a rather great and extremely multifold character, including and extremely complex in every respect a question such as problem of a life origin along with its destination in the general system of the universe.

The above phenomenon of *self–reproduction* is the most typical feature of animate nature and it is no wonder that the first attempts of cybernetic modelling have touched this process. To detailed study of a possibility to embody in an automaton the process of self-reproduction by the first, perhaps, has proceeded *J. Neumann* who has offered a certain *conceptual* approach to solution of this problem [15]. The results obtained further

on molecular genetics detect startling *analogies* between elements of the *Neumann* self-reproducing automaton and processes in an alive biologic cell. A lot of researchers have considered the *self–reproduction* problem from purely mathematical standpoint. Among these researchers can be noted such as *E. Moore, J. Myhill, A.R. Smith, S. Ulam, R. Laing, and others*.

However, *G. Herman* has shown, that this requirement does not protect from cases of a rather trivial self–reproduction [82,276]. This important circumstance speaks about necessity of more attentive approach to the choice of requirements to configurations of ***CA*** models which should be interpreted as certain biological processes and phenomena, whereas to the ***CA*** as a formal environment of modelling of biologic processes and phenomena. Therefore, the following important enough question seems a rather actual, namely: ***whether exists other quite satisfactory measure of complexity for the self–reproducing configurations in the CA models that is not based on the concept of universal computability?***

Certain interesting grapho–topological approaches in this direction are submitted in works [24,82], and also in connection with the complexity problem of finite configurations in ***CA*** models *(Chapter **4**)*. In spite of the certain successes in clearing up a question of the complexity concept of finite configurations in the ***CA*** models, we should well present the real problems put forward by the necessity of certain satisfactory biological interpretations of the self-reproduction phenomenon. So far that is open and important enough problem, whose decision in the near future, in all probability, is not foreknown.

As against *J. Neumann E.F. Moore* in research of self-reproduction on the base of ***CA*** models does not bind oneself by the universal computability [75]. The *E.F. Moore* definitions cover only the most general essence of the reproduction process, allowing to concentrate our attention only on it *(Section **3.2**)*. In the given direction a lot of rather original models has been received but all of them are of interest only from the most general standpoint on the complicated process of self-reproduction; in addition, these models can't receive satisfactory enough biological interpretation. The detailed discussion of these questions can be found in [24,41,82].

Particularly, existence of classical ***CA*** models for which on the basis of a finite configuration in aggregate can be generated all set of finite block configurations not only gives a rather interesting example of classical ***CA*** models possessing the property of universal reproducibility in the *Moore* sense but also brings an attention to the question concerning the *adequacy* of biologic interpretations of such extremely important process

as self-reproduction. Similar moments should be kept in mind in case of biological interpretations of ***CA*** models for their adequacy to various modeled processes. Multi–aspect discussion of a lot of other questions of simulation of biologic phenomenon of self–reproduction and use for these purposes of opportunities of classical ***CA*** models can be found in [7,24,43,82,102,106,278,286] and in references contained in them.

The above-mentioned and many other models of self–reproduction in a sense, in our opinion, are related to copying of the genetic information in a cell nucleus, but not to real self–reproduction of organisms [41,82]. Therefore in this direction rather serious researches still are in prospect. Depending on analysis of self–reproduction models existing today and approaches to modelling of this phenomenon follows, that they present the certain interest from standpoint of self–reproduction of robots, but not alive organisms. But that is our private standpoint at this question.

The phenomenon of growth in that or another extent is inherent in any evolutionary system. From biological standpoint the growth, perhaps, is one of the most simple components of the general development, but a lot of open questions exists and here. The growth is one of immanent properties of alive because for survival of any species the individuals making up it should reach a quite definite weight, without which the performance of all necessary vital functions by them is impossible. An individual has the finite sizes, achieved in process which is named the ***growth.*** The generally accepted standpoint – the sizes of organisms are fixed genetically. Now, there is not a common opinion about influence on process of growth of various factors such as metabolical, ecological, thermodynamic, etc., along with degree of abstraction from the partial phenomena. Without penetrating in deep essence of the process of the growth as one of the base components of the general development, at a formal level we shall consider only ***3*** basic problems characterizing the growth as an independent biologic phenomenon presenting indubitable interest from many standpoints.

One of the basic problems of the development – ***How can be reproduced a certain organism, using possibly least number of instructions?*** That is rather important from the standpoint of understanding of development in alive systems as the zygote should be somewhat simpler, than that organism itself to which it gives a life. The second problem touches the ***restrictions of the sizes of an organism,*** growing in various conditions, if such process is completely caused by a certain genotype of cells, self–reproducing during the growth. The third range of questions touches

study of such growth when a ***spatial differentiation*** during continuous self–reproduction of an initial set of instructions without influence of a certain external influence can take place.

For answer to that and other questions the various formal models of the growth have been suggested. The current variety of models of growth is explained by prevalence of mechanisms of the restriction of growth of a developing organism that are widely spread as well as itself process of self–reproduction. In addition, research of mechanisms of regulation of growth is urgent for comprehension of the *morphogenesis* phenomenon since the growth can be considered as one–dimensional analogue of the morphogenesis [2,5,8,24,31]. The reader can familiarize oneself with the widely enough presented problematics of continuous models of growth in the collective monograph [31] and very extensive bibliography cited in it, and also in the bibliography [7,24,82,102,106,278,286].

The certain simplest models of growth were investigated by means of computer modelling by *S. Ulam* and his colleagues which were among the first initiators of study of the growth phenomenon by the discrete apparatus, however much earlier this problem was being investigated by a number of researchers *(A. Thompson, L. Bertalanfi, etc.)* with use of the continuous apparatus of the modelling [7,8,82]. The discrete growth models studied by group of *S. Ulam* are the most suitable for description of certain abiotic systems similar to crystal structures, simple plants or simple organic molecules, than for real complex biological systems. In spite of that the work with similar models has allowed to clear up a lot of questions of the growth of forms in case of different restrictions such as logical, geometrical and certain others [143,144,220-222,272].

At working with discrete growth models of *S. Ulam* we have used the apparatus of classical ***2–CA*** models, that has allowed to receive a lot of new interesting enough properties of discrete process of growth that is subjected to various recurrent rules, allowing to study the phenomenon by formal means [7,24]. The further development of the ***CA*** concept as a basis of *discrete* modeling of the growth phenomenon has been received by *J. Buttler* and *S. Ntafos* [207]. From the standpoint of study of growth process the indubitable interest the problem of excitations spread in ***CA*** models with *refractority (**CAR**)* presents. On the basis of this class of ***CA*** models a number of interesting models of excitable environments has been proposed; part of them can be used for researches of processes of self–organizing in systems of cellular nature of various type; the more detailed information on this question can be found in [82,102,106,279].

For the purpose of modeling of phenomenon of discrete growth process *M. Apter* has used classical *Turing* machines and propositional calculus [273,274]. A number of interesting questions connected to the growing in ***CA*** models of spatial forms of various geometry has been considered in [107]. We have marked earlier, that the classical models well enough simulate growth processes on the basis of relatively simple generative rules and restrictions. However, such rules are insufficiently complex to model natural growth and a lot of other development phenomena of alive systems. So, often we are forced to use types of ***CA*** models distinct from classical for problems of discrete modelling.

Whereas the results already on *polygenic* ***CA*** models show that they can be enough successfully used for modelling of rather complex growing real systems which simulate some natural phenomena of growth. So, in [233] the polygenic ***2–CA*** models are successfully used for modelling of process of growth of inflorescences. On the basis of results of modelling of such kind a rather interesting comparative analysis of classifications of growth of inflorescences on classical botanical base and on the basis of ***2–CA*** models is submitted**;** furthermore, the competitiveness of ***CA*** models relative to ***L–***systems on a number of problems of modelling of growth and morphogenesis of plants has been shown. Meanwhile, the ***CA*** models of such type are insufficiently simple in order to supply the investigator with the convenient and visual apparatus of modelling of phenomena which themselves are complex enough. But use of *polygenic* ***CA*** models together with computer simulation, perhaps, can essentially improve this situation. For modelling tasks of phenomena of biological development the ***CA*** models with *storage (**CAS**)* have been suggested [8] which enough simply realize networks of the *Apter* growing automata**,** allowing to model the processes of growth of rather complex spatially**–**differentiated forms.

Very interesting problems of optimization arise in connection with the questions of restriction of process of growth. Indeed, the real biological organisms do not grow with no limits, but completely supervise own growth during all development and vital functions. In this connection *D. Gajski* and *H. Yamada* have investigated the rules of growth in the ***CA*** models, that allow to grow forms of the preset limited size [7,277]. The chief task here is reduced to revealing of the greatest possible size of the *passive* configurations generated by the classical ***CA*** models from some simple initial finite configurations. Rather interesting results concerning the lower estimations of sizes of such maximal *passive* configurations in

terms of various key parameters of the ***CA*** along with rather interesting discussions of biological interpretations of the results received in this direction can be found in [7]. Rather interesting questions of growing of chains of finite automata of the preset length can be found and in rather interesting works [148,278,280-282,286].

The works marked in this direction enough closely adjoin our results on the *Problem of Limited Growth (**PLG**)* considered in section ***3.2.*** The ***PLG*** concerns a class of minimax problems in the ***CA*** problematics, being of the certain interest from the standpoint of developing cellular systems of the various nature. Indeed, the growth process in the real biological systems is limited, is strictly controllable from within, and depends on genetic and of some external factors. Moreover, the ***PLG*** has a certain cognitive significance, allowing to estimate in a sense an quantity of the information required for growth of complex multicellular organisms. In more detail with the ***PLG*** and interpretation of the received results it is possible to familiarize in section ***3.2*** and in [24,43,82,102,106].

In view of more applied aspects it is necessary to mark utility of ***PLG*** for research of questions of information connection of the intercellular interactions of developing systems along with formation of the certain considerations about character of the genetic code. As distinct from the above ***CA*** models researching the ***PLG*** and explaining mechanisms of management by the process of *restriction* on the basis of the ***CA*** concept, there is a number of other ***CA*** models explaining the phenomenon from certain other standpoints such as similarity principle, thermodynamic laws, adaptation to external environment, mechanic stability, energetic expediency, etc. Diversity of such kind of interpretations is undoubtedly necessary and allows to carry out multifold research of the problems of growth and development as a whole. Thus, it in a certain extent can be considered as a biologic analogue of the principle of complementarity.

From cybernetical standpoint it has been shown that certain features of development such as growth and self**–**reproduction can be inherent in artificial systems too. Below, we shall try to briefly consider questions of discrete modeling and the more complex development phenomena **–** *differentiation*, *regulation* & *regeneration*. Differentiation of cells represents one of the major problems in the modern *developmental* biology. Despite of a rather huge number of works, devoted to those or other features of cellular differentiation, for today we do not have the general theory of *differentiation* **–** the majority of researches and hypotheses concern only molecular mechanisms of cellular differentiation. So, today there is not

quantitative theory of differentiation and the approaches to construction of this theory are even not clear. That is conditioned in great degree by both an ambiguity of the differentiation concept and absence of an exact criteria for receiving of its numeric estimations. In addition, the *forming (morphogenesis)* along with *spatial* differentiation is an essential enough feature of the biological development [7,12,28,31,33,43,82,102,106,284].

Thus, the biologic modelling is directed at illustration of a possibility of realization of different phenomena of development at a level of general enough assumptions. In addition, the ***CA*** models being created in this direction well illustrate the possibilities such as forming of hierarchical structures**;** control by processes of growth, regeneration and regulation, etc. For today, such models are intensively investigated not only from the biological point of view, but also within a new scientific discipline **–** *discrete synergetics* [7]. Along with that, certain control algorithms used in these models can appear rather useful to parallel computing systems [32,33]. Concrete elaborations of similar type at present time exist**;** they enough naturally use a lot of control algorithms, used in some discrete models of development on the basis of ***CA*** models [9,31]. While a rather detailed discussion on the model approach to study of the *differentiation* problem can be found in [7]. We pass now to question of formalization of the problem of development and regulation of a biological structure.

The central development problem is reduced to the following question**:** ***How of an egg which seems the completely undifferentiated and simple in the structural attitude a rather complex multicellular organism can next develop?*** In this respect, the assumption has been expressed, that the egg contains only some development program instead of complete specification of all organism that of him should develop. For example, in the *Waddington* formulation the formation problem of *spatial structure* consists in determination of the immediate reasons of separation of a homogeneous cellular area into separate parts that are located in space in strictly determined order [7,24,31,33,82,102,106,284].

Along with that, we inevitably should come to the conclusion, that the differentiation of cells at highly organized alive essences is direct result of activity of extremely complex regulator mechanisms. First of all, for us, apparently, the effective enough acting models are really necessary, whose purpose should be to help with formalization of the problem and apparently to discover a key to understanding of the basis approaches to the problem decision in languages of the exact science. In the future the experimental approach to this problem has allowed to formulate a

lot of concepts interesting and simplifying the problem**;** among them it is necessary to mark such principles as dominance and gradients [9,33]. The first rather serious attempt of creation of a working model capable to development and regulation of an axial structure was undertaken by *S. Rose* [283]. In further, a lot of the interesting enough models has been suggested, whose comprehensive review can be found in [24,31,33,102]. However, the most known formal model of differentiation, regulation and regeneration is the *French Flag's Problem (**FFP**)***,** offered by *L. Volpert.* In the most elementary form this problem is formulated as follows [284]**:**

There is a 1–dimension connected system from 3*m cells, each of which admits one of the states "red", "white" or "blue"; should be determined the rules of functioning of such cellular system whose the final state is the configuration of French flag (CFF) which to certain extent is stable to external influences and damages.

For solution of the ***FFP*** in its classical posing a lot of mathematical and automaton models has been offered, and their analysis from biological standpoint has been carried out [1,2,8,24-28,31,33,82,102]. In particular, discussions of the ***FFP*** formulation as a formal model of differentiation**,** regulation and regeneration of axial biological structures for concrete biological objects have been carried out.

For the solution and research of the ***FFP*** the ***CA*** models of a few types were used, putting before modelling a lot of tasks. In the first place, the question relative to the *minimal* complexity of a model that is capable to differentiation, regulation and regeneration interested us. It is shown, that at modelling of the ***FFP*** even on basis of polygenic ***1–CA*** models, an algorithm deciding the problem should be algorithm over alphabet ***A*** whose elements are symbols composing the ***CFF*** [24-28]. In addition, additional states of the model should admit a reasonable interpretation in the corresponding biological categories [24,31,33,43,82,102,106].

So, the second question is the revealing of those sufficient conditions that would promote a solution of the ***FFP*** along with their satisfactory biological interpretation. From this standpoint, a lot of models has been investigated on the basis of the ***CA–***concept. In particular, one of these models is able to a perfect enough regulation and slightly resembles the known model of *M. Arbib* [7,285]**,** however it is more simple and is free from a few defects of his model. Moreover, the basic properties of our model are absence of a gradient and thresholds along with presence in it of polarity, spontaneous self**–**limiting reactions and a bilateral stream of control information [31,33]. To this model in sense of its basic features

determining a solution of the ***FFP***, a model on the basis of a class of the ***1-CA**** models adjoins also, allowing to decide the ***FFP*** in its generalized formulation. An extension of the ***FFP*** can be determined as follows. In a ***1-CA**** model a finite configuration c_o of length r of states of elementary automata of the following kind is defined, namely:

$$c_o = \square x_1x_2x_3x_4 \ldots x_r \square; \qquad x_j \in A=\{0,1,2,3, \ldots, a-1\} \quad (j = 1 .. r)$$

Then, such generalized ***FFP*** is reduced to determination of a functional model algorithm, whose complexity does not depend on a number r of elementary automata of a differentiated chain, allowing to establish and support in the ***1–CA**** model a configuration of the following structural kind of the above states, namely:

$$C_f = \nabla b_{1_1} \ldots b_{1_q} b_{2_1} \ldots b_{2_q} \ldots b_{(a-3)_1} \ldots b_{(a-3)_q} b_{(a-2)_1} \ldots b_{(a-2)_q} b_{(a-1)_1} \ldots b_{(a-1)_k} \nabla$$

$$b_{p_j} = p; \; p = 1..(a-2); \; j = 1..q; \; q = [r/(a-1)]; \; b_{(a-1)_i} = a-1; \; i = 1..k; \; k = r-(a-2)q$$

Because of use for solution of the generalized ***FFP*** of ***CA–***approach we first of all would like to determine the most simple type of ***CA–***models allowing to solve this problem. In this direction there is the following result [24,31,33,82,102,106], namely:

The generalized FFP determined in a finite alphabet W of general kind cannot be decided by means of an one–dimensional polygenic structure determined in the same states alphabet W.

Hence, for solution of the *generalized* ***FFP*** even in the class of polygenic ***CA–***models we need to use an alphabet, expanded relative to its initial alphabet and, perhaps, along with some other assumptions. So, one of models on the basis of a ***1–CA**** model uses an elementary variant of the symbolical sorting allowing to solve the ***FFP*** in the *Volpert* formulation during no more than $t = 3*m$ steps; where m is length of a differentiated chain of automata of the model [31]. In addition, a sorting acts as one of kinds of a logical gradient whereas the model allows to make a number of interesting enough conclusions of biological nature. Along with that, the functional algorithm of a ***CA*–***model which decides the generalized ***FFP*** allows to formulate the following result [24,31,33,82,102,106].

Theorem 172*. There is a 1–CA* model with alphabet A={0,1, ..., a–1} and a functional algorithm, whose complexity does not depend on length h of a differentiated chain of elementary automata and which decides the generalized FFP during no more than t = [h/2] steps for sufficiently large values h. A set of all solutions of the generalized FFP that are minimal in temporal attitude is nonrecursive.*

The result presented by the theorem ***172*** is a solution of the generalized ***FFP*** which for today is the best in the time attitude. Of this theorem, in particular, follows**:** for sufficiently great values ***a*** and**/**or ***h*** decision time of the ***FFP*** asymptotically approaches half of length of a differentiated chain of elementary automata of a ***CA**** model. Consequently, a rather interesting question arises**:** ***Whether exist functional algorithms of any other type which decide this problem for the best time?*** In our opinion, a rather essential improvement of decision time of the generalized ***FFP*** defined by the theorem ***172*** not seems possible.

Along with that, an interesting question of research of the generalized ***FFP*** for case of the higher dimensionalities arises, when instead of the linear chains the ***d–***dimension networks of finite differentiable identical automata are considered. It is shown that, the results of solution of the generalized ***FFP*** rather essentially depend on the kind of ***d–***dimension ***CFF*** *(**d*** **≥ 2)* too [24,82]. The above ***CA*** models solving the ***FFP,*** in a great extent allow to make clear the questions such as properties of separate automata, nature of connections between them, input/output control impulses, along with a lot of other prerequisites giving rise to dividing of cellular system along axis onto segments, located in a certain order. A rather detailed analysis from biological standpoint of these and other ***CA*** models of differentiation**,** regulation**,** regeneration can be found in the above works. Analysis of the basic approaches to discrete modeling of processes of biological development with all evidence shows that at present the ***CA*** concept and ***L-***systems are among the main components of the apparatus of modelling in this field. In conclusion it is necessary to accent our attention on the modern advances of bioengineering that allow to receive completely different look at various aspects of previous models of development. Therefore, the model approach to study of the developmental biology should be essentially revised [24,41,82,102,106].

In the context of the biologic applications of the ***CA*** concept, one should also mention the directions and trends in biocomputer technologies that are being studied quite intensively. In this connexion in [7,9,12,13,24-28, 32-34,40-43,63,70,71,82-87,102,140,196,232] one can get acquainted with reviews of a number of biologically**-**oriented applications of ***CA*** models of different types and classes with an emphasis on computer science. In particular, more and more actively developed both the theoretical, and the applied study on neurocomputer architecture, composing nowadays a separate branch of the ***CA*** concept**,** directly adjoin this problematics.

Conclusion

It is a good idea to collect and publish from time to time the different surveys of interesting and rather useful results from some area of study. It helps to briefly summarize the current state of knowledge and to state the most important researches directions, taking into account the earlier received results. A number of reviews of our previous results on the ***CA*** problematics along with formulating of a number of unsolved problems can be found in works [11,14,24-28,40-43,82-87] and some others, while certain comments to this material can be found, in particular, in [7,278, 286] and in some other sources.

In [287]**,** in particular, a difference between our reviews of the unsolved problems in the ***CA*** problematics and published by *S. Wolfram (Physica Scripta T9, 1985)* has been characterized as follows**:** principal difference between the *Wolfram* paper and the others *(V. Aladjev et al.)* consists in that**,** the others concentrate themselves to the mathematical aspects of the ***CA*** problems whereas *S. Wolfram* is not on such level of abstraction**;** the problems presented here deal with questions of the constructibility, nonconstructability, hierarchical opportunities, modelling, complexity, decomposability, and configurations dynamics of classical ***CA*** models.

In the offered book both at a pithy level and in the form of strict enough mathematical formulations a number of results in the basic sections of mathematical theory of the ***CA*** models which have been received by us or earlier, or generalized along with the adjusted, and new results was submitted. By not covering all extensive problematics of this field of the modern mathematical cybernetics wholly, meanwhile, the represented results lay in the course of the basic modern directions, forming up an essential part of the modern study state in this direction. ***CA*** models are a bright example of generating of complex objects and their dynamics on the basis of simple enough initial elements and prerequisites. In this sense, homogeneous structures better answer the mathematical models used in more abstract fields of theoretical physics, discrete synergetics and mathematical development biology than to more practical models of computing sciences, basing on modern microelectronic technology. Although with the further development of technology, first of all, the nanotechnolody they, perhaps, can play more and more growing role in this field as formal models and prototypes of high**–**parallel systems of information processing. In recent years, the classical ***CA*** models are one of the most promising simulating environments for various highly

parallel discrete processes, objects and phenomena admitting reversible dynamics, that is rather important from a physical standpoint, above all.

Once again it is necessary to note that the ***CA*** concept in a great extent is an unique phenomenon **–** on the one hand, the ***CA*** concept is a base for formal modelling of manufold processes, phenomena and objects in a rather broad spectrum of fields, and, on the other hand, the concept has equivalent technical implementations, for example, such as *CAM* of *T. Toffoli*, networks of transputers, cellular processors, systolic structures, etc., doing the ***CA*** concept as rather attractive facility both in theoretical and applied researches in many fields, with a rather good reason that is raised this concept onto a new interdisciplinary scientifical level.

Hence, the ***CA*** **–** more than very useful abstraction since they possess a number of fundamental properties that can lead us to creation on base of reversible computing ***CA*** models of a new perspective architecture of high**–**efficiency computer systems and the control blocks of systems of an artificial intellect of the future generations and also to play a part of a perspective modelling environment for the broad area of appendices.

And again, we absolutely do not agree with the *Wolfram* standpoint on ***CA*** problematics as a new kind of science. Our rich experience in the ***CA*** problematics both on the theoretical**,** and especially applied level speaks fully other, namely**:** ***(1)*** the ***CA*** represent one of types of infinite abstract automata with specifical internal structure admitting a rather high level of parallel information processing and computation**;** ***CA*** models form a specific class of discrete dynamic systems operating in especially *parallel* manner on the basis of principle of local interaction**,** and ***(2)*** the ***CA*** can be considered as formal mathematic objects presenting undoubted self**–**dependent interest too. And further, the ***CA*** can be considered as a quite independent part of discrete mathematics, cybernetics, abstract infinite automata with specific internal organization**,** parallel discrete dynamic systems, but in any way no as a new type of science. Already name of the opus «*A New Kind of Science*» at once sets a trap without allowing to consider it as a rather serious scientific edition with a pretension to its certain its significance.

Thus, on examples of computer research of rather simple ***1-***dimensional cellular automata *S. Wolfram* draws "*deep*" scientific conclusions which were known for a long time. Furthermore, in the most cases he tries to present himself as their pioneer, ignoring their real authors. Let us say, he ascribed to himself many results and assumptions of the well-known scientist *K. Zuse* and others researchers mentioned in the present book.

And that already at the level of plagiarism by the highest standards not talkingabout full ignoring of historical justice. Particularly, if the *western* researchers still are sometimes mentioned by him, the *Soviet* researchers are completely ignored though they received not a little of fundamental results on ***CA*** theory and their applied aspects incommensurable on the importance with conclusions of the author of the tendentious opus [23].

It should be noted that our standpoint on the majority of "*fundamental*" conclusions of the above opus is completely consistent with numerous reviews of the opus [288]. I will give only excerpt from one review of *S. Wolfram* opus which is completely conformable to our opinion on it**:** *But he does know Goedel and Zuse and Turing. He must see that his own work is minor in comparison. Why does he desparately try to convince us otherwise? When I read Wolfram's first praise of the originality of his own ideas I just had to laugh. The tenth time was annoying. The hundredth time was boring. And that was my final feeling when I laid down this extremely repetitive book: viz. exhaustion and boredom. In hindsight I know I could have saved my time. But at least I can warn others.*

We also consider the above opus as a certain self**-**advertisement of both the writer and the ***CA*** theory regardless of the well**-**established facts and real situation. Consequently, in our opinion, the adequacy of the name «*A New Kind of Science*» to the essence of the book of the same name is of the same level as the name of well**-**known computer game "*Life*" that is based on a rather simple **2-**dimension binary cellular automaton to the life itself. Meantime we nevertheless recommend at a leisure to acquaint oneself with this opus at least for the general acquaintance with the ***CA*** problematics. Regardless of what has been said, thy opus had a positive meaning for attracting a certain circle of specialists and amateurs to the ***CA*** problematics along with its popularization.

Returning to our book, we note, the pithy level of statements which are illustrated by examples and separate proofs, allows to use the book by a rather wide audience of the readers of the various spheres of potential application of methods, results and the ***CA*** concept as a whole. For the more full acquaintance with the modern state of the ***CA*** problematics a rather extensive bibliography that, in turn, contains references to a lot of various publications in this direction is represented. In our opinion this book will present an indubitable interest for students, and persons why works for doctor's degree of the appropriate universities faculties along with teachers in disciplines such as mathematics, cybernetics, automata theory, physics, modelling, computer science, and a lot of others.

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During researches in the cellular automata theory the extensive enough bibliography of original sources of different level and directly as in the theory and in its numerous applications in different fields was collected by us. Naturally, the bibliography is not perfectly exhaustive, however it can present a certain interest for researchers in this field, first of all, of the beginners. Meantime, the reader has an opportunity to supplement the presented bibliography by the materials which are absent in it. We hope, that this bibliography will allow to outline better both the circle of researchers in this field and breadth of scope of problems considered by them. Above all, it concerns *Soviet* and *Russian* researchers who have received a lot of priority results of fundamental character with which English-speaking researchers are familiar insufficiently well or are not familiar entirely. Therefore certain of them have been rediscovered by other researchers. It is especially topical and for the reason, that some *Soviet* researchers directly stood at the beginnings of the forming up of this field of modern mathematical cybernetics. Presented bibliography is not annotated basically but the headings of a lot of publications give a rather defined comprehension concerning the contents of the quoted material. The more extensive bibliography of original sources on the ***CA*** theory and its numerous appendices can be found in [7,22,30,63,71,118, 137,140,149,196,197,232,286,286], while the interested reader is referred to the *Internet* with appropriate key phrases. Particularly, not a little of our works along with works of other writers can be found in the *Internet*. Whereas here an incomplete references list of our works concerning the material of the present book is represented.

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About the authors:

Professor Dr. ***Aladjev V.Z.*** was born on *June 14, 1942* in the town *Grodno (West Belarus)*. He is the First vice–president of the *International Academy of Noosphere (IAN, 1998)*, and academician–secretary of Baltic branch of the *IAN* whose scientific results have received *international* recognition, first, in the field of *cellular automata theory*. Dr. *Aladjev V.Z.* is known for the works on computer mathematical systems too. He is full member of the *Russian Academy of Cosmonautics (1994)*, *Russian Academy of Natural Sciences (1995)*, *International Academy of Noosphere (1998)*, and honorary member of *Russian Ecological Academy (1998)*. Prof. Dr. *Aladjev V.Z.* is the author of more than ***500*** scientific publications, including ***90*** books and monographs, published in many countries. He repeatedly participates as a member of the organizing committee and/or a guest lecturer in many international scientific forums in mathematics and cybernetics. In *2015* Dr. *Aladjev V.Z.* was awarded by Gold medal "*European Quality*" of the *European* scientifical and industrial consortium for works of scientifical and applied character.

Dr. ***Shishakov M.L.*** was born on *October 21, 1957* in Gomel area *(Belarus)*. For its scientific activity *Shishakov M.L.* has more than ***65*** publications on information technology in various application fields, including more ***25*** books and monographs. Fields of his main interests are cybernetics, theory of statistics, problems of *CAD* and designing of mobile software, artificial intelligence, computer telecommunication, along with applied software for solution of different tasks of technical and manufacturing nature. In *1998 M.L. Shishakov* was elected as a full member of the *IAN* on section of the information science and information technologies. At present, *M.L. Shishakov* is the director of the Belarusian–Swiss company "*TDF Ecotech*" that works in the field of "*green*" energy, combining the manufacturing activity with active scientifical researches.

Dr. ***Vaganov V.A.*** was born on *February 2, 1946* in the Primorye Territory *(Russia)*. Now Dr. *Vaganov V.A.* is the proprietor of the firms *Fortex* and *Sinfex* engaging of problems of delivery of industrial materials to firms of the Estonian republic. Simultaneously, Dr. *V.A. Vaganov* is executive director of the Baltic branch of the *IAN*. *Vaganov V.A.* is known for the investigations on automation of economical and statistical works. *V.A. Vaganov* is the honorary member of the *IANSD* and the author of more than ***60*** applied and scientifical publications, including ***8*** books.

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